Introspective Pushdown Analysis

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“grad-school Vietnam”

“charred remains”

“would-be Ph.D.s”

-Olin Shivers

“academic War on Terror”

“unwinnable”

“never-ending”

-Me, to my grad students

What is flow analysis?

What is flow analysis?

What is wrong with it?

What is flow analysis?

What is wrong with it?

How do we fix it?

How do we fix the fixes?

We use fixed points.

What is flow analysis?

What is control-flow analysis?

```
(f x)
```

What is `f`?

Why not run the program?

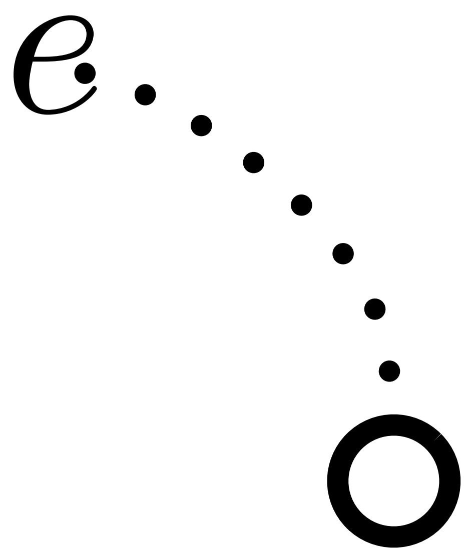
CEK

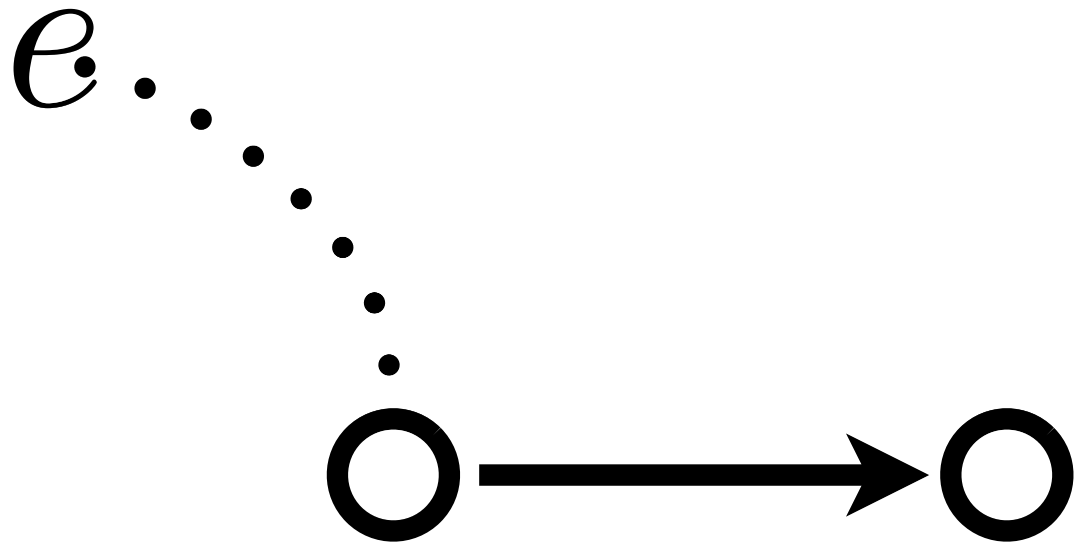
CEK

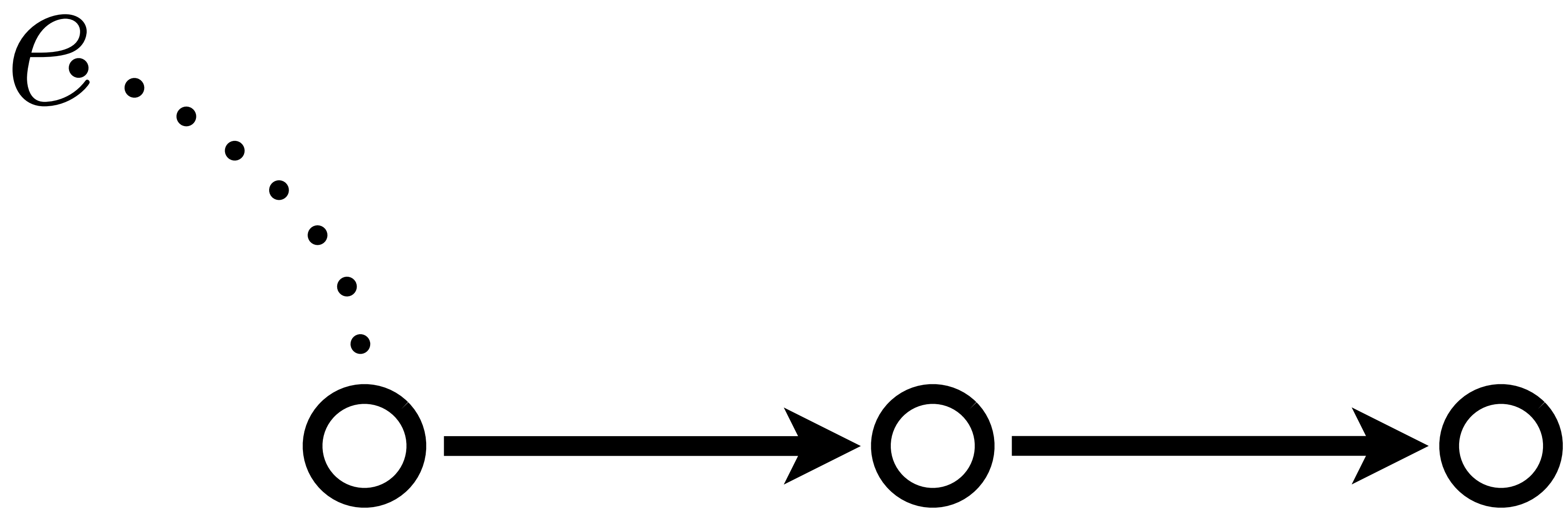
(Felleisen and Friedman, 1986)

e

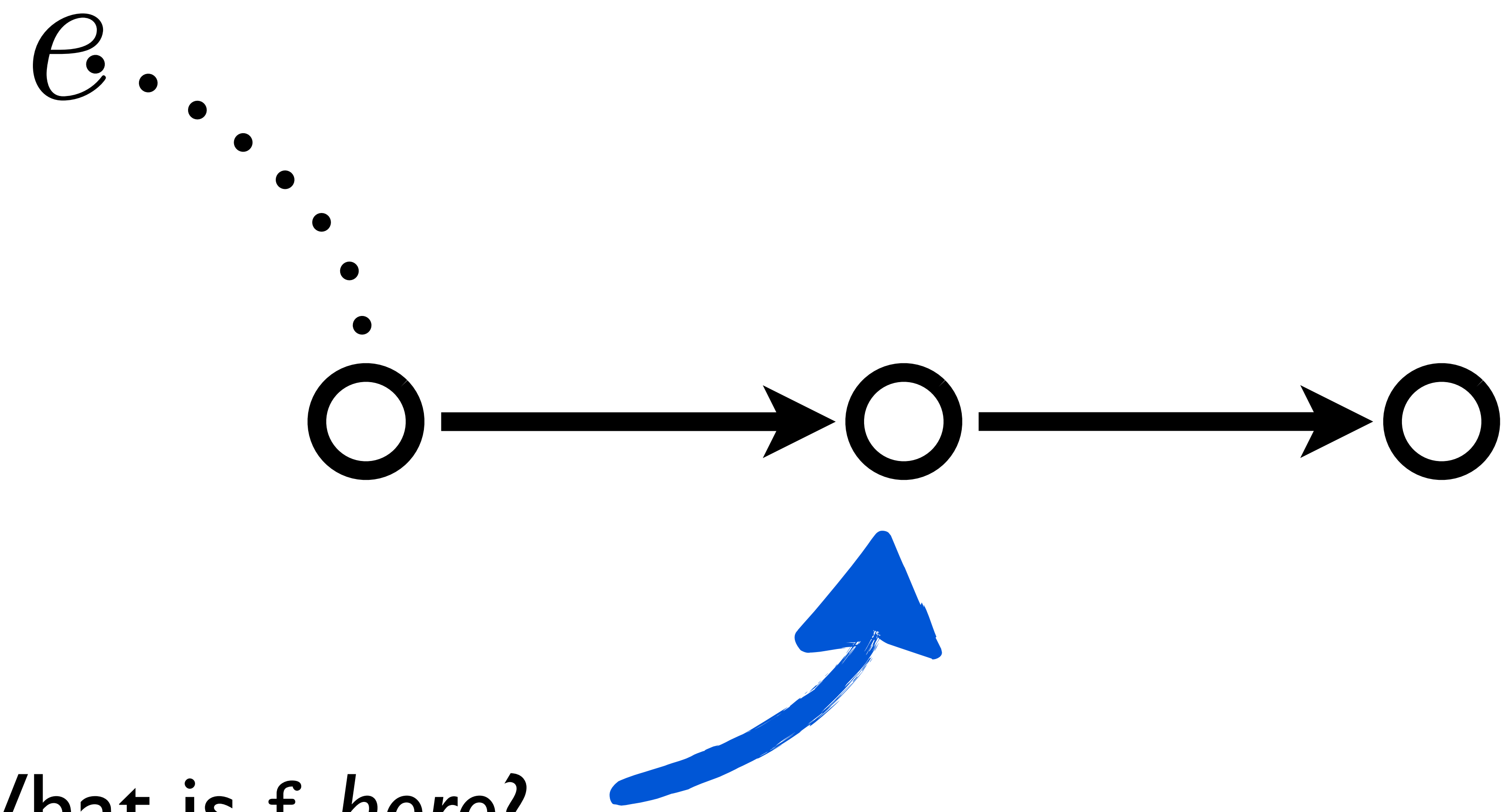
e


e

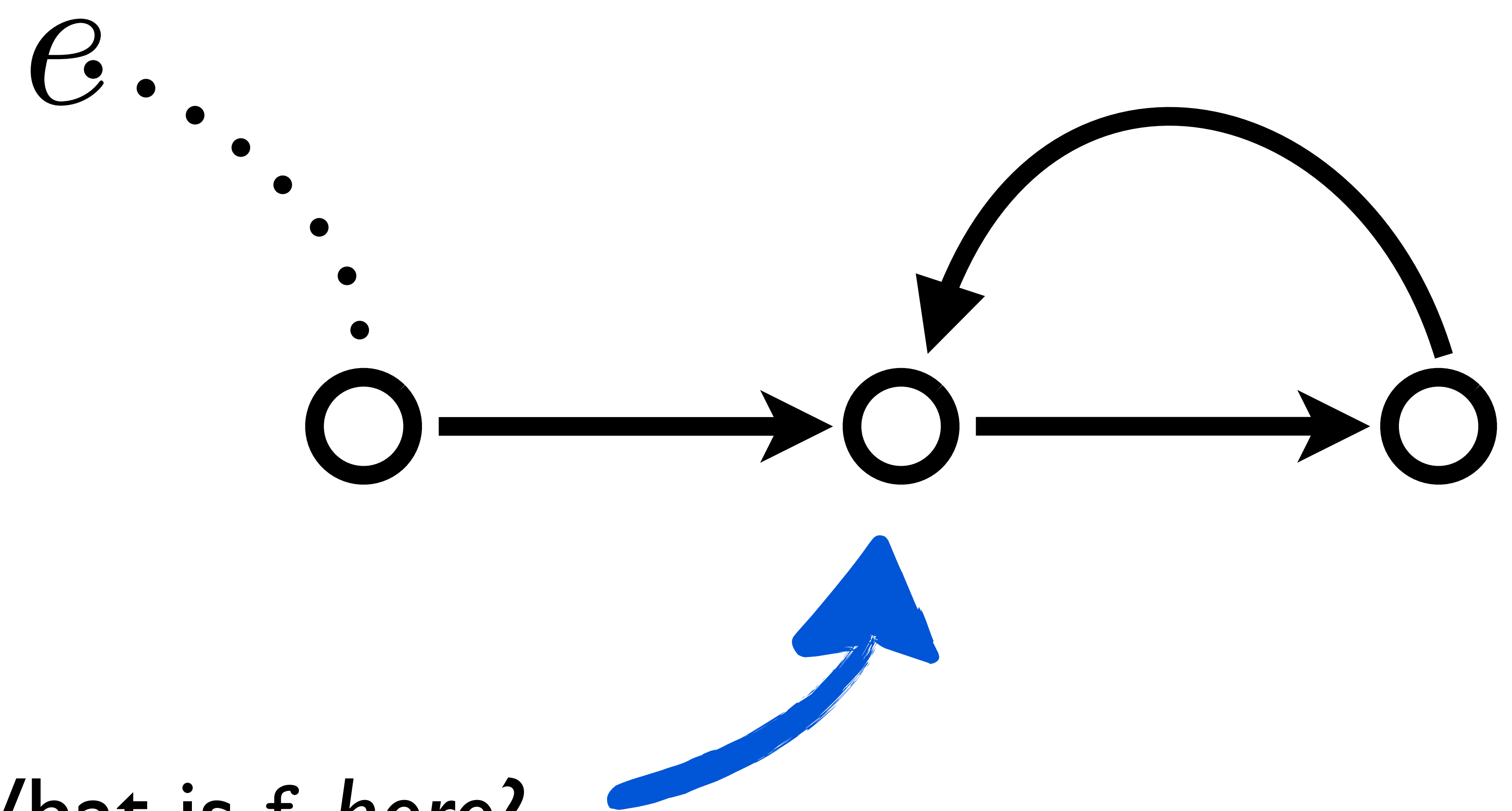

e



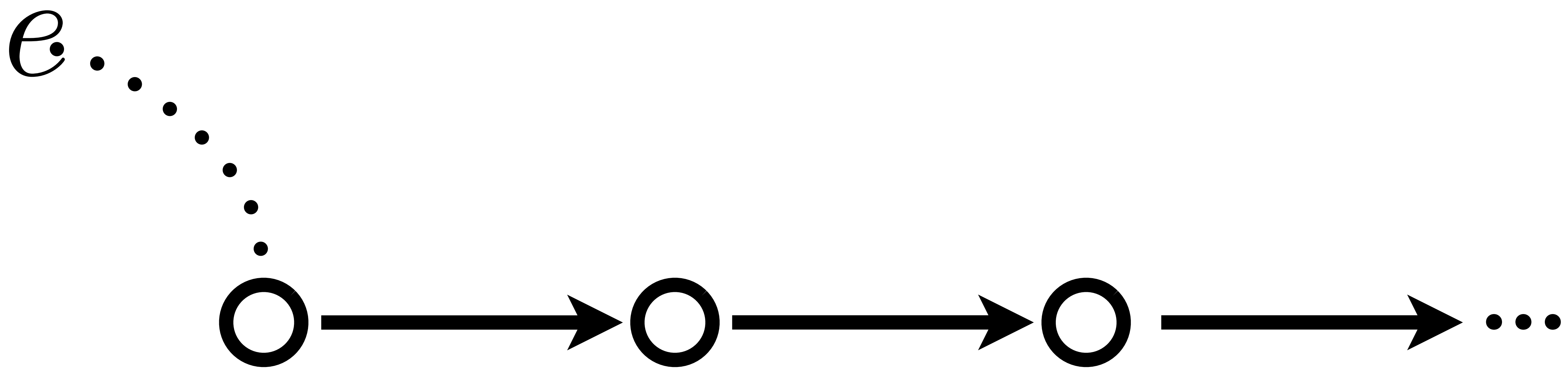
e



What is `f`, *here*?



What is `f`, *here*?


e

Make it terminate?

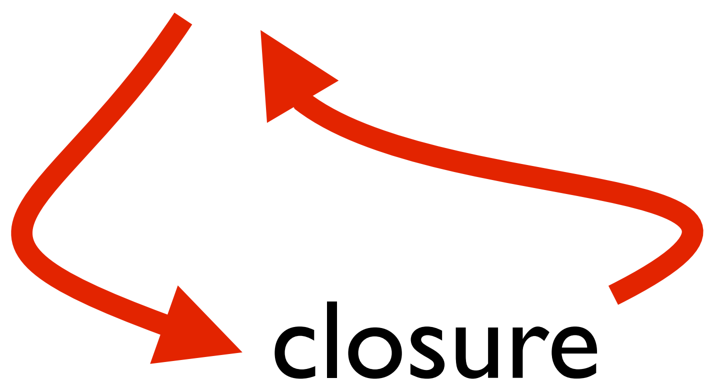
Make it finite.

Make it finite.

(Might, SAS 2010) (Van Horn and Might, ICFP 2010)

CEK

CEK



CEK

$$E = V \rightarrow \lambda\ x\ E$$

CEK

$$E = V \rightarrow \lambda x\ E$$

CEK

$$E = V \to A$$

CEK

CEK

CEK

CESK

CESK

(Felleisen and Friedman, 1987)

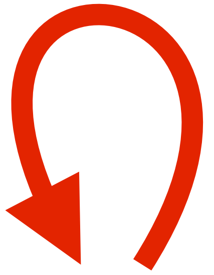
CESK

$$S = A \rightarrow \lambda \times E$$

CESK

$$S = A \to \lambda \times E$$

CESK



CESK

CESK*

CESK*

$$K^* \subset A$$

CESK*

$$S = A \rightarrow \lambda \times E + K$$

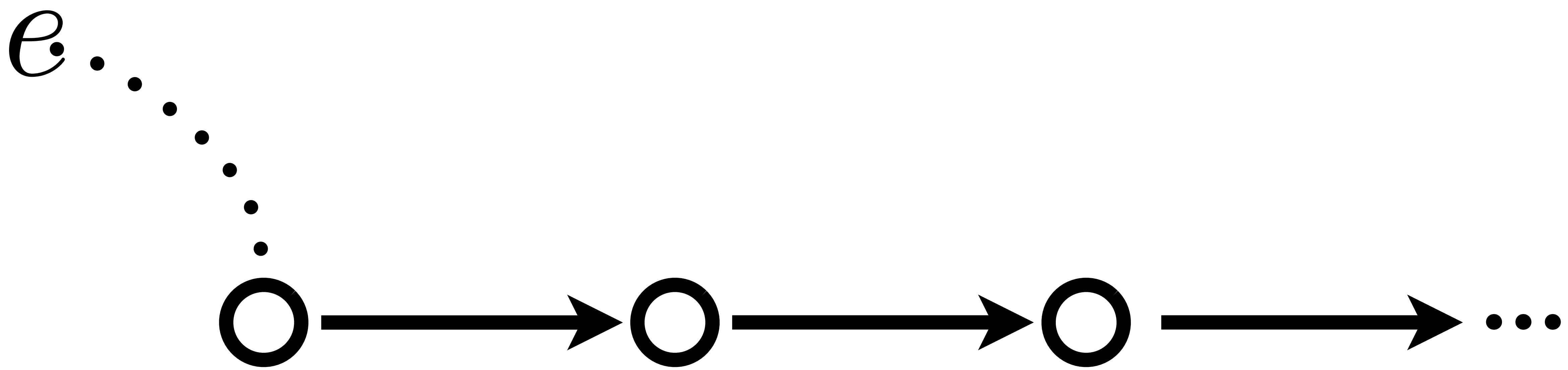
CESK*

CESK*

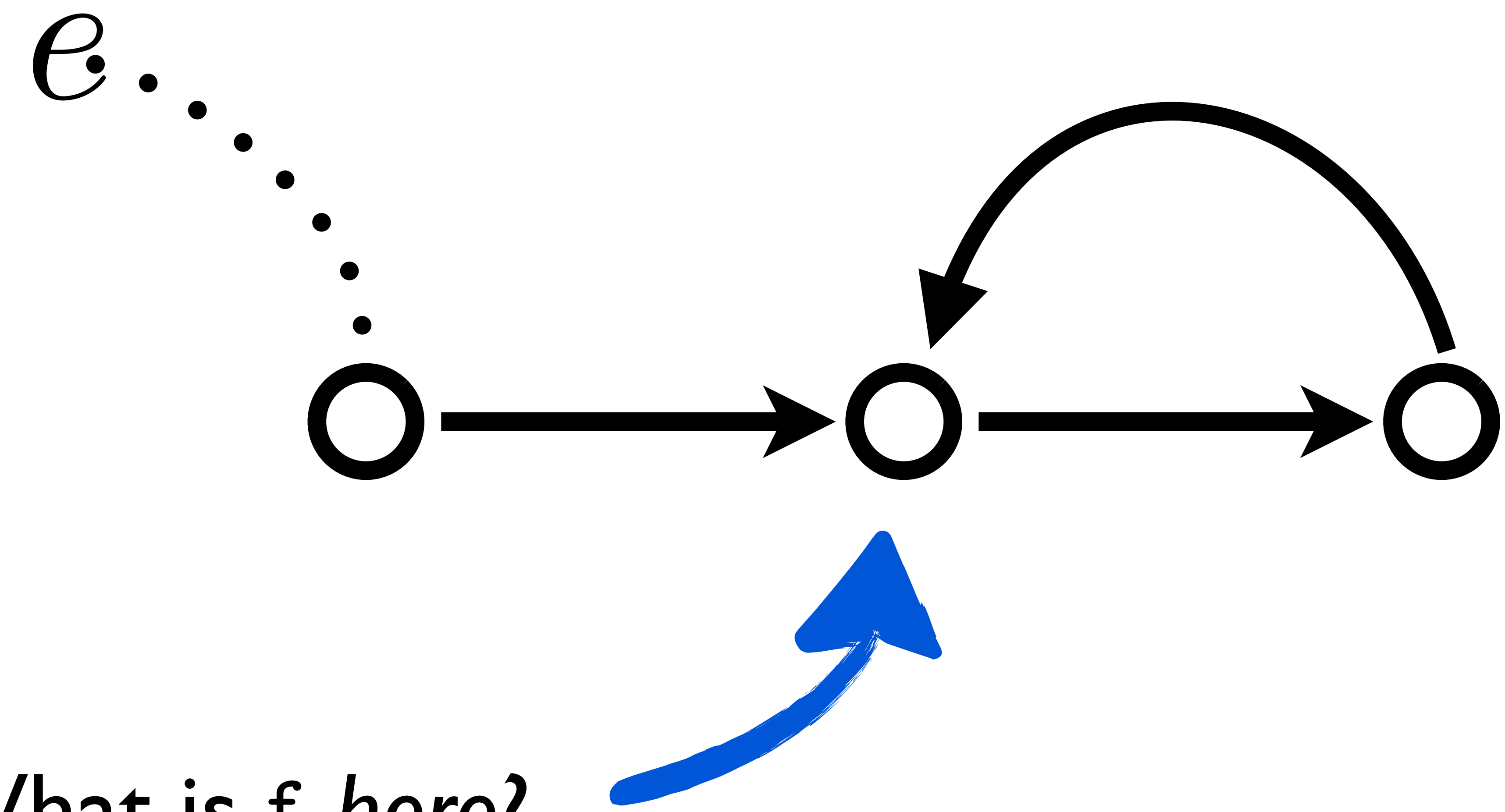
$C E \hat{S} K^*$

CESK*

e



e



What is $\mathtt{f}$, *here*?

What’s wrong with flow analysis?

Control-flow forks.

Data-flow merges.

Why?

$$S = A \rightarrow D$$

S = A → D

$$S = \hat{A} \longrightarrow D$$

$$S = \hat{A} \longrightarrow D$$

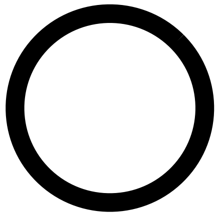
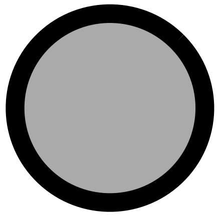
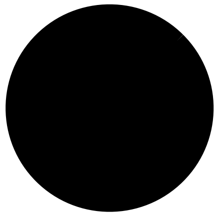
$$S = \hat{A} \rightarrow \hat{D}$$

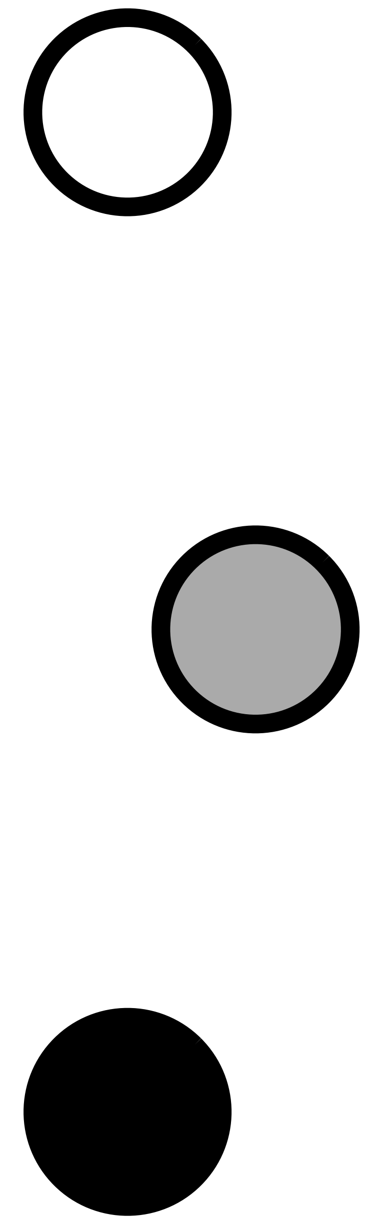
$$\widehat{S} = \widehat{A} \longrightarrow \mathcal{P}(\widehat{D})$$

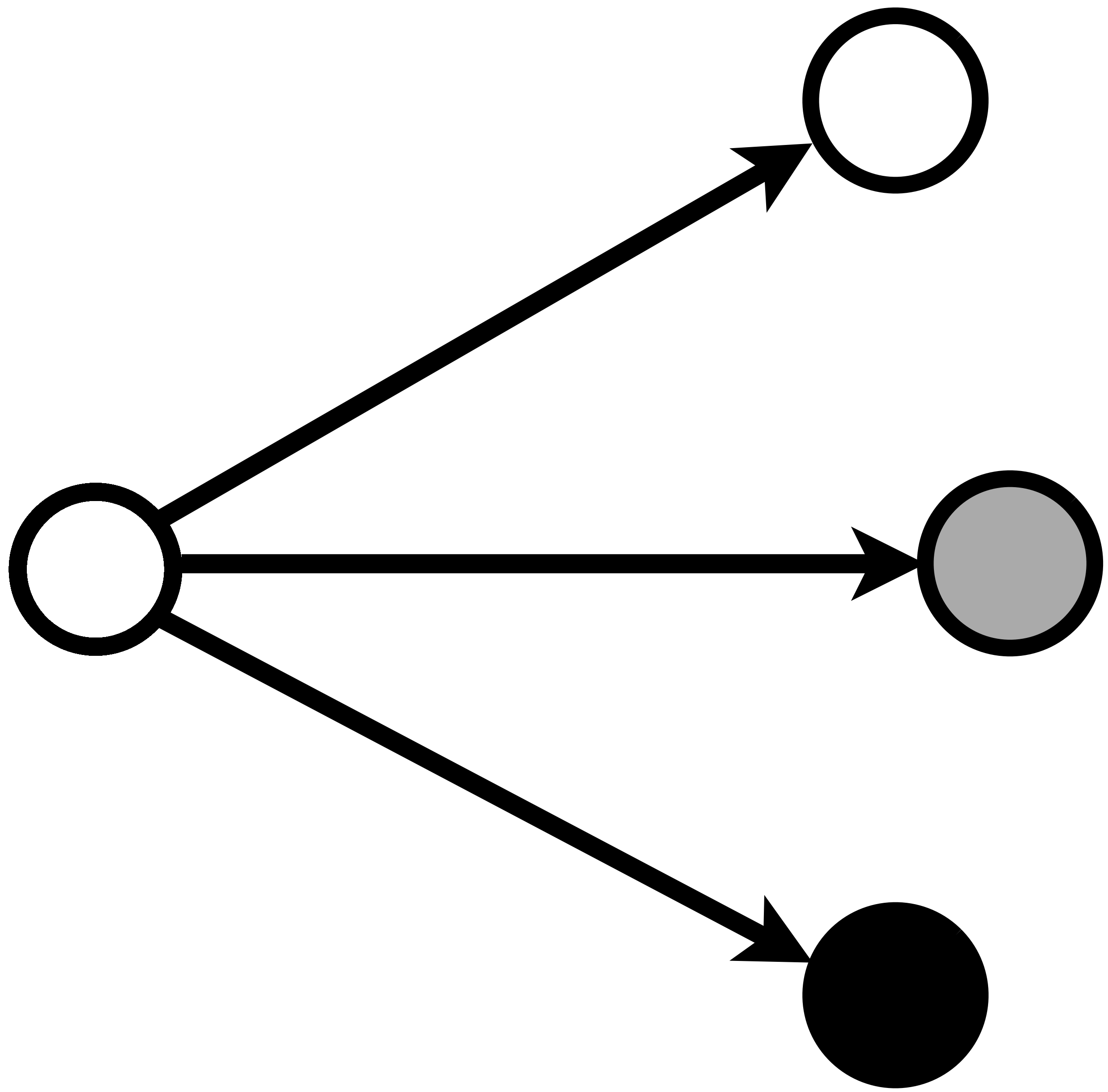
```
(f x)
```

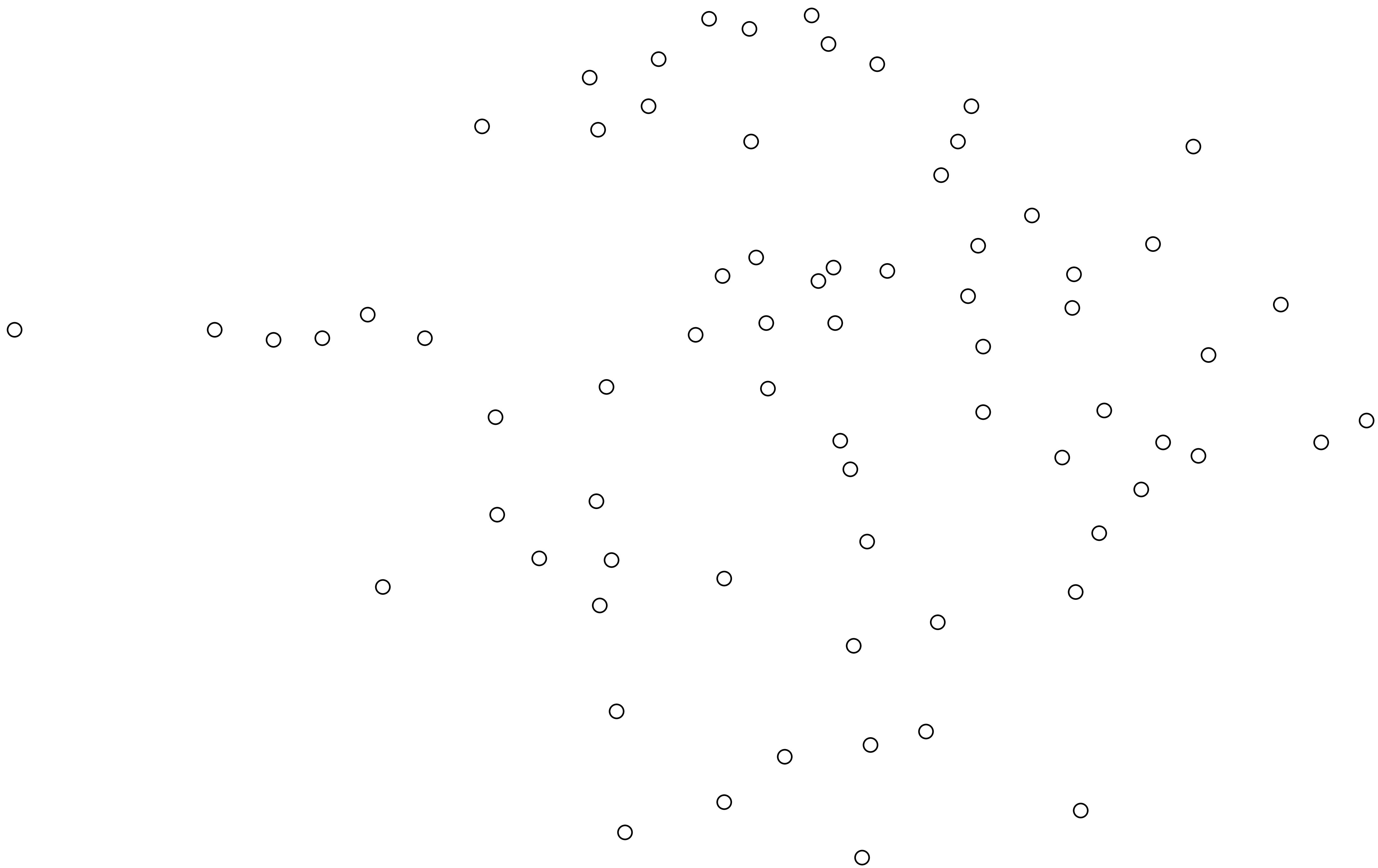
What is `f`?

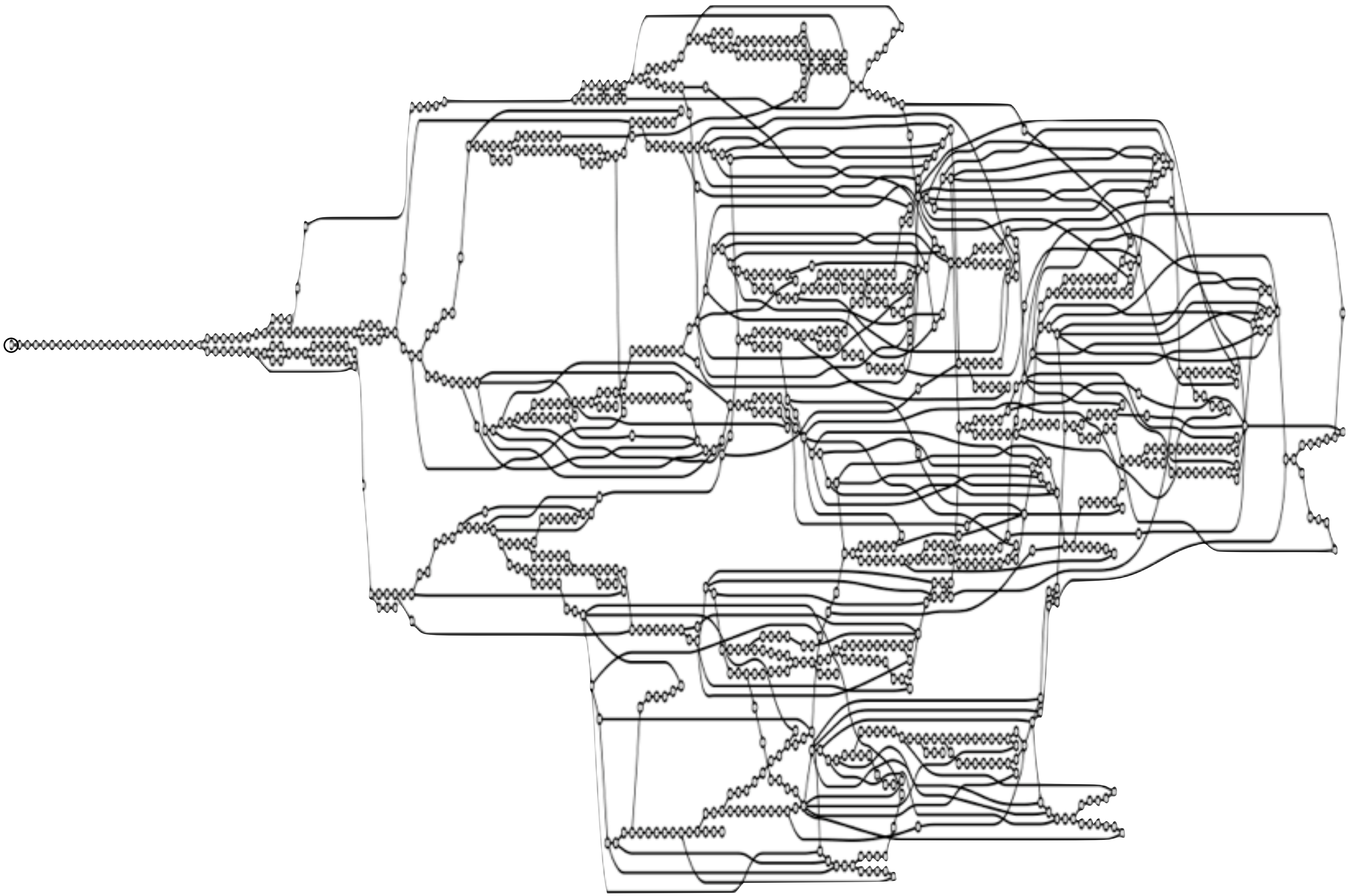
`f` is or or

`f` is  or  or 









Problem?

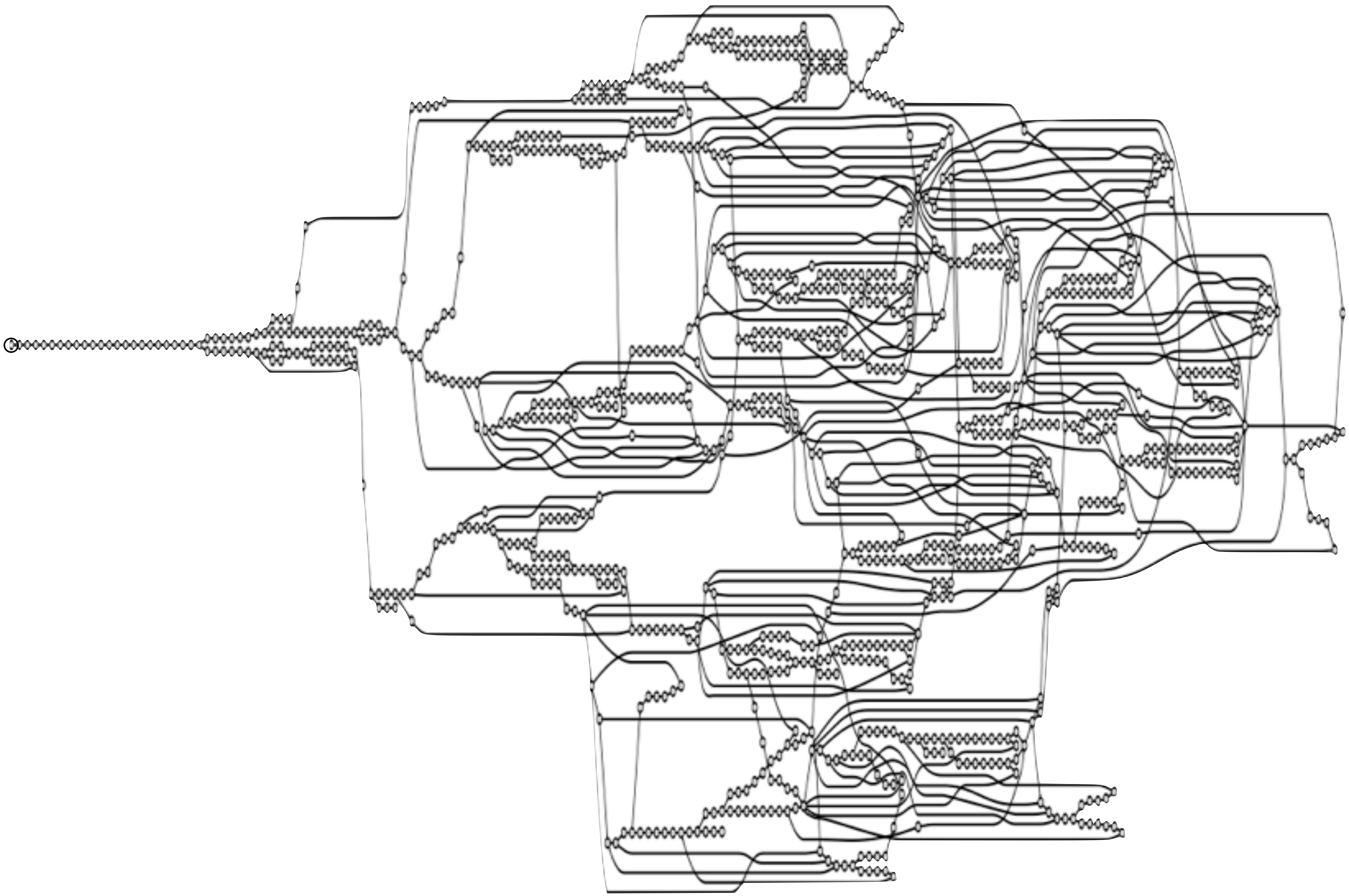
Finite store.

Solution?

