

# Fluorescence Microscopy Deconvolution Based on Bregman Iteration and Richardson-Lucy Algorithm with TV Regularization

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**Abstract.** Fluorescence microscopy has become an important tool in biological and medical sciences for imaging thin specimen, even living ones. Due to Out-of-focus blurring and noise the acquired images are degraded and therefore it can be difficult to analyse them. In the last decade many methods have been proposed to restore these images. One of the most popular methods to restore microscopy images is an iterative Richardson-Lucy Algorithm with Total Variation Regularization. Besides there are some new approaches based on Bregman Iteration to improve the quality of restored images in general. In this paper we formulate a new algorithm for restoring fluorescence microscope images using Bregman Iteration with special attention to the microscopy specific properties. We can prove that the quality of the restored images increases by using the I-divergence and the mean square error criteria.

## 1 Introduction

### 1.1 Image Formation Model

Any optical system degrades the acquired images due to the physical properties of the optical aperture and of light itself on the one side and due to the detection process on the other side. These degradations contain two separate parts: the blurring of the image which can be described by a PSF (point spread function) convolution and the additional noise in the image. The type of noise that can be found in the image is dependent on the acquisition process. For fluorescence microscopy it is well-known that there is Poisson noise in the acquired images because it is a low-photon imaginary technique. The Poisson noise will be represented by  $\phi(\cdot)$  in this paper. Thus the suitable image formation model where  $i$  is the observed image,  $o$  the original image,  $h$  represents the PSF and  $\otimes$  the convolution operator is given by:

$$i = \phi(o \otimes h) \tag{1}$$

In this paper we assume that the PSF is already known. It is possible to use a second image with beads to estimate the PSF in a preceding step or to calculate the PSF due to the physical properties of the used microscope.

## 1.2 Restoration of Fluorescence Microscope Images

To restore the degradation due to Poisson noise, as mentioned before, it is common to use a Richardson-Lucy (RL) algorithm consisting of an expectation maximization algorithm which calculates the maximum likelihood estimation [1, 2]. Basically the algorithm contains the iterative minimization of the following energy functional  $H$  applying the known variables from formula (1):

$$H(o, i) = \int o \otimes h - i \cdot \log(o \otimes h) \quad (2)$$

The basic RL algorithm has a huge disadvantage. It amplifies noise after several iterations while first improving the image. In order to overcome this problem one can denoise the image first and use the RL algorithm to deblurr the image afterwards but it is a better solution to add a separate regularization functional. This additional term is weighted with a regularization parameter. A very popular choice for a regularization term is the Tikonov-Miller (TM) regularization which can be combined with the RL energy functional [3].

$$R_{TM}(o) = \int |\nabla o|^2 \quad (3)$$

$\nabla$  is the gradient of the image  $o$  and  $|\cdot|$  describes the  $L^2$  norm. The TM regularization allows the RL algorithm to converge to a suitable solution but it smoothes the edges. This is a well-known problem and it is possible to use another regularization technique named Total Variation (TV) regularization which preserves the edges. A combined RL and TV algorithm for microscopy contains the TV term [4] which is again weighted with a regularization parameter. This method has another drawback because it rounds corners.

$$R_{TV}(o) = \int |\nabla o| \quad (4)$$

Both techniques are very popular and commonly used. The choice for the regularization term should depend on what one wants to analyse in the reconstructed image.

## 1.3 Restoration Approach Using the $L^2$ Norm and the Bregman Iteration

Another modified approach to reconstruct blurred and noisy images is using the Bregman distance introduced in [5]. This distance was added to a blind deconvolution algorithm [6] and later also used to reconstruct a high resolution image on basis of a set of low resolution images [7]. In both approaches a  $L^2$  norm based deblurring functional with TV is used. The blind deconvolution algorithm [6] consists of two independent parts where alternately the image is deblurred and the PSF is estimated. Here it is assumed that the PSF is already known and so only the deblurring of the image is of interest. To reconstruct the image first a common minimization of the combined energy functional containing the