A more efficient knowledge representation for Allen's algebra and point algebra

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Abstract. In many AI applications, one has incomplete qualitative knowledge about the order of occurring events. A common way to express knowledge about this temporal reasoning problem is Allen's interval algebra. Unfortunately, its main interesting reasoning tasks, consistency check and minimal labeling, are intractable (assuming $P \neq NP$). Mostly, reasoning tasks in tractable subclasses of Allen's algebra are performed with constraint propagation techniques. This paper presents a new reasoning approach that performs the main reasoning tasks much more efficient than traditional constraint propagation methods. In particular, we present a sound and complete O(n^2)-time algorithm for minimal labeling computation that can be used for the pointisable subclass of Allen's algebra.

1 Introduction

In many AI applications, one has incomplete qualitative knowledge about the order of occurring events. It is a temporal reasoning task to complete the event order as far as possible. A common way to express knowledge about this task is Allen's interval algebra $\mathcal{A}$ [1]. The algebra can express any possibly indefinite relationship between two intervals. Complete knowledge about their temporal relationship is expressible with one of the thirteen mutually exclusive basic relations depicted in Figure 1.

Relation	Symbol	Inverse	Meaning
A before B	b	bi	
A meets B	m	mi	
A overlaps B	o	oi	
A starts B	s	si	
A during B	d	di	
A ends B	f	fi	
A equals B	e	e	

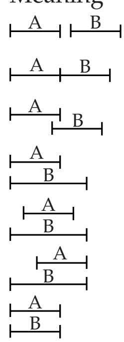

Figure 1. Basic interval relations

I. Imam et al. (Eds.): IEA/AIE-99, LNAI 1611, pp. 747–752, 1999.

Mainly, there are two reasoning tasks arising in $\mathcal{A}$:

- Consistency maintenance decides if new temporal knowledge incorporated into the actual knowledge base is consistent. In terms of constraint networks one has to check if there is a consistent scenario among the alternatively defined basic relations. In the following, we will call this problem ISAT.
- Question answering consists of providing answers to queries to the possible relative order between time relations. The main problem in terms of constraint networks is to determine for every network edge the subset of basic relations that is part of a consistent scenario. This task is called the minimal labeling or strongest implied relation problem ISI.

Unfortunately, [6] proof that ISAT($\mathcal{A}$) and ISI($\mathcal{A}$) are NP-complete. An alternative representation form is the less expressive point algebra $\mathcal{TP}$ [6]. This algebra has time points instead of intervals as its primitives and therefore contains only three basic relations (for two time points P_1, P_2 the possible relations are $P_1 < P_2$, $P_1 = P_2$, and $P_1 > P_2$). Like in $\mathcal{A}$, any disjunction of basic relations is allowed resulting in $2^3 = 8$ elements.

The restricted expressiveness of $\mathcal{TP}$ is rewarded with the tractability of ISAT and ISI, which are defined as in $\mathcal{A}$. Interestingly, [6] show that a subclass of $\mathcal{A}$, the pointisable algebra $\mathcal{P}$, can be expressed within $\mathcal{TP}$ (see [5] for an enumeration).

For ISAT($\mathcal{P}$), a $O(n^2)$-time algorithm (w.r.t. the number of time points) can be found in [4]. Additionally, [4] presents the so far best ISI($\mathcal{P}$) algorithm for minimal labeling computation which is $O(n^4)$-time in worst case.

This paper presents an alternative reasoning approach that solves ISI($\mathcal{P}$) in $O(n^2)$-time, too. In the remainder, we present the data structure called ‘ordered time line’ on which our reasoning takes place and outline the algorithm inserting time point algebra constraints into ordered time line. We conclude with an outlook to further research.

2 Instantiation Intervals

Our reasoning approach is influenced by van Beek’s instantiation algorithm for ISAT($\mathcal{P}$) [4]. Its first step is the transformation of all time interval constraints into constraints relating pairs of interval endpoints. These constraints can be expressed within $\mathcal{TP}$. Afterwards, van Beek finds a consistent instantiation of all constraint variables. The basic relations between them finally give an ISAT solution.

Unlike van Beek, we represent time points by intervals qualitatively constraining the time period in which time points can be instantiated. As an example, Figure 2 depicts the transformation and instantiation of $A\{b, m\}B \in \mathcal{P}$ into instantiation intervals. The built up total order of instantiation interval endpoints on an imaginary time line will be called ‘ordered time line’ OTL. Note that instantiation interval lengths and their position within OTL do not have any specific values. In Figure 2, they are chosen arbitrarily.

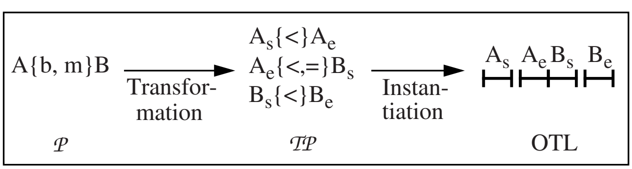

Figure 2. Transformation of *A{before, meets}B* into instantiation intervals.