Prediction of box failure from paper data for asymmetric corrugated board

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ABSTRACT: The well-known McKee formula was derived in 1963 for symmetric fluting constructions. This paper presents investigations showing the influence of asymmetric properties, either geometric or concerning the elastic moduli. Paper properties of the base papers used are taken to calculate the panel properties of corrugated boards. From these results, the box failure is predicted. To do this, a model approximating the corrugated medium by triangles is presented. With this assumption, the bending stiffness of corrugated board, no matter how many flutes, can be predicted from paper data. Machine direction and cross direction are treated separately. The theoretical considerations are verified by measuring paper parameters such as short compression test (SCT), thickness, and bending stiffness, as well as edge compression test (ECT) and bending stiffness of corrugated board and box compression test (BCT) of the boxes. It can be stated that the prediction of using paper data instead of corrugated board data may lead to even better prediction results, as the measuring of bending stiffness at the board may sometimes be influenced by sampling inconveniences. Finally, thickness of the used base papers, as well as thickness of the manufactured board, appear to be the main influencing parameters. The McKee formula is robust enough to be applied for asymmetric corrugated board as well.

Application: The results here help to predict the BCT values from paper data.

Since 1963, when McKee et al. [1] published a formula to predict box compression strength (BCT) from edge compression strength (ECT) and bending stiffness (S) of corrugated board, the manufacturers of corrugated boxes have used this equation as a simple, practical, and quite stable estimation of the quality of their end product. The formula was enhanced by Jonson and Ponton [2] in 1985 and Müller [3] in 2002 in order to be able to use paper data to predict box quality. Both proposed the short span compression strength (SCT value) to estimate the ECT value, which can further be used to calculate the BCT value.

In 1965, Shick [4] used the McKee formula for double-fluted board, but he had to state other fitting parameters. Pommier et al. [5] emphasize the importance of bending stiffness and discuss the ECT as too poor to be useful for BCT prediction, as ECT is highly dependent on grammage of the base papers. The authors point out that the McKee formula with the published constants is only applicable for symmetric board structure. They underline their comments with finite element calculations.

However, the McKee formula is subject to some restrictions. First, the connection of the flute to the liner is regarded to be ideal. Second, the ratio between box perimeter and box height must be higher than 7. The formulas have been derived from panels. In 1993, Batelka and Smith [6] indicated that they are most applicable to square boxes where all the panels are the same. Therefore, length and width of a box should not differ more than by a factor of 3. Finally, the corrugated board plate has to be assumed as symmetric in thickness (see also Pommier et al. [5]). This last restriction needs to be eliminated, as more and more board with two different flutes are produced. In Germany, the amount of corrugated board with more than one flute increased from 26% in 2000 to almost 33% in 2015. In addition, the grammage range of the papers used to produce corrugated boxes has changed to a higher variety since 1963. The corrugated base material may only have 80 g/m² or even less in manufacturing today. If coated liner is used, 180 g/m² is a common grammage in the upper range. A common grammage in 1963 was 120 g/m²; coated liner or white top liner was unknown at that time. To save material and costs, total mass of corrugated board declined from 531 g/m² in 2000 to 515 g/m² in 2015 [7].

In 1985, Carlsson et al. [8] published their work concerning the prediction of bending stiffness for asymmetric corrugated boards consisting of several layers. The way to estimate the contribution of flat layers to bending stiffness is the same as described later in this paper, which means that the same fundamental equations of mechanics are used. However, the contribution of the corrugated layers is calculated in a different manner. Finally, the authors do not extend the analytics to predict box failure.

Therefore, it is necessary to evaluate whether the McKee formula can also be used for low grammage corrugated papers, and whether the formula has to be enhanced to asymmetric corrugated boards with two or three flutes.

THE MCKEE FORMULA AND SOME THEORY

McKee used the theory of buckling plates. The critical load

P_{crit} causes buckling. The material can withstand the applied pressure until P_{max} (which means either break under pressure or total failure by buckling) is achieved. This leads to [1]:

$$\frac{P_{max}}{P_{crit}} = c \cdot \left(\frac{P_{Mat}}{P_{crit}}\right)^b$$
$$\text{respectively:} \quad P_{max} = c \cdot P_{Mat}^b \cdot P_{crit}^{1-b} \tag{1}$$

where P_{Mat} is a material property and will equal edge crush test (ECT) in this case, as well as P_{max} equals box compression test (BCT). The perimeter of the box Z and the bending stiffness S in machine direction (MD) and cross direction (CD) can be used to replace the critical load P_{crit} and leads to:

$$BCT = 2{,}028 \cdot ECT^{0{,}746} \cdot \sqrt{S_{MD} \cdot S_{CD}}^{\,0{,}254} \cdot Z^{0{,}492} \tag{2}$$

which is known as the original McKee formula. A second version uses ECT values and thickness of the board instead of bending stiffness. As thickness is much easier to measure than bending stiffness, this version is more often used in the corrugated board industries. However, Mühlenbein [9] and Maltenfort [10] discussed this approach in their publications. Mühlenbein recommended the four-point method for determination of bending stiffness, as described in ISO 5628:2012 "Paper and board — Determination of bending stiffness— General principles for two-point, three-point and four-point methods." He refused the use of thickness as an alternative, as he could not confirm the correlation between board thickness and BCT, as published by McKee et al. [1]. Maltenfort disagreed and showed that Mühlenbein had a mistake in his calculations. Maltenfort emphasised the correctness of the McKee approach, but it has to be mentioned that taking the bending stiffness to predict BCT, 80% of the tested boxes are in a range ± 10% around the calculated value but only 75% if the thickness is taken instead [1]. The numbers of the parameters have been derived from experiments with more than 100 boxes.

As bending stiffness seems to be the better parameter to predict BCT, some work dealing with this subject will be mentioned. Fellers and Carlsson [11] referred to the importance of bending stiffness for packaging board and summarized methods and theories for stiffness evaluation. It is important to know that the four point method delivers the possibility to calculate bending stiffness without regarding shear effects. Fellers, in his ongoing work [12], carried out a round robin study in 1997. He stated that variance of results increases with decreasing grammage.

Creeping and influence of climate changes on corrugated board and its bending stiffness have been subject of investigation as well (e.g., [13-14]), but this is not the focus of this paper.

Besides the semi-empirical approach to predict box strength offered by McKee, some authors published statistical, analytical, and numerical studies to predict paper and board properties. As an example, Lu and Carlsson in 1986 [15] used the Monte Carlo method to calculate paper formation and paper structure as a starting point to predict paper strength and stiffness properties and its variations. Biancoli and Brutti published a paper in 2003 dealing with the investigation of buckling by means of finite element analysis [16]. They found an excellent agreement of calculated values with experimental results for ECT as well as for BCT. This work was carried out for a single C-flute, and the results were also found to be in accordance with the prediction done by the McKee formula. Part of their approach is to replace the corrugated structure by a homogeneous layer. In 2013, Aboura et al. [17] presented an analytical model of single flute board. The approach is comparable to Carlsson et al. [8], as the shape of the flute is taken as a sine. Experimental results fit well to analytical predictions; however, the investigation is done in the elastic range of deformation. In 2014, Åslund et al. [18] did a numerical analysis of a single flute board plate regarded as a sandwich structure. They succeeded in getting insights not only about panel buckling, but also about local face buckling.

Despite many efforts to get a better prediction of BCT values, the McKee formula is still the favorite means of estimating the strength of a box from paper parameters. The main reason is probably its easy use for industrial requirements. Nevertheless, Coffin [19] addressed in his 2015 paper the necessity to decrease the number of boxes with unsatisfactory predictions of BCT. He suggested including independent investigations of material properties, edge restraints, and geometrical effect in the focus of research.

Concerning multilayered, asymmetric corrugated board constructions, an attempt is proposed in this paper to use different material properties of the base papers, as well as given geometric conditions, to calculate a box strength.

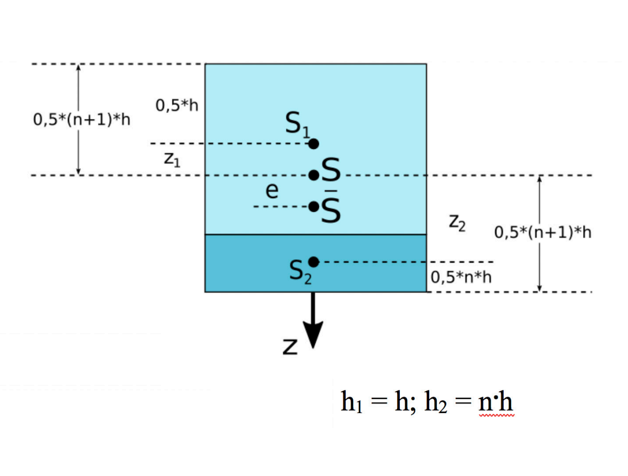
To start, a look back to the Euler theory of beams is helpful [20]. Although the McKee formulas have been derived by using plate theory, the beam theory is sufficient to understand the influence of asymmetry. In the Euler theory, the bending stiffness is given by multiplying the E-modulus of the material with the area moment of inertia, depending on the cross-section of the beam.

