

# Analysing Bidder Performance in Randomised and Fixed-Deadline Automated Auctions

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**Abstract.** The rule with which automated computerised auctions are closed play an important role in determining bidder strategies and auction outcomes. In this paper we examine two such rules: auctions with randomised closing times and fixed deadlines. To this end, stochastic models of auctions with discrete state-space representing the prices attained are developed and analysed. The models allow us to determine the stationary probabilistic outcomes of the auctions, which are used to examine the bidder performance, measured as the savings it makes with respect to the maximum payable or its payoff. For this purpose, one bidder is singled out as the “special bidder” (SB) and its performance is studied as a function of the speed with which it raises the price, or its bid rate. The results show that with random closures, the SB has incentives to place bids promptly to obtain high savings; on the other hand, with fixed deadline auctions, the SB should choose its bid rate with respect to the other system parameters in order to maximise payoffs.

**Keywords:** Analytical Models, Stochastic Processes, Automated Auctions.

## 1 Introduction

Automated entities engaging in computerised auctions on behalf of human counterparts of buyers and sellers have revolutionised trading activities over the Internet. The absence of geographical restrictions in participation, precise execution of predefined rules, fairness of outcomes, and the added advantage of computing power in monitoring information sources closely while performing demanding calculations have made automated auctions successful mechanisms. The rules for closing such auctions have significant influences on the bidder strategies and the closing prices [1,2,3]. Fixed deadlines or “hard closes” ensure auctions end in a predetermined time, and therefore neither participating party will be held up unpredictably. In contrast, “soft closes” allow for conditional extensions in the auction duration depending on certain criteria being fulfilled such as the incoming bids. Yet another important design feature is random closing times;

it is currently being used in auctioning financial securities, the London Stock Exchange [4] being a case in point.

Recently, [5] has examined probabilistic models of automated auctions with focus on a single bidder's performance, and has derived closed form solutions for the stationary probabilities of interest, including the winning probability, waiting time to win, and the savings made on the purchase. In [6] the model has been simplified using an approximation that yields tractable expressions for the measures of interest, at the cost of accuracy. In this paper, we continue the ongoing work, and analyse bidder performance in auctions with random closing times against bidder performance in auctions with fixed deadlines. The objective is to show how stochastic modelling tools that are commonly used in queueing systems may be used in developing mathematical models of repeated automated auctions, and employed to evaluate performance and determine bidder strategies. To this end, one bidder is tagged as the "special bidder" (SB), and the auction outcomes are evaluated as a function of the speed with which it raises the price.

Section 2 presents a model with random closures where groups of bidders participate; members within a group are statistically identical in behaviour, parameterised by the speed at which they raise the bids. The auction duration, in this setting, is random, and the model is a generalisation of that studied in [5,6]. Subsequently in Section 3 a model for fixed lengths of auction times is examined. Numerical examples are presented in discussing the model's results. The paper is concluded with summary, comments, and suggestions for future work in Section 4.

## 2 Random Closing Times

We consider ascending auctions with discrete unit price bid increments. Suppose a pool of bidders engage in an auction where each bidder raises the price by a unit at a particular speed called the bid rate. The auction may proceed in this regard until the maximum valuation for the good  $v$  is reached and the bidding stops. However, at any nonzero price, the auction may close after a memoryless random period described with the parameter  $\delta$ . This timeout is refreshed everytime a bid is placed before the expiry. A sale concludes the auction when the timeout expires without interruption from bids. After a random rest period with parameter  $r$ , the process restarts and repeats infinitely. All the random periods are drawn from exponential distributions with respective parameters.

### 2.1 The Mathematical Model

Formally, suppose  $k$  groups of bidders participate, and the  $i$ -th group consists of  $n_i$  bidders, each of whom raises the price at rate  $\beta_i$ , for  $i = 1, \dots, k$ . The system then is modelled as a continuous-time Markov process  $\{X_t : t \geq 0\}$  [9] with state space

$$X_t \in Y = \{0, B(i, l), A(i, l) : 1 \leq l \leq v, 1 \leq i \leq k\}, \quad (1)$$