Simulation Research on Stability for T-S Fuzzy System

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Abstract. In order to relax the conservatism of stability analysis methods of T-S fuzzy system, by employing the concept of the efficient maximal overlapped-rules group (EMORG), a theorem is proposed to reduce the difficulty of stability analysis for T-S fuzzy system, which can guarantee the quadratic stability of the open-loop T-S fuzzy systems globally. This theorem only requires finding a local common positive definite matrix in each EMORG. Finally, an example is conducted to show the effectiveness of the proposed method.

Keywords: stability condition, simulation research, EMORG.

1 Introduction

Recently, the stability analysis of T-S fuzzy system has been difficult and hot issue. Most of stability conditions based on the result proposed by Sugeno and Tanaka[1] need finding a common positive definite matrix **P** that satisfies a set of Lyapunov inequalities[2, 3]. However, if the number of fuzzy rules is large, it might be difficult to find a common matrix. In order to reduce this conservatism, Reference [4] proposed a new stability condition, which utilizes a set of local common matrices to satisfy the Lyapunov inequalities of rules included in the efficient maximal overlapped-rules group (EMORG) respectively, where G denotes the number of EMORG s. But $q_j \in$ even set and $q_j \in$ odd set are only considered (q_j denotes the number of the fuzzy partitions of the *j*th input variable) in Reference [4].

In this paper, $q_j \in$ others is considered by an example and the feasibility and the validity of the proposed approach are illustrated.

2 Preliminary

A T-S fuzzy continuous model can be written as follows:

$$R_i : \text{IF } x_1(t) \text{ is } M_1^i, \text{and} \cdots, \text{and } x_n(t) \text{ is } M_n^i, \text{ THEN } \dot{\boldsymbol{x}}(t) = \mathbf{A}_i \boldsymbol{x}(t), \tag{1}$$

where $i=1,2,\cdots,r$, r and n are the numbers of rules and state variables respectively, $\boldsymbol{x}^{\mathrm{T}}(t) = \left[x_1(t), x_2(t), \cdots, x_n(t)\right]$ is the state vector, M_j^i $(j=1,\cdots,n)$ is the

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fuzzy set, and $\mathbf{A}_i \in \mathbf{R}^{n\times n}$. By the singleton fuzzifier, the product inference engine and center average defuzzification, the final output of (1) is inferred as:

$$\dot{\boldsymbol{x}}(t)=\sum_{i=1}^{r} h_i\left(\boldsymbol{x}(t)\right)\mathbf{A}_i\boldsymbol{x}(t), \tag{2}$$

where $h_i\left(\boldsymbol{x}(\mathrm{t})\right)=\prod_{j=1}^{n} M_j^i\left(x_j(t)\right) \Big/ \sum_{i=1}^{r}\prod_{j=1}^{n} M_j^i\left(x_j(t)\right)$, and $M_j^i\left(x_j(t)\right)$ is the firing strength of membership function M_j^i. Moreover, $\sum_{i=1}^{r} h_i\left(\boldsymbol{x}(t)\right)=1$.

3 Stability Analysis

Definition 1. A set of maximal overlapped-rules groups (MORGs) is said to be an efficient maximal overlapped-rules group set (EMORGS) if the set is composed of the least number of MORGs which include all rules of a T-S fuzzy system. And each MORG included in an EMORGS is said to be an efficient maximal overlapped-rules group (EMORG).

Theorem 1. For a T-S fuzzy system described by (2), if input variables adopt SFPs, then the equilibrium point of the fuzzy system is asymptotically stable in the large if there exists a local common positive definite matrix $\mathbf{P}_l$ in lth EMORG such that

$$\mathbf{A}_i^{\mathrm{T}}\mathbf{P}_l+\mathbf{P}_l\mathbf{A}_i<0, \quad l=1,2,\cdots, g, \tag{3}$$

where $i \in$ {the sequence numbers of rules included in the lth EMORG}, g denotes the number of EMORGs,

$$g=\begin{cases} \prod_{j=1}^{n}\left(\dfrac{q_j+1}{2}\right) & q_j \in \text{odd set} \\ \prod_{j=1}^{n}\left(\dfrac{q_j}{2}\right) & q_j \in \text{even set} \\ \prod_{q_{j'}\in \text{even}}\left(\dfrac{q_{j'}}{2}\right)\prod_{q_{j''}\in \text{odd}}\left(\dfrac{q_{j''}+1}{2}\right),\ j',\, j'' \in \{1,2,\cdots,n\} & q_j \in \text{other} \end{cases}$$

and q_j denotes the number of the fuzzy partitions of the jth input variable.

Proof: The proof can be obtained in [4].

4 Numerical Example

In this section, for $q_j \in$ other a numerical example is given to illustrate the stability examination for an open-loop T-S fuzzy system in detail. An open-loop T-S fuzzy system is considered as follows: