

# Parallel Hybrid Metaheuristics for the Scheduling with Fuzzy Processing Times<sup>\*</sup>

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**Abstract.** In this paper, parallel simulated annealing with genetic enhancement algorithm (HSG) is presented and applied to permutation flow shop scheduling problem which has been proven to be  $\mathcal{NP}$ -complete in the strong sense. The metaheuristics is based on a clustering algorithm for simulated annealing but introduces a new mechanism for dynamic SA parameters adjustment based on genetic algorithms. The proposed parallel algorithm is based on the master-slave model with cooperation. Fuzzy arithmetic on fuzzy numbers is used to determine the minimum completion times  $C_{\max}$ . Finally, the computation results and discussion of the algorithms performance are presented.

## 1 Introduction

Practical machine scheduling problems are numerous and varied. They arise in diverse areas such as flexible manufacturing systems, production planning, communication, computer design, etc. A scheduling problem is to find sequences of jobs on given machines with the objective of minimizing some function. In a simpler version of the problem, flow shop scheduling, all jobs pass through all machines in the some order. In this paper, we deal with another special version of the problem called a permutation flow shop (PFS) scheduling problem where each machine processes the jobs in the same order. The PFS problem belongs to the NP-hard class problems, however a solution of such a problem is usually made using heuristic approach that converges to a locally optimal solution.

In recent studies, scheduling problems were fuzzificated by using the concept of fuzzy due date and processing times. In paper Dumitru and Luban [3] investigate the application of fuzzy sets on the problem of the production scheduling. Tsujimura et al. [12] present the branch and bound algorithm for the three machine flow shop problem when job processing times are described by triangular fuzzy numbers. Especially fuzzy logic application on the scheduling problems (by using fuzzy processing times) is presented in papers: Ishibuschi and Murata [6], Izzettin and Serpil [5] and Peng and Liu [9].

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In this study, flow shop scheduling problem of the typical situation of the flexible production systems which occupy a very important place in recent production systems are taken into consideration with fuzzy processing time.

## 2 Flow Shop Scheduling

The permutation flow shop problem can be formulated as follows. Each of  $n$  jobs from the set  $J = \{1, 2, \dots, n\}$  has to be processed on  $m$  machines  $1, 2, \dots, m$  in that order. Job  $j \in J$ , consists of a sequence of  $m$  operations; operation  $O_{jk}$  corresponds to the job  $j$  processing on machine  $k$  during an uninterrupted processing time  $p_{jk}$ . Assumptions:

- (a) for each job only one operation can be processed on a machine,
- (b) each machine can process only one job at a time,
- (c) the processing order is the same on each machine
- (d) all jobs are available for machine processing simultaneously at time zero.

We want to find a schedule such that the processing order is the same on each machine and the maximum completion time is minimal.

The flow shop problem is NP-complete and thus it is usually solved by approximation or heuristic methods. The use of simulated annealing is presented, e.g., in Osman and Potts [8], Bożejko and Wodecki [1] (parallel algorithm), tabu search in Nowicki and Smutnicki[7], Grabowski and Wodecki [4], and genetic algorithm in Reeves [11].

Each schedule of jobs can be represented by the permutation  $\pi = (\pi(1), \pi(2), \dots, \pi(n))$  on the set  $J$ . Let  $\Pi$  denote the set of all such permutations. We wish to find such permutation  $\pi^* \in \Pi$ , that

$$C_{\max}(\pi^*) = \min_{\pi \in \Pi} C_{\max}(\pi),$$

where  $C_{\max}(\pi)$  is the time required to complete all jobs on the machines.

## 3 Flow Shop Scheduling with Fuzzy Processing Times

Let us suppose that processing times of the jobs on machines are not deterministic but they are given by fuzzy numbers.

In this paper the fuzzy processing times  $p_{i,j}$  ( $i = 1, 2, \dots, m$ ,  $j = 1, 2, \dots, n$ ) are represented by a triangular membership function  $\mu$  (i.e. 3-tuple  $\tilde{p}_{i,j} = (p_{i,j}^{\min}, p_{i,j}^{\text{med}}, p_{i,j}^{\max})$ ) ( $i = 1, 2, \dots, m$ ,  $j = 1, 2, \dots, n$ ) with the following properties:

- i)  $(p_{i,j}^{\min} \leq p_{i,j}^{\text{med}} \leq p_{i,j}^{\max})$ ,
- ii)  $\mu(a) = 0$  for  $a \leq p_{i,j}^{\min}$  or  $a \geq p_{i,j}^{\max}$ ,
- iii)  $\mu(p_{i,j}^{\text{med}}) = 1$ ,
- iv)  $\mu$  is increasing on  $[p_{i,j}^{\min}, p_{i,j}^{\text{med}}]$  and decreasing on  $[p_{i,j}^{\text{med}}, p_{i,j}^{\max}]$ .

The addition of fuzzy numbers  $\tilde{a} = (a_1, a_2, a_3)$  and  $\tilde{b} = (b_1, b_2, b_3)$ , can be derived from the extension principle and it is as follows (see [2])

$$\tilde{a} + \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3).$$