DPIN 3.0
A PROGRAM FOR DECOMPOSING PRODUCTIVITY INDEX NUMBERS

by

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1. INTRODUCTION

An index number is a measure of change in a variable, or group of variables, over time or space. The DPIN computer program uses the aggregate-quantity framework developed by O'Donnell (2008) to compute and decompose *productivity* index numbers. The O'Donnell (2008) methodology does not rely on the availability of price data and does not require any assumptions concerning either the degree of competition in product markets or the optimizing behaviour of firms. Thus, DPIN can be used to analyse the drivers of productivity change even when prices are unavailable and/or industries are non-competitive. The program uses data envelopment analysis (DEA) linear programs (LPs) to estimate the production technology and *levels* of productivity and efficiency. The program then decomposes *changes* in productivity into measures of

(a) technical change (measuring movements in the production frontier);
(b) technical efficiency change (movements towards or away from the frontier);
(c) scale efficiency change (movements around the frontier surface to capture economies of scale); and
(d) mix efficiency change (movements around the frontier surface to capture economies of scope).

This Guide outlines the methodological framework and provides a guide to installing and running the program.

2. TOTAL FACTOR PRODUCTIVITY INDEXES

The productivity of a single-output single-input firm is almost always defined as the output-input ratio. O'Donnell (2008) generalizes this idea to the multiple-output multiple-input case by formally defining the *total factor productivity* (TFP) of a firm to be the ratio of an aggregate output to an aggregate input. Let $x_{it} = (x_{1it}, ..., x_{Kit})'$ and $q_{it} = (q_{1it}, ..., q_{Jit})'$ denote the input and output quantity vectors of firm *i* in period *t*. Then the TFP of the firm is

(1) $$TFP_{it} \equiv \frac{Q_{it}}{X_{it}}$$ (total factor productivity)

where $Q_{it} = Q(q_{it})$ is an aggregate output, $X_{it} = X(x_{it})$ is an aggregate input, and $Q(.)$ and $X(.)$ are non-negative, non-decreasing and linearly homogeneous aggregator functions. The associated index number that measures the TFP of firm *i* in period *t* relative to the TFP of firm *h* in period *s* is

(2) $$TFP_{hs,it} \equiv \frac{TFP_{it}}{TFP_{hs}} = \frac{Q_{it} / X_{it}}{Q_{hs} / X_{hs}} = \frac{Q_{hs,it}}{X_{hs,it}}$$ (TFP index)

where $Q_{hs,it} = Q_{it} / Q_{hs}$ is an is an output quantity index and $X_{hs,it} = X_{it} / X_{hs}$ is an input quantity index. Thus, TFP growth can be expressed as a measure of output growth divided by a measure of input growth.

Different aggregator functions give rise to different TFP indexes. The class of non-negative, non-decreasing and linearly homogeneous aggregator functions includes

(3) $Q(q) = p'_{hs}q$ (Laspeyres)

(4) $Q(q) = p'_{it}q$ (Paasche)

(5) $Q(q) = (p'_{it}qq'p_{hs})^{1/2}$ (Fisher)

(6) $Q(q) = p'_0 q$ (Lowe)

(7) $Q(q) = D_O(x_{hs}, q, s)$ (Malmquist-*hs*)

(8) $Q(q) = D_O(x_{it}, q, t)$ (Malmquist-*it*)

(9) $Q(q) = [D_O(x_{hs}, q, s)D_O(x_{it}, q, t)]^{1/2}$ (Hicks-Moorsteen)

(10) $Q(q) = D_O(x_0, q, t_0)$ (Färe-Primont)

(11) $X(x) = w'_{hs}x$ (Laspeyres)

(12) $X(x) = w'_{it}x$ (Paasche)

(13) $X(x) = (w'_{hs}xx'w_{it})^{1/2}$ (Fisher)

(14) $X(x) = w'_0 x$ (Lowe)

(15) $X(x) = D_I(x, q_{hs}, s)$ (Malmquist-*hs*)

(16) $X(x) = D_I(x, q_{it}, t)$ (Malmquist-*it*)

(17) $X(x) = [D_I(x, q_{hs}, s)D_I(x, q_{it}, t)]^{1/2}$ (Hicks-Moorsteen) and

(18) $X(x) = D_I(x, q_0, t_0)$ (Färe-Primont)

where $w_{it} = (w_{1it},..., w_{Kit})'$ and $p_{it} = (p_{1it},..., p_{Jit})'$ are vectors of input and output prices; p_0, w_0, q_0 and x_0 are vectors of representative prices and quantities; t_0 denotes a representative time period; and $D_O(.)$ and $D_I(.)$ are Shephard (1953) output and input distance functions. The aggregator functions (3) to (18) are so-named because when they are substituted into (1) and (2) they give rise to the following TFP indexes:

(19) $TFP_{hs,it} = \frac{p'_{hs}q_{it}}{p'_{hs}q_{hs}} \frac{w'_{hs}x_{hs}}{w'_{hs}x_{it}}$ (Laspeyres)

(20) $TFP_{hs,it} = \frac{p'_{it}q_{it}}{p'_{it}q_{hs}} \frac{w'_{it}x_{hs}}{w'_{it}x_{it}}$ (Paasche)

(21) $TFP_{hs,it} = \left(\frac{p'_{it}q_{it}}{p'_{it}q_{hs}} \frac{p'_{hs}q_{it}}{p'_{hs}q_{hs}} \frac{w'_{hs}x_{hs}}{w'_{hs}x_{it}} \frac{w'_{it}x_{hs}}{w'_{it}x_{it}} \right)^{1/2}$ (Fisher)

(22) $TFP_{hs,it} = \frac{p'_0 q_{it}}{p'_0 q_{hs}} \frac{w'_0 x_{hs}}{w'_0 x_{it}}$ (Lowe)

(23) $TFP_{hs,it} = \frac{D_O(x_{hs}, q_{it}, s)}{D_O(x_{hs}, q_{hs}, s)} \frac{D_I(x_{hs}, q_{hs}, s)}{D_I(x_{it}, q_{hs}, s)}$ (Malmquist-*hs*)

(24) $TFP_{hs,it} = \frac{D_O(x_{it}, q_{it}, t)}{D_O(x_{it}, q_{hs}, t)} \frac{D_I(x_{hs}, q_{it}, t)}{D_I(x_{it}, q_{it}, t)}$ (Malmquist-*it*)

(25) $TFP_{hs,it} = \left(\frac{D_O(x_{hs}, q_{it}, s)}{D_O(x_{hs}, q_{hs}, s)} \frac{D_I(x_{hs}, q_{hs}, s)}{D_I(x_{it}, q_{hs}, s)} \frac{D_O(x_{it}, q_{it}, t)}{D_O(x_{it}, q_{hs}, t)} \frac{D_I(x_{hs}, q_{it}, t)}{D_I(x_{it}, q_{it}, t)} \right)^{1/2}$ (Hicks-Moorsteen) and

(26) $TFP_{hs,it} = \frac{D_O(x_0, q_{it}, t_0)}{D_O(x_0, q_{hs}, t_0)} \frac{D_I(x_{hs}, q_0, t_0)}{D_I(x_{it}, q_0, t_0)}$ (Färe-Primont).

The Laspeyres, Paasche and Fisher indexes defined by (19) to (21) are well-known in the productivity literature. O'Donnell (2010b) refers to the index (22) as a Lowe TFP index because the component output quantity and input quantity indexes have been traced back to Lowe (1823). This Guide refers to the indexes (23) and (24) as Malmquist-*hs* and Malmquist-*it* indexes because the component output quantity and input quantity indexes are the firm-specific Malmquist indexes defined by Caves, Christensen and Diewert (1982, p. 1396,1400). The index defined by (25) was first proposed by Bjurek (1996) but is commonly known as a Hicks-Moorsteen index because it is the geometric average of two indexes that Diewert (1992, p. 240) attributed to Hicks (1961) and Moorsteen (1961). Finally, the index defined by (26) was first proposed by O'Donnell (2011a) but is referred to in this Guide as a Färe-Primont index because it can be written as the ratio of two indexes defined by Färe and Primont (1995).

Lowe and Färe-Primont indexes are economically-ideal in the sense that they satisfy all economically-relevant axioms and tests from index number theory, including an identity axiom and a transitivity test. This means they can be used to make reliable multi-temporal (i.e., many period) and/or multi-lateral (i.e., many firm) comparisons of TFP and efficiency. Laspeyres, Paasche, Fisher, Malmquist-*hs*, Malmquist-*it* and Hicks-Moorsteen indexes all fail the transitivity test and can generally only be used to make a *single* binary comparison (i.e., to compare two observations only). For more details on the importance of index number axioms and tests, see O'Donnell (2011b).

3. MEASURES OF EFFICIENCY

Most, if not all, economic measures of efficiency can be defined as ratios of measures of TFP. Examples include (O'Donnell (2008))

(27) $$TFPE_{it} = \frac{TFP_{it}}{TFP_t^*} \leq 1$$ (TFP efficiency)

(28) $$OTE_{it} = \frac{Q_{it} / X_{it}}{\bar{Q}_{it} / X_{it}} = \frac{Q_{it}}{\bar{Q}_{it}} = D_O(x_{it}, q_{it}, t) \leq 1$$ (output-oriented technical efficiency)

(29) $$OSE_{it} = \frac{\bar{Q}_{it} / X_{it}}{\tilde{Q}_{it} / \tilde{X}_{it}} \leq 1$$ (output-oriented scale efficiency)

(30) $$OME_{it} = \frac{\bar{Q}_{it} / X_{it}}{\hat{Q}_{it} / X_{it}} = \frac{\bar{Q}_{it}}{\hat{Q}_{it}} \leq 1$$ (output-oriented mix efficiency)

(31) $$ROSE_{it} = \frac{\hat{Q}_{it} / X_{it}}{TFP_t^*} \leq 1$$ (residual output-oriented scale efficiency)

(32) $$ITE_{it} = \frac{Q_{it} / X_{it}}{Q_{it} / \bar{X}_{it}} = \frac{\bar{X}_{it}}{X_{it}} = D_I(x_{it}, q_{it}, t)^{-1} \leq 1$$ (input-oriented technical efficiency)

(33) $$ISE_{it} = \frac{Q_{it} / \bar{X}_{it}}{\tilde{Q}_{it} / \tilde{X}_{it}} \leq 1$$ (input-oriented scale efficiency)

(34) $$IME_{it} = \frac{Q_{it} / \bar{X}_{it}}{Q_{it} / \hat{X}_{it}} = \frac{\hat{X}_{it}}{\bar{X}_{it}} \le 1$$ (input-oriented mix efficiency)

(35) $$RISE_{it} = \frac{Q_{it} / \hat{X}_{it}}{TFP_t^*} \le 1$$ (residual input-oriented scale efficiency) and

(36) $$RME_{it} = \frac{\tilde{Q}_{it} / \tilde{X}_{it}}{TFP_t^*} \le 1$$ (residual mix efficiency)

where TFP_t^* denotes the maximum TFP that is possible using the technology available in period t; $\bar{Q}_{it} \equiv Q_{it} D_O(x_{it}, q_{it}, t)^{-1}$ is the maximum aggregate output possible when using x_{it} to produce a scalar multiple of q_{it}; $\bar{X}_{it} \equiv X_{it} D_I(x_{it}, q_{it}, t)^{-1}$ is the minimum aggregate input possible when using a scalar multiple of x_{it} to produce q_{it}; $\hat{Q}_{it}$ is the maximum aggregate output possible when using x_{it} to produce *any* output vector; $\hat{X}_{it}$ is the minimum aggregate input possible when using *any* input vector to produce q_{it}; and $\tilde{Q}_{it}$ and $\tilde{X}_{it}$ are the aggregate output and input obtained when TFP is maximized subject to the constraint that the output and input vectors are scalar multiples of q_{it} and x_{it} respectively.

