Arity-Monotonic Extended Aggregation Operators

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Abstract. A class of extended aggregation operators, called impact functions, is proposed and their basic properties are examined. Some important classes of functions like generalized ordered weighted averaging (OWA) and ordered weighted maximum (OWMax) operators are considered. The general idea is illustrated by the Producer Assessment Problem which includes the scientometric problem of rating scientists basing on the number of citations received by their publications. An interesting characterization of the well known h-index is given.

Keywords: aggregation, extended aggregation function, OWA, OWMax, h-index, scientometrics.

1 Introduction

Aggregation plays a central role in many areas of the human activity, including not only statistics, engineering, computer science or physics but also decision making, economy and social sciences. It appears always when the reasoning requires merging several values into a single one which may represent a kind of synthesis for all individual inputs. Such functions projecting multidimensional numerical space of input values into one dimension are generally called *aggregation operators*.

Apart from particular applications the theory of aggregation operators is a rapidly developing mathematical domain (we refer the reader to [6] for the recent state of art monograph).

Classically, aggregation operators are usually considered for a fixed number of arguments. For some applications it may be to restrictive. We face such a situation in the so-called Producer Assessment Problem where given alternatives are rated not only with respect to the quality of delivered items but also to their productivity. As a typical example we may indicate the problem of rating scientists by their publications' citations.

This is the reason that the aggregation operators defined for arbitrary number of arguments are of interest. In the paper we propose the axiomatic approach to such a class of extended aggregation operators, called impact functions, and discuss some interesting properties of such functions for different arities. We also study the properties of the

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generalized versions of some well known classes of aggregation operators like generalized ordered weighted averaging (OWA) and ordered weighted maximum (OWMax) operators. It is worth noting that well-known Hirsch h-index turns out to be a particular example of the latter family.

2 Preliminaries

2.1 Basic Notation

We adopt the notational convention from the recent monograph [6].

Let $\mathbb{I} = [a, b]$ denote any nonempty closed interval of extended real numbers $\bar{\mathbb{R}} = [-\infty, \infty]$. In this paper we assume that $0 \cdot \infty = 0$. Unless stated otherwise, $n, m \in \mathbb{N}$.

Let $\mathbb{N}_0 = \{0, 1, 2, \dots\}$ denote the set of all nonnegative integers. Moreover, let $[n] := \{1, 2, \dots, n\}$.

The set of all vectors of arbitrary length with elements in $\mathbb{I}$, i.e. $\bigcup_{n=1}^{\infty} \mathbb{I}^n$, will be denoted by $\mathbb{I}^{1,2,\cdots}$.

Given any $\mathbf{x} = (x_1, \dots, x_n)$, $\mathbf{y} = (y_1, \dots, y_n) \in \mathbb{I}^n$, we write $\mathbf{x} \leq \mathbf{y}$ iff $(\forall i \in [n])$ $x_i \leq y_i$. Moreover, $(n * x)$ is equivalent to $(x, x, \dots, x) \in \mathbb{I}^n$.

Let $x_{(i)}$ denote the ith-smallest value of $\mathbf{x} = (x_1, \dots, x_n)$. For simplicity of notation, we assume that $x_{(n+j)} := x_{(n)}$ for $j = 1, 2, \dots$.

For any $\mathbf{x} \in \mathbb{I}^n$ and $\mathbf{y} \in \mathbb{I}^m$ and any function f defined on $\mathbb{I}^{n+m}$ the notation $\mathsf{f}(\mathbf{x}, \mathbf{y})$ stands for $\mathsf{f}(x_1, \dots, x_n, y_1, \dots, y_m)$.

If $\mathsf{f} : X \to Y$ and $X' \subset X$ then a function $\mathsf{f}|_{X'} : X' \to Y$ such that $(\forall x \in X')$ $\mathsf{f}|_{X'}(x) = \mathsf{f}(x)$ is called a *restriction* of f to X'. Furthermore, if $\mathcal{F}$ is a family of functions mapping X to Y, then $\mathcal{F}|_{X'} := \{\mathsf{f}|_{X'} : \mathsf{f} \in \mathcal{F}\}$.

2.2 Aggregation Functions

Let us recall the notion of the aggregation function, which is often considered in the literature. Note that it is a particular sublass of the very broad family of aggregation operators. Here is a slightly modified version of the definition given in [6].

Definition 1. *An* ***aggregation function*** *in* $\mathbb{I}^n = [a, b]^n$ *is any function* $\mathsf{a}^{(n)} : \mathbb{I}^n \to \bar{\mathbb{R}}$ *which*

(nd) *is nondecreasing in each variable, i.e.* $(\forall \mathbf{x}, \mathbf{y} \in \mathbb{I}^n)$ $\mathbf{x} \leq \mathbf{y} \Rightarrow \mathsf{a}^{(n)}(\mathbf{x}) \leq \mathsf{a}^{(n)}(\mathbf{y})$,

(bl) *fulfills the lower boundary condition:* $\inf_{\mathbf{x} \in \mathbb{I}^n} \mathsf{a}^{(n)}(\mathbf{x}) = a$,

(bu) *fulfills the upper boundary condition:* $\sup_{\mathbf{x} \in \mathbb{I}^n} \mathsf{a}^{(n)}(\mathbf{x}) = b$.

Typical examples of aggregation functions are: sample minimum, maximum, arithmetic mean, median, and OWA operators. On the other hand, generally sample size, sum and constant function are not aggregation functions in the above sense.

It is worth noticing that axioms (nd) and (bl) imply $\mathsf{a}^{(n)}(n * a) = a$. We also have $\mathsf{a}^{(n)}(n * b) = b$ by (nd) and (bu).

From now on, let $\mathcal{E}(\mathbb{I})$ be the set of all functions $\mathsf{F} : \mathbb{I}^{1,2,\cdots} \to \bar{\mathbb{R}}$.

Now let us extend the class of aggregation functions to any number of arguments. Our definition agrees with the one given in [6]. Note that quite a different extension was proposed by Mayor and Calvo in [11].