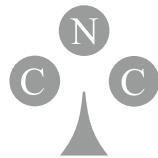


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Thomas Jansen

# Analyzing Evolutionary Algorithms

The Computer Science Perspective



Springer

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*I dedicate this book to Ingo Wegener, my  
Doktorvater.*



# Preface

Evolutionary algorithms are general randomized search heuristics, inspired by the concept of natural evolution. They are easy to implement and easy to apply, thought to be robust problem solvers, popular in circumstances where there is no good problem-specific algorithm known and where there is not enough time or expertise to develop one. Their development and application is motivated by practical needs. Therefore, the theory of evolutionary algorithms seems to be a contradiction. In fact, many people are very skeptical toward evolutionary algorithm theory and reject it as impractical, useless, or even just plain wrong. Admittedly, there were times when evolutionary algorithm theory was dominated by unfounded claims, risky hypotheses, and overgeneralizations. But evolutionary algorithm theory has seen a change in the last two decades, a change that itself was more revolutionary than evolutionary and that has lead to a rigorous, sound, and arguably useful theory. At the heart of this novel theory for evolutionary algorithms is the insight that evolutionary algorithms are in fact randomized algorithms and that consequently they should be analyzed as randomized algorithms. This establishes the study of evolutionary algorithms as an important topic in the area of computer science. Analyzing evolutionary algorithms as randomized algorithms is by far the most important and fruitful form of theory in the area of evolutionary computation today.

The analysis of evolutionary algorithms is a lively and very active field of research. Many different and tremendously useful tools and methods for analyzing evolutionary algorithms have been and continue to be developed. In this book, an introduction to this field of research is presented that makes it accessible by introducing the most important and fundamental analytical methods, presenting them in a rigorous manner, including complete proofs, and demonstrating how they are applied by means of a number of instructive examples. This should enable anyone to enter this fascinating and fruitful field of research: people interested in further developing the theory of randomized search heuristics, practitioners in evolutionary computation who appreciate the importance of a solid theoretical foundation, and people who teach evolutionary computation and prefer solid and proven facts over rules of thumb. Since there is nothing more practical than a good theory, it is hoped that this book on analyzing evolutionary algorithms is seen as

not merely a theoretical exercise but as a useful guide into practical evolutionary computation.

Many people have contributed to this book in different forms. I avoid missing any of them by extending my sincere thanks to all of them without naming them: the research groups at the TU Dortmund, George Mason University, the Max-Planck-Institut für Informatik, and the University of Birmingham and everybody who participated in discussions at Dagstuhl seminars about evolutionary algorithm theory, at FOGA workshops, ThRaSH workshops, workshops at GECCO and PPSN, or conferences—they all deserve to be acknowledged for contributing insights, expertise, and motivation. Moreover, I gladly acknowledge the generous support by the Deutsche Forschungsgemeinschaft (DFG), the German Academic Exchange Service (DAAD), and Science Foundation Ireland (SFI).

Cork, Ireland

Thomas Jansen

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