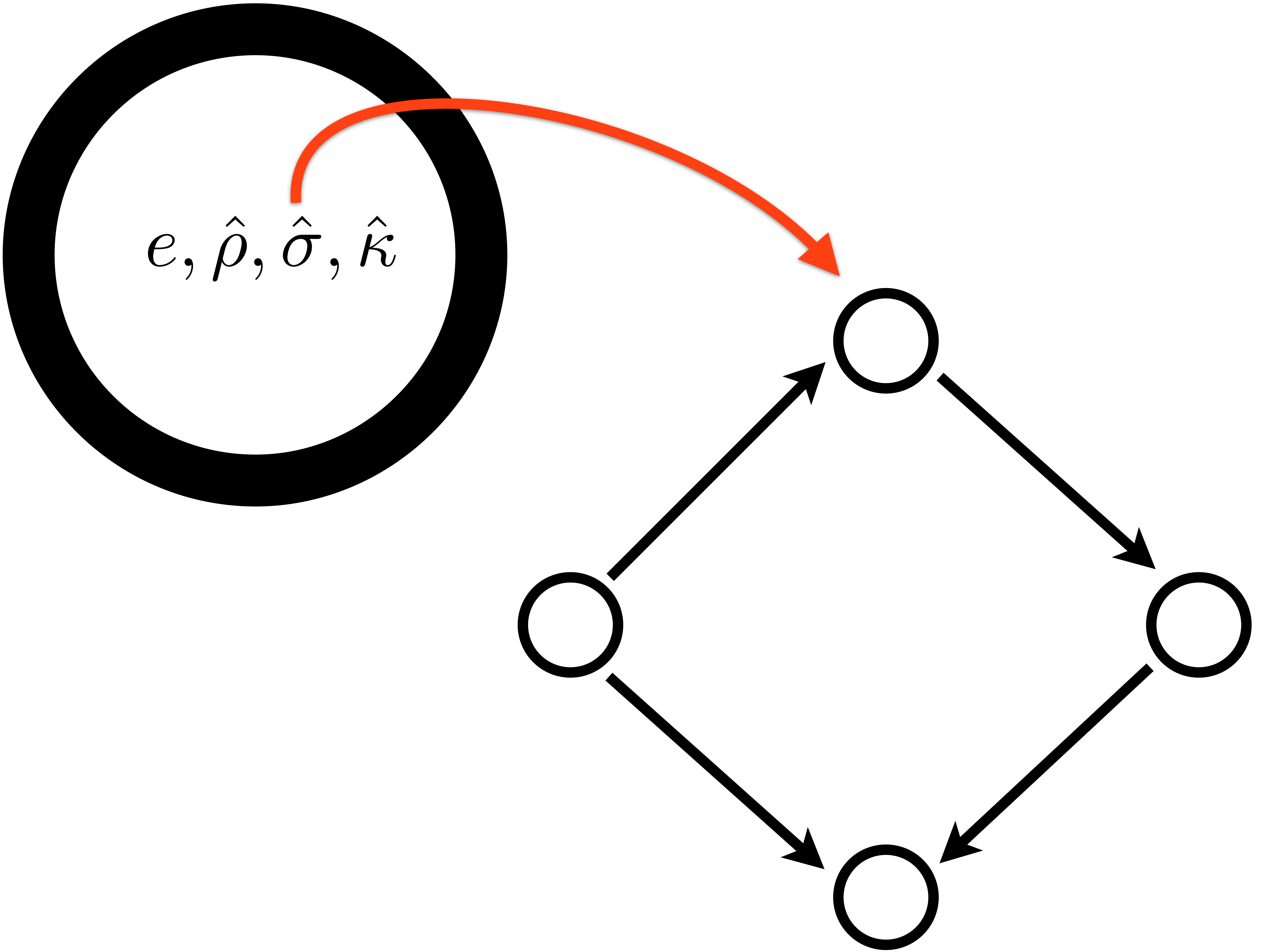
Toss garbage.

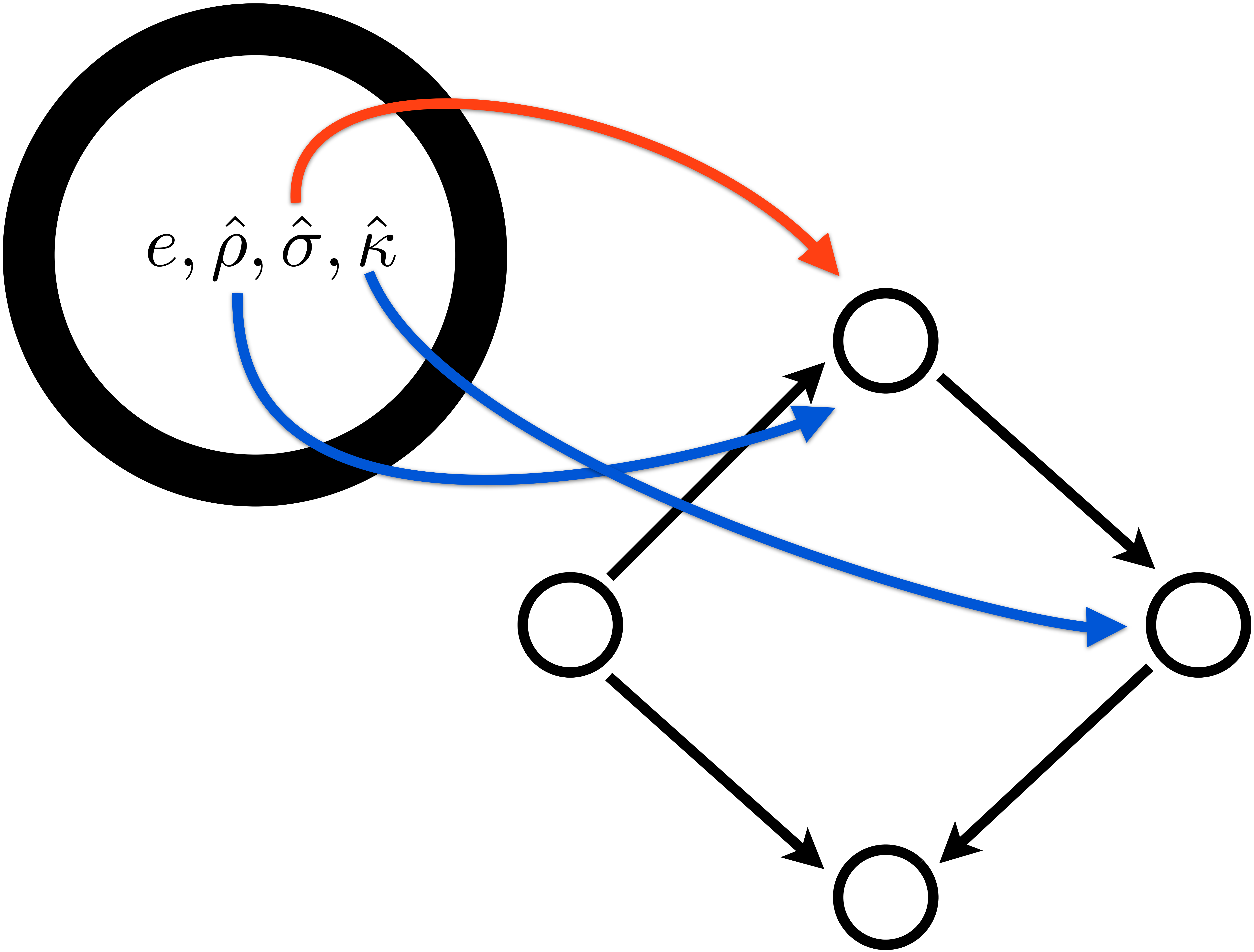
Toss garbage.

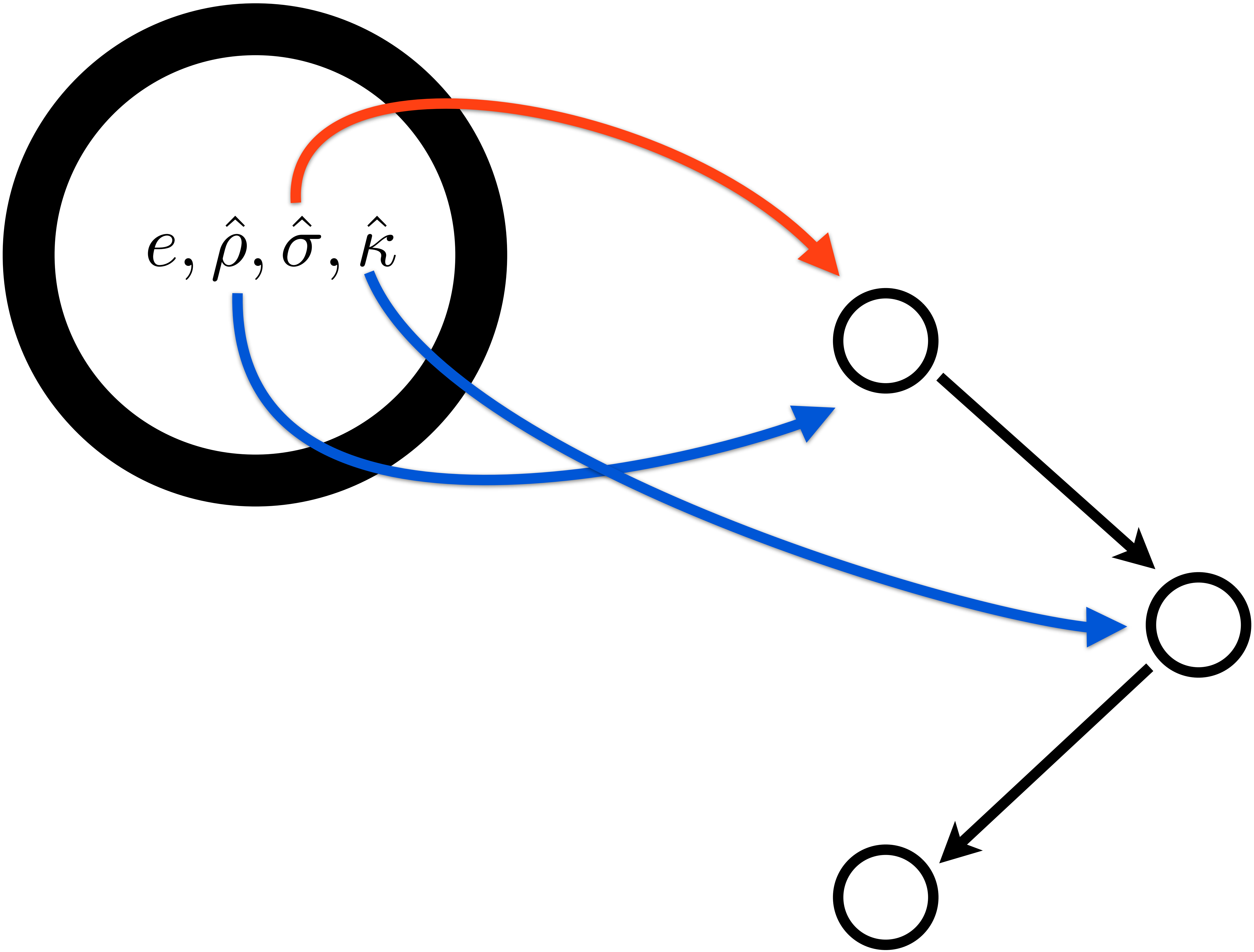
(Might & Shivers, ICFP 2006)



