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THE SLATER AND SUB-k-DOMINATION NUMBER OF A GRAPH WITH APPLICATIONS TO DOMINATION AND k-DOMINATION

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Abstract

In this paper we introduce and study a new graph invariant derived from the degree sequence of a graph G, called the *sub-k-domination number* and denoted $\text{sub}_k(G)$. This invariant serves as a generalization of the *Slater number*; in particular, we show that $\text{sub}_k(G)$ is a computationally efficient sharp lower bound on the k-domination number of G, and improves on several known lower bounds. We also characterize the sub-k-domination numbers of several families of graphs, provide structural results on sub-k-domination, and explore properties of graphs which are $\text{sub}_k(G)$-critical with respect to addition and deletion of vertices and edges.

Keywords: Slater number, domination number, sub-k-domination number, k-domination number, degree sequence index strategy.

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References

[1] Y. Caro and R. Pepper, *Degree sequence index strategy*, Australas. J. Combin. **59** (2014) 1–23.

[2] Y. Caro and Y. Roditty, *A note on the k-domination number of a graph*, Int. J. Math. Math. Sci. **13** (1990) 205–206.
doi:10.1155/S016117129000031X

[3] E. DeLaViña, C.E. Larson, R. Pepper and B. Waller, *Graffiti.pc on the* 2*-domination number of a graph*, Congr. Numer. **203** (2010) 15–32.

[4] W.J. Desormeaux, T.W. Haynes and M.A. Henning, *Improved bounds on the domination number of a tree*, Discrete Appl. Math. **177** (2014) 88–94.
doi:10.1016/j.dam.2014.05.037

[5] M. Dorfling, W.D. Goddard and M.A. Henning, *Domination in planar graphs with small diameter* II, Ars Combin. **78** (2006) 237–255.

[6] Z. Du and A. Ilić, *A proof of the conjecture regarding the sum of domination number and average eccentricity*, Discrete Appl. Math. **201** (2016) 105–113.
doi:10.1016/j.dam.2015.08.002

[7] O. Favaron, M. Mahěo and J.F. Saclě, *On the residue of a graph*, J. Graph Theory **15** (1991) 39–64.
doi:10.1002/jgt.3190150107

[8] O. Favaron, A. Hansberg and L. Volkmann, *On the k-domination and minimum degree in graphs*, J. Graph Theory **57** (2008) 33–40.
doi:10.1002/jgt.20279

[9] J.F. Fink and M.S. Jacobson, *n-domination in graphs*, Graph Theory with Applications to Algorithms and Computer Science (Kalamazoo, Mich., 1987) 283–300.

[10] M. Gentner and D. Rautenbach, *Some comments on the Slater number*, (2016). preprint arXiv:1608.04560v1

[11] R. Glebov, A. Liebenau and T. Szabó, *On the concentration of the domination number of the random graph*, SIAM J. Discrete Math. **29** (2015) 1186–1206.
doi:10.1137/12090054X

[12] A. Hansberg, *Bounds on the connected k-domination number in graphs*, Discrete Appl. Math. **158**(2010) 1506–1510.
doi:10.1016/j.dam.2010.05.021

[13] A. Hansberg and R. Pepper, *On k-domination and j-independence in graphs*, Discrete Appl. Math. **161** (2013) 1472–1480.
doi:10.1016/j.dam.2013.02.008

[14] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Domination in Graphs: Advanced Topics (Marcel Decker, New York, 1998).

[15] T. Haynes, S.T. Hedetniemi and P.J. Slater, Fundamentals of Domination in Graphs (Marcel Decker, New York, 1998).

[16] M.A. Henning and A. Yeo, Total Domination in Graphs (Springer Monographs in Mathematics, 2013).
doi:10.1007/978-1-4614-6525-6

[17] M.S. Jacobson and K. Peters, *Complexity questions for n-domination and related parameters*, Congr. Numer. **68** (1989) 7–22.

[18] A.P. Kazemi, *On the total k-domination number of graphs*, Discuss. Math. Graph Theory **32** (2012) 419–26.
doi:10.7151/dmgt.1616

[19] M. Lemańska, *Lower bound on the domination number of a tree*, Discuss. Math. Graph Theory **24** (2004) 165–169.
doi:10.7151/dmgt.1222

[20] R. Pepper, Binding Independence, PhD Thesis (University of Houston 2004).

[21] R. Pepper, *Implications of some observations about the k-domination number*, Congr. Numer. **206** (2010) 65–71.

[22] D. Rautenbach and L. Volkmar, *New bounds on the k-domination number and the k-tuple domination number*, Appl. Math. Lett. **20** (2007) 98–102.
doi:10.1016/j.aml.2006.03.006

[23] P.J. Slater, *Locating dominating sets and locating-dominating sets*, in: Graph Theory, Combinatorics, and Applications, Proceedings of the 7th Quadrennial International Conference on the Theory and Applications of Graphs **2** (1995) 1073–1079.

[24] D. Stevanović, M. Aouchiche and P. Hansen, *On the spectral radius of graphs with a given domination number*, Linear Algebra Appl. **428** (2008) 1854–1864.
doi:10.1016/j.laa.2007.10.024

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