Discussiones Mathematicae
Graph Theory 35 (2015) 589–594
doi:10.7151/dmgt.1825

Note

CHARACTERIZATION OF LINE-CONSISTENT SIGNED GRAPHS

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Abstract

The line graph of a graph with signed edges carries vertex signs. A vertex-signed graph is *consistent* if every circle (cycle, circuit) has positive vertex-sign product. Acharya, Acharya, and Sinha recently characterized *line-consistent* signed graphs, i.e., edge-signed graphs whose line graphs, with the naturally induced vertex signature, are consistent. Their proof applies Hoede's relatively difficult characterization of consistent vertex-signed graphs. We give a simple proof that does not depend on Hoede's theorem as well as a structural description of line-consistent signed graphs.

Keywords: line-consistent signed graph, line graph, consistent vertex-signed graph, consistent marked graph.

2010 Mathematics Subject Classification: 05C22, 05C76.

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Received 28 May 2014
Revised 14 December 2014
Accepted 22 January 2015