The technical efficiency measures given by (28) and (32) are usually attributed to Farrell (1957). The scale efficiency measures given by (29) and (33) are the conventional measures defined by, for example, Balk (1998, p. 20, 23). The remaining measures of efficiency were first defined by O'Donnell (2008) – TFP efficiency is a measure of overall productive performance, while measures of residual scale and mix efficiency are measures of productive performance associated with economies of scale and scope. Other important measures of efficiency include (O'Donnell (2010b))

(37) $$OSME_{it} = OME_{it} \times ROSE_{it} = OSE_{it} \times RME_{it} \le 1$$ (output-oriented scale-mix efficiency) and

(38) $$ISME_{it} = IME_{it} \times RISE_{it} = ISE_{it} \times RME_{it} \le 1$$ (input-oriented scale-mix efficiency).

To illustrate the relationship between measures of productivity and efficiency, several of the measures defined by (27) to (38) are depicted in Figures 1 and 2. In these figures, the curve passing through point D is what O'Donnell (2008) refers to as a mix-restricted frontier – it is the boundary of the set of all technically-feasible aggregate input-output combinations that have the same input and output mix as the firm operating at point A. The curve passing through point E is an unrestricted production frontier – it is the boundary of the production possibilities set that is available to firms when all mix restrictions are relaxed. O'Donnell (2008) shows how measures of TFP and efficiency can be expressed in terms of slopes of rays in aggregate quantity space. For example, the TFP of the firm operating at point A in Figure 1 is $TFP_{it} = Q_{it} / X_{it} = \text{slope 0A}$, the measure of TFP efficiency defined by (27) is $TFPE_{it} = TFP_{it} / TFP_t^* = \text{slope 0A} / \text{slope 0E}$, and the measure of residual output-oriented scale efficiency defined by (31) is $ROSE_{it} = (\hat{Q}_{it} / X_{it}) / TFP_t^* = \text{slope 0V} / \text{slope 0E}$. For more details see O'Donnell (2008).

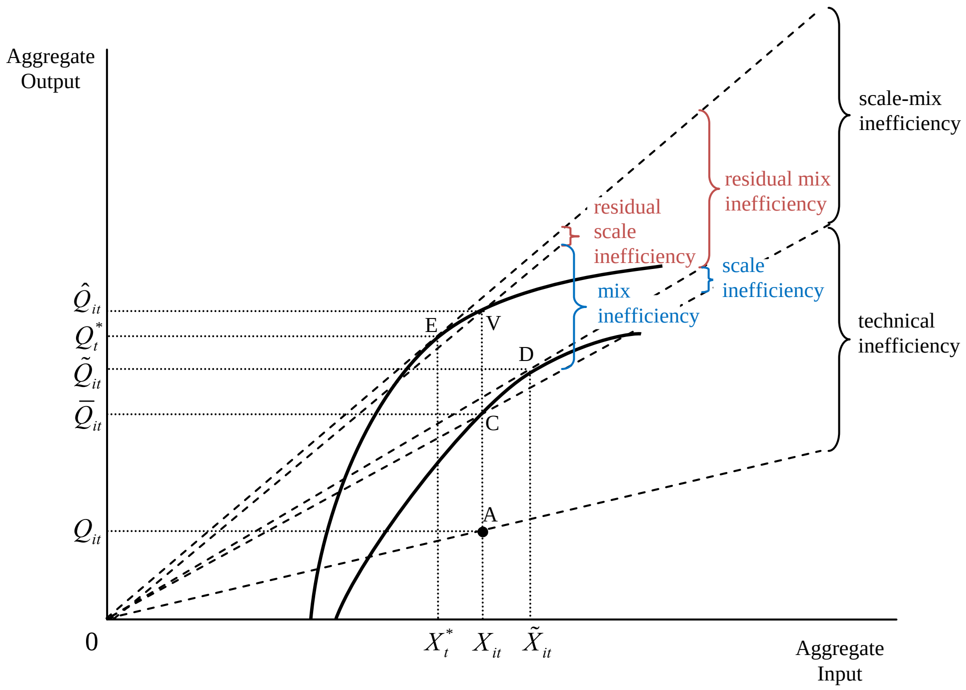

Figure 1. Output-Oriented Measures of Efficiency for a Multiple-Input Multiple-Output Firm

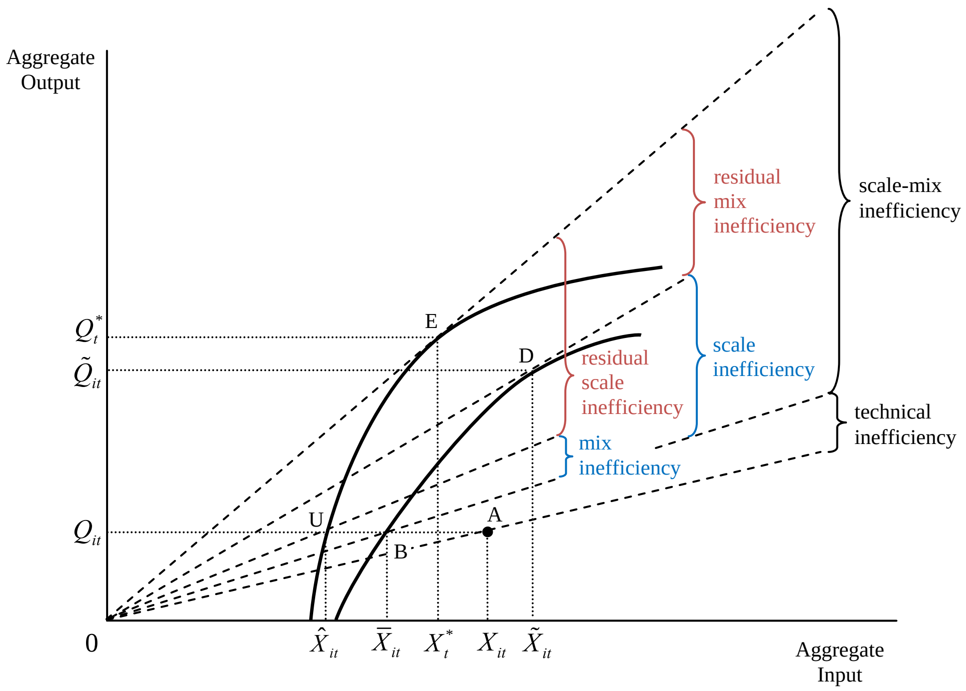

Figure 2. Input-Oriented Measures of Efficiency for a Multiple-Input Multiple-Output Firm

Associated with the aggregate output and input quantities Q_{it} and X_{it} are (implicit) aggregate output and input prices $P_{it} \equiv p_{it}'q_{it} / Q_{it}$ and $W_{it} \equiv w_{it}'x_{it} / X_{it}$. If prices are available then DPIN can also be used to compute the following measures of efficiency:

(39) $$RE_{it} = \frac{P_{it}Q_{it}}{r(x_{it}, p_{it}, t)} \le 1$$ (revenue efficiency)

(40) $$RAE_{it} = \frac{P_{it}\bar{Q}_{it}}{r(x_{it}, p_{it}, t)} \le 1$$ (revenue-allocative efficiency)

(41) $$CE_{it} = \frac{c(w_{it}, q_{it}, t)}{W_{it}X_{it}} \le 1$$ (cost efficiency) and

(42) $$CAE_{it} = \frac{c(w_{it}, q_{it}, t)}{W_{it}\bar{X}_{it}} \le 1$$ (cost-allocative efficiency)

where $r(x_{it}, p_{it}, t)$ is the maximum revenue that can be obtained in period t when the input vector is x_{it} and the output price vector is p_{it}, and $c(w_{it}, q_{it}, t)$ is the minimum cost of producing q_{it} in period t when the input price vector is w_{it}.

4. THE COMPONENTS OF TFP CHANGE

O'Donnell (2008) refers to TFP indexes that can be expressed in terms of aggregate quantities as in equation (2) as being *multiplicatively-complete*. All such TFP indexes can be decomposed into a measure of technical change and various measures of efficiency change. The simplest way to see this is to rewrite equation (27) as $TFP_{it} = TFP_t^* \times TFPE_{it}$. A similar equation holds for firm h in period s: $TFP_{hs} = TFP_s^* \times TFPE_{hs}$. It follows that the TFP index (2) can be decomposed as

(43) $$TFP_{hs,it} = \left(\frac{TFP_t^*}{TFP_s^*}\right)\left(\frac{TFPE_{it}}{TFPE_{hs}}\right).$$

The first term in parentheses on the right-hand side of (43) measures the change in the maximum TFP over time – this is a natural measure of technical change. The second term is a measure of overall efficiency change. Equations (28) to (42) can be used to effect an even finer decomposition of TFP change than the simple decomposition given by equation (43). For example, three finer output-oriented decompositions are

(44) $$TFP_{hs,it} = \left(\frac{TFP_t^*}{TFP_s^*}\right)\left(\frac{OTE_{it}}{OTE_{hs}}\right)\left(\frac{OSME_{it}}{OSME_{hs}}\right)$$

(45) $$TFP_{hs,it} = \left(\frac{TFP_t^*}{TFP_s^*}\right)\left(\frac{OTE_{it}}{OTE_{hs}}\right)\left(\frac{OSE_{it}}{OSE_{hs}}\right)\left(\frac{RME_{it}}{RME_{hs}}\right)$$ and

(46) $$TFP_{hs,it} = \left(\frac{TFP_t^*}{TFP_s^*}\right)\left(\frac{OTE_{it}}{OTE_{hs}}\right)\left(\frac{OME_{it}}{OME_{hs}}\right)\left(\frac{ROSE_{it}}{ROSE_{hs}}\right).$$

5. ESTIMATION USING DEA

If prices are available then computing Laspeyres, Paasche, Fisher and Lowe indexes is straightforward using equations (19) to (22). However, decomposing these indexes into measures of technical change and efficiency change involves estimating the production technology. Whether or not prices are available, estimating (and decomposing) Malmquist-*hs*, Malmquist-*it*, Hicks-Moorsteen and Färe-Primont indexes also involves estimating the technology. DPIN estimates the production technology (and associated measures of productivity and efficiency) using DEA LPs. DEA is underpinned by the assumption that the (local) output and input distance functions representing the technology available in period *t* take the form (e.g., O'Donnell (2011b))

(47) $D_O(x_{it}, q_{it}, t) = (q'_{it}\alpha)/(\gamma + x'_{it}\beta)$ and

(48) $D_I(x_{it}, q_{it}, t) = (x'_{it}\eta)/(q'_{it}\phi - \delta)$.

