Cayley-Dixon Resultant Matrices of Multi-univariate Composed Polynomials

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Abstract. The behavior of the Cayley-Dixon resultant construction and the structure of Dixon matrices are analyzed for composed polynomial systems constructed from a multivariate system in which each variable is substituted by a univariate polynomial in a distinct variable. It is shown that a Dixon projection operator (a multiple of the resultant) of the composed system can be expressed as a power of the resultant of the *outer* polynomial system multiplied by powers of the leading coefficients of the univariate polynomials substituted for variables in the outer system. The derivation of the resultant formula for the composed system unifies all the known related results in the literature. A new resultant formula is derived for systems where it is known that the Cayley-Dixon construction does not contain any extraneous factors. The approach demonstrates that the resultant of a composed system can be effectively calculated by considering only the resultant of the outer system.

1 Introduction

Problems in many application domains, including engineering and design, graphics, CAD-CAM, geometric modeling, etc. can be modelled using polynomial systems [1–8]. Often a polynomial system arising from an application has a structure. Particularly in engineering and design applications and in geometric modeling, a polynomial system can be expressed as a composition of two distinct polynomial systems, each of which is of much lower degree in comparison to the original system. Furthermore, if the structure of given polynomials is not known a priori, one can efficiently check if they can be decomposed [9].

* Supported by NSF grant no. CCR-0203051 and a grant from the Computer Science Research Institute at the Sandia National Labs.
⋆ Also supported by NSF grant CCF 0430741.

V.G. Ganzha, E.W. Mayr, and E.V. Vorozhtsov (Eds.): CASC 2005, LNCS 3718, pp. 125–137, 2005.

This paper addresses the resultant computation for such composed polynomial systems [10–14]. The resultant of a polynomial system with symbolic parameters is a necessary and sufficient condition on its parameters for the polynomial system to have a common solution[1]. Resultant computations have been found useful in many application domains including engineering and design, robotics, inverse kinematics, manufacturing, design and analysis of nano devices in nanotechnology, image understanding, graphics, solid modeling, implicitization, CAD-CAM design, geometric construction, drug-design, and control theory.

The focus in this paper is on the Cayley-Dixon formulation for multivariate resultants which have been shown to be efficient (both experimentally and theoretically) for computing resultants by simultaneously eliminating many variables from a polynomial system [16]. The behavior of the Cayley-Dixon resultant construction is analyzed for composed polynomial systems constructed from a multivariate system in which each variable is substituted by a univariate polynomial in a distinct variable, referred to as multi-univariate composition in [9]. It is shown that the resultant of the composed system can be expressed as a power of the resultant of the *outer* polynomial system, multiplied by powers of the leading coefficients of the univariate polynomials substituted for variables in the outer system. It is important to point out that the techniques used for deriving resultant formulas in the current paper are different from the techniques used in previous works (such as [10–13,17,18]), which seemed not applicable.

A new resultant formula is derived for multi-univariate composed polynomials where it is known that the Cayley-Dixon resultant formulation does not produce any extraneous factors for the outer system. The derivation unifies all known related results in the literature [18,19]. Such systems include n-degree [8], bivariate corner cut [20] and generalized corner cut systems [21]. Even when extraneous factors are present, a similar formula is derived showing that the extraneous factor of the outer system will be "amplified" in the extraneous factor of composed system. Hence exploiting the composed structure of a polynomial system can reduce the extraneous factors in the resultant computation. Furthermore, it demonstrates that the resultant of a composed system can be effectively calculated by considering only the resultant of the outer system. For practical applications, that is what is needed.

Below, we first state the main result of the paper. This is followed by a section on preliminaries and notation; the generalized Cayley-Dixon formulation as proposed by Kapur, Saxena and Yang [8] is briefly reviewed. Since the Cayley-Dixon formulation involves two disjoint sets of variables, the bilinear form representation of a polynomial in disjoint sets of variables is useful. In section 2, we discuss how bilinear forms are affected by polynomial operations, particularly when two polynomials are multiplied, a polynomial is composed with other polynomials by substituting variables by polynomials etc. To express these relations among bilinear forms, a series of matrix operations is introduced.

We assume that the reader is familiar with the notion of resultant with respect to a given variety (see for example [15]). This notion includes classic resultants

[1] Resultant depends on an algebraic set for which solutions are sought [15].