Bending stiffness in cross direction (CD)

The area moment of inertia is easy to calculate for symmetric, rectangular structures. However, for example, a combination of B- and C-flute or even B- and E-flute will lead to a structure comparable to **Fig. 1**.

Layer 1 and layer 2 have different heights, h_1 and h_2, as well as different E-moduli, E_1 and E_2. The ratio of h_2 to h_1 is n. S_1 is the position of the neutral axis of layer 1, S_2 of layer 2, S the middle axis and $\overline{S}$ the neutral axis of the whole structure. With the parallel axis theorem, the area moment of inertia of the two layers can be calculated. Therefore, the distance between the neutral axis of each layer to the neutral axis of the whole structure has to be calculated with the help of z_1, z_2, and eccentricity e.

The values of z_1 and z_2 may be derived from geometric con-



1. Asymmetrical structures [21].

ditions. The eccentricity is influenced by the elastic moduli E_1 and E_2 of the layers. Letting *m* be the relation of E_2 to E_1, eccentricity *e* results in:

$$e = \frac{n \cdot h_1 \cdot (m-1)}{2 \cdot (n \cdot m + 1)} \tag{3}$$

Using this and applying the parallel axis theorem leads to:

$$\overline{E \cdot I} = E_1 \left\{ \frac{b h_1^3}{12} + b h_1 \cdot \left[\frac{1}{2} n h_1 + e \right]^2 + m \cdot \left(\frac{b h_1^3}{12} n^3 + n b h_1 \cdot \left[\frac{1}{2} h_1 - e \right]^2 \right) \right\} \tag{4}$$

as the bending stiffness of the whole structure. The width of the corrugated plate is *b*. These steps may be carried out for more than two layers as well and will lead to bigger formulas. With two different liners and one fluting medium, three layers are already given [21].

The next step is to calculate a substitutional E-modulus for the flute. Sine shapes cannot be calculated analytically; therefore the sine flute is approximated by a triangular shape (**Fig 2**). This appears to be reasonable, as the flute-shape very often is not an exact sine in the industrial corrugators.

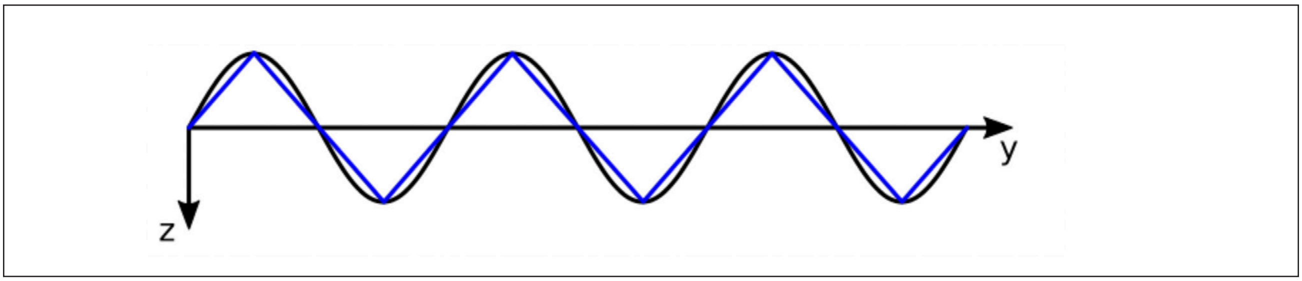
Assuming that the width of a corrugated board is much larger than the pitch of the flute, the triangles may be displaced in a manner shown in **Fig. 3**. This will still lead to the same bending stiffness across the y-axis as the structure in Fig. 2.

Now, the bending stiffness of the flute-layer is easy to calculate knowing the E-modulus of the fluting paper in cross direction ($E_{CD,fl}$) and the geometry (height of the flute h_{fl}, pitch p, width of the flute-strip w and thickness of the fluting-paper δ_{fl}):

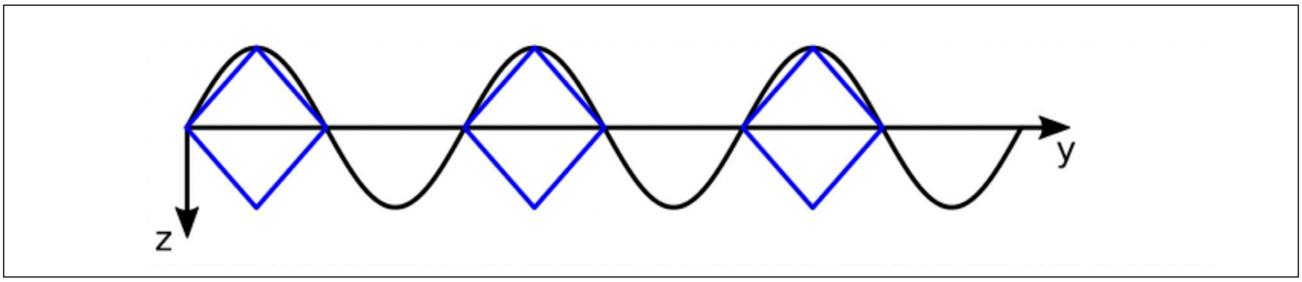
$$\overline{E_{CD,fl} \cdot I_{fl}} = E_{CD,fl} \cdot \frac{w}{p} \cdot \frac{1}{6} \delta_{fl} \cdot h_{fl}^2 \cdot \sqrt{\frac{1}{4} p^2 + h_{fl}^2} \tag{5}$$

This bending stiffness can be used to derive a substitutional E-modulus in CD, $E_{CD,sub}$. Therefore, imagine to substitute the flute-layer by a homogeneous layer with the height h_{fl} and the width w (which may be measured). The moment of inertia $I_{CD,sub}$ can then be calculated as:

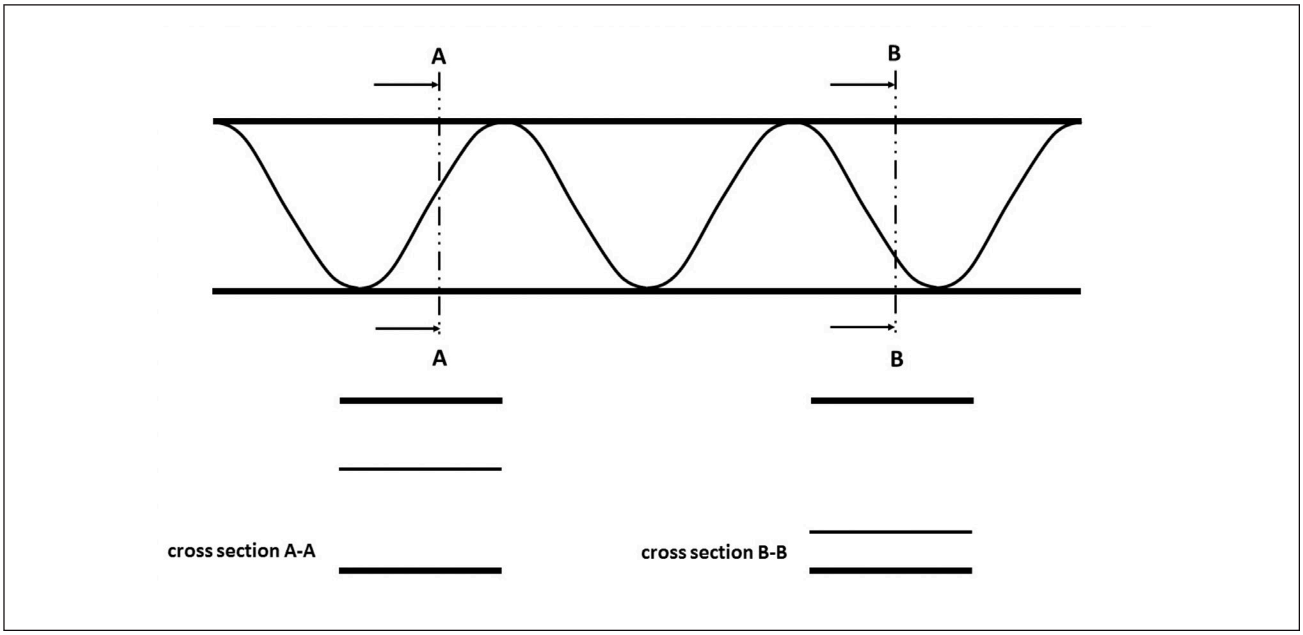
$$I_{CD,sub} = \frac{w \cdot h_{fl}^3}{12} \tag{6}$$