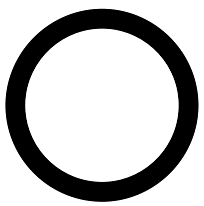

$e, \hat{\rho}, \hat{\sigma}, \hat{\kappa}$

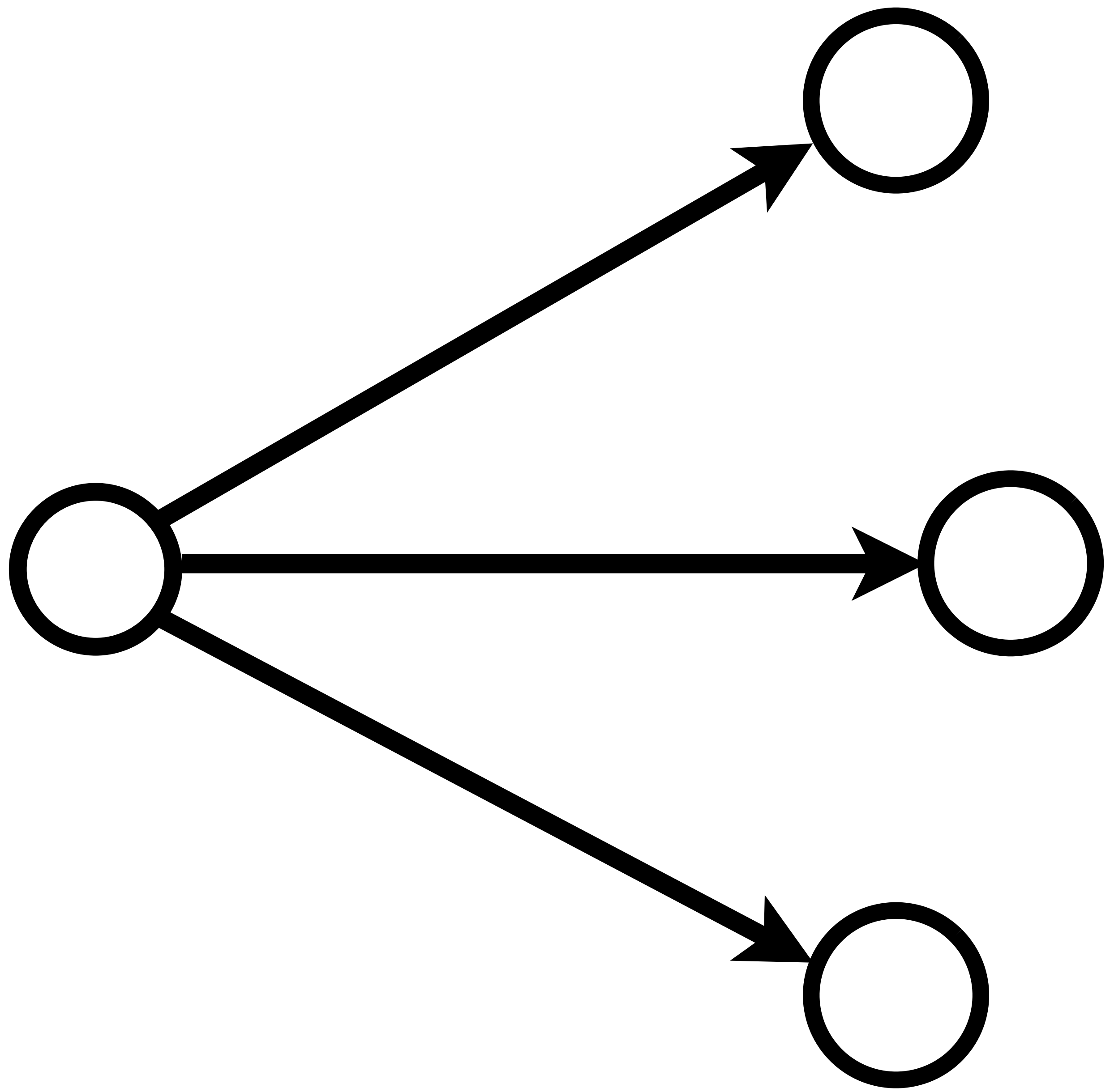

$e, \hat{\rho}, \hat{\sigma}, \hat{\kappa}$

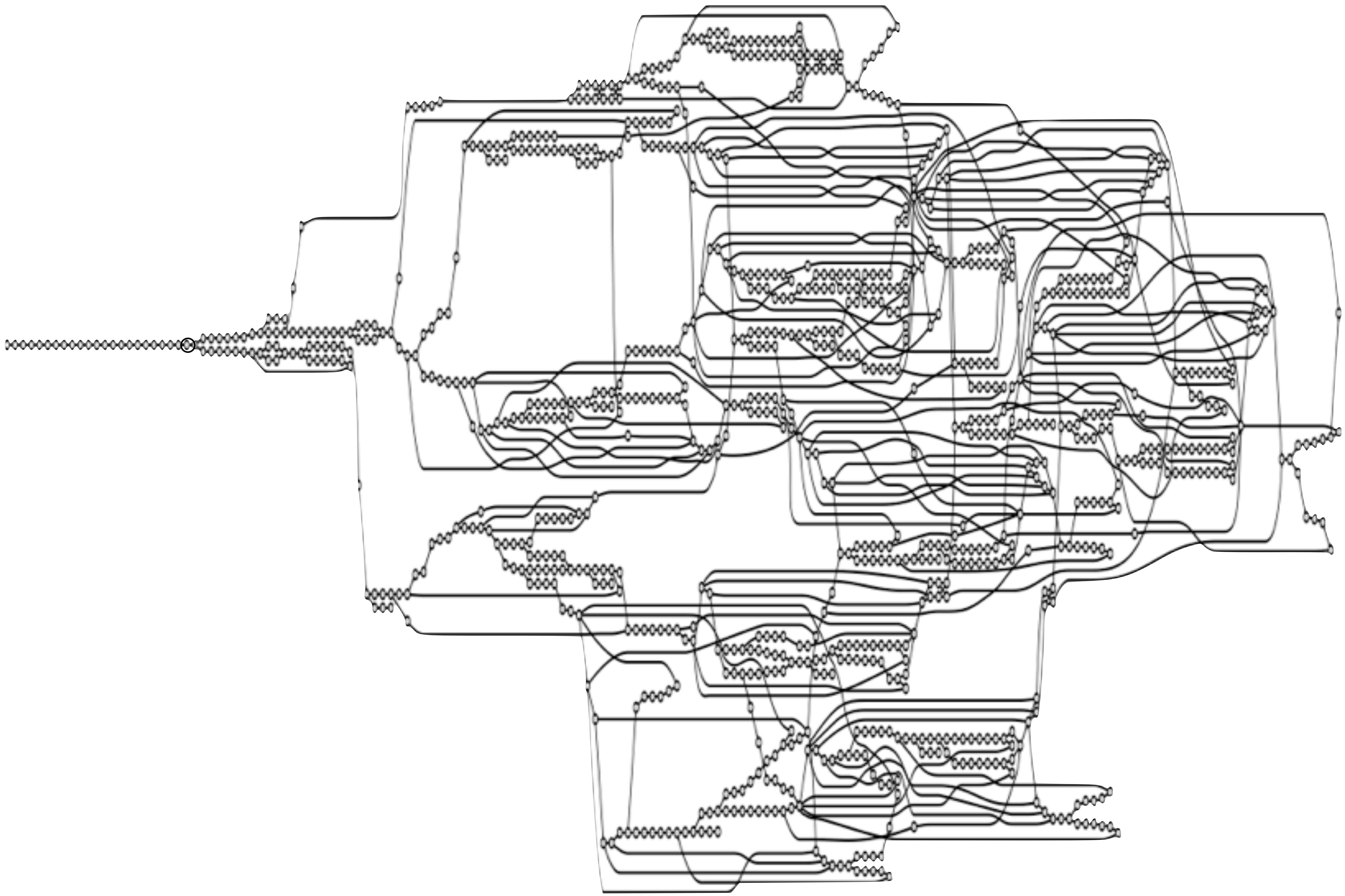

$e, \hat{\rho}, \hat{\sigma}, \hat{\kappa}$

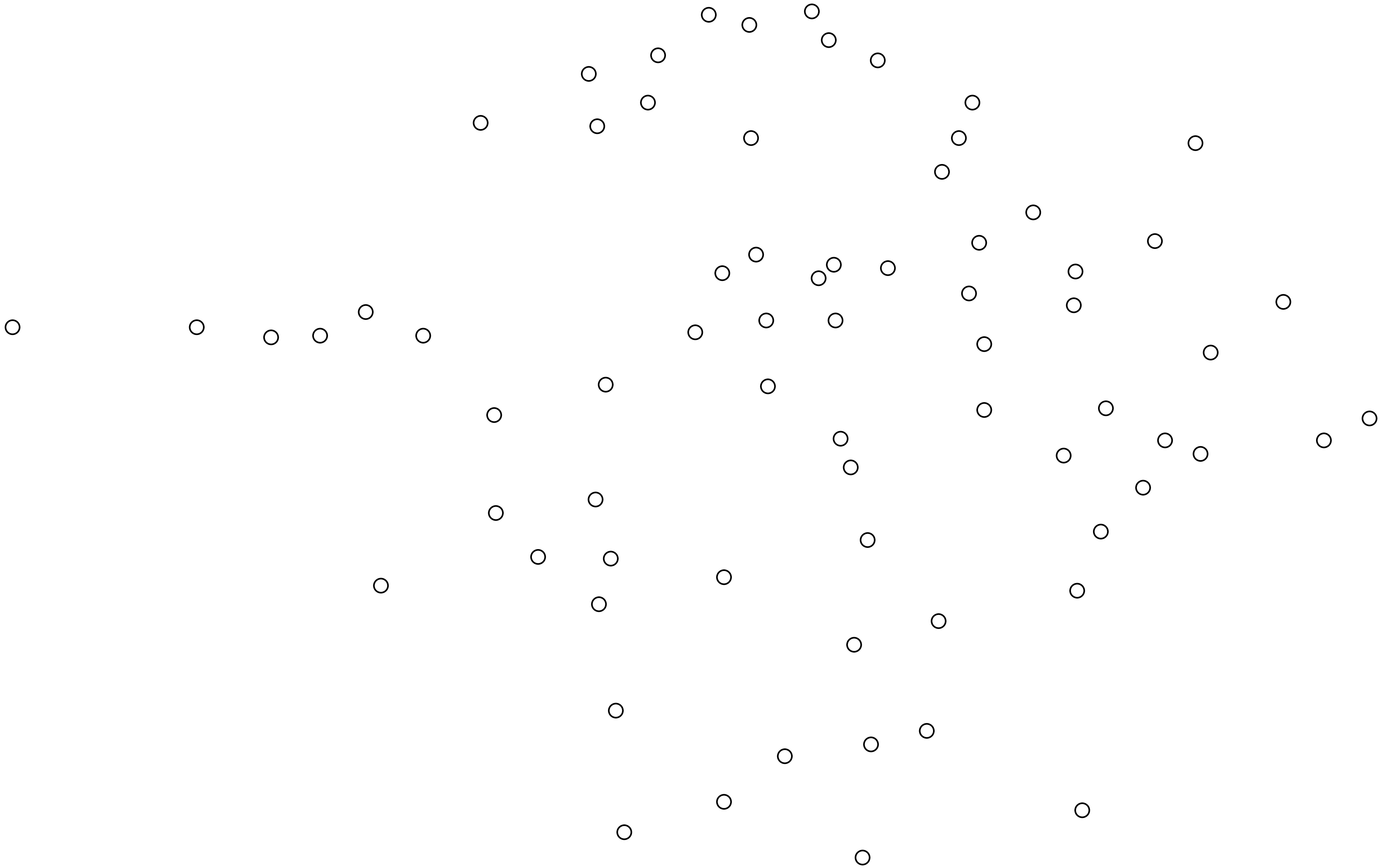

$e, \hat{\rho}, \hat{\sigma}, \hat{\kappa}$

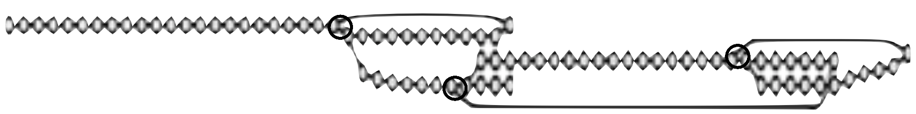

$e, \hat{\rho}, \hat{\sigma}', \hat{\kappa}$









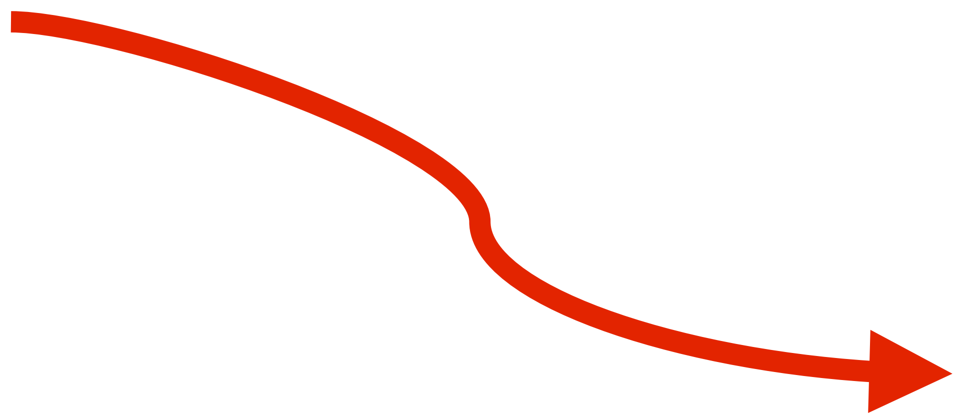


Control-flow forks.

Return-flow forks.

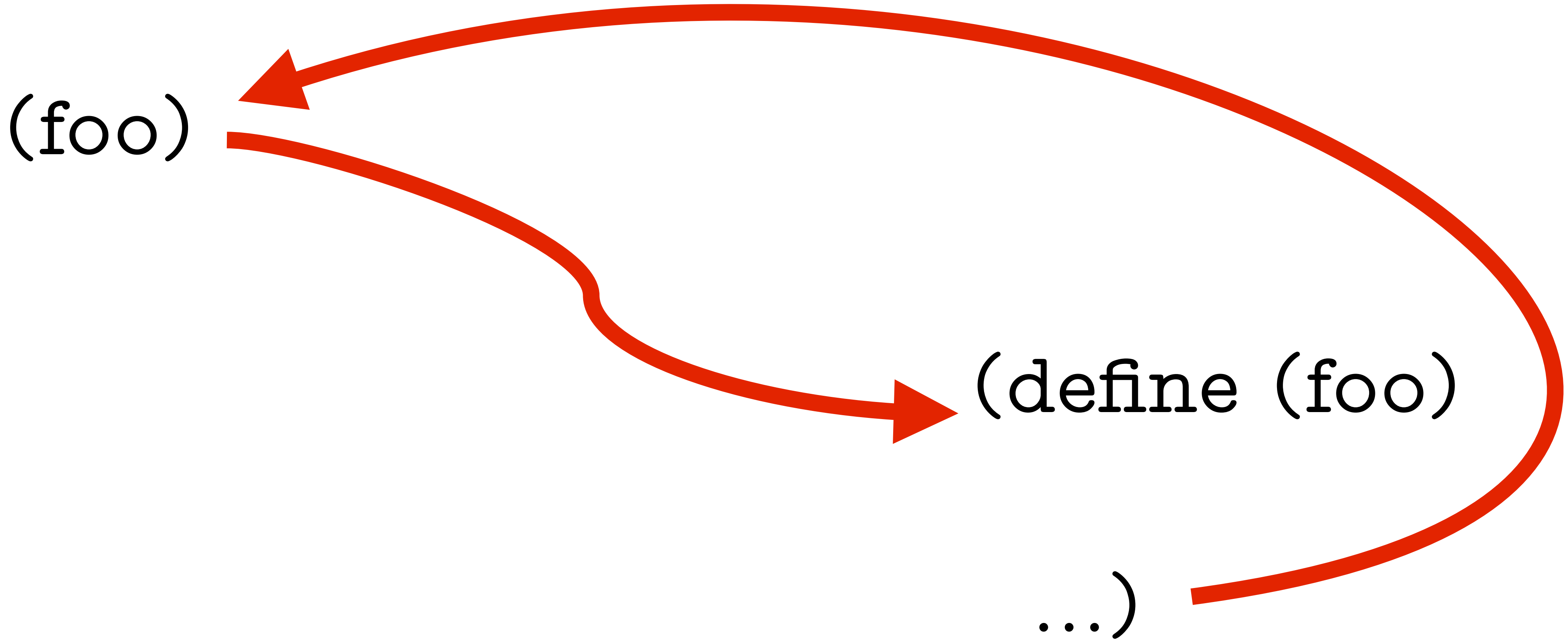
```
(foo)

                    (define (foo)
                      ...)

(foo)
```

```
(foo)
```

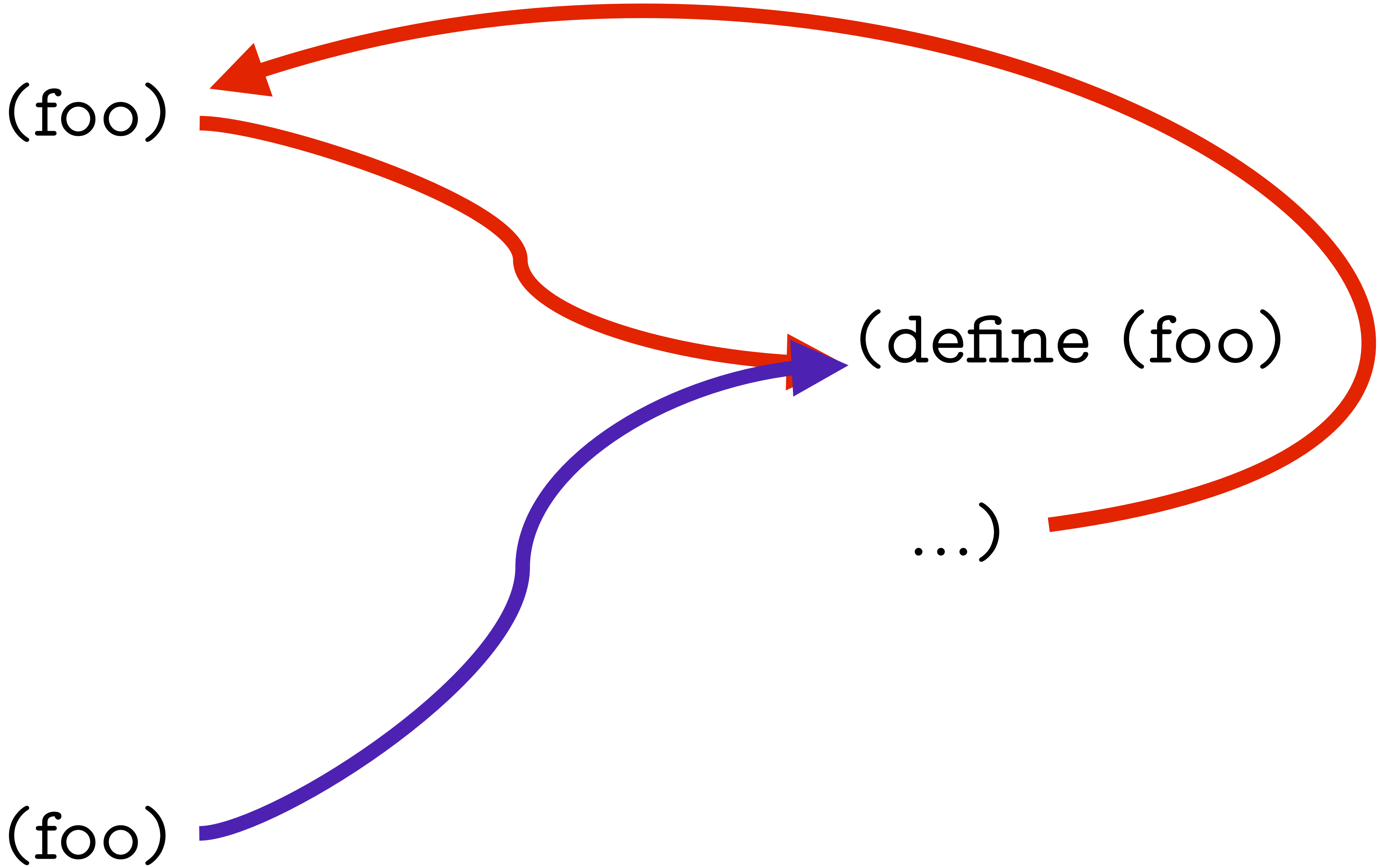


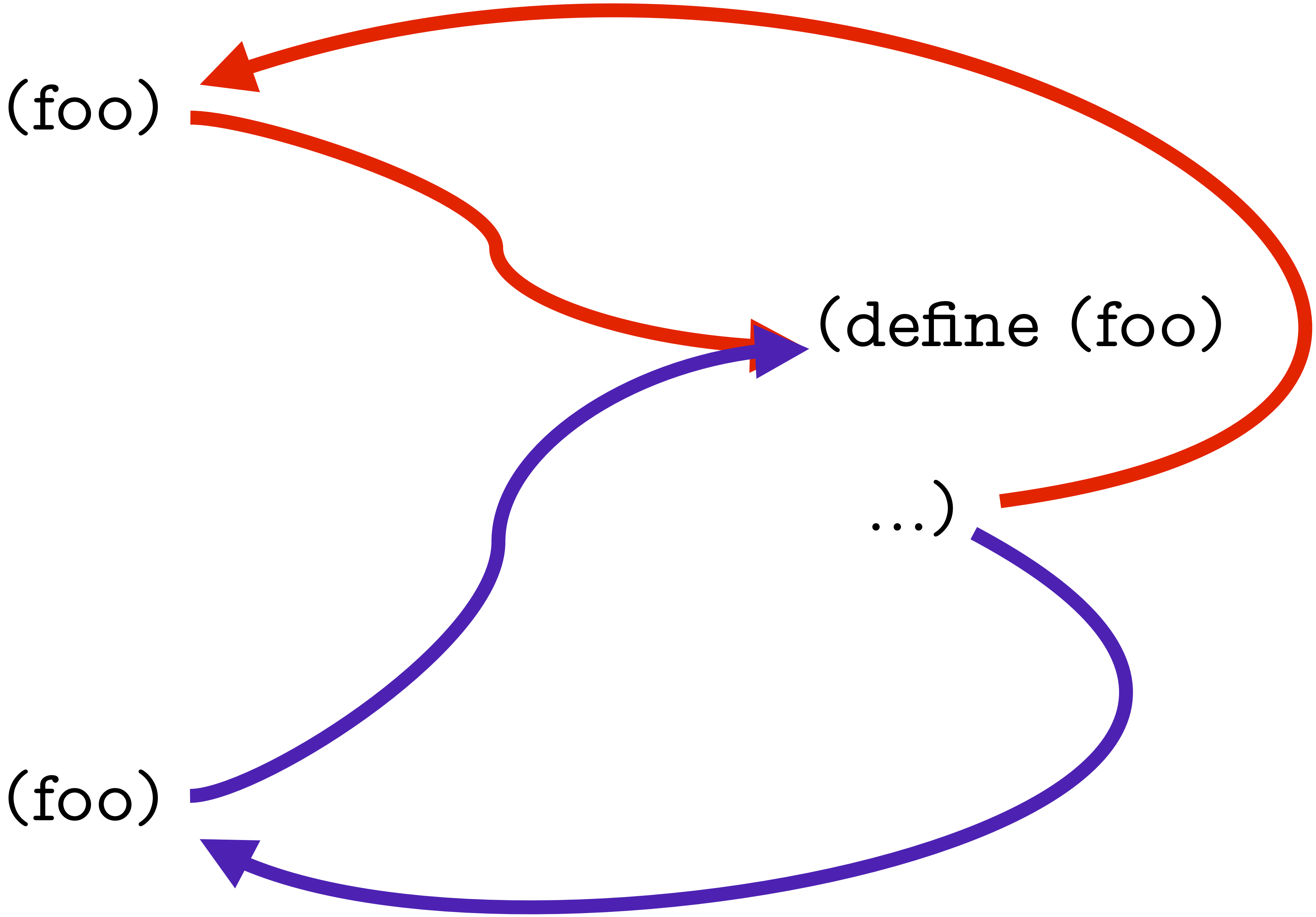
```
(define (foo)
  ...)
```

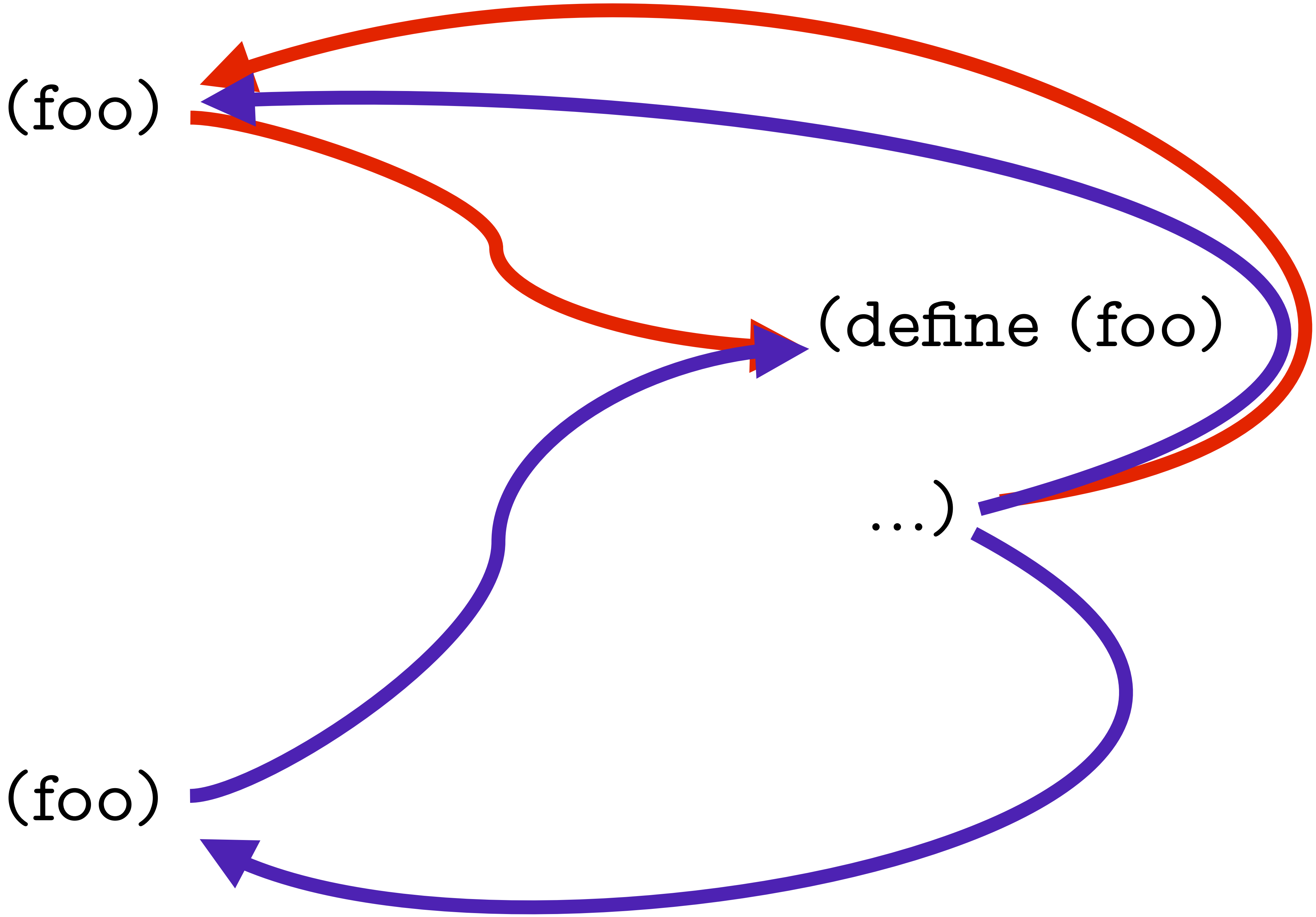
```
(foo)
```




```
(foo)
```


(foo)
(define (foo)
...)
(foo)


(foo)
(define (foo)
...)
(foo)


(foo)
(define (foo)
...)
(foo)

$$\widehat{S} = \widehat{A} \longrightarrow \mathcal{P}(\widehat{D})$$

$$\widehat{S} = \widehat{A} \longrightarrow \mathcal{P}(\lambda \times \widehat{E} + \widehat{K})$$

