Fuzzy Spatial Relationships for Model-Based Pattern Recognition in Images and Spatial Reasoning Under Imprecision

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Abstract. We show in this paper that mathematical morphology provides a unified and consistent framework to express different types of spatial relationships and to answer different questions about them, with good properties. We show then how to use these fuzzy relationships in model-based pattern recognition and spatial reasoning under imprecision. Two examples are presented, one where recognition of face features is expressed as non bijective correspondence between graphs representing regions and spatial relations, and one where anatomical expert knowledge involving spatial relationships is used to guide the recognition of brain structures.

1 Introduction

Fuzzy set theory provides a good theoretical basis to represent imprecision of information, at different levels of representation, in particular in image processing and interpretation. It constitutes a unified framework for representing and processing both numerical and symbolic information, as well as structural information (constituted mainly by spatial relationships in image processing). The interest of spatial relationships between objects has been highlighted in very different types of works. Indeed, the spatial arrangement of objects in images provides important information for recognition and interpretation tasks, in particular when the objects are embedded in a complex environment like in medical or remote sensing images. We distinguish between relationships that are mathematically well defined and relationships that are intrinsically vague. Topological relationships (such as set relationships and adjacency) and distances belong to the first class. If the objects are precisely defined, their relationships can be defined and computed in a numerical (purely quantitative) setting. But if the objects are imprecise, as is often the case if they are extracted from images, then the semi-quantitative framework of fuzzy sets proved to be useful for their representation, as spatial fuzzy sets. Definitions of relationships have then to be extended to be applicable on fuzzy objects. Results can also be semi-quantitative, and provided in the form of intervals or fuzzy numbers. Some metric relationships, like relative directional position, belong to the second class. Even for crisp objects, fuzzy definitions are then appropriate.

V. Di Gesú, F. Masulli, and A. Petrosino (Eds.): WILF 2003, LNAI 2955, pp. 164–173, 2006.

We show in this paper that mathematical morphology provides a unified and consistent framework to express different types of spatial relationships and to answer different questions about them, with good properties (Section 2). We show then through two examples how to use these fuzzy relationships in model-based pattern recognition and spatial reasoning under imprecision (Section 3).

2 Mathematical Morphology as a Unified Framework for Defining Spatial Relationships

In this Section we address the problem of modeling spatial relationships in the fuzzy set framework. This framework is interesting here for several reasons:

- the objects of interest can be imprecisely defined, for instance due to the segmentation step;
- some relations are imprecise, such as *to be left of*, and find a more suitable definition in the fuzzy set framework;
- the type of knowledge available about the structures or the type of question we would like to answer can be imprecise too.

We consider here adjacency (as one example of topological relation), distances, and directional relative position. Some of them have led to a rich literature in the fuzzy set community, like distances which have been defined using a lot of different approaches, while others have not raised so much attention. We summarize here our work based on fuzzy mathematical morphology [1], which allows us to represent in a unified way various spatial relationships [2].

Two types of questions are important for applications in structural pattern recognition:

1. given two objects (possibly fuzzy), assess the degree to which a relation is satisfied;
2. given one reference object, define the area of the space in which a relation to this reference is satisfied (to some degree).

Our approach provides answers to these two types of questions. The second one will be illustrated only for distances and directional position here (see [2] for the other relations).

We consider the general case of a 3D space $\mathcal{S}$ (typically $\mathbb{R}^3$ or $\mathbb{Z}^3$ in the digital case), where objects can have any shape and any topology, and can be crisp or fuzzy.

Adjacency. Adjacency has a large interest in image processing and pattern recognition, since it denotes an important relationship between image objects or regions [3], widely used as a feature in model-based pattern recognition. In the crisp case, it is defined based on the digital connectivity $n_c(x, y)$ defined on the image: two subsets X and Y in $\mathcal{S}$ are adjacent according to the c-connectivity if: $X \cap Y = \emptyset \ \ and \ \ \exists x \in X, \exists y \in Y : n_c(x, y)$. This definition can also be expressed equivalently in terms of morphological dilation, as: $X \cap Y = \emptyset \ \ and \ \ D_B(X) \cap Y \neq$