COMPUTATIONAL POWER OF ONE-WAY MULTIHEAD FINITE AUTOMATA

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In this paper we sketch our results concerning one-way multihead finite automata (1-MFA). The full version with complete proofs can be found in a series of papers ([3],[4],[5],[6]).

1-MFA belong to the weakest models of computational devices. Despite that, they recognize many interesting and important languages. They work in linear time, so the algorithms running on 1-MFA are in some sense practical. Unfortunately, many important questions concerning 1-MFA have turned out to be hard to answer, despite the simplicity of the computational model. We get the results which answer some of such open questions. Before we proceed, we recall shortly the definition of 1-MFA.

A 1-MFA consists of an input tape, some number of read-only heads and a control unit with finitely many internal states (see figure 1). Input words are placed on the input tape, each symbol occupying one cell. The heads are placed initially at the first from the left input symbol. During a computation the heads move independently on the tape (no moves to the left are allowed) and read different symbols of the input word.

The computation consists of several steps, during which the internal state can change and the heads can move to the right. These actions are determined at each step by:

- the internal state of the automaton,
- the symbols currently read by the heads.

To express it more precisely, we associate with each automaton M so called transition function δ_M. If at some moment of a computation automaton M is in state q, the heads $H_1, H_2, ..., H_k$ of M see symbols $a_1, a_2, ..., a_k$ on the tape and $\delta_M(q, a_1, ..., a_k) = (p, d_1, ..., d_k)$ then during this step:

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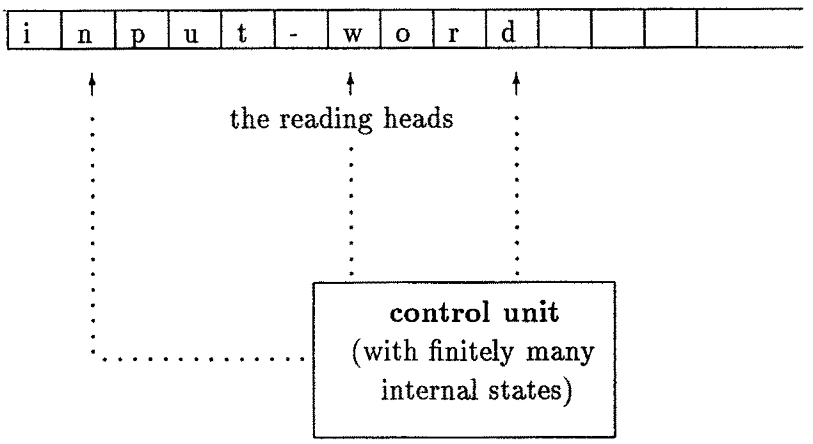

Figure 1: A 4-head one-way finite automaton.

- the internal state of M changes from q to p,
- for each $i \leq k$ head H_i moves d_i positions to the right ($d_i \in \{0, 1\}$).

Notice that the computation is deterministic and the heads do not see each other (the "non-sensing model"). A word x is accepted by a 1-MFA M if during the computation on x automaton M reaches an accepting state.

From the very beginning of the studies on 1-MFA the following problem has been investigated:

Problem. *Given a language L which can be recognized by some 1-MFA. Find the minimal number k such that L can be recognized by some k-head 1-MFA.*

We consider this problem trying to find how the number of heads influences the computational power of 1-MFA.

Most intensively studied was the behaviour of 1-MFA on so called bounded languages. We say that a language L is bounded if there is $m \in \mathbf{N}$ such that each word $x \in L$ has the form $x = 1^{a_1} * 1^{a_2} * \dots * 1^{a_m}$ for some $a_1, \dots, a_m \in \mathbf{N}$. In the above situation we say also that language L is m-bounded.