The standard output-oriented DEA problem involves selecting values of the unknown parameters in (47) in order to minimize $OTE_{it}^{-1} = D_O(x_{it}, q_{it}, t)^{-1}$. The input-oriented problem involves selecting values of the unknown parameters in (48) in order to maximise $ITE_{it} = D_I(x_{it}, q_{it}, t)^{-1}$. The resulting linear programs are (e.g., O'Donnell (2011b))

(49) $D_O(x_{it}, q_{it}, t)^{-1} = OTE_{it}^{-1} = \min_{\alpha,\gamma,\beta}\{\gamma + x'_{it}\beta : \gamma\iota + X'\beta \geq Q'\alpha; q'_{it}\alpha = 1; \alpha \geq 0; \beta \geq 0\}$ and

(50) $D_I(x_{it}, q_{it}, t)^{-1} = ITE_{it} = \max_{\phi,\delta,\eta}\{q'_{it}\phi - \delta : Q'\phi \leq \delta\iota + X'\beta; x'_{it}\eta = 1; \phi \geq 0; \eta \geq 0\}$

where Q is a $J \times M_t$ matrix of observed outputs, X is a $K \times M_t$ matrix of observed inputs, ι is an $M_t \times 1$ unit vector, and M_t denotes the number of observations used to estimate the frontier in period *t*. DPIN uses variants of these two LPs to compute productivity indexes and measures of efficiency (change).

Productivity Indexes

DPIN estimates Malmquist-*hs*, Malmquist-*it*, Hicks-Moorsteen and Färe-Primont aggregates by first solving the following variants of LPs (49) and (50) (O'Donnell (2011b)):

(51) $D_O(x_{hs}, q_{it}, s)^{-1} = \min_{\alpha,\gamma,\beta}\{\gamma + x'_{hs}\beta : \gamma\iota + X'\beta \geq Q'\alpha; q'_{it}\alpha = 1; \alpha \geq 0; \beta \geq 0\}$ (Malmquist-*hs*)

(52) $D_I(x_{it}, q_{hs}, s)^{-1} = \max_{\phi,\delta,\eta}\{q'_{hs}\phi - \delta : Q'\phi \leq \delta\iota + X'\eta; x'_{it}\eta = 1; \phi \geq 0; \eta \geq 0\}$ (Malmquist-*hs*)

(53) $D_O(x_{it}, q_{hs}, t)^{-1} = \min_{\alpha,\gamma,\beta}\{\gamma + x'_{it}\beta : \gamma\iota + X'\beta \geq Q'\alpha; q'_{hs}\alpha = 1; \alpha \geq 0; \beta \geq 0\}$ (Malmquist-*it*)

(54) $D_I(x_{hs}, q_{it}, t)^{-1} = \max_{\phi,\delta,\eta}\{q'_{it}\phi - \delta : Q'\phi \leq \delta\iota + X'\eta; x'_{hs}\eta = 1; \phi \geq 0; \eta \geq 0\}$ (Malmquist-*it*)

(55) $D_O(x_0, q_0, t_0)^{-1} = \min_{\alpha,\gamma,\beta}\{\gamma + x'_0\beta : \gamma\iota + X'\beta \geq Q'\alpha; q'_0\alpha = 1; \alpha \geq 0; \beta \geq 0\}$ (Färe-Primont)

(56) $D_I(x_0, q_0, t_0)^{-1} = \max_{\phi,\delta,\eta}\{q'_0\phi - \delta : Q'\phi \leq \delta\iota + X'\eta; x'_0\eta = 1; \phi \geq 0; \eta \geq 0\}$ (Färe-Primont)

Aggregate outputs and inputs are then estimated as (O'Donnell (2011b))

(57) $Q_{it} = (q_{it}'\alpha_{hs})/(\gamma_{hs} + x_{hs}'\beta_{hs})$ (Malmquist-*hs*)

(58) $X_{it} = (x_{it}'\eta_{hs})/(q_{hs}'\phi_{hs} - \delta_{hs})$ (Malmquist-*hs*)

(59) $Q_{hs} = (q_{hs}'\alpha_{it})/(\gamma_{it} + x_{it}'\beta_{it})$ (Malmquist-*it*)

(60) $X_{hs} = (x_{hs}'\eta_{it})/(q_{it}'\phi_{it} - \delta_{it})$ (Malmquist-*it*)

(61) $Q_{it} = (q_{it}'\alpha_0)/(\gamma_0 + x_0'\beta_0)$ (Färe-Primont) and

(62) $X_{it} = (x_{it}'\eta_0)/(q_0'\phi_0 - \delta_0)$ (Färe-Primont)

where α_{hs}, β_{hs}, γ_{hs}, ϕ_{hs}, δ_{hs} and η_{hs} solve (51) and (52), α_{it}, β_{it}, γ_{it}, ϕ_{it}, δ_{it} and η_{it} solve (53) and (54), and α_0, β_0, γ_0, ϕ_0, δ_0 and η_0 solve (55) and (56). DPIN uses sample mean vectors as representative output and input vectors in LPs (55) and (56). The representative technology in these two LPs is the technology obtained under the assumption of no technical change (i.e., M_t is the sample size). Each of the LPs (51) to (56) allows the technology to exhibit variable returns to scale (VRS). If the technology is assumed to exhibit constant returns to scale (CRS) then DPIN sets $\gamma = \delta = 0$.

Technical and Scale Efficiency

DPIN obtains measures of technical efficiency by solving the following dual LPs:

(63) $OTE_{it} = Q_{it}/\bar{Q}_{it} = D_O(x_{it}, q_{it}, t) = \min_{\lambda,\theta}\left\{\lambda^{-1} : \lambda q_{it} \le Q\theta; X\theta \le x_{it}; \theta'\iota = 1; \theta \ge 0\right\}$ and

(64) $ITE_{it} = \bar{X}_{it}/X_{it} = D_I(x_{it}, q_{it}, t)^{-1} = \min_{\rho,\theta}\left\{\rho : Q\theta \ge q_{it}; \rho x_{it} \ge X\theta; \theta'\iota; \theta \ge 0\right\}$

where θ is an $M_t \times 1$ vector. To estimate measures of technical efficiency under a CRS assumption, DPIN removes the constraint $\theta'\iota = 1$ and solves

(65) $OTE_{it}^{CRS} = H_O(x_{it}, q_{it}, t) = \min_{\lambda,\theta}\left\{\lambda^{-1} : \lambda q_{it} \le Q\theta; X\theta \le x_{it}; \theta \ge 0\right\}$ and

(66) $ITE_{it}^{CRS} = H_I(x_{it}, q_{it}, t) = \min_{\rho,\theta}\left\{\rho : Q\theta \ge q_{it}; \rho x_{it} \ge X\theta; \theta \ge 0\right\}$.

Measures of scale efficiency are then computed as

(67) $OSE_{it} = OTE_{it}^{CRS}/OTE_{it}$ and

(68) $ISE_{it} = ITE_{it}^{CRS}/ITE_{it}$.

Mix Efficiency

Measures of mix efficiency are defined by equations (30) and (34). Estimates of $\hat{Q}_{it}$, $\hat{X}_{it}$, $\hat{Q}_{hs}$ and $\hat{X}_{hs}$ are obtained by solving the following LPs:

(69) $\hat{Q}_{it} = \max_{\theta,q} \{Q(q) : q \le Q\theta; X\theta \le x_{it}; \theta'\iota = 1; \theta \ge 0\}$ and

(70) $\hat{X}_{it} = \min_{\theta,x} \{X(x) : Q\theta \ge q_{it}; x \ge X\theta; \theta'\iota = 1; \theta \ge 0\}$.

For any aggregator function, LP (69) gives the maximum aggregate output that can be produced using x_{it} (i.e., the maximum aggregate output that firm *i* in period *t* could produce using its input vector), while LP (70) gives the minimum aggregate input that can produce q_{it} (i.e., the minimum aggregate input that could be used by firm *i* in period *t* to produce its output vector). Paasche, Laspeyres and Lowe estimates of $\hat{Q}_{it}$, $\hat{X}_{it}$, $\hat{Q}_{hs}$ and $\hat{X}_{hs}$ are obtained by replacing $Q(q)$ and $X(x)$ in (69) and (70) with the aggregator functions (3), (4), (6), (11), (12) and (14). Fisher estimates are obtained by taking the geometric average of the Paasche and Laspeyres estimates. Malmquist-*hs*, Malmquist-*it* and Färe-Primont estimates are obtained by replacing $Q(q)$ and $X(x)$ with functions (57) to (62). Hicks-Moorsteen estimates are obtained by taking the geometric average of the Malmquist-*hs* and Malmquist-*it* estimates.

Other Efficiency and Productivity Measures

DPIN computes the maximum TFP in period *t* as

(71) $TFP_t^* = \max_i Q_{it} / X_{it}$.

Other efficiency and productivity measures are computed residually:

(72) $TFP_{it} = Q_{it} / X_{it}$

(73) $TFPE_{it} = TFP_{it} / TFP_t^*$

(74) $ROSE_{it} = (\hat{Q}_{it} / X_{it}) / TFP_t^*$

(75) $RISE_{it} = (Q_{it} / \hat{X}_{it}) / TFP_t^*$

(76) $OSME_{it} = OME_{it} \times ROSE_{it}$

(77) $ISME_{it} = IME_{it} \times RISE_{it}$ and

(78) $RME_{it} = \dfrac{TFPE_{it}}{OTE_{it} OSE_{it}}$.

