A Generalization of the Pignistic Transform for Partial Bet

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Abstract. The Transferable Belief Model is a powerful interpretation of belief function theory where decision making is based on the pignistic transform. Smets has proposed a generalization of the pignistic transform which appears to be equivalent to the Shapley value in the transferable utility model. It corresponds to the situation where the decision maker bets on several hypotheses by associating a subjective probability to non-singleton subsets of hypotheses. Naturally, the larger the set of hypotheses is, the higher the Shapley value is. As a consequence, it is impossible to make a decision based on the comparison of two sets of hypotheses of different size, because the larger set would be promoted. This behaviour is natural in a game theory approach of decision making, but, in the TBM framework, it could be useful to model other kinds of decision processes. Hence, in this article, we propose another generalization of the pignistic transform where the belief in too large focal elements is normalized in a different manner prior to its redistribution.

1 Introduction

The Transferable Belief Model [1] (TBM) is based on the decomposition of the problem into two stages: the **credal level**, in which the pieces of knowledge are aggregated under the formalism of belief functions, and the **pignistic level**, where the decision is made by applying the Pignistic Transform (PT): It converts the final belief function (resulting from the fusions of the credal level) into a probability function. Then, a classical probabilistic decision is made.

The manner in which belief functions allow to deal with compound hypotheses (i.e. set of several singleton hypotheses) is one of the main interests of the TBM. On the other hand, the decision making in the TBM only allows betting on singletons. Hence, at the decision making level, part of the belief function flexibility is lost. Of course, it is made on purpose, as, betting on a compound

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hypothesis is equivalent to remain hesitant among several singletons. It would mean no real decision is made, or equivalently, that no bet is booked, which seems curious, as the PT is based on betting ("pignistic" is derived from the Latin word for "bet").

Nevertheless, there are situations in which it could be interesting to bet on compound hypotheses. From the TBM point of view, it means generalising the PT so that it can handles compound bets. Smets has already presented such a generalisation [2], and it appears [3] to corresponds to the situation of a "n-person games" [4] presented by Shapley in the Transferable Utility Model in 1953. This work on game theory considers the case of a coalition of gamblers who wants to share fairly the gain with respect to the involvement of each. Once the formula is transposed to the TBM, the purpose is to share a global belief between several compound hypotheses. Obviously, one expects the transform to promote the hypotheses the cardinality of which is the greatest... Roughly, it means that, if for the same book, it is possible to bet on the singleton hypothesis $\{h_1\}$ or on the compound hypothesis $\{h_1, h_2\}$, then, this latter must be preferred (even if the chances for h_1 are far more interesting than for h_2). Practically, this intuitive behaviour looks perfectly accurate, and of course, the generalization proposed by Smets behaves so.

On the other hand, there are yet other situations, where it should be encouraged to bet on singleton hypotheses when possible, whereas it should remain allowed to bet on compound hypotheses when it is impossible to be more accurate. Hence, we depict a "progressive" decision process, where it is possible to remain slightly hesitant, and to manually tune the level between hesitation and bet. Let us imagine such a situation: the position of a robot is modelled by a state-machine, and its trajectory along a discrete time scale is modelled by a lattice. At each iteration of the discrete time, the sensors provide information to the robot, and these pieces of information are processed in the TBM framework: they are fused together (the credal level) and the state of the robot is inferred by a decision process (the pignistic level). At this point several stances are possible:

- the classical PT is used. Unfortunately, as the sensors are error-prone, the inferred state is not always the right one. Finally, the inferred trajectory is made of right and wrong states with respect to the ground-truth (Fig. 1). Of course, the TBM provides several tools to filter such trajectories [5,6,7], and, in spite of a relative computational cost, they are really efficient.
- Instead of betting on a single state at each iteration of the time, it is safer to bet on a compound hypothesis (i.e. on a group of several states, knowing that, the more numerous, the less chance to make a mistake). Unfortunately, the risk is now to face a situation where no real decision is made and the inferred trajectory is too imprecise (Fig. 2).
- The balance between these two extreme stances would be to automatically tune the level of hesitation in the bet: When the decision is difficult to make, a compound hypothesis is assessed to avoid a mistake, and otherwise, a singleton hypothesis is assessed, to remain accurate (Fig. 3).