CESK

CESK*

CESK*

CESK

CES K

$\widehat{CES}$ $\hat{K}$

$\widehat{CES}\ \hat{K}$

control state

$\widehat{CES}\ \widehat{K}$

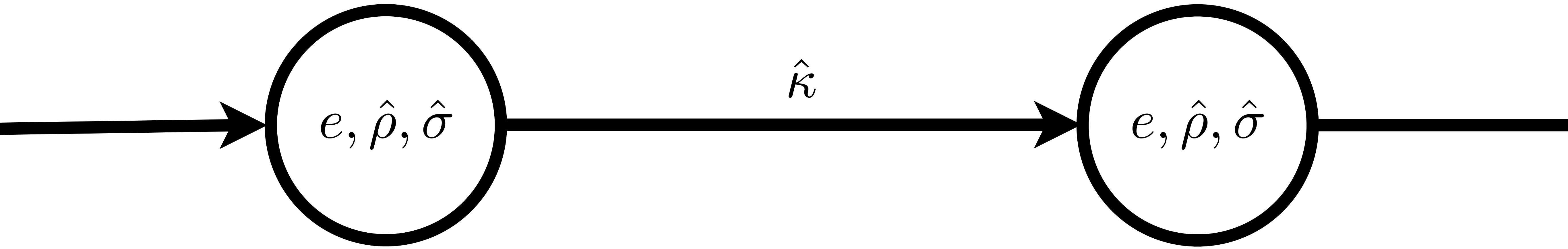
control state stack

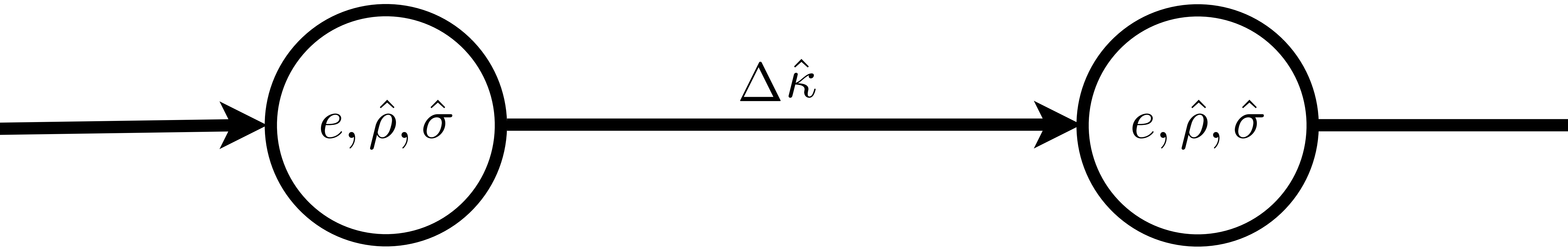
control state + stack = pushdown

control state + stack = pushdown

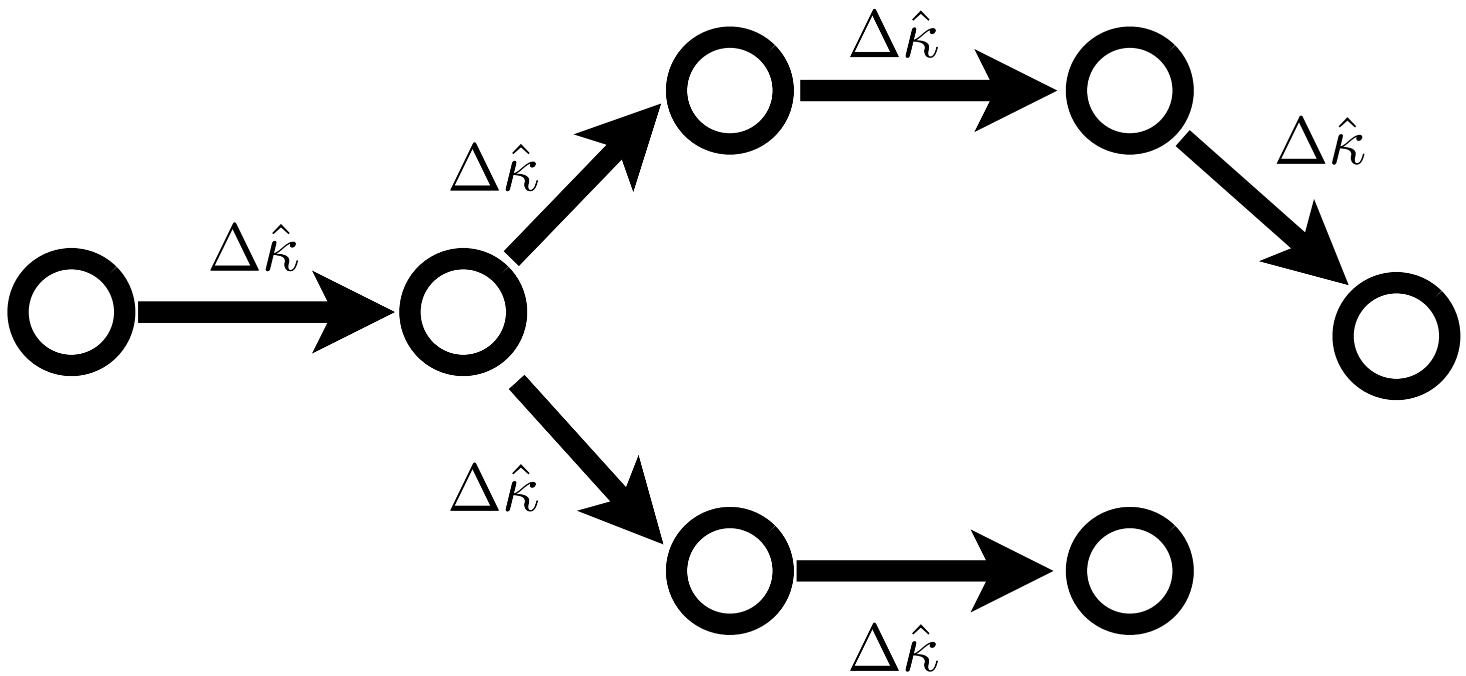
(Vardoulakis and Shivers, CFA2)


$e, \hat{\rho}, \hat{\sigma}, \hat{\kappa}$
$e, \hat{\rho}, \hat{\sigma}, \hat{\kappa}$

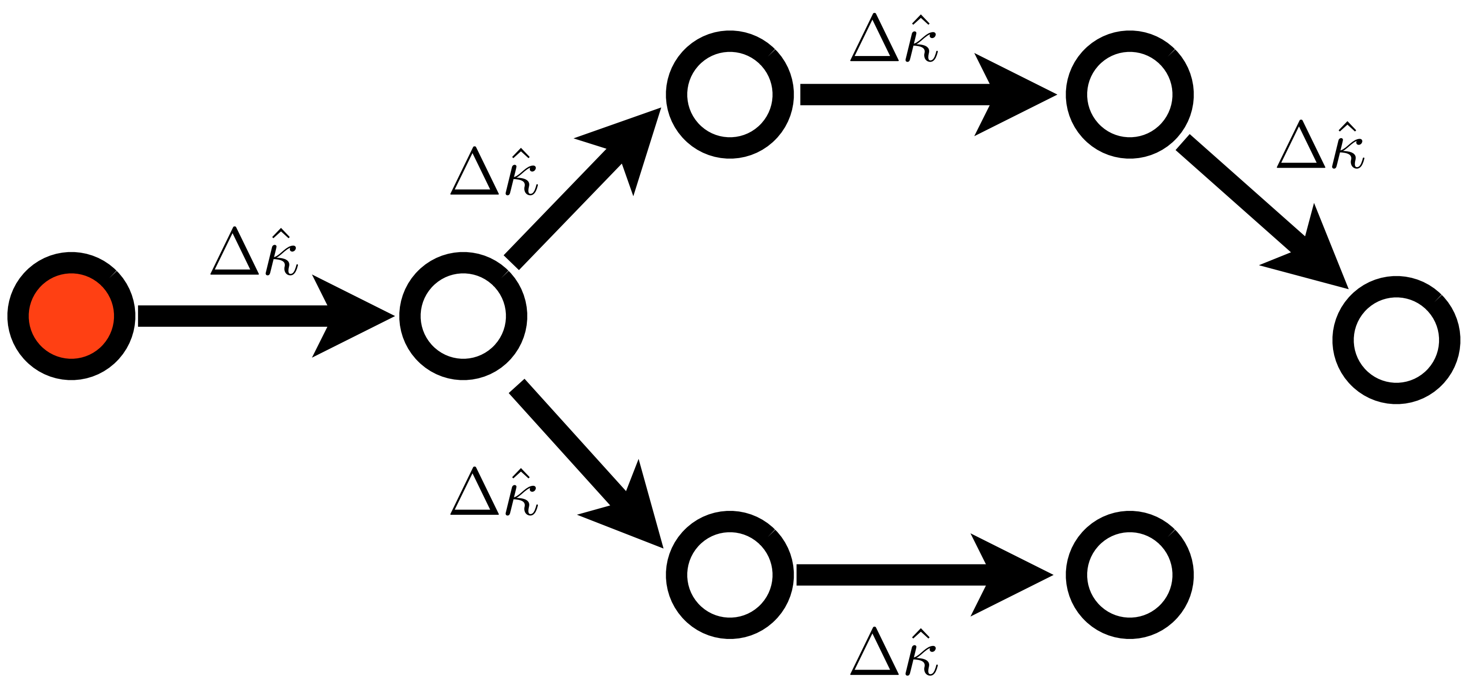

$e, \hat{\rho}, \hat{\sigma}$
$\hat{\kappa}$
$e, \hat{\rho}, \hat{\sigma}$



$e, \hat{\rho}, \hat{\sigma}$
$\Delta\hat{\kappa}$
$e, \hat{\rho}, \hat{\sigma}$



$\Delta\hat{\kappa}$
$\Delta\hat{\kappa}$
$\Delta\hat{\kappa}$
$\Delta\hat{\kappa}$
$\Delta\hat{\kappa}$
$\Delta\hat{\kappa}$



$\Delta\hat{\kappa}$
$\Delta\hat{\kappa}$
$\Delta\hat{\kappa}$
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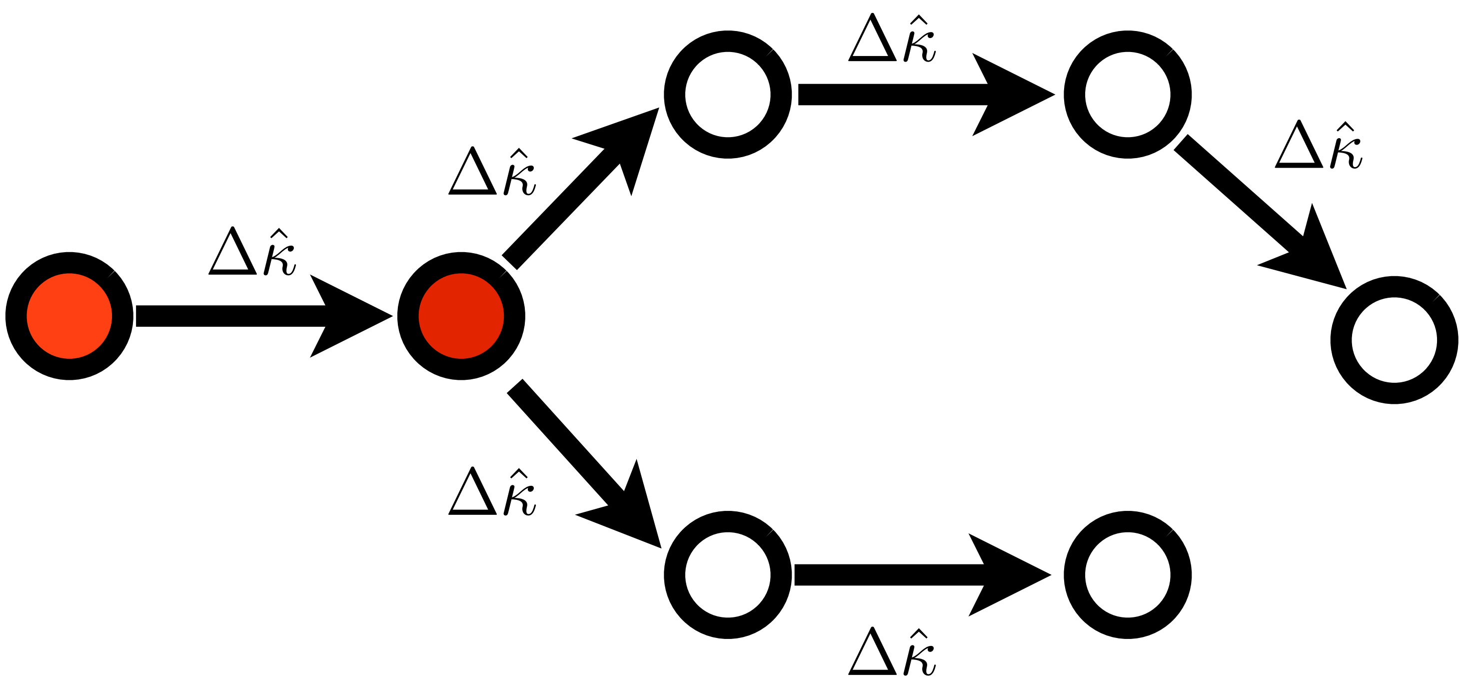
$\Delta\hat{\kappa}$
$\Delta\hat{\kappa}$