Again, results for Fisher and Hicks-Moorsteen indexes are computed as geometric averages of the Laspeyres, Paasche, Malmquist-*hs* and Malmquist-*it* indexes as appropriate. If the Paasche aggregator function is used to measure efficiency and TFP then output- and input-oriented measures of mix efficiency for firm *i* in period *t* will be measures of revenue-allocative efficiency (RAE_{it}) and cost-allocative efficiency (CAE_{it}) respectively (O'Donnell (2010b)). EXCEL commands can then be used to compute

(79) $RE_{it} = OTE_{it} \times RAE_{it}$ (revenue efficiency) and

(80) $CE_{it} = ITE_{it} \times CAE_{it}$. (cost efficiency)

If prices are available then DPIN also computes

(81) $P_{it} = REV_{it} / Q_{it}$ (aggregate output price)

(82) $W_{it} = COST_{it} / X_{it}$ (aggregate input price)

(83) $TT_{it} = P_{it} / W_{it}$ (terms of trade) and

(84) $PROF_{it} = REV_{it} / COST_{it}$ (profitability)

where $REV_{it} = p'_{it} q_{it}$ denotes the total revenue of firm i in period t and $COST_{it} = w'_{it} x_{it}$ denotes total cost. Finally, if prices are available then DPIN decomposes profitability change into the product of an index measuring the change in the terms-of-trade (i.e., ratio of output prices to input prices) and the change in TFP (i.e., ratio of output quantity to input quantity) (e.g., O'Donnell (2010a)):

(85) $$PROF_{hs,it} \equiv \frac{PROF_{it}}{PROF_{hs}} = \frac{REV_{hs,it}}{COST_{hs,it}} = \left(\frac{P_{hs,it}}{W_{hs,it}}\right)\left(\frac{Q_{hs,it}}{X_{hs,it}}\right) = TT_{hs,it} \times TFP_{hs,it}$$ (profitability index)

where $REV_{hs,it} \equiv REV_{it} / REV_{hs}$, $COST_{hs,it} \equiv COST_{it} / COST_{hs}$, $P_{hs,it} \equiv P_{it} / P_{hs}$, $W_{hs,it} \equiv W_{it} / W_{hs}$ and $TT_{hs,it} = P_{hs,it} / W_{hs,it}$ are revenue, cost, output price, input price and terms-of-trade indexes respectively.

Shadow Prices

The derivatives of output and input distance functions with respect to outputs and inputs can be interpreted as revenue- and cost-deflated output and input shadow prices (e.g., Grosskopf, Margaritis and Valdmanis (1995)). For example, the first-derivatives of the (local) output and input distance functions (47) and (48) are

(86) $p^*_{it} = \partial D_O(x_{it}, q_{it}, t) / \partial q_{it} = \alpha / (\gamma + x'_{it}\beta)$ and

(87) $w^*_{it} = \partial D_I(x_{it}, q_{it}, t) / \partial x_{it} = \eta / (q'_{it}\phi - \delta).$

DPIN evaluates these derivatives (shadow prices) at the values of α, β, γ, ϕ, δ and η that solve LPs (49) and (50). By way of further example, the derivatives of $D_O(x_0, q_0, t_0)$ and $D_I(x_0, q_0, t_0)$ are

(88) $p^*_0 = \partial D_O(x_0, q_0, t_0) / \partial q_0 = \alpha / (\gamma + x'_0\beta)$ and

(89) $w^*_0 = \partial D_I(x_0, q_0, t_0) / \partial x_0 = \eta / (q'_0\phi - \delta).$

DPIN evaluates these shadow prices at the values of α, β, γ, ϕ, δ and η that solve LPs (55) and (56). Observe from equations (6), (14), (61) and (62) that Färe-Primont indexes are identical to Lowe indexes whenever $p_0 = p^*_0$ and $w_0 = w^*_0$. Similar relationships exist between Malmquist-*hs* and Laspeyres indexes, and between Malmquist-*it* and Paasche indexes (O'Donnell (2011b)).

6. INSTALLING AND RUNNING THE DPIN SOFTWARE

DPIN is written in C++ and is designed to run on a Windows XP or Vista platform. The program and associated files can be downloaded from DPIN website at http://www.uq.edu.au/economics/cepa/dpin.htm. DPIN is available two editions: the Standard edition will compute and decompose Malmquist-*hs*, Malmquist-*it*, Hicks-Moorsteen and Färe-Primont TFP indexes and is available free-of-charge; the Professional edition will also compute and decompose Paasche, Laspeyres, Fisher and Lowe TFP indexes and is available on payment of an annual license fee. Both editions are hard-wired to analyze up to 5000 observations. Further details concerning the functionality of the two editions and the pricing of the Professional edition are available on the DPIN website. Irrespective of the edition, installing DPIN involves downloading a .zip file from the DPIN website and extracting the contents of the file into any directory. Installing the Professional edition also involves purchasing a license key by following the instructions on the DPIN website. Running DPIN then involves creating an input file and running the executable file.

Creating the DPIN Input File

DPIN input files can be created within Microsoft EXCEL and must be saved in a .csv (comma-delimited) format. The input file contains both commands and data. To illustrate, Figure 3 is a screenshot of the input file Eg1_input.csv required for the example discussed in O'Donnell (2011b). The rows before the end command are "command rows"; the row immediately after the end command is a "header row"; and the remaining rows are "data rows". DPIN will read the text and data in columns A and B of every row until it encounters an end command. It will then skip the end command and the header row and start reading the data. The DPIN program is case-sensitive and will only recognize certain text strings in the command rows. The main commands are

Firms	to specify the number of firms
Periods	to specify the number of periods
Outputs	to specify the number of outputs
Inputs	to specify the number of inputs
Prices	included if and only if the input file contains prices as well as quantities

The command rows (i.e., the rows before the end command) can be listed in any order (e.g., the number of outputs can be specified first, second, or last). The program will not read the output and input variable names in the header row (i.e., the row immediately after the end command). The output and input data must be stored in the data rows in a particular format:

- the observation identifier must be stored in column A; the firm and period identifiers must be stored in columns B and C; all identifiers must be numeric (e.g., 1, 2, ..., not Jan, Feb ...);
- the first output quantity variable must be stored in column D; all the output quantity variables must be stored first, followed by the input quantity variables (e.g., q1, q2, q3, x1, x2);

Eg1_input.csv

	A	B	C	D	E	F	G	H	I	J
1	Firms	4								
2	Periods	2								
3	Inputs	2								
4	Outputs	1								
5	Prices									
6	end									
7	Obs	Firm	Period	q	x1	x2	p	w1	w2	
8	1	1	1	30	60	120	3	6	12	
9	2	2	1	20	60	30	2	6	3	
10	3	3	1	20	20	60	2	2	6	
11	4	4	1	10	50	20	1	5	2	
12	5	1	2	30	60	120	2	2	6	
13	6	2	2	20	60	30	2	2	6	
14	7	3	2	20	20	60	2	2	6	
15	8	4	2	10	40	20	2	2	6	

Figure 3. Example Input File for Computing and Decomposing Lowe TFP Indexes

- if prices are available then the output and input quantity variables must be stored first, followed by the output price variables, followed by the input price variables (e.g., q1, q2, q3, x1, x2, p1, p2, p3, w1, w2);
- the data must be a balanced panel; if the panel is unbalanced then it can be artificially balanced by replacing any missing observations with any other observations from the same time period (this will not affect measures of OTE, ITE, OSE or ISE, but Malmquist-*hs*, Malmquist-*it*, Hicks-Moorsteen and Färe-Primont estimates of mix efficiency and TFP may be sensitive to the choice of replacement observations);
- **the data must be sorted first by period and then by firm** (i.e., all observations from period 1 must be listed first, followed by all observations from period 2, and so on);

The default TFP index in the Standard edition of DPIN is the Färe-Primont index. The default index in the Professional edition is the Lowe index when prices are included in the input file and the Färe-Primont index otherwise. The default TFP index can be changed using one of the following commands:

Laspeyres	for Laspeyres indexes (available in Professional edition only)
Paasche	for Paasche indexes (available in Professional edition only)
Fisher	for Fisher indexes (available in Professional edition only)
Lowe	for Lowe indexes (available in Professional edition only)
Malmquist-hs	for Malmquist-*hs* indexes
Malmquist-it	for Malmquist-*it* indexes
HicksMoorsteen	for Hicks-Moorsteen indexes
FarePrimont	for Färe-Primont indexes

The default DPIN settings are to estimate the technology allowing for technical regress, technical progress and variable returns to scale. These and other default settings can be changed using the following commands:

Base	to specify the reference observation when computing (transitive) Lowe or Färe-Primont indexes; the reference observation number must be provided as a numeric value in column B of the same row as the Base command (the default value of Base is 1).
CRS	to impose constant returns to scale (the default is variable returns to scale)
NoTechChange	to prohibit technical change.
NoTechRegress	to prohibit technical regress.
UnitMeans	to rescale the data (i.e., change units of measurement) so that all output and input quantity variables have unit means. This option can be used to avoid numerical problems when quantity variables are of very different orders of magnitude. Some LP software packages recommend that LP variables should always be measured in units such that no value is greater than 1E+5 or less than 1E-4 (Winston, 2004, p.167). Malmquist-*hs*, Malmquist-*it*, Hicks-Moorsteen and Färe-Primont indexes (i.e., indexes that involve solving LPs) may be sensitive to rescaling.
Window	(available in Professional edition only) to estimate the production technology using all observations in a moving window of time periods; the window length must be provided as a numeric value in column B of the same row as the Window command.

Figures 4 and 5 illustrate the use of some of these options. Figure 4 shows how the command rows in the input file in Figure 3 should be modified to compute and decompose Paasche TFP indexes under the assumption of CRS. Observe that it is possible to add text to cells in columns D and E of any command row (i.e., any row before the end command). Figure 5 is a partial screenshot of the file NE_input.csv used for analysing a subset of the US agricultural data compiled by Ball, Hallahan and Nehring (2004).

Running the Executable File

The DPIN executable file is DPIN.exe. Double-clicking this file will open windows similar to those depicted in Figures 6 to 8. Figure 6 is a command window that reports program and run-time information; Figure 7 is a window used to browse and select the input file; and Figure 8 is the command window as it appears shortly after the input file NE_input.csv has been selected. DPIN performs six sets of computations in the decomposition of TFP indexes, and the command window reports how the program is progressing through each of these sets. Warnings and common runtime errors are also reported in the command window.

