Fuzzy-Rough Modus Ponens and Modus Tollens as a Basis for Approximate Reasoning

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Abstract. We have proposed a fuzzy rough set approach without using any fuzzy logical connectives to extract gradual decision rules from decision tables. In this paper, we discuss the use of these gradual decision rules within modus ponens and modus tollens inference patterns. We discuss the difference and similarity between modus ponens and modus tollens and, moreover, we generalize them to formalize approximate reasoning based on the extracted gradual decision rules. We demonstrate that approximate reasoning can be performed by manipulation of modifier functions associated with the gradual decision rules.

1 Introduction

Rough set theory deals mainly with the ambiguity of information caused by granular description of objects, while fuzzy set theory treats mainly the uncertainty of concepts and linguistic categories. Because of the difference in the treatment of uncertainty, fuzzy set theory and rough set theory are complementary and their various combinations have been studied by many researchers (see for example [1], [3], [6], [7], [8], [9], [10], [12], [16], [17], [18]). Most of them involved some fuzzy logical connectives (t-norm, t-conorm, fuzzy implication) to define fuzzy set operations. It is known, however, that selection of the "right" fuzzy logical connectives is not an easy task and that the results of fuzzy rough set analysis are sensitive to this selection. The authors [4] have proposed fuzzy rough sets without using any fuzzy logical connectives to extract gradual decision rules from decision tables. Within this approach, lower and upper approximations, are defined using modifier functions following from a given decision table.

This paper presents results of a fundamental study concerning utilization of knowledge obtained by the fuzzy rough set approach proposed in [4]. Since the obtained knowledge is represented by gradual decision rules, we discuss inference pat-

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terns (modus ponens and modus tollens) for gradual decision rules. We discuss the difference and the similarity between modus ponens and modus tollens under some monotonicity conditions. Moreover, we discuss inference patterns of the generalized modus ponens and modus tollens as a basis for approximate reasoning. The results demonstrate that approximate reasoning can be performed by manipulation of modifier functions associated with the extracted gradual decision rules.

In the next section, we review gradual decision rules extracted from a decision table and underlying fuzzy rough sets. We describe fuzzy-rough modus ponens and modus tollens with respect to the extracted gradual decision rules in Section 3. We discuss the difference and the similarity between fuzzy-rough modus ponens and modus tollens. In Section 4, we generalize the modus ponens and modus tollens in order to make inference using different fuzzy sets in the gradual decision rules. We demonstrate that all inference can be done by manipulation of modifier functions. Finally, we give concluding remarks in Section 5.

2 Gradual Decision Rules Extracted from a Decision Table

In a given decision table, we may found some gradual decision rules of the following types [4]:

- *lower-approximation rules with positive relationship* (LP-rule): "if condition X has credibility $C(X)\geq\alpha$, then decision Y has credibility $C(Y)\geq f_{Y|X}^{+}(\alpha)$";
- *lower-approximation rules with negative relationship* (LN-rule): "if condition X has credibility $C(X)\leq\alpha$, then decision Y has credibility $C(Y)\geq f_{Y|X}^{-}(\alpha)$";
- *upper-approximation rule with positive relationship* (UP-rule): "if condition X has credibility $C(X)\leq\alpha$, then decision Y could have credibility $C(Y)\leq g_{Y|X}^{+}(\alpha)$";
- *upper-approximation rule with negative relationship* (UN-rule): "if condition X has credibility $C(X)\geq\alpha$, then decision Y could have credibility $C(Y)\leq g_{Y|X}^{-}(\alpha)$",

where X is a given condition (premise), Y is a given decision (conclusion) and $f_{Y|X}^{+}:[0,1]\to[0,1]$, $f_{Y|X}^{-}:[0,1]\to[0,1]$, $g_{Y|X}^{+}:[0,1]\to[0,1]$ and $g_{Y|X}^{-}:[0,1]\to[0,1]$ are functions relating the credibility of X with the credibility of Y in lower- and upper-approximation rules, respectively. Those functions can be seen as modifier functions (see, for example, [8]). An LP-rule can be regarded as a gradual decision rule [2]; it can be interpreted as: "the more object x is X, the more it is Y". In this case, the relationship between credibility of premise and conclusion is positive and certain. LN-rule can be interpreted in turn as: "the less object x is X, the more it is Y", so the relationship is negative and certain. On the other hand, the UP-rule can be interpreted as: "the more object x is X, the more it could be Y", so the relationship is positive and possible. Finally, UN-rule can be interpreted as: "the less object x is X, the more it could be Y", so the relationship is negative and possible.

Example 1. Let us consider a decision table about hypothetical car selection problem in which the mileage is used for evaluation of cars. We may define a fuzzy set X of *gas_saving_cars* by the following membership function: