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ON $(p, 1)$-TOTAL LABELLING OF SOME 1-PLANAR GRAPHS[1]

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Abstract

A graph is 1-planar if it can be drawn on the plane so that each edge is crossed by at most one other edge. In this paper, it is proved that the $(p, 1)$-total labelling number ($p \geq 2$) of every 1-planar graph G is at most $\Delta(G) + 2p - 2$ provided that $\Delta(G) \geq 6p + 7$ or $\Delta(G) \geq 4p + 6$ and G is triangle-free.

Keywords: 1-planar graph, total coloring, $(p, 1)$-total labelling, structural theorem.

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