Eg1_input.csv

	A	B	C	D	E	F	G	H	I	J
1	Firms	4		Text can be added to columns D and E of any command row.						
2	Periods	2								
3	Inputs	2								
4	Outputs	1								
5	Prices									
6	Paasche			These last two commands instruct DPIN to compute Paasche indexes						
7	CRS			(and associated measures of RAE and CAE) under the assumption of CRS.						
8	end									
9	Obs	Firm	Period	q	x1	x2	p	w1	w2	
10	1	1	1	30	60	120	3	6	12	
11	2	2	1	20	60	30	2	6	3	
12	3	3	1	20	20	60	2	2	6	
13	4	4	1	10	50	20	1	5	2	
14	5	1	2	30	60	120	2	2	6	
15	6	2	2	20	60	30	2	2	6	
16	7	3	2	20	20	60	2	2	6	
17	8	4	2	10	40	20	2	2	6	

Figure 4. Example Input File for Computing and Decomposing Paasche TFP Indexes

NE_input.csv

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Periods	30		USDA agricultural data for 11 states in the Northeast Farm Production Region from 1960 - 1989. For a									
2	Firms	11		description of the data see									
3	Outputs	3											
4	Inputs	4			Ball, V. E., C. Hallahan and R. Nehring (2004). "Convergence of Productivity: An Analysis								
5	NoTechRegress				of the Catch-Up Hypothesis Within a Panel of States." American Journal of Agricultural								
6	Prices				Economics 86(5): 1315-1321.								
7	FarePrimont												
8	Window	3											
9	Base	5											
10	UnitMeans			This last option instructs DPIN to rescale quantity variables to have unit means.									
11	end												
12	Obs	State	Year	Q1 = Lives	Q2 = Crop:	Q3 = Othe	X1 = Capit	X2 = Land	X3 = Labor	X4 = Mate	P1 = Lives	P2 = Crops	P3 = Othe
13	1	6	1960	206391	181812	21207.34	118921.2	77614.56	677535.1	198095.6	0.494678	0.365515	0.207144
14	2	7	1960	186637.3	103970.5	18272.64	52374.17	39547.89	259322.2	210460.4	0.427883	0.377675	0.193301
15	3	17	1960	247701.9	230353.5	30402.2	122367	68646.06	1104077	222335.8	0.447073	0.32149	0.201944
16	4	18	1960	531124.7	330768.3	62663.46	266428.4	234554	1213558	505603.9	0.363264	0.40137	0.199179
17	5	19	1960	238832	273749.1	46441.77	137093.6	84352.38	803982.2	345988.5	0.52906	0.374929	0.203024
18	6	28	1960	117943.1	52748.54	14058.77	60953.23	52150.35	384957.7	104259.5	0.412663	0.363311	0.179036
19	7	29	1960	406044.6	494693.2	50888.07	185685.2	199111.8	1011455	433554.3	0.408633	0.310838	0.177526
20	8	32	1960	2102617	1339371	206987.6	902940.5	528261.5	4449151	1700272	0.303839	0.329621	0.187
21	9	36	1960	1770840	1116555	162621.9	874220.5	626763.2	5055331	1563069	0.358341	0.376793	0.210743
22	10	37	1960	30225.79	27786.62	1666.237	18666.41	10945.13	121694.1	30021.78	0.459293	0.368777	0.220187
23	11	44	1960	381124.6	133151.9	49158.37	133899.5	104063	892418.3	261663.5	0.304003	0.351911	0.162421
24	12	6	1961	204307.3	179208.6	20730.33	115893.7	74596.71	622402.5	191773.5	0.471273	0.3696	0.209427
25	13	7	1961	182865.6	100525.7	18073.69	51667.85	39113.7	241924.4	192520.5	0.374393	0.401373	0.195988

Figure 5. Input File for Analysing Productivity in the Northeast (NE) Farm Production Region of the United States

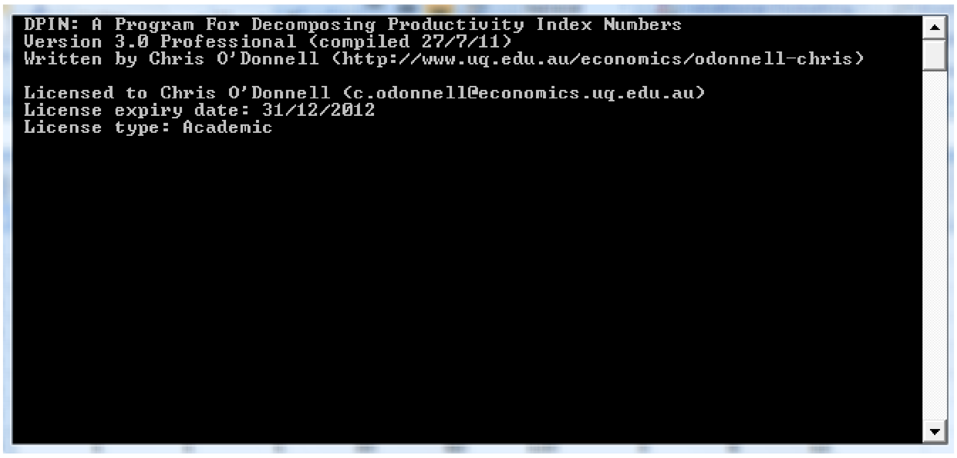

Figure 6. Initial Command Window

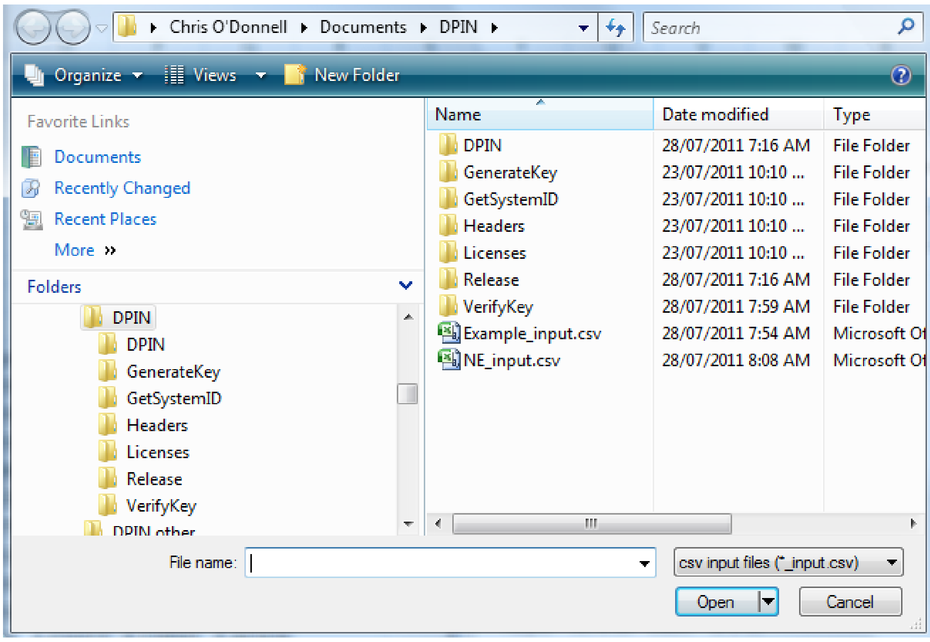

Figure 7. Input File Selection Window

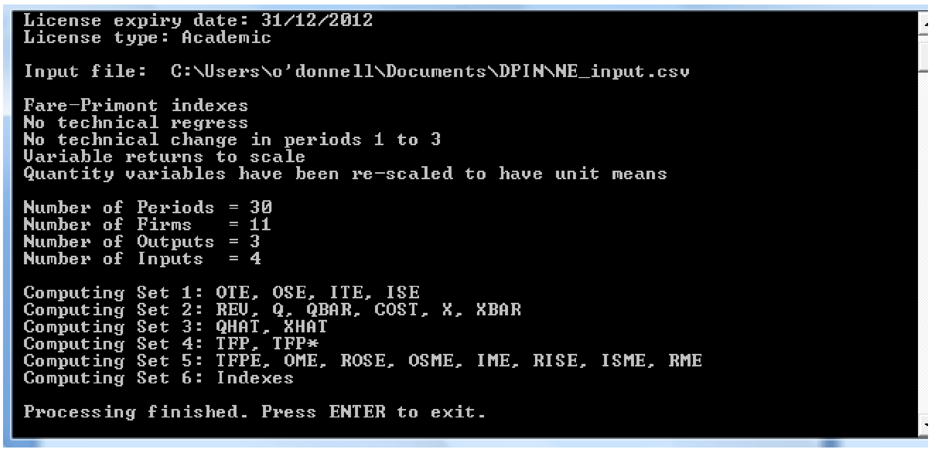

Figure 8. Command Window After Data Processing

7. DPIN OUTPUT FILES

DPIN computes and reports estimated levels of TFP and various types efficiency for every firm in every time period in the sample. It also computes and reports indexes of TFP change and efficiency change. DPIN results are written to EXCEL output files having the following extensions (and contents):

_output_indexes.csv — This file reports indexes measuring *changes* in aggregate output (labelled dQ) , aggregate input (dX), total factor productivity (dTFP), the technology (dTech = dTFP*) and various types of efficiency (dTFPE, dOTE, dOSE, dOME, dROSE, dOSME, dITE, dIME, dRISE, dISME and dRME). If prices are available then the Professional Edition of DPIN also reports indexes measuring changes in revenue (dRev), cost (dCost), profitability (dProf), the aggregate output price (dP), the aggregate input price (dW) and the terms of trade (dTT). If an intransitive index (e.g., Hicks-Moorsteen) is used then DPIN reports comparisons between firm *i* in period *t* and firm *i* in period *t*-1 (these are the only comparisons that are meaningful – see Section 2). If a transitive index (e.g., Färe-Primont) is used then the default comparison is between firm *i* in period *t* and firm 1 in period 1 (this reference observation can be easily changed using the Base command).

_output_levels.csv — This file reports estimated *levels* of aggregate output (Q), aggregate input (X), total factor productivity (TFP), the maximum TFP possible in each period (TFP*) and various types of efficiency (TFPE, OTE, OSE, OME, ROSE, OSME, ITE, IME, RISE, ISME and RME). Again, if prices are available then the Professional Edition of DPIN also reports revenue (Rev), cost (Cost), profitability (Prof), the aggregate output price (P), the aggregate input price (W) and the terms of trade (TT). Note that if an intransitive index (e.g., Hicks-Moorsteen) is used then it is not generally possible to use the reported estimates of Q, X, TFP, TFP*, OME, ROSE, OSME, IME, ISME or RME to obtain the corresponding index values reported in the _output_indexes.csv file (see the example below).