$\Delta\hat{\kappa}$
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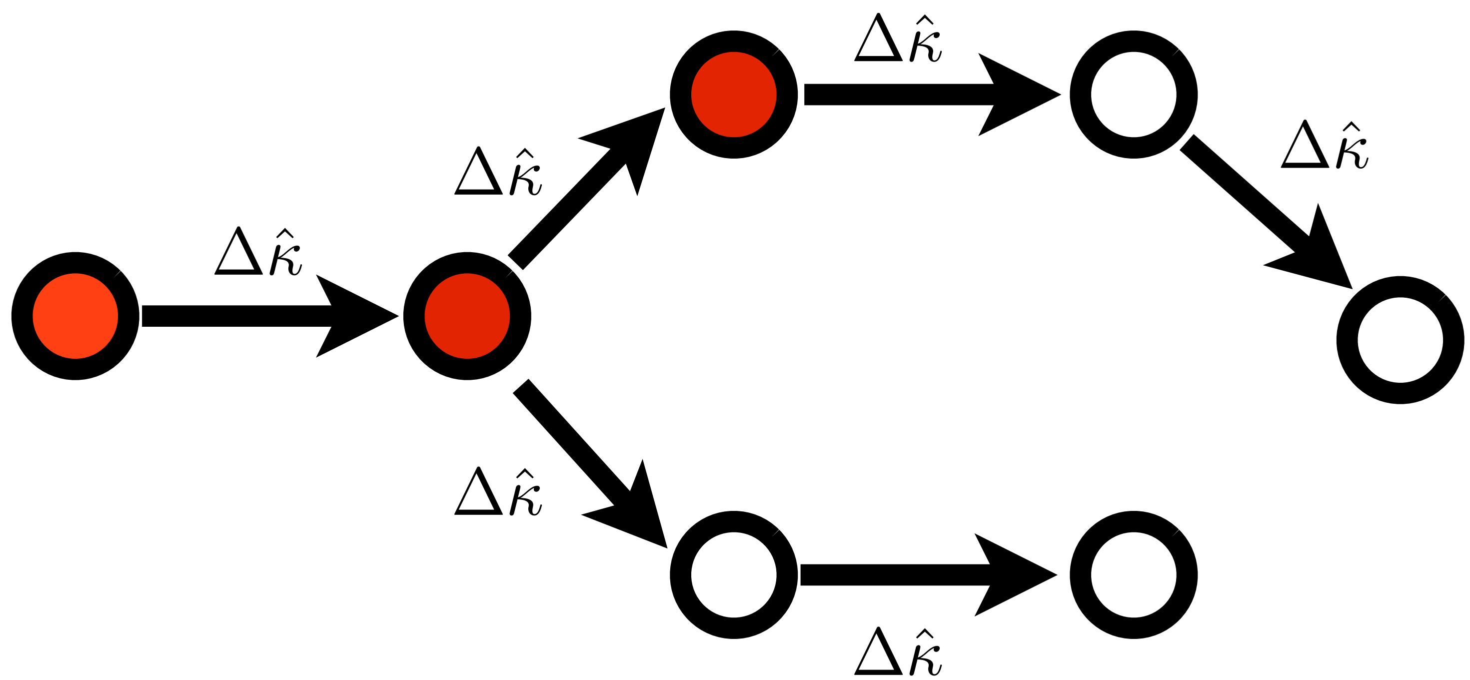
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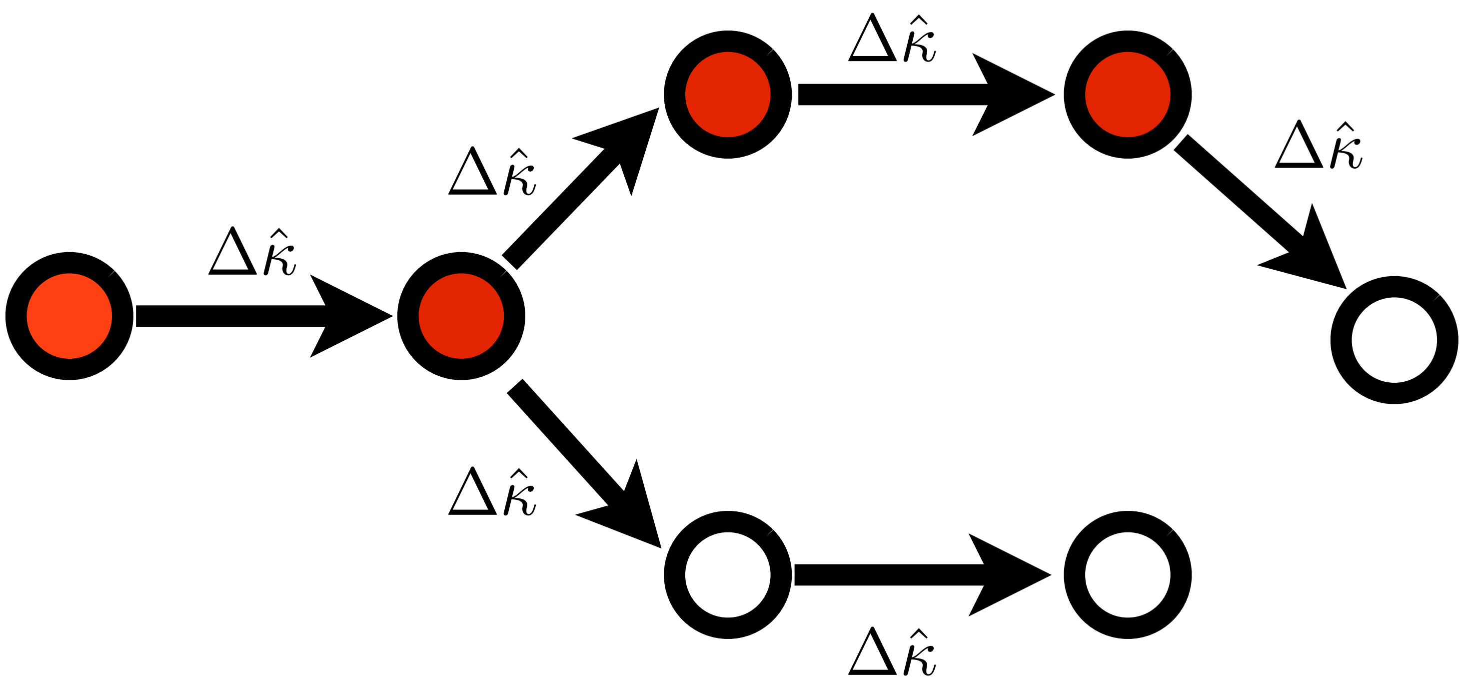
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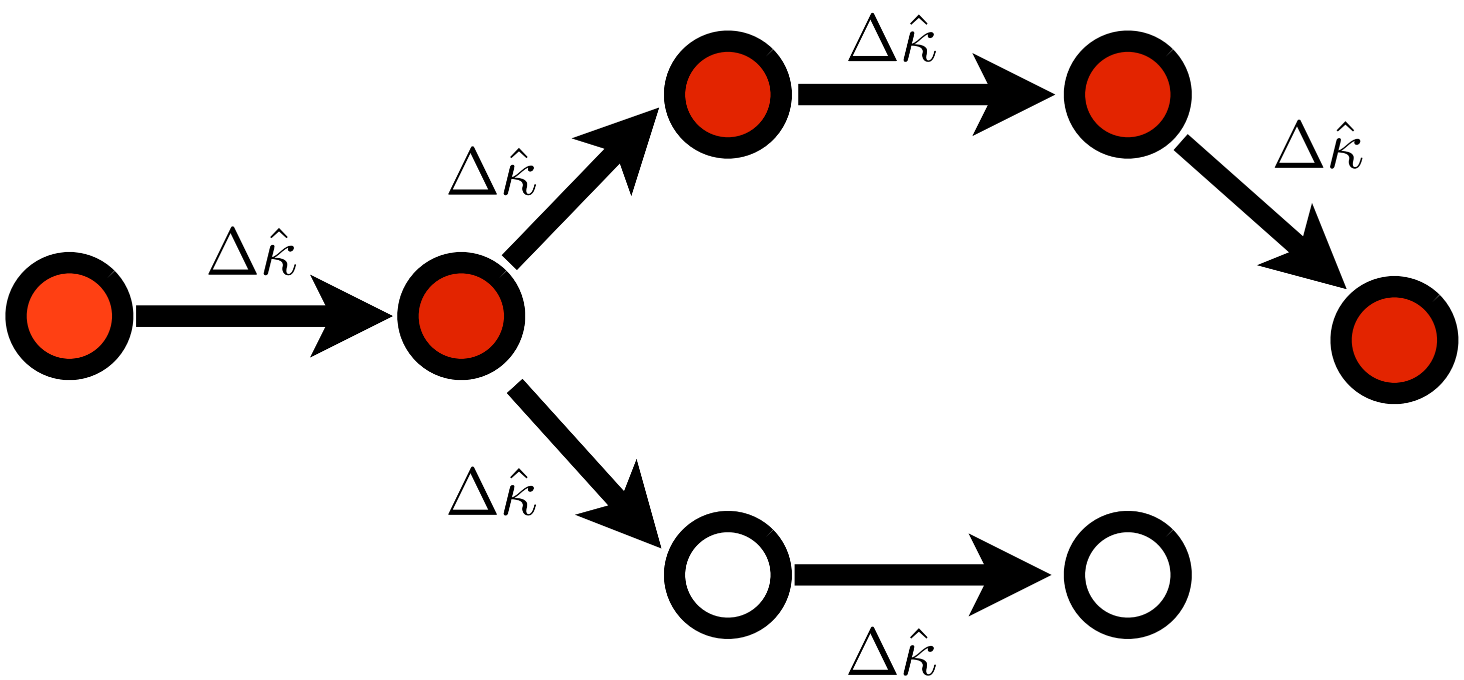
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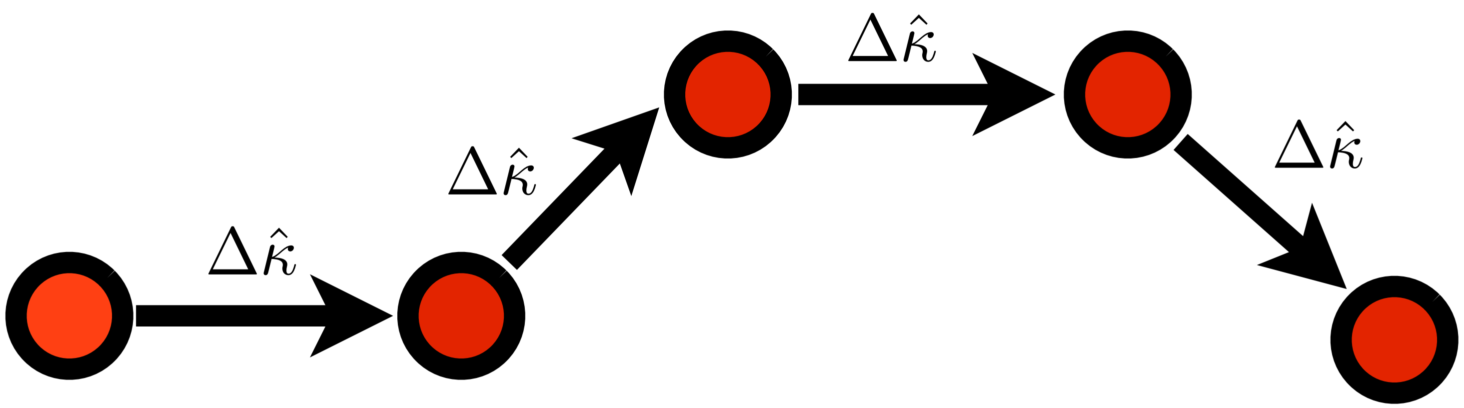
$\Delta\hat{\kappa}$
$\Delta\hat{\kappa}$
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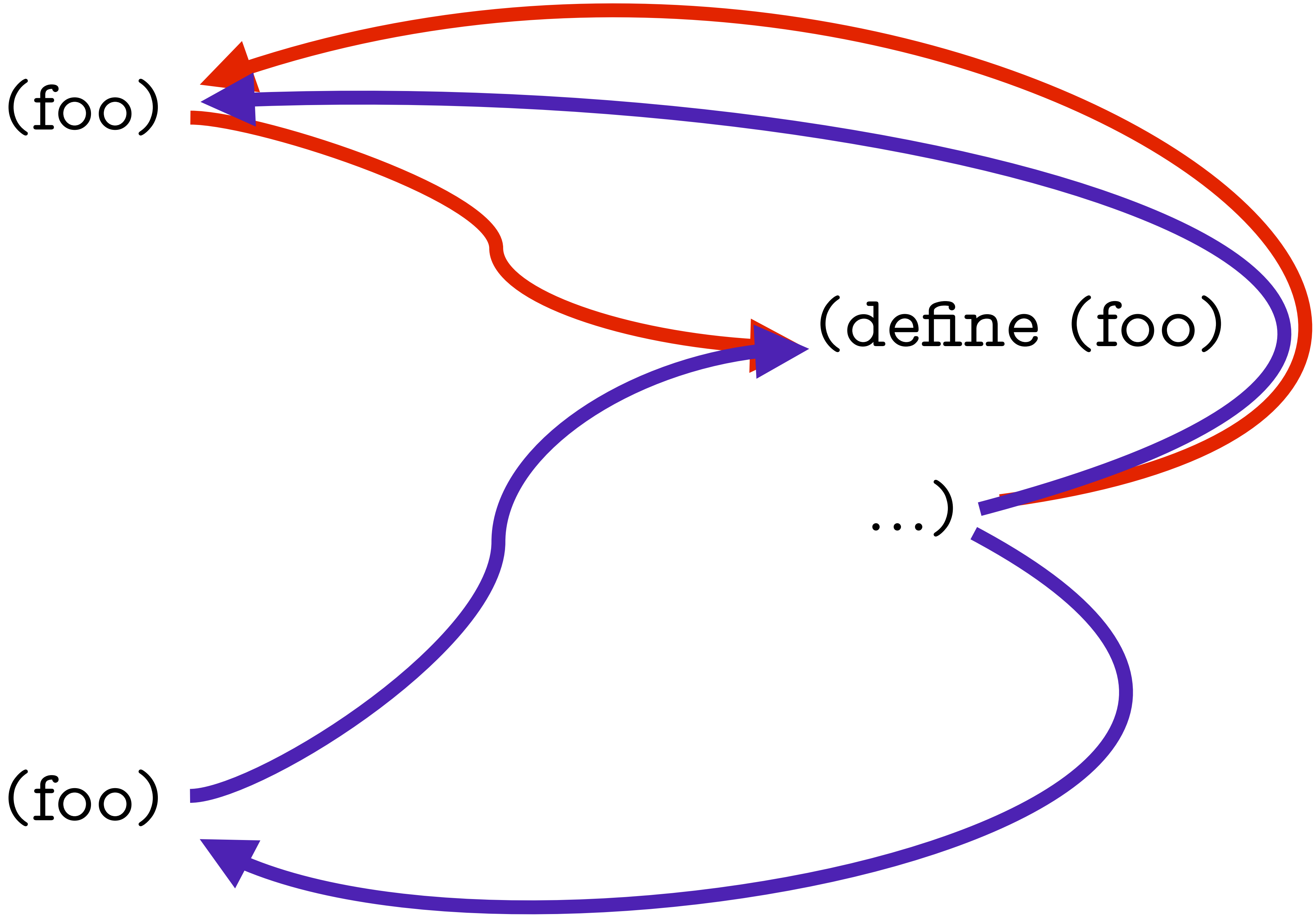
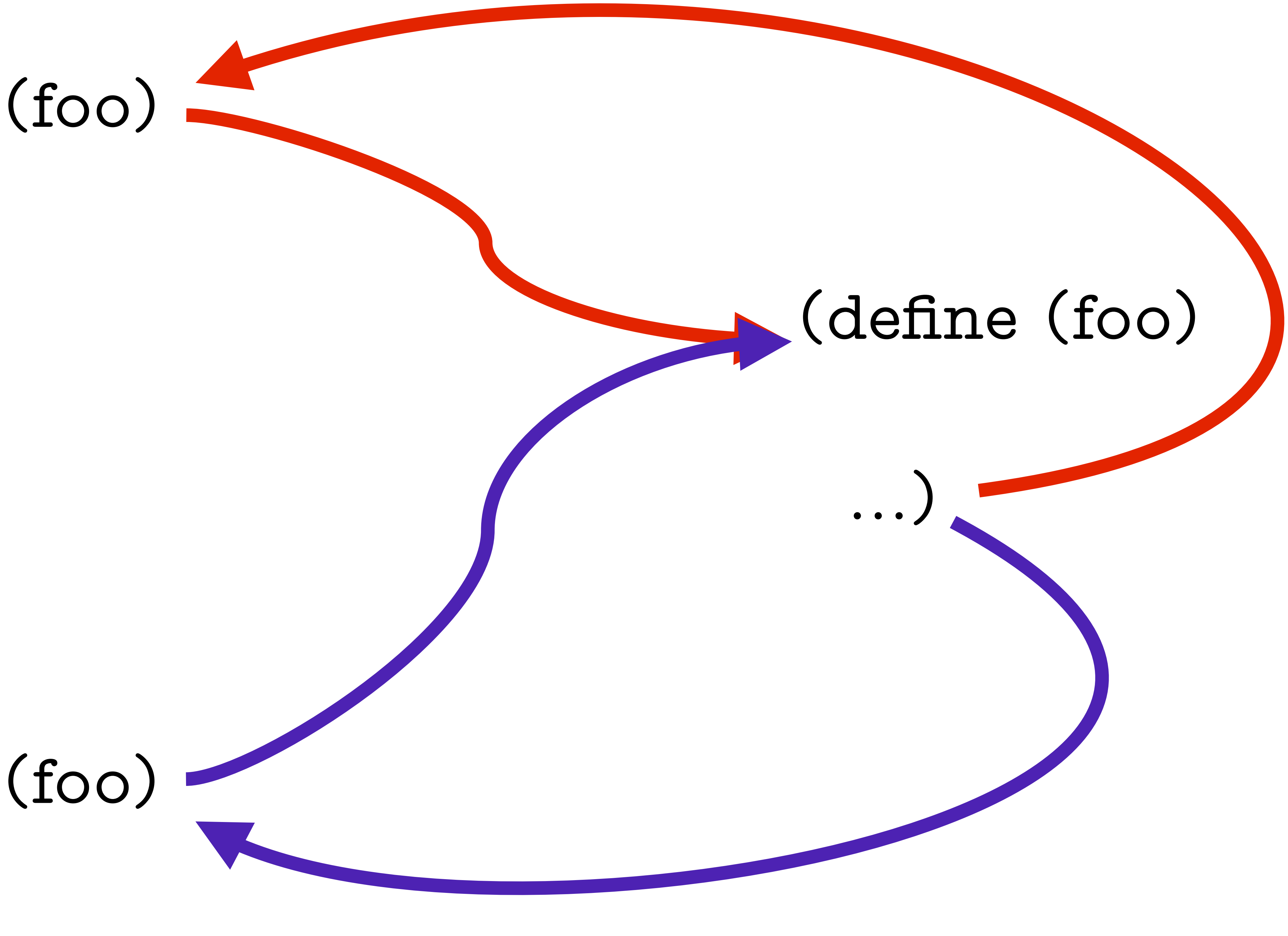


(foo)
(define (foo)
...)
(foo)


(foo)
(define (foo)
...)
(foo)

GC & pushdown?

Halt!

$$\delta : Q \times \Delta\Gamma \rightarrow \mathcal{P}(Q)$$

Control state

Next states

$$\delta : Q \times \Delta\Gamma \to \mathcal{P}(Q)$$

Stack change

$$\delta : Q \times \Delta\Gamma \times \Gamma^* \rightarrow \mathcal{P}(Q)$$

Entire stack

$$\delta : Q \times \Delta\Gamma \times \Gamma^* \rightarrow \mathcal{P}(Q)$$

$$\delta : Q \times \Delta\Gamma \times \mathcal{P}(\Gamma^*) \rightarrow \mathcal{P}(Q)$$

Infinite stacks

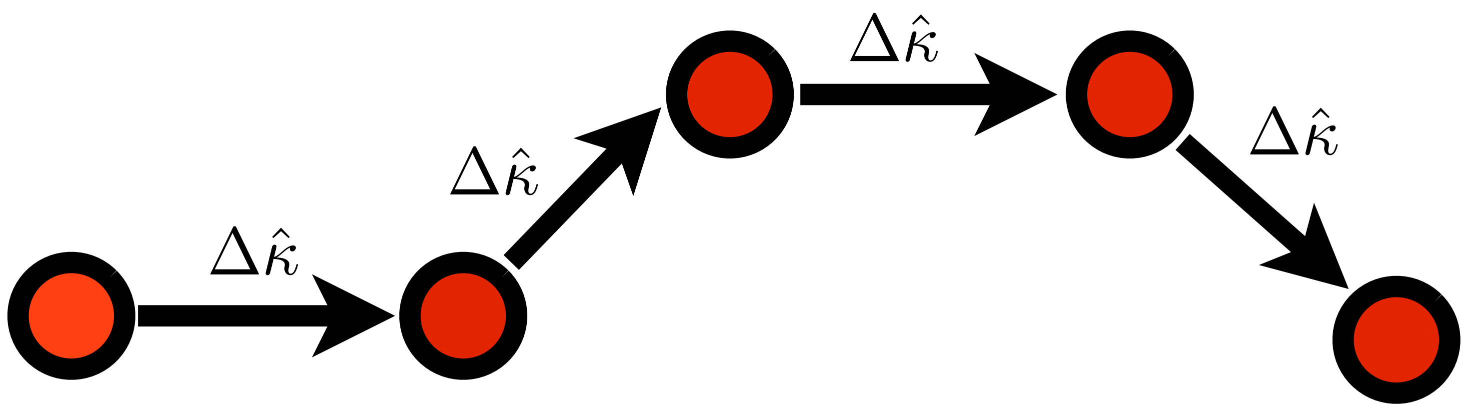
↓

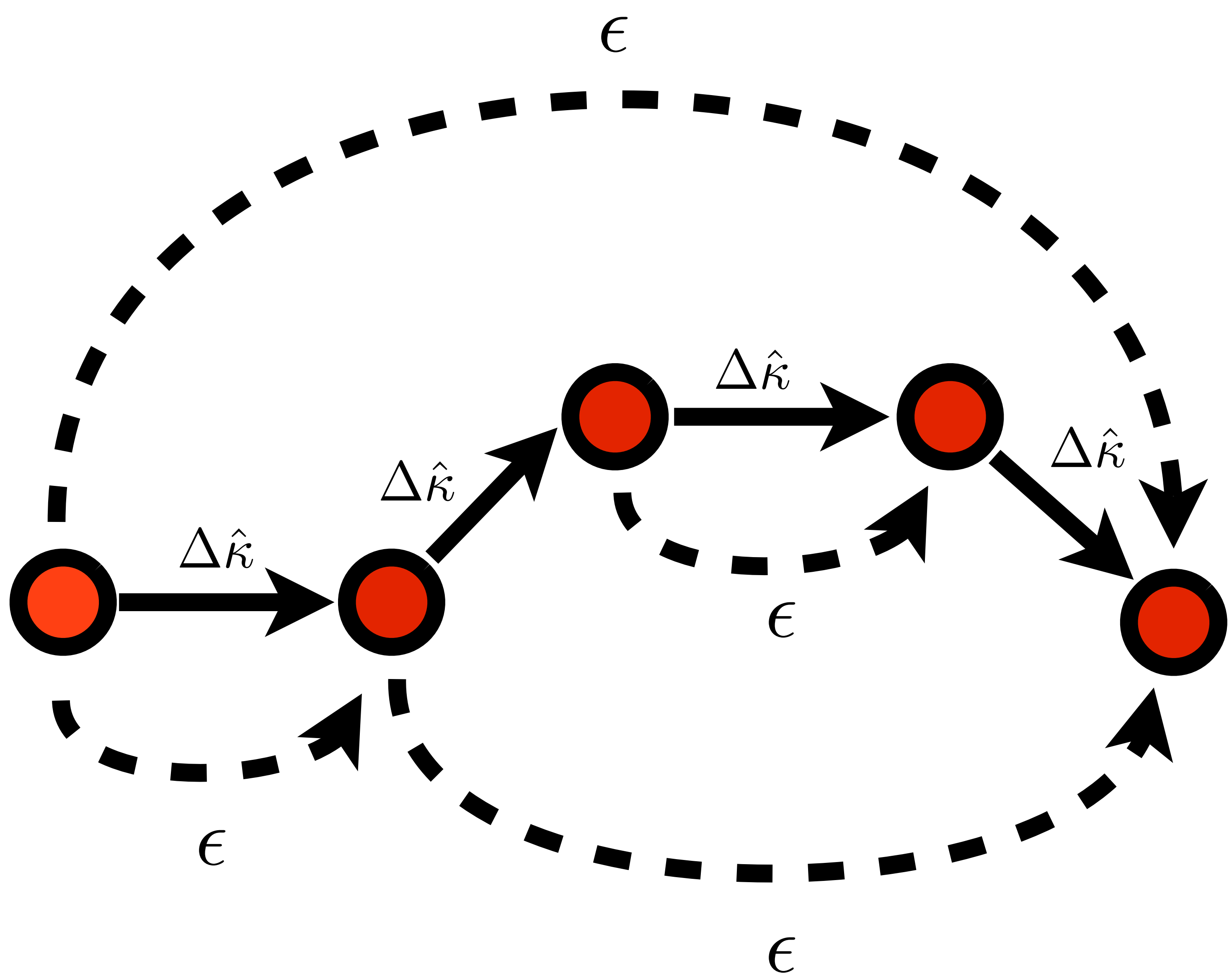
$$\delta : Q \times \Delta\Gamma \times \mathcal{P}(\Gamma^*) \rightarrow \mathcal{P}(Q)$$

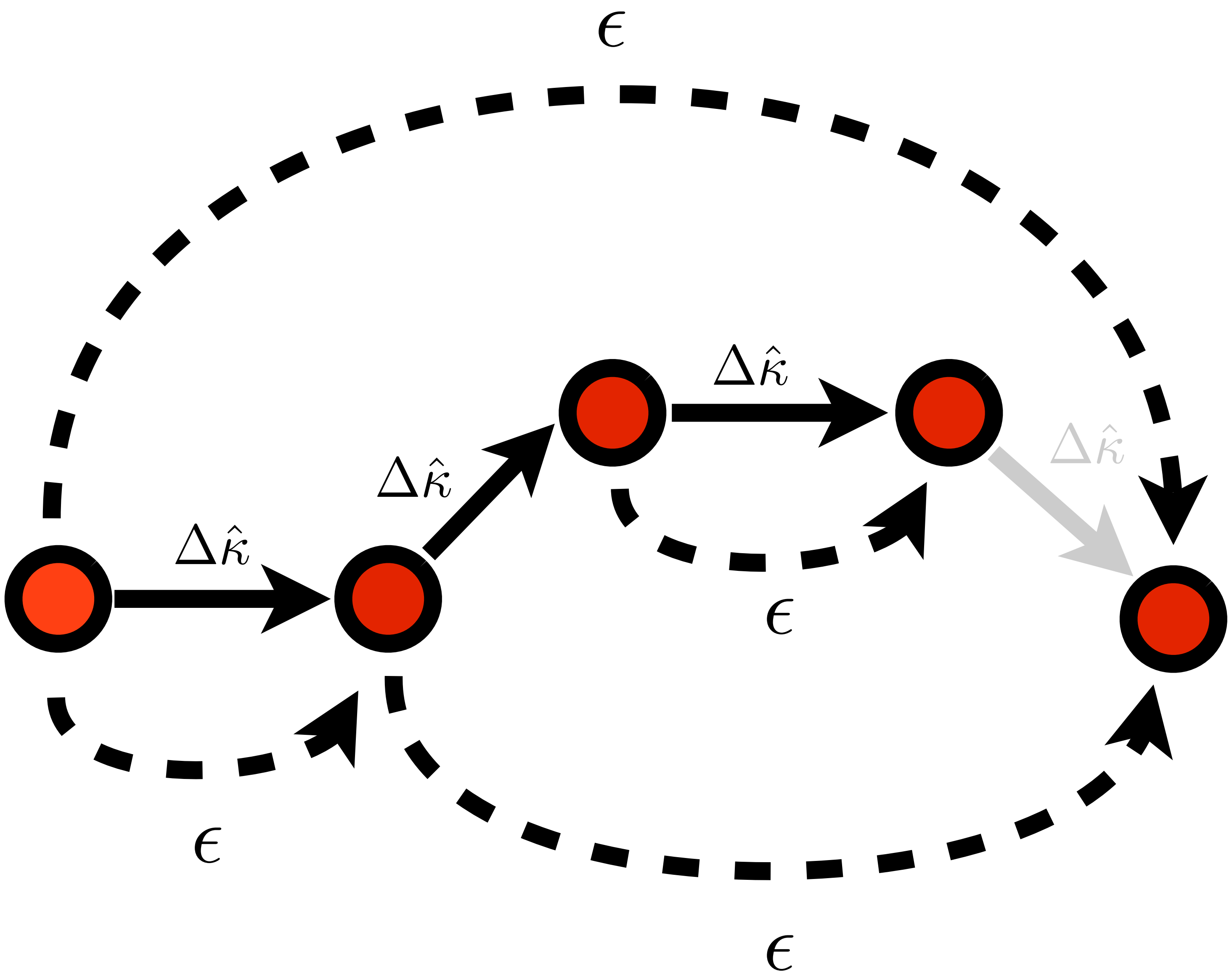
How do we fix the fixes?

