TRANSFORMATIONS OF DERIVATION SEQUENCES IN GRAPH GRAMMARS

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ABSTRACT: Basing on the Church-Rosser Theorems in /EK 76b/ analysis and synthesis of parallel derivations in graph grammars are introduced. This allows specific, transparent transformations of derivation sequences, which can be used as elementary steps of algorithms acting on derivations, and the calculation rules for transformations presented in this paper are needed to prove the correctness of such algorithms. One example of this kind is given: the equivalence of derivations in graph grammars is defined and canonical derivations representing the equivalence classes are specified. For graph grammars canonical derivations will be as important as leftmost derivations and respective concepts for classical Chomsky grammars to the design of correct parsing algorithms.

1. INTRODUCTION

In /EK 76b/ is announced that each derivation in a graph grammar can be transformed into an equivalent canonical one, which is, above all, a unique representative of its equivalence class. But the proof is omitted. To fill the gap a framework of specific transformations acting on graph grammar derivations is presented in this paper. Basing on the Church-Rosser Theorems in /EK 76b/ (cf. also /Ro 75a, ER 76a/) analysis, synthesis and shift of parallel derivations are defined as elementary transformations in section 2, and their calculation rules are studied in section 2 partly and in section 3. In particular, in Analysis and Synthesis Theorem (2.7) the results of the Church-Rosser Theorems and closely related considerations interpreted for the new notation of transformations are summarized. Furthermore we can show, that transformations preserve independence of derivations, which corresponds to the applicability of specific syntheses and shifts (3.1), and that transformations of parallel productions can be replaced by transformations of the corresponding components (3.2). Finally it results that each two shifts applied to a parallel derivation can be continued to a common derivation by applying further shifts. This Church-Rosser property of shifts is the main theorem of the present paper, because it is as useful as other Church-Rosser properties in several fields of Computer Sciences (cf. e.g. /Ro 73, Ro 74, FKZ 76, ER 76a, EK 76b/).

For example, it is the main step to prove the uniqueness of canonical derivations in section 4, where besides this the equivalence of graph grammar derivations is defined and a procedure is presented, which transforms an arbitrary derivation into an equivalent canonical one using shifts.

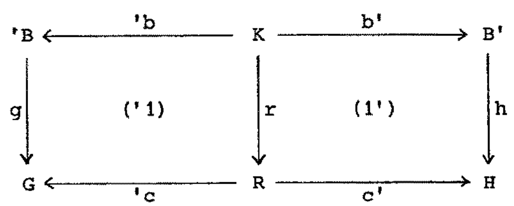
The framework of transformations and the resulting canonical derivations correspond to leftmost derivations and left and right interchanges for classical Chomsky grammars studied by Griffiths /Gr 68/ for instance. Hence, hopefully, canonical derivations are able to become a powerful tool for the design of parsing algorithms of graph grammars.

This and other possible applications of the presented results to problems of syntax analysis, semantic of recursively defined functions, record handling and semantic networks (cf. Fr 75, FKZ 76, ER 76a, ER 76b, Sch 76, EK 76b/) can not be pointed out explicitly. Some aspects on this line are mentioned in /Eh 77/.

2. BASIC CONCEPTS

2.1 DERIVATIONS IN GRAPH GRAMMARS

Labelled graphs are a well-known, frequently used description of multidimensional information structures in several fields of Computer Science. Manipulations of these linked data structures like deleting and updating of some information can be formalized by applying a production in a graph grammar. Following the algebraic approach to graph grammars introduced in /EPS 73/ and /Ro 75b/ a direct derivation $G \overset{p}{\Longrightarrow} H$ via a production $p=({}'B \xleftarrow{{}'b} K \xrightarrow{b'} B')$ is defined by two gluing diagrams (i.e. pushouts in the category of graphs)



where 'c and c' are required to be injective in the case of binatural derivations assumed in the following for simplicity as well as uniqueness of all considered constructions. Given a production p and its (left) occurrence $g:{}'B \longrightarrow G$ the diagrams ('1) and (1') can be constructed uniquely using a suitable gluing condition (see /EK 76a/).

As sequences of direct derivations $G_0 \overset{p_1}{\Longrightarrow} G_1 \overset{p_2}{\Longrightarrow} G_2 \cdots\cdots \overset{p_n}{\Longrightarrow} G_n$, abreviated $p_1\, p_2 \cdots\cdots p_n$, if $G_0, \ldots\ldots, G_n$ are clear from the context, we obtain derivations in graph grammars.

On the other hand a simultaneous application of productions $p_1, \ldots, p_n$ is defined as a direct derivation via the corresponding parallel production $p_1 + \ldots + p_n$, i.e. the disjoint union or coproduct of $p_1, \ldots, p_n$. Hence, parallel derivations are simply