_output_shadowprices.csv — (Professional edition only) This file reports the estimated shadow prices given by equations (86) and (87) (labelled pstar1, ..., pstarJ and wstar1, ..., wstarK).

_output_xtras.csv — (Professional edition only) This file reports selected computations that may be useful for diagnostic purposes. Specifically, it reports estimates of the aggregate quantities $\hat{Q}_{it}$, $\bar{Q}_{it}$, Q_{it}, $\hat{X}_{it}$, $\bar{X}_{it}$, X_{it}, $\hat{Q}_{hs}$, $\bar{Q}_{hs}$, Q_{hs}, $\hat{X}_{hs}$, $\bar{X}_{hs}$ and X_{hs} (labelled QHATt, QBARt, Qt, XHATt, XBARt, Xt, QHATs, QBARs, Qs, XHATs, XBARs and Xt respectively) as well as estimates of $D_I(x_{it}, q_{hs}, s)$, $D_I(x_{hs}, q_{it}, t)$, $D_I(x_{it}, q_{it}, t)$, $H_I(x_{it}, q_{it}, t)$, $D_O(x_{hs}, q_{it}, s)$, $D_O(x_{it}, q_{hs}, t)$,

$D_O(x_{it}, q_{it}, t)$ and $H_o(x_{it}, q_{it}, t)$ (labelled DItss, DIstt, DIttt, HIttt, DOsts, DOtst, DOttt and HOttt respectively). If Malmquist-*hs*, Malmquist-*it* or Hicks-Moorsteen indexes are used then the file reports the values of α and η that solve LPs (51) to (54). If Färe-Primont indexes are used then the file reports estimates of $D_O(x_0, q_0, t_0)$ and $D_I(x_0, q_0, t_0)$ (labelled DO000 and DI000 respectively), the values of α_0, β_0, γ_0, ϕ_0, δ_0 and η_0 that solve (55) and (56), and the vectors of output and input shadow prices given by equations (88) and (89) (labelled pstar0 and wstar0).

8. EXAMPLES

To illustrate the decomposition of intransitive TFP indexes, Figures 9 to 12 present partial screenshots of the output files obtained after running DPIN on the input file Eg1_input.csv presented in Figure 4. This input file instructs DPIN to compute and decompose a Paasche TFP index. Caution must be exercised when interpreting reported levels of productivity and efficiency associated with intransitive indexes like the Paasche. Consider the Paasche TFP index $TFP_{41,42}$ that compares firm 4 in period 2 with firm 4 in period 1. The computations involved in computing this index are as follows:

$Q_{42} = p_{42}q_{42} = (2)(10) = 20$	(cell F11 in Fig. 9 and cell J10 in Fig. 10)
$X_{42} = w'_{42}x_{42} = (2)(40) + (6)(20) = 200$	(cell I11 in Fig. 9 and cell K10 in Fig. 10)
$Q_{41} = p_{42}q_{41} = (2)(10) = 20$	(cell L11 in Fig. 9)
$X_{41} = w'_{42}x_{41} = (2)(50) + (6)(20) = 220$	(cell O11 in Fig. 9)
$Q_{41,42} = Q_{42} / Q_{41} = 20/20 = 1$	(cell J10 in Fig. 11)
$X_{41,42} = X_{42} / X_{41} = 200/220 = 0.9091$	(cell K10 in Fig. 11)
$TFP_{41,42} = Q_{41,42} / X_{41,42} = 1/0.9091 = 1.1$	(cell L10 in Fig. 11)

Note that the base period aggregate output $Q_{41} = 20$ and the base period aggregate input $X_{41} = 220$ are *not* reported in Fig. 9 (we might expect to see them in row 6, but these entries are aggregates computed using different aggregator functions that use period 1 prices as weights). This illustrates that reported levels of output, input and productivity associated with (intransitive) Paasche, Laspeyres, Fisher, Malmquist-*hs*, Malmquist-*it* and Hicks-Moorsteen indexes cannot be blindly used to construct measures of output, input and productivity change – **if an intransitive index is used then it is generally only meaningful to compare values that are reported in the same row of the DPIN output files**. For example, it is meaningful to use the entries in row 8 of Figure 10 as follows:

$$REV_{42} = P_{42}Q_{42} = (1)(20) = 20$$
$$COST_{42} = W_{42}X_{42} = (1)(200) = 200$$
$$TT_{42} = P_{42} / W_{42} = 1/1 = 1$$
$$TFP_{42} = Q_{42} / X_{42} = 20/200 = 0.1$$

$$TFPE_{42} = TFP_{42} / TFP_2^* = 0.1/0.1333 = 0.75$$
$$TFPE_{42} = OTE_{42} \times OSME_{42} = 0.75 \times 1 = 0.75$$
$$OSME_{42} = RAE_{42} \times ROSE_{42} = 1 \times 1 = 1$$

The finding that $TFPE_{42} = 0.75$ tells us that the productivity of firm 4 is 25% less than the maximum productivity that is possible using the technology available in period 2. The finding that $OTE_{42} = 0.75$ and $OSME_{42} = 1$ tells us that the entire productivity shortfall is due to technical inefficiency. In terms of productivity change, it is meaningful to use the entries in Row 8 of Figure 9 as follows:

$$PROF_{41,42} = REV_{41,42} / COST_{41,42} = 2/0.6897 = 2.9$$
$$PROF_{41,42} = TT_{41,42} \times TFP_{41,42} = 2.6364 \times 1.1 = 2.9$$
$$TT_{41,42} = P_{41,42} / W_{41,42} = 2/0.7586 = 2.6364$$
$$TFP_{41,42} = Q_{41,42} / X_{41,42} = 1/0.9091 = 1.1$$
$$TFP_{41,42} = (TFP_2^* / TFP_1^*) \times TFPE_{41,42} = dTech \times TFPE_{41,42} = 1 \times 1.1 = 1.1$$
$$TFPE_{41,42} = OTE_{41,42} \times OSME_{41,42} = 1 \times 1.1 = 1.1$$
$$OSME_{41,42} = RAE_{41,42} \times ROSE_{41,42} = 1 \times 1.1 = 1.1$$

Thus, we find that the profitability of firm 4 has increased almost three-fold $(PROF_{41,42} = 2.9)$ as a result of a significant improvement in the terms of trade $(TT_{41,42} = 2.6364)$ and a 10% increase in productivity $(TFP_{41,42} = 1.1)$. The improvement in the terms of trade is due to a doubling of output prices $(P_{41,42} = 2)$ and a 25% fall in input prices $(W_{41,42} = 0.7586)$. Finally, all of the increase in productivity is due to an increase in residual output-oriented scale efficiency $(ROSE_{41,42} = 1.1)$ (a residual measure that captures productivity changes associated with changes in both inputs and outputs).

To illustrate the decomposition of transitive TFP indexes, Figures 13 to 16 present partial screenshots of the output files obtained after running DPIN on the input file NE_input.csv presented in Figure 5. The Färe-Primont index is transitive, so it is meaningful to compare cells in any columns or rows of these tables. For example, the TFP of state 6 in 1960 is 0.5891 (cell L3 in Fig. 13) and the TFP of state 19 in 1960 is 0.5499 (cell L7 in Fig. 13). Thus, the TFP index that compares state 6 with state 19 is 0.5891/0.5499 = 1.0714 (cell L3 in Fig. 14). Observe from row 24 in Figure 13 that state 44 achieved the maximum TFP possible in 1961 (indeed, because there is no technical change in the first three years, state 44 achieved the maximum TFP possible in any of the years 1960-1962). Thus, the efficiency scores reported in Table 1 can all be viewed as indexes that compare efficiency in each state in each year with the efficiency of state 44 in 1961. When it comes to an examination of the components of productivity change, it is convenient to use the Data Sort facility in EXCEL to sort the results first by state and then by year, and to then use the Insert Line (graph) option to plot different measures of interest. For example, Figures 17 and 18 present decompositions of profitability change and TFP change in state 6 over the period 1960 to 1989. For further examples of the computation and interpretation of transitive TFP indexes, see O'Donnell, Fallah-Fini and Triantis (2011) (US interstate highway maintenance), O'Donnell (2010b) (US agriculture) and O'Donnell (2011b) (US manufacturing sectors).

9. RUNTIME ERRORS

Error diagnostics are written to text files having extensions _output_xxerrors.txt. Common error codes reported in these files (and the command window) are:

-1 this error code indicates that a linear program cannot be solved. Some program may fail to solve because of numerical errors that occur when input and output variables are of very different orders of magnitude (e.g., some variables are measured in thousands of units and others are measured in tenths of a unit). Numerical errors often lead to values of 1.#IND or 1.#INF in the DPIN output file and can generally be avoided by scaling all input and output quantity variables to have unit means (use the UnitMeans command). Other linear programs may fail to solve simply because they are infeasible – see O'Donnell (2010a).

5 this error code indicates that the maximum number of simplex iterations has been exceeded. The maximum number of iterations may be reached if the linear program is degenerate. For details on degeneracy and cycling in linear programs see Winston (2004, p. 168-171).

Note that the Hicks-Moorsteen index sometimes takes the value zero. This is *not* an error – the Hicks-Moorsteen index that compares the TFP of firm i in period t with the TFP of firm i in period s will always take the value zero when, for example, firm i uses less of one input in period t than any other firm in the sample, and if it fails to produce an output in period t that it had produced in period s. Finally, values of 1.#IND or 1.#INF mean that DPIN has either exceeded the finite limits of floating point arithmetic (e.g., generated a number that is infinitely large) or attempted to obtain a result that is simply undefined (e.g. division by zero). In such cases it is often worthwhile checking for outlier observations (e.g., observations where all inputs are zero or where one or more variables are extremely large).

Eg1_output_xtras.csv

Selected computations for Paasche Laspeyres Fisher or Lowe indexes

Obs	Firm	Period	QHATt	QBARt	Qt	XHATt	XBARt	Xt	QHATs	QBARs	Qs	XHATs	XBARs	Xs	DIttt	HIttt	DOttt	HOttt
1	1	1	132	132	90	1080	1227	1800	132	132	90	1080	1227	1800	1.467	1.467	0.6818	0.6818
2	2	1	40	40	40	300	450	450	40	40	40	300	450	450	1	1	1	1
3	3	1	40	40	40	300	400	400	40	40	40	300	400	400	1	1	1	1
4	4	1	13.33	13.33	10	110	217.5	290	13.33	13.33	10	110	217.5	290	1.333	1.333	0.75	0.75
5	1	2	88	88	60	450	572.7	840	88	88	60	450	572.7	840	1.467	1.467	0.6818	0.6818
6	2	2	40	40	40	300	300	300	40	40	40	300	300	300	1	1	1	1
7	3	2	40	40	40	300	400	400	40	40	40	300	400	400	1	1	1	1
8	4	2	26.67	26.67	20	150	150	200	26.67	26.67	20	150	165	220	1.333	1.333	0.75	0.75

Figure 9. Paasche Example: Selected Computations (Eg1_output_xtras.csv)

Eg1_output_levels.csv

Levels Computed Using Paasche Aggregator Functions

Obs	Firm	Period	Rev	Cost	Prof	P	W	TT	Q	X	TFP	TFP*	TFPE	OTE	OSE	RAE	ROSE	OSME	ITE	ISE
1	1	1	90	1800	0.05	1	1	1	90	1800	0.05	0.0833	0.6	0.6818	1	1	0.88	0.88	0.6818	1
2	2	1	40	450	0.0889	1	1	1	40	450	0.0889	0.1333	0.6667	1	1	1	0.6667	0.6667	1	1
3	3	1	40	400	0.1	1	1	1	40	400	0.1	0.1333	0.75	1	1	1	0.75	0.75	1	1
4	4	1	10	290	0.0345	1	1	1	10	290	0.0345	0.0909	0.3793	0.75	1	1	0.5057	0.5057	0.75	1
5	1	2	60	840	0.0714	1	1	1	60	840	0.0714	0.1333	0.5357	0.6818	1	1	0.7857	0.7857	0.6818	1
6	2	2	40	300	0.1333	1	1	1	40	300	0.1333	0.1333	1	1	1	1	1	1	1	1
7	3	2	40	400	0.1	1	1	1	40	400	0.1	0.1333	0.75	1	1	1	0.75	0.75	1	1
8	4	2	20	200	0.1	1	1	1	20	200	0.1	0.1333	0.75	0.75	1	1	1	1	0.75	1

Figure 10. Paasche Example: Levels of Productivity and Efficiency (Eg1_output_levels.csv)

Eg1_output_indexes.csv

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
1	Paasche Indexes Comparing Firm i in Period t with Firm i in Period t-1																				
2	Obs	Firm	Period	dRev	dCost	dProf	dP	dW	dTT	dQ	dX	dTFP	dTech	dTFPE	dOTE	dOSE	dRAE	dROSE	dOSME	dITE	dISE
3	1	1	1																		
4	2	2	1																		
5	3	3	1																		
6	4	4	1																		
7	5	1	2	0.6667	0.4667	1.4286	0.6667	0.4667	1.4286	1	1	1	1	1	1	1	1	1	1	1	1
8	6	2	2	1	0.6667	1.5	1	0.6667	1.5	1	1	1	1	1	1	1	1	1	1	1	1
9	7	3	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	8	4	2	2	0.6897	2.9	2	0.7586	2.6364	1	0.9091	1.1	1	1.1	1	1	1	1.1	1.1	1	1
11																					

Figure 11. Paasche Example: Indexes of Productivity and Efficiency Change Under CRS (Eg1_output_indexes.csv)

Eg1_output_shadowprices.csv

	A	B	C	D	E	F	G	H	I	J
1	Revenue-deflated Output Shadow Prices and Cost-deflated Input Shadow Prices									
2	Obs	Firm	Period	pstar1	wstar1	wstar2				
3	1	1	1	0.02273	0.006667	0.008889				
4	2	2	1	0.05	0.01	0.01333				
5	3	3	1	0.05	0.01	0.01333				
6	4	4	1	0.075	0	0.06667				
7	5	1	2	0.02273	0.006667	0.008889				
8	6	2	2	0.05	0.01	0.01333				
9	7	3	2	0.05	0.01	0.01333				
10	8	4	2	0.075	0.02	0.02667				

Figure 12. Paasche Example: Revenue-deflated Output Shadow Prices and Cost-deflated Input Shadow Prices (Eg1_output_shadowprices.csv)

NE_output_levels.csv

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
1	Levels Computed Using Fare-Primont Aggregator Functions																				
2	Obs	State	Year	Rev	Cost	Prof	P	W	TT	Q	X	TFP	TFP*	TFPE	OTE	OSE	OME	ROSE	OSME	ITE	ISE
3	1	6	1960	172945.1	202552.5	0.8538	555244.4	383114.8	1.4493	0.3115	0.5287	0.5891	0.7662	0.7689	0.8932	0.9984	0.902	0.9544	0.8608	0.8938	0.9977
4	2	7	1960	122658	109117.7	1.1241	487152.7	254354.8	1.9152	0.2518	0.429	0.5869	0.7662	0.766	1	1	0.9964	0.7687	0.766	1	1
5	3	17	1960	190936.8	211407.5	0.9032	491285.4	379347.1	1.2951	0.3886	0.5573	0.6974	0.7662	0.9102	0.9895	0.9986	0.9704	0.9479	0.9199	0.9891	0.999
6	4	18	1960	338180	355362.7	0.9516	449389.5	261129.6	1.7209	0.7525	1.3609	0.553	0.7662	0.7217	0.9131	0.956	0.9511	0.831	0.7904	0.9036	0.9661
7	5	19	1960	238421.7	212954.4	1.1196	553803.6	271989	2.0361	0.4305	0.783	0.5499	0.7662	0.7176	0.9022	0.9978	0.9384	0.8476	0.7954	0.9023	0.9978
8	6	28	1960	70351.84	93914.38	0.7491	446072.3	321837.8	1.386	0.1577	0.2918	0.5405	0.7662	0.7054	0.8151	0.9643	0.9788	0.8842	0.8654	0.8513	0.9232
9	7	29	1960	328726.4	299427.3	1.0979	473892.7	267137.4	1.774	0.6937	1.1209	0.6189	0.7662	0.8077	1	0.9993	1	0.8077	0.8077	1	0.9993
10	8	32	1960	1119049	1184943	0.9444	383619.1	279932.6	1.3704	2.9171	4.233	0.6891	0.7662	0.8994	1	0.9714	0.9799	0.9178	0.8994	1	0.9714
11	9	36	1960	1089546	1355098	0.804	448412.6	328428.3	1.3653	2.4298	4.126	0.5889	0.7662	0.7686	0.8868	0.9787	0.9954	0.8706	0.8667	0.8852	0.9805
12	10	37	1960	24496.44	29908.87	0.819	563218.1	375372.6	1.5004	0.0435	0.0797	0.5459	0.7662	0.7124	0.9278	0.9304	0.9948	0.7718	0.7678	0.9795	0.8813
13	11	44	1960	170705.2	186657.6	0.9145	342003.7	275738.7	1.2403	0.4991	0.6769	0.7373	0.7662	0.9623	0.9921	0.9986	0.9782	0.9916	0.97	0.9912	0.9995
14	12	6	1961	166861.5	196731.5	0.8482	542663.4	384333.8	1.412	0.3075	0.5119	0.6007	0.7662	0.784	0.9165	0.9977	0.8973	0.9533	0.8554	0.917	0.9972
15	13	7	1961	112354	103566	1.0849	456009.2	258791.9	1.7621	0.2464	0.4002	0.6157	0.7662	0.8035	1	1	0.992	0.81	0.8035	1	1
16	14	17	1961	180976.4	207473.5	0.8723	475686	385320	1.2345	0.3805	0.5384	0.7066	0.7662	0.9222	0.982	0.9996	0.9939	0.9448	0.939	0.9818	0.9998
17	15	18	1961	336369.4	355314.1	0.9467	446266.6	260879.7	1.7106	0.7537	1.362	0.5534	0.7662	0.7223	0.9334	0.9494	0.9454	0.8185	0.7738	0.9253	0.9577
18	16	19	1961	205046.6	216469.5	0.9472	438901.3	276568.1	1.587	0.4672	0.7827	0.5969	0.7662	0.779	1	1	0.9429	0.8262	0.779	1	1
19	17	28	1961	67908.67	88463.2	0.7676	422917.9	312668.5	1.3526	0.1606	0.2829	0.5675	0.7662	0.7407	0.8392	0.9818	0.994	0.8879	0.8827	0.8668	0.9506
20	18	29	1961	323457.3	289065.9	1.119	487909.6	279025.6	1.7486	0.6629	1.036	0.6399	0.7662	0.8352	1	1	0.9865	0.8466	0.8352	1	1
21	19	32	1961	1121114	1165632	0.9618	376618.3	278408.8	1.3528	2.9768	4.1868	0.711	0.7662	0.9279	1	1	1	0.9279	0.9279	1	1
22	20	36	1961	1092570	1322216	0.8263	445947.7	322731.3	1.3818	2.45	4.097	0.598	0.7662	0.7805	0.897	0.9856	0.9911	0.878	0.8701	0.8962	0.9864
23	21	37	1961	23146.03	28286.41	0.8183	543406.9	360611.6	1.5069	0.0426	0.0784	0.543	0.7662	0.7087	0.9592	0.8971	0.9721	0.7601	0.7388	0.9926	0.8669
24	22	44	1961	169869.7	187303.4	0.9069	331232.5	279845.2	1.1836	0.5128	0.6693	0.7662	0.7662	1	1	1	1	1	1	1	1
25	23	6	1962	164348.2	202424.3	0.8119	546187.6	412246.6	1.3249	0.3009	0.491	0.6128	0.7662	0.7998	0.9427	0.998	0.9017	0.9409	0.8484	0.943	0.9976
26	24	7	1962	118742.3	110624	1.0734	478456.5	277009.5	1.7272	0.2482	0.3994	0.6215	0.7662	0.8111	1	1	1	0.8111	0.8111	1	1
27	25	17	1962	180271.6	209978.5	0.8585	481040.3	406203.8	1.1842	0.3748	0.5169	0.725	0.7662	0.9462	1	1	1	0.9462	0.9462	1	1
28	26	18	1962	350401.2	381087.9	0.9195	454773.1	279745.6	1.6257	0.7705	1.3623	0.5656	0.7662	0.7382	0.9198	0.9664	0.9644	0.8322	0.8025	0.9103	0.9765
29	27	19	1962	207586.8	218650.7	0.9494	444361.3	283766.4	1.5659	0.4672	0.7705	0.6063	0.7662	0.7913	1	1	0.9705	0.8153	0.7913	1	1
30	28	28	1962	65116.1	85535.63	0.7613	418938.8	309104.2	1.3553	0.1554	0.2767	0.5617	0.7662	0.7331	0.8255	0.9859	0.9889	0.8981	0.8881	0.8501	0.9573
31	29	29	1962	308649.2	286204.2	1.0784	471590	276452	1.7059	0.6545	1.0353	0.6322	0.7662	0.8251	1	1	0.9828	0.8395	0.8251	1	1
32	30	32	1962	1102181	1195372	0.922	376695.2	289873.7	1.2995	2.9259	4.1238	0.7095	0.7662	0.926	1	1	1	0.926	0.926	1	1
33	31	36	1962	1043263	1321664	0.7894	441452	334462.3	1.3199	2.3633	3.9516	0.598	0.7662	0.7805	0.93	0.9568	0.9679	0.8671	0.8393	0.9274	0.9595

