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Continuous Tensor Field Approximation of Diffusion Tensor MRI data

Sinisa Pajevic[1], Akram Aldroubi[2], and Peter J. Basser[3]

[1] National Institutes of Health, MSCL/CIT, Bethesda, MD 20892-5620, USA
`pajevic@nih.gov`

[2] Dept. of Mathematics, Vanderbilt University, Nashville, TN 37240-0001, USA
`aldroubi@math.vanderbilt.edu`

[3] National Institutes of Health, STBB/NICHD, Bethesda, MD 20892-5772, USA
`pjbasser@helix.nih.gov`

Summary. Diffusion Tensor MRI (DT-MRI) measurements are a discrete noisy sample of an underlying macroscopic effective diffusion tensor field, $\underline{\mathbf{D}}(\mathbf{x})$, of water. This field is presumed to be piecewise continuous/smooth at a gross anatomical length scale. Here we describe a mathematical framework for obtaining an estimate of this tensor field from the measured DT-MRI data using a spline-based continuous approximation. This methodology facilitates calculation of new structural quantities and provides a framework for applying differential geometric methods to DT-MRI data. A B-spline approximation has already been used to improve robustness of DT-MRI fiber tractography. Here we propose a piecewise continuous approximation based on Non-Uniform Rational B-Splines (NURBS), which addresses some of the shortcomings of the previous implementation.

18.1 Introduction

Diffusion tensor MRI provides a measurement of an effective diffusion tensor of water, $\underline{\mathbf{D}}^{\text{eff}}$, in each voxel within an imaging volume [1]. These diffusion measurements are inherently discrete, noisy and voxel-averaged. Here we treat DT-MRI data as discrete noisy samples of an underlying macroscopic piecewise continuous diffusion tensor field, $\underline{\mathbf{D}}(\mathbf{x})$, where, $\mathbf{x} = (x, y, z)$ are the spatial coordinates in the laboratory frame of reference. This field is presumed to be piecewise continuous or smooth at a gross anatomical length scale, an assumption based on the known anatomy of many soft fibrous tissues, including white matter, muscles, ligaments, and tendons. One of our objectives is to develop a mathematical framework to estimate this piecewise continuous field, $\underline{\mathbf{D}}(\mathbf{x})$, from discrete noisy DT-MRI measurements. A reliable estimate of this field enables us to use differential geometric methods directly. Additionally, it enables computation and display of intrinsic architectural or microstructural

MRI features based upon tissue fiber geometry [2, 2]. Some previously suggested characteristics are curvature and torsion of the individual fiber tracts, as well as the properties of the tangent field, e.g. twisting, bending, and diverging [4]. Here we focus on estimating curvature of the fiber tracts (tangent field) but also show the architectural features of the tensor field itself. Estimating such quantities accurately using measured diffusion tensor data and interpolation is difficult, since their evaluation requires spatial differentiation of noisy tensor quantities. Below we show that they can be calculated more reliably and robustly using continuous tensor field approximation.

Originally, estimating the tensor field from sample tensor data was performed using B-spline approximation [5]. It was used with DT-MRI data to elucidate fiber tract trajectories, which can be done by integrating the fiber direction (vector) field [9]. Other methods for fiber tracking at the time utilized interpolation or directly followed the local fiber orientation [6, 7, 8], with exception of Poupon et al. [11] who used a regularization method. Integrating a noisy direction vector field can result in fiber trajectories that wander off course. Using a smoothed representation of the direction field, obtained from the continuous representation of $\underline{\mathbf{D}}(\mathbf{x})$, however, can improve the fidelity of tract following [9]. Establishing connectivity and continuity of neural pathways can also benefit from the development of this specialized tensor field processing methodology. These tasks require determining continuous links between different regions of the brain, or assessing disjunctions between them. Finally, there are a number of generic image processing tasks one would like to perform on high dimensional DT-MRI data, since no signal processing framework currently exists for these. These include: filtering noise, sharpening edges, detecting boundaries; compressing, storing and transmitting large image files; interpolating and extrapolating tensor data; resampling data at different resolutions (e.g., rebinning); extracting textural features, segmenting images, clustering data, and classifying tissues; and detecting statistical outliers. The B-spline approximation provides the mathematical underpinnings for performing these tasks both rapidly and efficiently [10]. However, the problem with it is that it introduces smoothing in the data uniformly and isotropically and is incapable of dealing with discontinuities. The smaller structures as well as sudden or rapid changes (edges, high curvatures, etc.) will be distorted at the levels of approximation/smoothing required to alleviate the noise effects. To achieve a more efficient approximation we use Non-Uniform Rational B-Splines (NURBS). They allow for discontinuities and can describe complex piecewise continuous geometrical shapes with many fewer parameters than the original B-spline approximation.

Although there are other approaches for finding an approximate tensor field, in this chapter we focus on a mathematical framework for continuous approximation based on splines. A number of other methods for tensor field approximation exist, for example see references [11, 12, 13, 14]. Also, Chap. 17 by Moakher and Batchelor and Chap. 19 by Weickert and Welk present novel and sophisticated ways of interpolating and regularizing tensor fields.