

COMPARING ECCENTRICITY-BASED GRAPH INVARIANTS

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Abstract

The first and second Zagreb eccentricity indices (EM_1 and EM_2), the eccentric distance sum (EDS), and the connective eccentricity index (CEI) are all recently conceived eccentricity-based graph invariants, some of which found applications in chemistry. We prove that $EDS \geq EM_1$ for any connected graph, whereas $EDS > EM_2$ for trees. Moreover, in the case of trees, $EM_1 \geq CEI$, whereas $EM_2 > CEI$ for trees with at least three vertices. In addition, we compare EDS with EM_2 , and compare EM_1 , EM_2 with CEI for general connected graphs under some restricted conditions.

Keywords: eccentricity (of vertex), Zagreb eccentricity index, eccentric distance sum, connective eccentricity index.

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