2. Approximation of a sine-shape flute by triangles.



3. Displacing the triangles.



4. Contribution of fluting paper, depending on cutting position.

As the imaginary homogeneous layer shall have the same bending stiffness as the real layer, the substitutional E-modulus can now be calculated as well [21]:

$$E_{CD,sub} = E_{CD,fl} \cdot \frac{2 \cdot \delta_{fl}}{p \cdot h_{fl}} \cdot \sqrt{\frac{1}{4}p^2 + h_{fl}^2} \quad (7)$$

The layer of liner paper is characterized by the thickness of the liner δ_{lin}, which equals the height of the liner layer h_{lin}, and the liner E-modulus in cross direction, $E_{CD,lin}$. Going back to Eq. 4, E_1 may be taken as E_{sub}, h_1 as h_{fl}, in consequence E_2 equals E_{lin} and h_2 equals h_{lin}. So, one liner layer connected to one flute is described.

The results may be taken, and analogous to the described procedure, a second liner layer can be added. Therefore, the obtained E of the liner fluting combination will be regarded as a new E_1 for Eq. 4, and the new h_1 will be $h_{lin} + h_{fl}$. The other E_2 is given by the E-modulus of the second liner layer, and the other h_2 by the thickness of the second liner layer. Now, a complete corrugated board consisting of two liners and one flute is described.

This result can be enhanced by adding the next fluting layer, calculated in the proposed way with a substitutional E-modulus, and so on. By this, it is possible to calculate the bending stiffness of multilayer corrugated boards in CD just by knowing the flute type and by measuring the thicknesses and the E-moduli of the papers used.

Bending stiffness in machine direction (MD)

The same procedure for the CD has to be carried out in the MD. However, the contribution of the fluting paper to the bending stiffness is dependent on the position where the cross section is selected (**Fig. 4**).

Paper Grade Applied for Indicated Corrugated Board	δ, mm	E_{MD}, N/mm²	E_{CD}, N/mm²
Liner 1 (used for B-flute)	0.172	3477	1188
Liner 2 (used for B-flute)	0.172	3458	1150
Flute (used for B-flute)	0.139	3503	1151
Liner 1 (used for C-flute)	0.175	3001	1164
Liner 2 (used for C-flute)	0.151	3700	1271
Flute (used for C-flute)	0.142	3157	1449
Liner 1 (used for BC-flute)	0.280	3824	1267
Liner 2 (used for BC-flute)	0.151	3681	1383
Liner 3 (used for BC-flute)	0.248	3699	1613
Flute C (used for BC-flute)	0.153	3359	1096
Flute B (used for BC-flute)	0.145	3798	1625
Liner 1 (used for EB-flute)	0.156	6109	1847
Liner 2 (used for EB-flute)	0.154	3954	1224
Liner 3 (used for EB-flute)	0.149	3919	1556
Flute E (used for EB-flute)	0.144	4104	1379
Flute B (used for EB-flute)	0.151	3522	1467

I. Paper data.

To overcome the problem resulting from the unknown position of the flute, the influence is estimated by using the thickness of the paper and the width of the strip. This covers the situation when the fluting paper is in the center between the two liners. The fluting layer is used to create a distance between the liner papers; its contribution to bending resistance is low. So, the mistake made by this assumption is expected to be low, as well.

$$I_{MD,fl} = \frac{w \cdot \delta_{fl}^3}{12} \tag{8}$$

Analogue to cross direction, a substitutional E-modulus $E_{MD,sub}$ can be calculated in MD:

$$E_{MD,sub} = E_{MD,fl} \cdot \left(\frac{\delta_{fl}}{h_{fl}}\right)^3 \tag{9}$$

Now, the same procedure applies as described in cross direction. With help of Eq. 4, the resulting bending stiffness of a liner flute combination can be calculated, and the next liner layer may be added and so on.

The goal of this calculation is to take the obtained bending stiffness of the whole, asymmetric structure in MD and CD and use it in Eq. 2, the McKee formula. The calculated BCT should match to the measured one.

