✓ 891 ✓

April 11, 1974

Neil:

Your ~~addendum~~ addendum to the Handbook has sparked some discoveries.

Consider the words $W_n = w_1 w_2 \cdots w_n$ with $w_i = 1(1)i$; there are $n!$ words of length n; e.g. $n=2$ 11, 12; $n=3$ 111, 112, 113, 121, 122, 123

Count the total appearances of each number; W_{nj} is the count for number j and $W_n(x) = \sum_1 W_{nj} x^j$. For $n=3$ the count by position is

no.\pos	1	2	3	
1	6	3	2	11
2		3	2	5
3			2	2

and it is easy to see that

$$W_{n1} = n!\left(1 + \frac{1}{2} + \cdots + \frac{1}{n}\right)$$
$$W_{n2} = n!\left(\frac{1}{2} + \cdots + \frac{1}{n}\right)$$
$$W_{nj} = n!\left(\frac{1}{j} + \cdots + \frac{1}{n}\right)$$

A table for W_{nj} is enclosed, marked with the Handbook sequence numbers. Thus I was surprised to note that W_{n1} is the sequence of signless Stirling I nos $|s_{n+1,2}|$. The other duplications of the Handbook all have the caption. Generalized Stirling Numbers PEF 77 762, Notice that $W_{n2} = W_{n1} - n! = |s_{n+1,2}| - n!$

The same count for level codes of ballots with say B_{nj} = no of appearances of j, gives $B_{n1} = C_{n+1} - C_n$ ($C_n = \frac{1}{n+1}\binom{2n}{n}$) for sequence 1130 and $B_{n2} = C_{n+1} - 2C_n$ for sequence 1415 Partitions of a Polygon by No. of Parts Cayley 13 95.

Yours

John

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Words $W_n = w_1 \dots w_n$ $w_i = 1(1)i$ by Totals

$W_n(x) = \sum W_{ni} x^i$

	1	2	3	4	5	6	7 ~~8~~	8
#1165	1	3	11	50	274	1764	13068	109584
# 1125		1	5	26	154	1044	8028	69264
2x#1873			2	14	94	684	5508	49104
6x#1990				6	54	444	3828	35664
					24	264	2568	25584
						120	1560	17520
							720	10800
								5040