Some Behavioural Aspects of Net Theory

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0 Introduction

Net theory was initiated by C.A. Petri in the early 60's [P1]. The subject matter of the theory is distributed systems and processes. The key aspect of net theory is that the three fundamental relationships that can exist between the occurrences of a pair of actions at a state are clearly separated from each other at all levels of the theory. These three relationships are:

(i) At the state s, the action a_2 can occur only after the action a_1 has occurred (causality).

(ii) a_1 can occur or a_2 can occur at s but not both (conflict, choice, indeterminacy).

(iii) At the state s both a_1 and a_2 can occur but with *no order* over their occurrences (concurrency).

Another important feature of net theory is that states and changes-of-states (called transitions) are viewed as two interwined but distinct entities; they are treated on an "equal" footing by the theory.

Over the years net theory has evolved along many directions. It is difficult to give an overview of the whole theory in one place. Hence we shall attempt to do something more modest here. We shall first convey the basic concerns of net theory by presenting a simple system model called elementary net systems. Then we shall give a brief sketch of some of the tools that have been proposed to describe the *behaviour* of elementary net systems. We shall concentrate on those tools that have either directly come out of net theory or which have been prodded into existence by the insistence of net theory that causality, conflict and concurrency should be clearly separated from each other in behavioural descriptions of distributed systems.

In our presentation we will concentrate on motivations and basic definitions at the expense of stating theorems. The few results that we present are stated without proofs. The proofs can be found in [NRT]. We shall however leave a trail of pointers to the literature using which the interested reader can get a reasonably broad overview of net theory and closely related topics.

In the next section the elementary net system model is presented. Using this model we then define the basic concepts of net theory. This sets the stage for developing the behavioural tools that can capture the essential features of distributed systems as defined by the elementary net system model. Section 2 develops some notation and introduces a purely sequential mode of behavioural description called firing sequences. In the next section the theory of traces which have an independent existence is used to recover information concerning concurrency from the firing sequences. In section 4 the notion of non-sequential processes is introduced. Non-sequential processes are a behavioural tool developed within net theory to describe the non-sequential stretches of behaviour of an elementary net system.

Both trace theory and the theory of non-sequential processes represent concurrency directly but handle information concerning conflict in an indirect fashion. One must work with the whole set of traces or non-sequential processes in order to talk about conflicts and that too in an indirect fashion. This disadvantage can be overcome with the help of behavioural tools called unfoldings and labelled event structures that are presented in section 5. The unfolding of an elementary net system is a single object in which all the basic behavioural features of the system are represented in a transparent fashion. Labelled event structures are direct descendents of unfoldings and they are more pleasing mathematical objects.

1 Elementary Net Systems

Elementary net systems, as the name suggests, are meant to be the simplest system model of net theory. They may be viewed as transition systems obeying a particular principle of change. This view of elementary net systems is explained in more detail in [T]. Here, for the sake of brevity, we shall make a direct presentation.

Definition 1.1

A *net* is a triple $N = (S, T, F)$ where S and T are sets and $F \subseteq (S \times T) \cup (T \times S)$ are such that

(i) $S \cap T = \emptyset$

(ii) domain$(F) \cup$ range$(F) = S \cup T$ where

domain$(F) = \{x \mid \exists y.(x, y) \in F\}$ and
range$(F) = \{y \mid \exists x.(x, y) \in F\}$.

$\square$

Thus a net may be viewed as a directed bipartite graph with no isolated elements. Note that we admit the *empty* net $N_\emptyset = (\emptyset, \emptyset, \emptyset)$.

S is the set of *S-elements*, T is the set of *T-elements* and F is the *flow relation* of the net $N = (S, T, F)$. In diagrams the S-elements will be drawn as circles, the