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THE DISTINGUISHING NUMBER AND DISTINGUISHING INDEX OF THE LEXICOGRAPHIC PRODUCT OF TWO GRAPHS

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Abstract

The distinguishing number (index) $D(G)$ ($D'(G)$) of a graph G is the least integer d such that G has a vertex labeling (edge labeling) with d labels that is preserved only by the trivial automorphism. The lexicographic product of two graphs G and H, $G[H]$ can be obtained from G by substituting a copy H_u of H for every vertex u of G and then joining all vertices of H_u with all vertices of H_v if $uv \in E(G)$. In this paper we obtain some sharp bounds for the distinguishing number and the distinguishing index of the lexicographic product of two graphs. As consequences, we prove that if G is a connected graph with $\mathrm{Aut}(G[G]) = \mathrm{Aut}(G)[\mathrm{Aut}(G)]$, then for every natural number k, $D(G) \leq D(G^k) \leq D(G) + k - 1$ and all lexicographic powers of G, G^k ($k \geq 2$) can be distinguished by two edge labels, where $G^k = G[G[\ldots]]$.

Keywords: distinguishing index, distinguishing number, lexicographic.

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