Figure 13. NE Farm Production Example: Levels of Productivity and Efficiency (NE_output_levels.csv)

NE_output_indexes.csv

Fare-Primont Indexes Comparing Observation i to Observation 5

Obs	State	Year	dRev	dCost	dProf	dP	dW	dTT	dQ	dX	dTFP	dTech	dTFPE	dOTE	dOSE	dOME	dROSE	dOSME	dITE	dISE
1	6	1960	0.7254	0.9512	0.7626	1.0026	1.4086	0.7118	0.7235	0.6753	1.0714	1	1.0714	0.99	1.0006	0.9612	1.126	1.0823	0.9906	1
2	7	1960	0.5145	0.5124	1.004	0.8796	0.9352	0.9406	0.5848	0.5479	1.0674	1	1.0674	1.1084	1.0022	1.0618	0.907	0.963	1.1083	1.0022
3	17	1960	0.8008	0.9927	0.8067	0.8871	1.3947	0.6361	0.9027	0.7118	1.2683	1	1.2683	1.0967	1.0007	1.034	1.1184	1.1565	1.0962	1.0012
4	18	1960	1.4184	1.6687	0.85	0.8115	0.9601	0.8452	1.748	1.7381	1.0057	1	1.0057	1.0121	0.9581	1.0135	0.9804	0.9936	1.0015	0.9683
5	19	1960	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
6	28	1960	0.2951	0.441	0.6691	0.8055	1.1833	0.6807	0.3663	0.3727	0.9829	1	0.9829	0.9034	0.9663	1.043	1.0432	1.088	0.9435	0.9253
7	29	1960	1.3788	1.4061	0.9806	0.8557	0.9822	0.8712	1.6113	1.4316	1.1255	1	1.1255	1.1084	1.0015	1.0656	0.9529	1.0154	1.1083	1.0016
8	32	1960	4.6936	5.5643	0.8435	0.6927	1.0292	0.673	6.7758	5.4064	1.2533	1	1.2533	1.1084	0.9735	1.0442	1.0828	1.1307	1.1083	0.9736
9	36	1960	4.5698	6.3633	0.7182	0.8097	1.2075	0.6706	5.6439	5.2698	1.071	1	1.071	0.9829	0.9808	1.0607	1.0272	1.0896	0.981	0.9827
10	37	1960	0.1027	0.1404	0.7315	1.017	1.3801	0.7369	0.101	0.1018	0.9927	1	0.9927	1.0284	0.9324	1.0601	0.9106	0.9653	1.0856	0.8833
11	44	1960	0.716	0.8765	0.8168	0.6176	1.0138	0.6092	1.1594	0.8646	1.341	1	1.341	1.0996	1.0007	1.0423	1.17	1.2195	1.0985	1.0018
12	6	1961	0.6999	0.9238	0.7576	0.9799	1.413	0.6935	0.7142	0.6538	1.0925	1	1.0925	1.0159	0.9999	0.9561	1.1247	1.0754	1.0163	0.9994

Figure 14. NE Farm Production Example: Indexes of Productivity and Efficiency Change Relative to State 19 in 1960 (NE_output_indexes.csv)

NE_output_shadowprices.csv

Revenue-deflated Output Shadow Prices and Cost-deflated Input Shadow Prices

Obs	State	Year	pstar1	pstar2	pstar3	wstar1	wstar2	wstar3	wstar4
1	6	1960	1.609	0.8139	0	0	0.03176	0.1464	2.528
2	7	1960	1.661	2.017	0	0	2.23	1.286	0
3	17	1960	0	0.7827	0.7907	0	0.07901	0	2.199
4	18	1960	0.3144	0.1217	0.3652	0	0	0.7237	0.1687
5	19	1960	0.2831	0.5458	0.4021	0	0.1196	0.1046	1.374
6	28	1960	0	0	2.442	0	0	0	5.67
7	29	1960	0	0.4349	0.4141	0.2161	0	0.7849	0
8	32	1960	0	0	0.2035	0	0	0.1155	0.1337
9	36	1960	0.2404	0.07028	0	0	0	0	0.3637

Figure 15. NE Farm Production Example: Revenue-deflated Output Shadow Prices and Cost-deflated Input Shadow Prices (NE_output_shadow prices.csv)

NE_output_xtras.csv

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
1	Selected computations for Fare-Primont indexes																			
2																				
3	DO000	0.8275																		
4	alpha0	0.6631	0.2428	0.09417																
5	beta0	0.1905	0.06218	0	0.8842															
6	gamma0	0.07145																		
7	q0	1	1	1																
8	pstar0	0.5487	0.2009	0.07793																
9																				
10	DI000	1.222																		
11	phi0	0.5369	0.2763	0.02712																
12	eta0	0.2178	0.1508	0	0.6314															
13	delta0	0.02199																		
14	x0	1	1	1	1															
15	wstar0	0.2661	0.1843	0	0.7717															
16																				
17	Obs	State	Year	QHATt	QBARt	Qt	XHATt	XBARt	Xt	QHATs	QBARs	Qs	XHATs	XBARs	Xs	DIttt	HIttt	DOttt	HOttt	
18	1	6	1960	0.3866	0.3487	0.3115	0.4357	0.4725	0.5287	0.5085	0.4772	0.4305	0.7038	0.7064	0.783	1.119	1.121	0.8932	0.8917	
19	2	7	1960	0.2527	0.2518	0.2518	0.3511	0.429	0.429	0.5085	0.4772	0.4305	0.7038	0.7064	0.783	1	1	1	1	
20	3	17	1960	0.4048	0.3928	0.3886	0.5452	0.5512	0.5573	0.5085	0.4772	0.4305	0.7038	0.7064	0.783	1.011	1.012	0.9895	0.9881	
21	4	18	1960	0.8665	0.8241	0.7525	1.055	1.23	1.361	0.5085	0.4772	0.4305	0.7038	0.7064	0.783	1.107	1.146	0.9131	0.873	
22	5	19	1960	0.5085	0.4772	0.4305	0.7038	0.7064	0.783	0.5085	0.4772	0.4305	0.7038	0.7064	0.783	1.108	1.111	0.9022	0.9003	
23	6	28	1960	0.1977	0.1935	0.1577	0.2301	0.2484	0.2918	0.5085	0.4772	0.4305	0.7038	0.7064	0.783	1.175	1.272	0.8151	0.7859	
24	7	29	1960	0.6937	0.6937	0.6937	1.093	1.121	1.121	0.5085	0.4772	0.4305	0.7038	0.7064	0.783	1	1.001	1	0.9993	
25	8	32	1960	2.977	2.917	2.917	4.233	4.233	4.233	0.5085	0.4772	0.4305	0.7038	0.7064	0.783	1	1.029	1	0.9714	
26	9	36	1960	2.753	2.74	2.43	3.423	3.652	4.126	0.5085	0.4772	0.4305	0.7038	0.7064	0.783	1.13	1.152	0.8868	0.8679	
27	10	37	1960	0.04712	0.04688	0.04349	0.07627	0.07805	0.07968	0.5085	0.4772	0.4305	0.7038	0.7064	0.783	1.021	1.158	0.9278	0.8633	
28	11	44	1960	0.5143	0.5031	0.4991	0.6656	0.6709	0.6769	0.5085	0.4772	0.4305	0.7038	0.7064	0.783	1.009	1.009	0.9921	0.9907	
29	12	6	1961	0.3739	0.3355	0.3075	0.4308	0.4694	0.5119	0.5085	0.4772	0.4305	0.7038	0.7064	0.783	1.09	1.094	0.9165	0.9144	

Figure 16. NE Farm Production Example: Selected Computations (NE_output_xtras.csv)

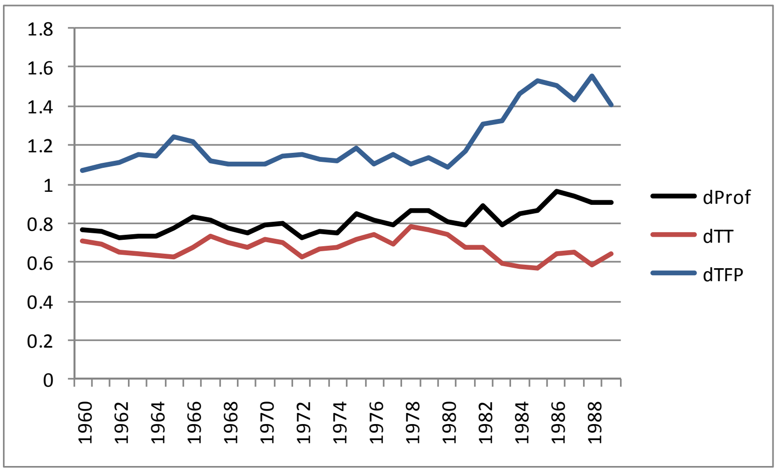

Figure 17. NE Farm Production Example: The Components of Profitability Change in State 6

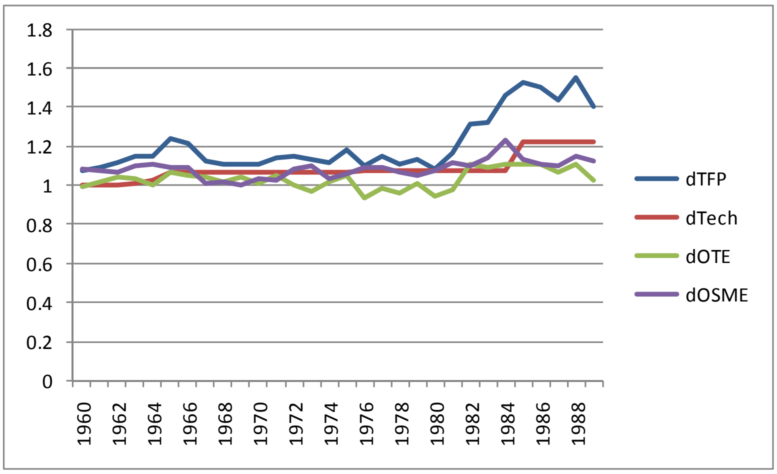

Figure 18. NE Farm Production Example: The Components of TFP Change in State 6

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