$$\delta : Q \times \Delta\Gamma \times \mathcal{P}(\Gamma^*) \rightarrow \mathcal{P}(Q)$$

$$\mathcal{P}(\Gamma^*)$$

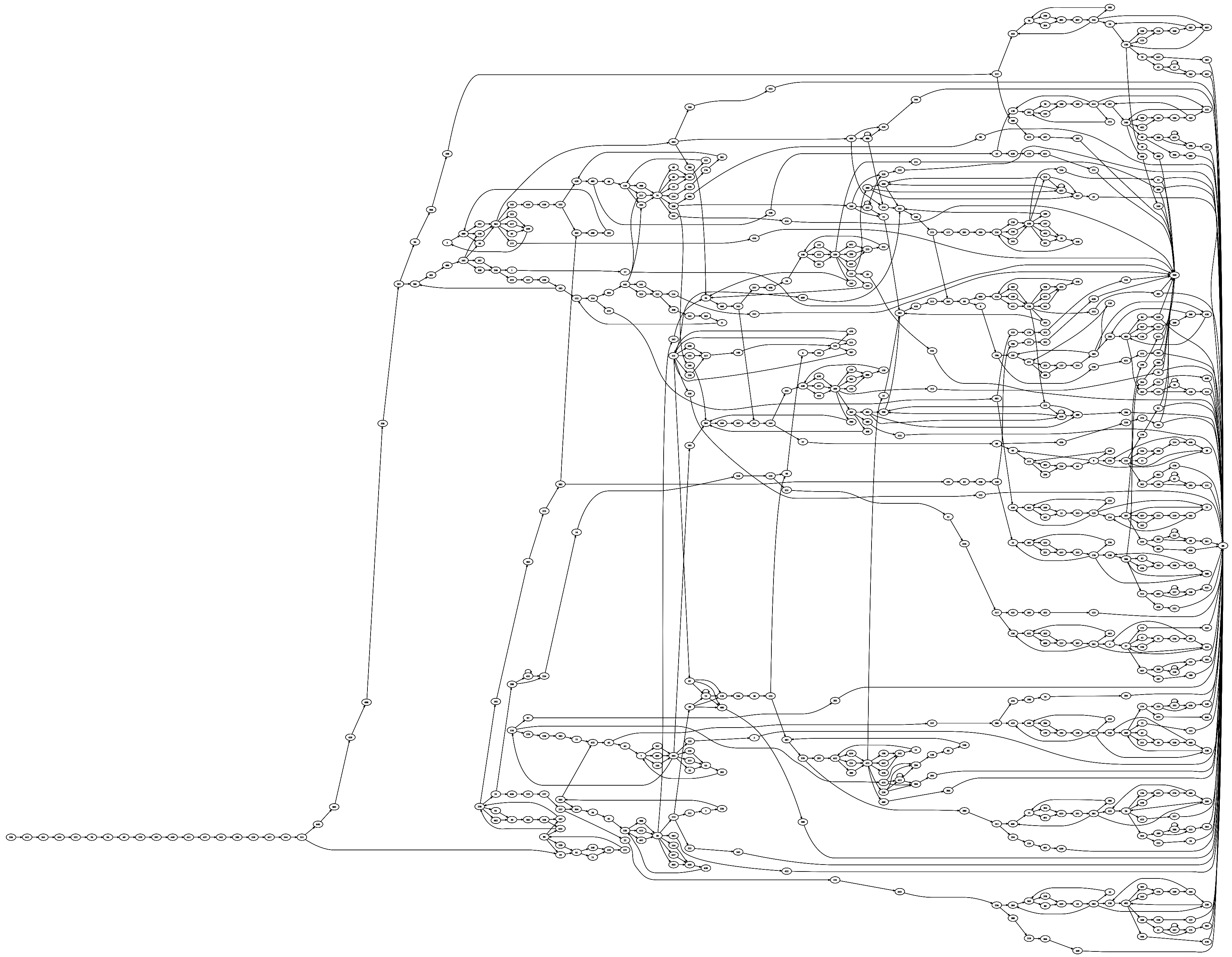

$\Delta\hat{\kappa}$
$\Delta\hat{\kappa}$
$\Delta\hat{\kappa}$
$\Delta\hat{\kappa}$

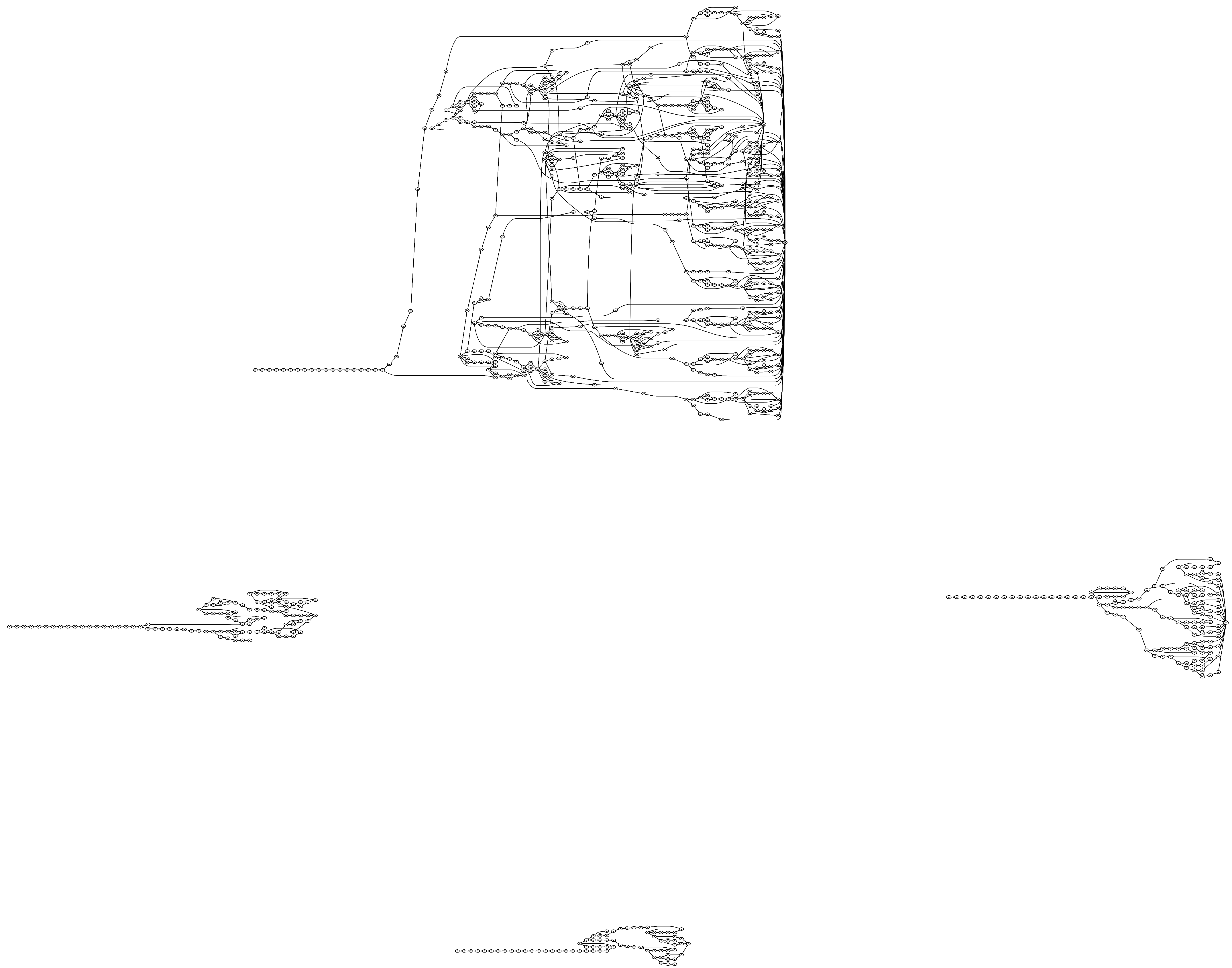

ϵ
Δκ̂
Δκ̂
Δκ̂
Δκ̂
ϵ
ϵ
ϵ

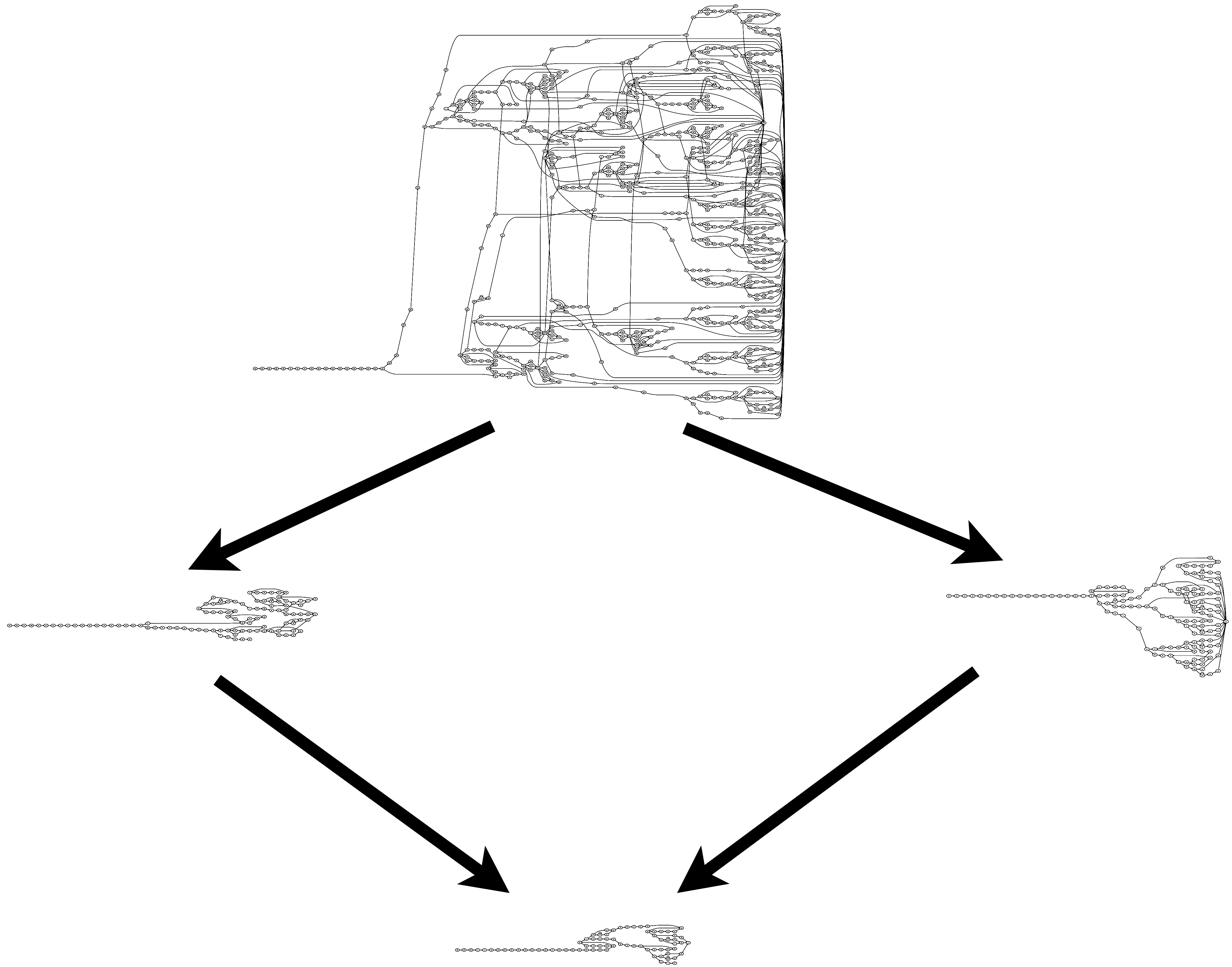

ϵ
$\Delta\hat{\kappa}$
$\Delta\hat{\kappa}$
$\Delta\hat{\kappa}$
$\Delta\hat{\kappa}$
ϵ
ϵ
ϵ

$$\mathcal{P}(\Gamma^*)$$

Result?







$$g \in \Gamma_{\pm} ::= \epsilon \mid \gamma_{+} \mid \gamma_{-}$$

$$(q, \vec{\gamma}) \underset{M}{\overset{\epsilon}{\longmapsto}} (q', \vec{\gamma}) \text{ iff } q \overset{\epsilon}{\rightarrowtail} q' \in \delta$$

$$(q, \gamma : \vec{\gamma}) \underset{M}{\overset{\gamma_{-}}{\longmapsto}} (q', \vec{\gamma}) \text{ iff } q \overset{\gamma_{-}}{\rightarrowtail} q' \in \delta$$

$$(q, \vec{\gamma}) \underset{M}{\overset{\gamma_{+}}{\longmapsto}} (q', \gamma : \vec{\gamma}) \text{ iff } q \overset{\gamma_{+}}{\rightarrowtail} q' \in \delta$$

$$\lfloor \vec{g}\, \gamma_{+} \gamma_{-}\, \vec{g}\,' \rfloor = \lfloor \vec{g}\, \vec{g}\,' \rfloor \qquad \lfloor \vec{g}\, \epsilon\, \vec{g}\,' \rfloor = \lfloor \vec{g}\, \vec{g}\,' \rfloor.$$

$$\lceil \gamma_{+} \gamma'_{+} \ldots \gamma^{(n)}_{+} \rceil = \langle \gamma^{(n)}, \ldots, \gamma', \gamma \rangle$$

$$c_0 \underset{M}{\overset{g_1 \ldots g_n}{\longmapsto\!\!\!\rightarrow}} c_n \text{ iff } c_0 \underset{M}{\overset{g_1}{\longmapsto}} c_1 \underset{M}{\overset{g_2}{\longmapsto}} \cdots \underset{M}{\overset{g_{n-1}}{\longmapsto}} c_{n-1} \underset{M}{\overset{g_n}{\longmapsto}} c_n$$

$$\left\{ q : q_0 \underset{M}{\longmapsto\!\!\!\rightarrow} q \right\}$$

$$c \underset{M}{\overset{g}{\longmapsto\!\!\!\rightarrow}} c' \text{ iff } (q_0, \langle\rangle) \underset{M}{\overset{*}{\longmapsto}} c \text{ and } c \underset{M}{\overset{g}{\longmapsto}} c'.$$

$$q \underset{M}{\longmapsto\!\!\!\rightarrow} q' \text{ iff } q \underset{M}{\overset{g}{\longmapsto\!\!\!\rightarrow}} q' \text{ for some action } g.$$

$$q \underset{M}{\overset{g}{\longmapsto\!\!\!\rightarrow}} q' \text{ iff } (q, \vec{\gamma}) \underset{M}{\overset{g}{\longmapsto\!\!\!\rightarrow}} (q', \vec{\gamma}\,') \text{ for some stacks } \vec{\gamma}, \vec{\gamma}\,'$$

$$\mathcal{A}(lam, \rho, \sigma) = (lam, \rho)$$
$$\mathcal{A}(v, \rho, \sigma) = \sigma(\rho(v))$$

$c \in Conf = \mathsf{Exp} \times Env \times Store \times Kont$
$\rho \in Env = \mathsf{Var} \rightharpoonup Addr$
$\sigma \in Store = Addr \to Clo$
$clo \in Clo = \mathsf{Lam} \times Env$
$\kappa \in Kont = Frame^{*}$
$\phi \in Frame = \mathsf{Var} \times \mathsf{Exp} \times Env$
$a \in Addr$ is an infinite set of addresses

$e \in \mathsf{Exp} ::= \texttt{(let ((}v\ call\texttt{))}\ e\texttt{)}$
$\quad\mid\ call$
$\quad\mid\ æ$
$f, æ \in \mathsf{Atom} ::= v \mid lam$
$lam \in \mathsf{Lam} ::= \texttt{(}\lambda\ \texttt{(}v\texttt{)}\ e\texttt{)}$
$call \in \mathsf{Call} ::= \texttt{(}f\ æ\texttt{)}$
$v \in \mathsf{Var}$ is a set of identifiers

$$\mathcal{E}(e) = \{c : \mathcal{I}(e) \Rightarrow^{*} c\}$$

$$\hat{c}_0 = \hat{\mathcal{I}}(e) = (e, [], [], \langle\rangle).$$

$$\overbrace{([\![\texttt{(let ((}v\ call\texttt{))}\ e\texttt{)}]\!], \rho, \sigma, \kappa)}^{c} \Rightarrow \overbrace{(call, \rho, \sigma, (v, e, \rho) : \kappa)}^{c'}.$$

$$Addr = \mathbb{N}$$
$$alloc(v, (e, \rho, \sigma, \kappa)) = 1 + \max(dom(\sigma))$$

$$\overbrace{([\![\texttt{(}f\ æ\texttt{)}]\!], \rho, \sigma, \kappa)}^{c} \Rightarrow \overbrace{(e, \rho'', \sigma', \kappa)}^{c'}, \text{ where}$$
$$([\![\texttt{(}\lambda\ \texttt{(}v\texttt{)}\ e\texttt{)}]\!], \rho') = \mathcal{A}(f, \rho, \sigma)$$
$$a = alloc(v, c)$$
$$\rho'' = \rho'[v \mapsto a]$$
$$\sigma' = \sigma[a \mapsto \mathcal{A}(æ, \rho, \sigma)].$$

$$\overbrace{(æ, \rho, \sigma, (v, e, \rho') : \kappa)}^{c} \Rightarrow \overbrace{(e, \rho'', \sigma', \kappa)}^{c'}, \text{ where}$$
$$a = alloc(v, c)$$
$$\rho'' = \rho'[v \mapsto a]$$
$$\sigma' = \sigma[a \mapsto \mathcal{A}(æ, \rho, \sigma)].$$

$$(\hat{\sigma} \sqcup \hat{\sigma}')(\hat{a}) = \hat{\sigma}(\hat{a}) \cup \hat{\sigma}'(\hat{a}).$$

$$\hat{\mathcal{E}}(e) = \left\{ \hat{c} : \hat{\mathcal{I}}(e) \leadsto^{*} \hat{c} \right\}$$

$$\hat{\mathcal{A}}(lam, \hat{\rho}, \hat{\sigma}) = \{(lam, \hat{\rho})\}$$
$$\hat{\mathcal{A}}(v, \hat{\rho}, \hat{\sigma}) = \hat{\sigma}(\hat{\rho}(v))$$

$$\overbrace{(æ, \hat{\rho}, \hat{\sigma}, (v, e, \hat{\rho}') : \hat{\kappa})}^{\hat{c}} \leadsto \overbrace{(e, \hat{\rho}'', \hat{\sigma}', \hat{\kappa})}^{\hat{c}'}, \text{ where}$$
$$\hat{a} = \widehat{alloc}(v, \hat{c})$$
$$\hat{\rho}'' = \hat{\rho}'[v \mapsto \hat{a}]$$
$$\hat{\sigma}' = \hat{\sigma} \sqcup [\hat{a} \mapsto \hat{\mathcal{A}}(æ, \hat{\rho}, \hat{\sigma})].$$

