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April 11, 1974

Neil:

Your addendum to the Handbook has sparked some discoveries.

Consider the words $W_n = w_1 w_2 \dots w_n$ with $w_i = 1(i) i$; there are $n!$ words of length n ; eg- $n=2$ $\begin{matrix} 11 \\ 12 \end{matrix}$ $n=3$ $\begin{matrix} 111 \\ 112 \\ 123 \end{matrix}$ $\begin{matrix} 121 \\ 122 \\ 123 \end{matrix}$

Count the total appearances of each number; W_{nj} is the count for number j and $W_n(x) = \sum W_{nj} x^j$. For $n=3$ the counts

by position is

no.	pos	1	2	3
1		6	3	2
2			3	2
3				2

and it is easy to see that

$$W_{n1} = n! \left(1 + \frac{1}{2} + \dots + \frac{1}{n} \right)$$

$$W_{n2} = n! \left(\frac{1}{2} + \dots + \frac{1}{n} \right)$$

$$W_{nj} = n! \left(\frac{1}{j} + \dots + \frac{1}{n} \right)$$

A table for W_{nj} is enclosed, marked with the Handbook sequence numbers. Thus I was surprised to note that W_{n1} is the sequence of signless Stirling I nos $|s(n,1)|$. The other duplications of the Handbook all have the caption, Generalized Stirling Numbers P&F 77 762. Notice that $W_{n2} = W_{n1} - n! = |s(n,2)| - n!$

The same count for level codes of ballots

with say B_{nj} = no of appearances of j , gives $B_{n1} = C_{n+1} - C_n$ ($C_n = \frac{1}{n!} \binom{2n}{n}$) for sequence 1130 and $B_{n2} = C_{n+1} - 2C_n$ for sequence 1+15 Partitions of a Polygon by No. of Parts Cayley 13 95.

Yours
John

April 11, 1957

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How important is the standard deviation?

Consider the number of trials $n = 100$ and the number of successes $X = 50$.
The standard deviation is $\sqrt{npq} = \sqrt{100 \cdot 0.5 \cdot 0.5} = 5$.

Let X be the number of successes in n trials.
The mean is $\mu = np = 50$ and the standard deviation is $\sigma = \sqrt{npq} = 5$.

Let $Z = \frac{X - \mu}{\sigma}$ be the standardized variable.
Then Z has a mean of 0 and a standard deviation of 1.

By the Central Limit Theorem, the distribution of Z approaches a standard normal distribution as n increases.
For $n = 100$, the distribution is very close to normal.

Done

Vertical grid with handwritten notes on the left side.

Handwritten notes on the right margin, including '1900' and '1910'.