EXPERIMENTS

Prediction of bending stiffness of corrugated board from paper quality

During the production of corrugated board, samples were taken in a board mill. They include paper as well as board. E-moduli E of the papers (**Table I**) were derived using bending tests according to ISO 2493-1:2010 "Paper and board—Determination of bending resistance—Part 1: Constant rate of deflection."

In addition, the bending stiffness divided by the width of the tested stripes (called S) of the corrugated board was measured according to ISO 5628. With the derived equations, the bending stiffness S (related to width) of the board was calculated from the E-moduli of the papers. The results are demonstrated in **Table II**.

The height of the B-flute was 2.43 mm, pitch was 6.09 mm, and take-up factor was 1.3 mm. The C-flute was 3.21 mm high, pitch was 7.92 mm, and take-up factor was 1.34 mm. The values of the E-flute were: height 1.27 mm, pitch 3.23 mm, and take-up factor 1.32 mm. As documented in Table II, the results for calculated and measured bending stiffness of the board fit quite well for the BC- and EB-combination, but less well for the single flutes. It has to be mentioned that a slight curl could be observed with the single flutes. This is assumed to be the reason of the high difference. Box I and box II made with C-flute are different in dimension. Due to this, no sample in CD could be cut from box II, because it was too small. For further calculations, CD values of S are taken from box I.

Prediction of BCT from paper quality and from ECT

According to Jonson and Ponton [2], the SCT values of different papers were measured and used to calculate the ECT value of the corresponding corrugated board. As described in the "Prediction of bending stiffness of corrugated board from paper quality" section, the bending stiffness in MD and CD of the papers were calculated and used to derive the bending stiffness of the board. From this board, boxes were manufactured, and the BCT value was measured and calculated both according to the McKee formula (Eq. 2), with ECT-values and

Board Type	S_{MD} Board Calculated, Nmm	S_{MD} Board Measured, Nmm	Difference, %	S_{CD} Board Calculated, Nmm	S_{CD} Board Measured, Nmm	Difference, %
B-flute	2023	3260	-37.9	844	1330	-36.5
C-flute (box I)	3073	4350	-29.4	1430	2040	-29.9
C-flute (box II)	3073	4620	-33.5	./.	./.	./.
BC-flute	30487	28180	8.2	14578	13100	11.3
EB-flute	8373	7930	+ 5.6	4083	3900	+ 4.7

II. Calculated board stiffness and measured board stiffness, S, from paper data for machine direction (S_{MD}) and cross direction (S_{CD}).

Board	BC-Flute, kN	EB-Flute, kN	C-Flute I, kN	C-Flute II, kN	B-Flute, kN
BCT measured	5.47	2.69	1.08	1.85	1.28
McKee results with measured ECT (old method)	5.22	2.49	1.74	1.76	1.14
McKee results with new method	5.34	2.73	1.44	1.44	1.51
BCT = board compression test; ECT = edge compression test.					

III. Comparison of measured BCT using McKee formula to calculated BCT [20].

stiffness values S measured from samples cut from the box, and according to the theory demonstrated in the previous sections of this paper (**Table III**). Again, the double flute boards fit better than the single flute boards. The McKee formula was least applicable to the C-flutes (box I: Z/h >7; box II: Z/h < 7). This may be regarded as an outlier, as it is well known that the McKee formula fits well to single layer, symmetric boards. On the other hand, these results show that more data is necessary and that better fits for single-flute boards may be found.

CONCLUSIONS

Both theory and the experiments lead to the result that asymmetric board structures, caused by the papers used or due to double flute structures, do not influence the mechanics so much that the McKee formula becomes invalid. Predictions are still expected in the range of ± 10%.

The thicknesses of the papers used, and even more significantly the thicknesses of the different flute-layers, are the main influencing parameters of the total stiffness and the final box strength. Hence, the McKee formula is robust enough to be applied when using low grammage papers, as well as multilayer board constructions. **TJ**

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ABOUT THE AUTHORS

Research on the topic of box failure is necessary for the packaging industry. The work here complements previous research in that it considers asymmetric paper and board properties.

The most difficult aspect of this research was the analytical derivation of usable equations, which we addressed as a team. It was interesting to discover that lightweight packaging papers still correspond to the McKee equations. Surprisingly, as long as the thickness of the board is high, bending stiffness may be predicted quite well from paper stiffness.

The information here may help corrugated board mills better select the necessary paper quality. Our next step is to gather more data on a wider range of furnishes.

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