$\hat{c} \in \widehat{Conf} = \mathsf{Exp} \times \widehat{Env} \times \widehat{Store} \times \widehat{Kont}$
$\hat{\rho} \in \widehat{Env} = \mathsf{Var} \rightharpoonup \widehat{Addr}$
$\hat{\sigma} \in \widehat{Store} = \widehat{Addr} \to \mathcal{P}\left(\widehat{Clo}\right)$
$\widehat{clo} \in \widehat{Clo} = \mathsf{Lam} \times \widehat{Env}$
$\hat{\kappa} \in \widehat{Kont} = \widehat{Frame}^{*}$
$\hat{\phi} \in \widehat{Frame} = \mathsf{Var} \times \mathsf{Exp} \times \widehat{Env}$
$\hat{a} \in \widehat{Addr}$ is a *finite* set of addresses

$$\overbrace{([\![\texttt{(let ((}v\ call\texttt{))}\ e\texttt{)}]\!], \hat{\rho}, \hat{\sigma}, \hat{\kappa})}^{\hat{c}} \leadsto \overbrace{(call, \hat{\rho}, \hat{\sigma}, (v, e, \hat{\rho}) : \hat{\kappa})}^{\hat{c}'}.$$

$$\overbrace{([\![\texttt{(}f\ æ\texttt{)}]\!], \hat{\rho}, \hat{\sigma}, \hat{\kappa})}^{\hat{c}} \leadsto \overbrace{(e, \hat{\rho}'', \hat{\sigma}', \hat{\kappa})}^{\hat{c}'}, \text{ where}$$
$$([\![\texttt{(}\lambda\ \texttt{(}v\texttt{)}\ e\texttt{)}]\!], \hat{\rho}') \in \hat{\mathcal{A}}(f, \hat{\rho}, \hat{\sigma})$$
$$\hat{a} = \widehat{alloc}(v, \hat{c})$$
$$\hat{\rho}'' = \hat{\rho}'[v \mapsto \hat{a}]$$
$$\hat{\sigma}' = \hat{\sigma} \sqcup [\hat{a} \mapsto \hat{\mathcal{A}}(æ, \hat{\rho}, \hat{\sigma})].$$

$$\widehat{Addr} = \mathsf{Var}$$
$$alloc(v, \hat{c}) = v.$$

$$\widehat{Addr} = \mathsf{Var} \times \mathsf{Exp}$$
$$alloc(v, (e, \hat{\rho}, \hat{\sigma}, \hat{\kappa})) = (v, e).$$

$$O((|\Gamma| m^2) \times (|\Gamma_{\pm}|^3 m^3)) = O(|\Gamma|^4 m^5).$$

$$\widehat{Addr} = \mathsf{Var} + \mathsf{Var} \times \mathsf{Exp}$$
$$alloc(v, ([\![\texttt{(}f\ æ\texttt{)}]\!], \hat{\rho}, \hat{\sigma}, \hat{\kappa})) = \begin{cases} (v, [\![\texttt{(}f\ æ\texttt{)}]\!]) & f \text{ is let-bound} \\ v & \text{otherwise.} \end{cases}$$

$$\widehat{Addr} = \mathsf{Var} \times \mathsf{Exp}^{k}$$
$$\widehat{alloc}(v, \langle (e_1, \hat{\rho}_1, \hat{\sigma}_1, \hat{\kappa}_1), \ldots \rangle) = (v, \langle e_1, \ldots, e_k \rangle).$$

$$\hat{\rho} \sqsubseteq \hat{\rho}' \text{ iff } \hat{\rho}(v) = \hat{\rho}'(v) \text{ for all } v \in dom(\hat{\rho});$$

$$\hat{\sigma} \sqsubseteq \hat{\sigma}' \text{ iff } \hat{\sigma}(\hat{a}) \sqsubseteq \hat{\sigma}'(\hat{a}) \text{ for all } \hat{a} \in dom(\hat{\sigma});$$

$$(lam, \hat{\rho}) \sqsubseteq (lam, \hat{\rho}') \text{ iff } \hat{\rho} \sqsubseteq \hat{\rho}'; \qquad (v, e, \hat{\rho}) \sqsubseteq (v, e, \hat{\rho}') \text{ iff } \hat{\rho} \sqsubseteq \hat{\rho}';$$

$$\langle \hat{\phi}_1, \ldots, \hat{\phi}_n \rangle \sqsubseteq \langle \hat{\phi}'_1, \ldots, \hat{\phi}'_n \rangle \text{ iff } \hat{\phi}_i \sqsubseteq \hat{\phi}'_i;$$

$$(e, \hat{\rho}, \hat{\sigma}, \hat{\kappa}) \sqsubseteq (e, \hat{\rho}', \hat{\sigma}', \hat{\kappa}') \text{ iff } \hat{\rho} \sqsubseteq \hat{\rho}' \text{ and } \hat{\sigma} \sqsubseteq \hat{\sigma}' \text{ and } \hat{\kappa} \sqsubseteq \hat{\kappa}'.$$

$$\alpha(e, \rho, \sigma, \kappa) = (e, \alpha(\rho), \alpha(\sigma), \alpha(\kappa))$$
$$\alpha(\rho) = \lambda v. \alpha(\rho(v))$$
$$\alpha(\sigma) = \lambda \hat{a}. \bigsqcup_{\alpha(a) = \hat{a}} \{\alpha(\sigma(a))\}$$
$$\alpha \langle \phi_1, \ldots, \phi_n \rangle = \langle \alpha(\phi_1), \ldots, \alpha(\phi_n) \rangle$$
$$\alpha(v, e, \rho) = (v, e, \alpha(\rho))$$
$\alpha(a)$ is determined by the allocation functions.

Theorem 4.1. *If:*
$$\alpha(c) \sqsubseteq \hat{c} \text{ and } c \Rightarrow c',$$
then there must exist $\hat{c}' \in \widehat{Conf}$ *such that:*
$$\alpha(c') \sqsubseteq \hat{c}' \text{ and } \hat{c} \leadsto \hat{c}'.$$

$$\widehat{\mathcal{PDS}}(e) = (Q, \Gamma, \delta, q_0), \text{ where}$$
$$Q = \mathsf{Exp} \times \widehat{Env} \times \widehat{Store}$$
$$\Gamma = \widehat{Frame}$$
$$(q, \epsilon, q') \in \delta \text{ iff } (q, \hat{\kappa}) \leadsto (q', \hat{\kappa}) \text{ for all } \hat{\kappa}$$
$$(q, \hat{\phi}_{-}, q') \in \delta \text{ iff } (q, \hat{\phi} : \hat{\kappa}) \leadsto (q', \hat{\kappa}) \text{ for all } \hat{\kappa}$$
$$(q, \hat{\phi}'_{+}, q') \in \delta \text{ iff } (q, \hat{\kappa}) \leadsto (q', \hat{\phi}' : \hat{\kappa}) \text{ for all } \hat{\kappa}$$
$$(q_0, \langle\rangle) = \hat{\mathcal{I}}(e).$$

$$\hat{G}(\overbrace{e, \hat{\rho}, \hat{\sigma}, \hat{\kappa}}^{\hat{c}}) = (e, \hat{\rho}, \hat{\sigma} | Reachable(\hat{c}), \hat{\kappa}),$$

$$Reachable(\overbrace{e, \hat{\rho}, \hat{\sigma}, \hat{\kappa}}^{\hat{c}}) = \left\{ \hat{a} : \hat{a}_0 \in Root(\hat{c}) \text{ and } \hat{a}_0 \underset{\hat{\sigma}}{\overset{*}{\multimap}} \hat{a} \right\}$$

$$Root(e, \hat{\rho}, \hat{\sigma}, \hat{\kappa}) = range(\hat{\rho}) \cup StackRoot(\hat{\kappa}).$$

$$StackRoot \langle (v_1, e_1, \hat{\rho}_1), \ldots, (v_n, e_n, \hat{\rho}_n) \rangle = \bigcup_{i} range(\hat{\rho}_i),$$

$$\hat{a} \underset{\hat{\sigma}}{\multimap} \hat{a}' \text{ iff there exists } (lam, \hat{\rho}) \in \hat{\sigma}(\hat{a}) \text{ such that } \hat{a}' \in range(\hat{\rho}).$$

$$(\leadsto_{\mathrm{GC}}) = (\leadsto) \circ \hat{G}$$

$$q \underset{M}{\overset{g}{\longmapsto\!\!\!\rightarrow}} q' \text{ iff there exists } \vec{g} \text{ such that } q_0 \underset{M}{\overset{\vec{g}}{\longmapsto\!\!\!\rightarrow}} q \text{ and } (q, [\vec{g}], g, q') \in \delta,$$
$$\text{where } q \underset{M}{\overset{\langle g_1, \ldots, g_n \rangle}{\longmapsto\!\!\!\rightarrow}} q' \text{ iff } q \underset{M}{\overset{g_1}{\longmapsto\!\!\!\rightarrow}} q_1 \underset{M}{\overset{g_2}{\longmapsto\!\!\!\rightarrow}} \cdots \underset{M}{\overset{g_n}{\longmapsto\!\!\!\rightarrow}} q'.$$

$$q_0 \underset{M}{\overset{g}{\longmapsto\!\!\!\rightarrow}} q \text{ iff } (q_0, \langle\rangle, g, q) \in \delta.$$

$$S = \left\{ q : q_0 \underset{M}{\overset{\vec{g}}{\longmapsto\!\!\!\rightarrow}} q \text{ for some stack-action sequence } \vec{g} \right\}$$

$$E = \left\{ q \overset{g}{\rightarrowtail} q' : q \underset{M}{\overset{g}{\longmapsto\!\!\!\rightarrow}} q' \right\}$$

$$\widehat{\mathcal{IPDS}}(e) = (Q, \Gamma, \delta, q_0)$$
$$Q = \mathsf{Exp} \times \widehat{Env} \times \widehat{Store}$$
$$\Gamma = \widehat{Frame}$$
$$(q, \hat{\kappa}, \epsilon, q') \in \delta \text{ iff } \hat{G}(q, \hat{\kappa}) \leadsto (q', \hat{\kappa})$$
$$(q, \hat{\phi} : \hat{\kappa}, \hat{\phi}_{-}, q') \in \delta \text{ iff } \hat{G}(q, \hat{\phi} : \hat{\kappa}) \leadsto (q', \hat{\kappa})$$
$$(q, \hat{\kappa}, \hat{\phi}_{+}, q') \in \delta \text{ iff } \hat{G}(q, \hat{\kappa}) \leadsto (q', \hat{\phi} : \hat{\kappa})$$
$$(q_0, \langle\rangle) = \hat{\mathcal{I}}(e).$$

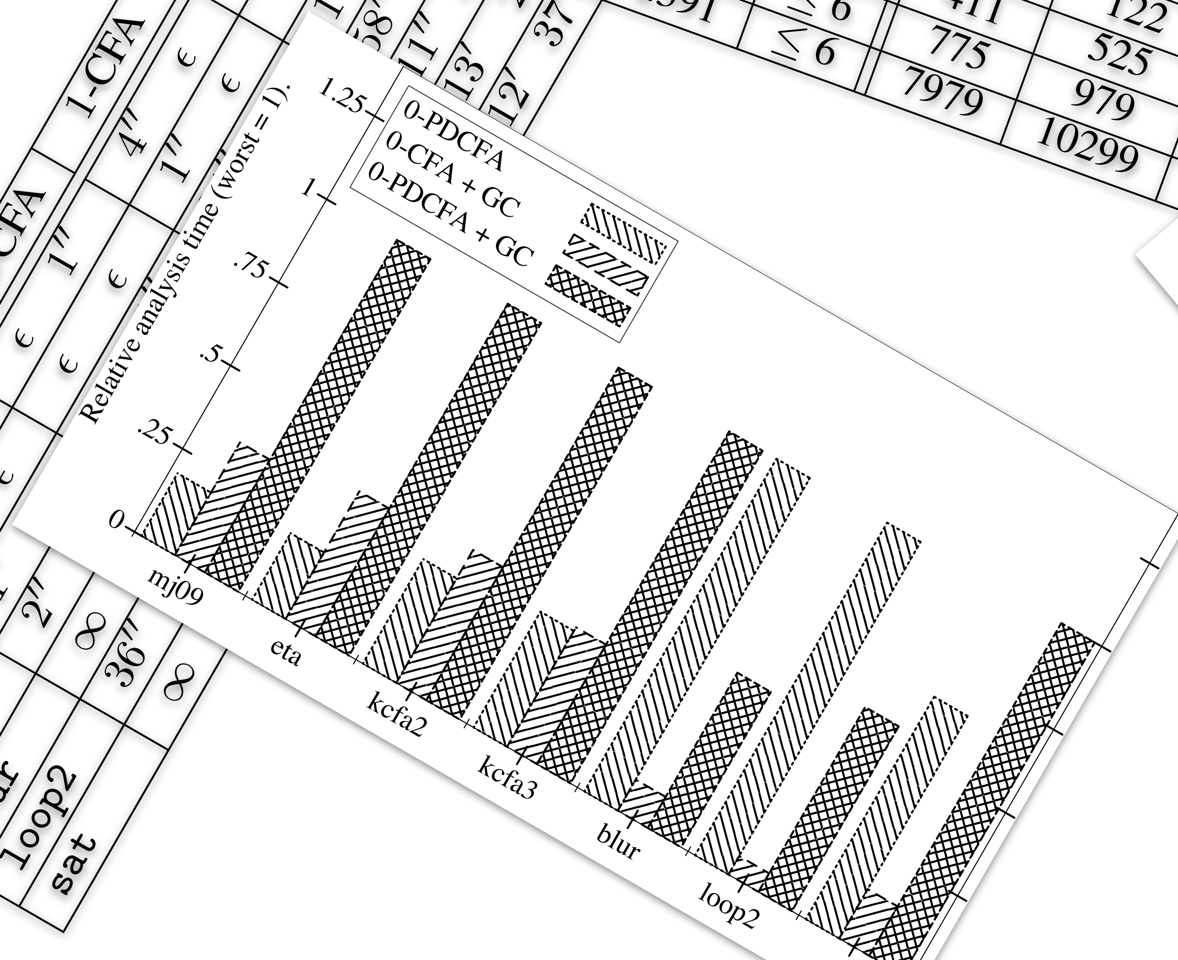
$$Stacks(\overbrace{S, \Gamma, E, s_0}^{M})(s) = (S, \Gamma, \delta, s_0, \{s\}), \text{ where}$$
$$(s', \gamma, s'') \in \delta \text{ if } (s', \gamma_{+}, s'') \in E$$
$$(s', \epsilon, s'') \in \delta \text{ if } s' \underset{M}{\overset{\vec{g}}{\longmapsto\!\!\!\rightarrow}} s'' \text{ and } [\vec{g}] = \epsilon.$$

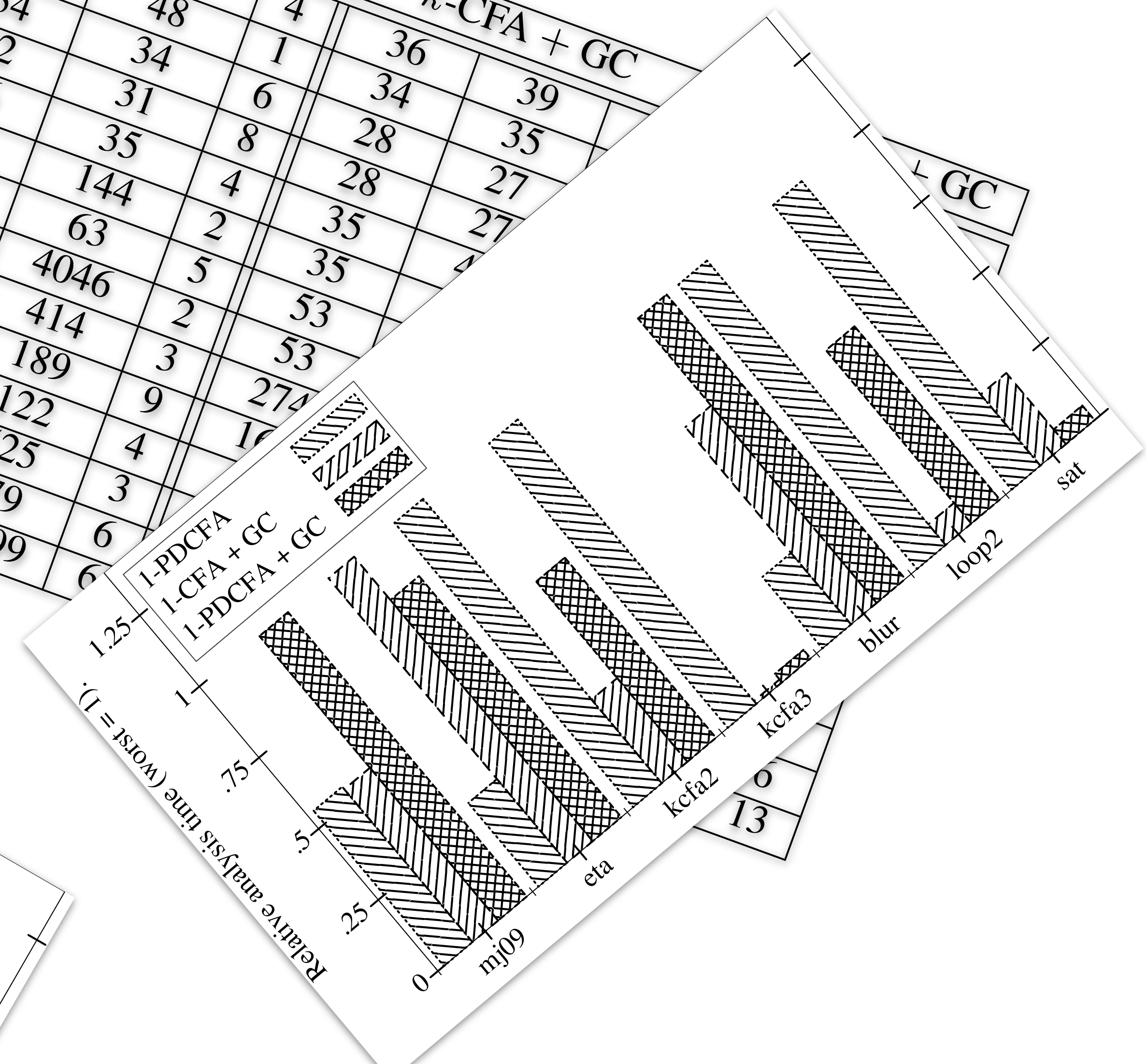
$$\mathcal{F}(M) = f, \text{ where}$$
$$M = (Q, \Gamma, \delta, q_0)$$
$$f(S, \Gamma, E, s_0) = (S', \Gamma, E', s_0), \text{ where}$$
$$S' = S \cup \left\{ s' : s \in S \text{ and } s \underset{M}{\longmapsto\!\!\!\rightarrow} s' \right\} \cup \{s_0\}$$
$$E' = E \cup \left\{ s \overset{g}{\rightarrowtail} s' : s \in S \text{ and } s \underset{M}{\overset{g}{\longmapsto\!\!\!\rightarrow}} s' \right\}.$$

Theorem 7.1. $\mathcal{DSG}(M) = \mathrm{lfp}(\mathcal{F}(M)).$

Program	Exp	Var	k	k-CFA			k-PDCFA			k-CFA + GC			k-PDCFA + GC		
mj09	19	8	0	83	107	4	38	38	4	36	39	[illegible]	[illegible]	[illegible]	[illegible]
			1	454	812	1	44	48	1	34	35	[illegible]	[illegible]	[illegible]	[illegible]
eta	21	13	0	63	74	4	34	34	6	28	35	[illegible]	[illegible]	[illegible]	[illegible]
			1	33	33	8	32	31	8	28	27	[illegible]	[illegible]	[illegible]	[illegible]
kcfa2	20	10	0	194	236	3	36	35	4	35	27	[illegible]	[illegible]	[illegible]	[illegible]
			1	970	1935	1	87	144	2	35	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
kcfa3	25	13	0	272	327	4	58	63	5	53	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
			1	> 7119	> 14201	≤ 1	1761	4046	2	53	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
blur	40	20	0	> 1419	> 2435	≤ 3	280	414	3	274	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
			1	261	340	9	177	189	9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
loop2	41	14	0	228	252	4	113	122	4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
			1	> 10867	> 16040	≤ 3	411	525	3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
sat	63	31	0	> 5362	> 7610	≤ 6	775	979	6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	6
			1	> 8395	> 12391	≤ 6	7979	10299	6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	13

Program	0-CFA		0-PDCFA		1-CFA		1-PDCFA	
mj09	1″	ϵ	ϵ	1″	4″	ϵ	ϵ	ϵ
eta	ϵ	ϵ	ϵ	ϵ	1″	ϵ	ϵ	ϵ
kcfa2	1″	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1″	ϵ
kcfa3	2″	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	58″	2″
blur	∞	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	11″	55″
loop2	36″	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	13′	2′
sat	∞	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	12′	37″





http://github.com/ilyasergey/reachability http://github.com/ilyasergey/reachability

CRAPL

Finitization is double-edged.

Progress attacks finitization.

We can skirt inside decidability.

Tak.

Complexity?

- Monovariant, global store? Polynomial.
- Polyvariant? Exponential.
- Flat environments, global store? Polynomial.

Alternative?

$$\delta : Q \times \Delta\Gamma \times \mathcal{P}(\Gamma^*) \rightarrow \mathcal{P}(Q)$$

$$\delta : Q \times \Delta\Gamma \times \mathcal{P}(\Delta\Gamma) \rightarrow \mathcal{P}(Q)$$

$$\delta : Q \times \mathcal{P}(\Delta\Gamma) \times \Delta\Gamma \to \mathcal{P}(Q)$$

$$\delta : \overbrace{Q \times \mathcal{P}(\Delta\Gamma)}^{\text{control states}} \times \Delta\Gamma \rightarrow \mathcal{P}(Q)$$