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Income-Oriented Cutting Budget
Tulotavoitteeseen perustuva hakkuulaskelma

Pekka Kilkki

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INCOME-ORIENTED CUTTING BUDGET

TULOTAVOITTEESEEN PERUSTUVA HAKKUULASKELMA

PEKKA KILKKI

To be presented, with the permission of the Faculty of Agriculture and Forestry of the University of Helsinki for public criticism in Auditorium XII at 12 noon on 23rd November, 1968

HELSINKI 1968

Helsinki 1968. Kunnallispaino

PREFACE

The start of this study goes back to the year 1964. Looking at the recent advancement within the field of forest management, Professor AARNE NYYSSÖNEN, my principal at the Institute of Forest Mensuration and Management, University of Helsinki, suggested that new methods be studied for the preparation of cutting budgets. He has also offered valuable help during the course of my work.

Part of the work was carried out at the University of California, Berkeley. Professor DENNIS E. TEEGUARDEN spent a great deal of his time in reading and discussing the study. Several valuable suggestions were also made by Professor WILLIAM S. JEWELL and Dr WILLIAM G. O'REGAN.

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Mrs MARJATTA MÄÄTTÄ and Mr LARS REHNSTRÖM took part in various phases of the office work, and the figures were drawn by Mrs IRMA NYLANDER. The English text was checked by Mr F. A. FEWSTER.

I wish to express my sincere thanks to all these persons. The stimulating working environment provided by the Institute of Forest Mensuration and Management also contributed greatly to achievement of the task set.

My special thanks are due to the Computer Centres of the University of Helsinki and the University of California for their granting free computer time; without this it would have proved impossible to complete this study.

For the grant I received during this study I wish to express my thanks to the University of Helsinki. I am grateful to the W. K. Kellogg Foundation and the U.S. Institute of International Education in Finland, whose economic support enabled me to study and continue this work in the U.S.A. I am also indebted to the Society of Forestry in Finland for the provision of a travel grant to Norway, and including this study in its series of publications.

Helsinki, September 1968.

Pekka Kilkki

CONTENTS

1 INTRODUCTION

11 DEVELOPMENT OF CUTTING BUDGET METHODS

One of the key tasks in forest planning is the scheduling of timber harvests over a forest area for a specified time period. The result of such planning is termed a cutting budget, and forms the basis for the working plan. Consequently, a cutting budget — no matter how simple or sophisticated it may be — is always needed in the practice of planned forestry.

In Finland, cutting budgets have attracted interest from foresters since the early part of this century (Ericsson 1906; Lindholm 1909; Lönnroth 1919; 1927). Lihtonen (1943; 1959) developed the »rental cut method», which has been widely applied both in practice and in scientific works (Lihtonen 1946; Ilvessalo 1956; Kallio 1958; Linnamies 1959). Later on, Kuusela (1958a; 1958b; 1959) developed methods of making increment forecasts in conjunction with the cutting budget and of determining the largest permanent allowable cut. These methods were further developed in the work done to estimate the cutting potentialities in the forests of Finland (Heikurainen, Kuusela, Linnamies, and Nyyssönen 1960).

A method termed the »cutting budget for a desirable growing stock» was developed by Kuusela and Nyyssönen (1962). This method has subsequently almost completely superseded the rental cut method; in this method, the cutting budget is generally prepared for a 20 year period. The growing stock is divided into age or treatment classes. The allowable cut is determined by the original situation in each class, by the growth rate of each class, and by the target given for each class. One important factor is the size of the regeneration area, as the final goal is a relatively homogeneous age or development class structure.

In Norway, work has been done to develop methods for determination of the largest permanent allowable cut (balansekvantum) (Nersten and Delbeck 1965). Nersten (1965) has also made some tests involving substitution of the detailed cutting budget by equations calculated by the application of regression analysis. A similar approach was later used by Kilkki (1966) and Roiko-Jokela (1966).

In different countries, cutting budget methods often include features arising from the circumstances in which they are applied. Logging techniques, utilizable wood, and other special characteristics have shaped the cutting budget methods. However, during recent years more general methods, with the employment of electronic computers, have in many countries taken the place of the traditional cutting budget methods. It has been noticed that the preparation of a cutting budget is a task which can be effected by means of new tools of the management sciences. Several alternative methods have been suggested, such as simulation (Gould and O'Regan 1965; Majevskis 1965; Kilkki 1966), linear programming (Stridsberg 1959; Curtis 1962; Leak 1964; Loucks 1964; Wardle 1965; Kidd, Thompson, and Hoepner 1966; Höfle 1967; Kilkki 1967; Liittschwager and Tcheng 1967; v. Malmborg 1967), and dynamic programming (Arimizu 1958; Hool 1966).

It seems evident that the new methods can be applied to forest planning problems. Nevertheless, a number of difficulties arise from such items as inadequate data, unusually long planning period, and the magnitude of many problems. All these difficulties must be surmounted before the new tools can be applied efficiently.

Moreover, it has become apparent that cutting budgets can no longer be based upon volumes alone. Several authors have also taken into account the financial yield (e.g. Oswald 1931; Gould and O'Regan 1965; Nersten and Delbeck 1965; Kilkki 1966). The cutting budget has further been seen as a part of the greater problem involved in planning the whole forest undertaking (v. Malmborg 1967). Accordingly, the entire planning situation must be considered when cutting budgets are under preparation.

The recent development in planning methods and needs has led to a situation in which the traditional cutting budget methods are no longer adequate. Methods are needed that are faster, more accurate, and provide more information than those applied today.

12 ECONOMIC FRAMEWORK

121 Planning situation

The object of this study is a pure forest undertaking of which the property consists solely of the land and growing stock. For the purposes of analysis, it is assumed that (1) the forest will be grown forever; (2) prices of timber and the productivity of soil will not change; (3) all the timber is sold as stumpage; (4) a constant amount of money is paid annually for management costs; (5) reforestation costs are paid at the end of the rotation, and no other silvicultural costs are entailed; (6) thinning and clear cutting with reforestation are the only activities that can occur in the forest (reforestation is considered as an obligatory silvicultural measure following clear cutting); (7) no uncertainties are involved.

The owner of the forest undertaking has to decide what kind of cutting policy he will practise in the future. It is assumed that the main interest of the forest owner lies

in the net income from his forest undertaking. This can be expressed by the following function:

$$NCR = f(t) \qquad t = 1, \ldots \tag{1}$$

where NCR = annual net cash receipts
t = year

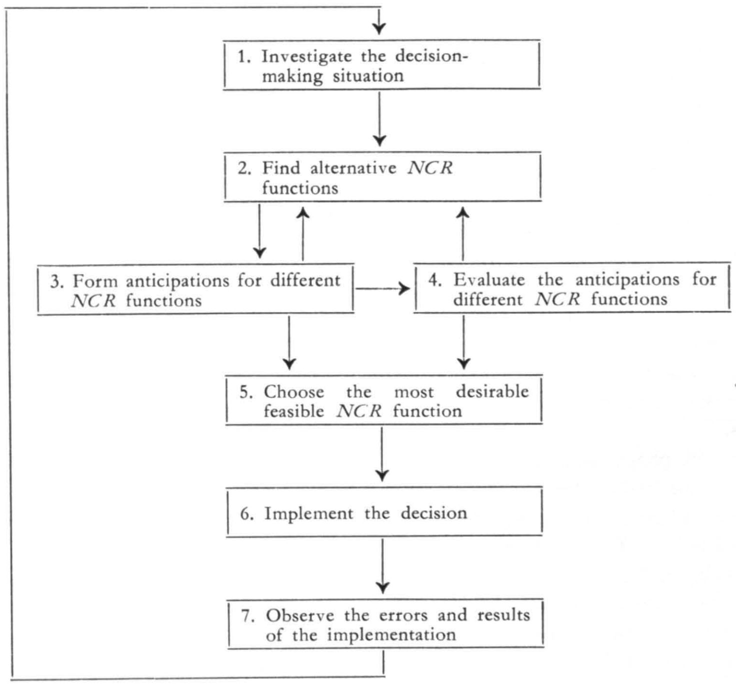
The sum of the annual net cash receipts and the annual disbursements determines the stumpage value of timber to be cut in one year:

$$GCR = NCR + C_m + C_r \tag{2}$$

where GCR = gross annual income from sales on the stump
C_m = annual management costs
C_r = annual reforestation costs

The decision-making process the forest-owner has to undergo can be illustrated by a diagram based on papers by Törnqvist (1963, pp. 1—5) and Niitamo (1967, Appendix 2). This diagram is presented in Figure 1.

Figure 1. Structure of the decision-making process of the forest owner.



First, the forest owner has to investigate the decision-making situation. If he is to be able to do this, adequate information on the forest must be at hand. This information is obtained by forest inventories, and by the use of growth and yield tables. Also, he has to study his own economic situation, to arrive at an approximation of his expectations of income from the forest, and the measures he is able and willing to adopt in the forest.

Secondly, the forest owner has to define some alternative *NCR* functions which might be satisfactory.

Thirdly, the forest owner has to form anticipations for different *NCR* functions. An anticipation shows what the development of the forest will be in the future, and how the cuts should be scheduled to satisfy a given *NCR* function. It may appear that an *NCR* function is not feasible, as it would result in a situation with more timber being cut than the law or the total amount of growing stock allows. It is then necessary to revert to phase two, and reduce the net cash receipts requirements.

In the fourth phase of the decision-making process, the forest owner has to evaluate the anticipations for different *NCR* functions. Since there is an infinite number of different cutting schedules, this evaluation leads to an iterative procedure in which the most desirable *NCR* function is arrived at by trial and error experimentation.

The following four main aspects have to be considered by the forest owner in his decision on the *NCR* function he wishes to follow: (1) profitability, (2) readiness to pay, (3) security, and (4) non-market values (Runeberg 1959, p. 37). Profitability here relates to the long-term economic result, and may be expressed by the net present-value of the future income. Readiness to pay corresponds to the net cash receipts during a given period. Security aspects, such as the relationship between the entrepreneur's and borrowed capital may also exercise an influence upon the decisions made by the forest owner. There may also be some non-market values, such as aesthetic ones the forest owner wishes to preserve.

Profitability is certainly the factor of greatest interest to most forest owners. However, many forest owners are not economically independent of the level of cuts in their forests; thus they cannot follow the cutting policy which would lead to the highest net present-value of future income, but must time the cuts in accordance with their cash needs. Moreover, there are forest owners who themselves work in their own forest, and forest owners who use the timber in their own woodworking plants. All of these aspects must be taken into account in evaluation of the anticipations for different *NCR* functions.

In the fifth phase, the forest owner has to choose the most desirable feasible *NCR* function on the basis of the anticipations and their evaluation.

The sixth phase comprises implementation of the cutting plan, based upon the most desirable feasible *NCR* function.

As a rule, an *NCR* function apparently best at one time cannot be followed for a very long time. The reasons which lead to adjustment of the *NCR* function are attributable either to changes in the economic situation of the forest owner, or to inaccurate forecasting methods. Moreover, the changes in the general economic situation reflect

on the planning situation, for instance on timber prices and costs. Forest inventories provide the method for checking the accuracy of the anticipations. Phase seven, which includes the inspection of working plans, always leads to the beginning of the decision-making process.

122 Forest model

The forest area which forms the basis for planning is characterized by the following properties: (1) it is divided into forest stands identified by site, age, and area; (2) there are only three different sites (1=MT, 2=VT, 3=CT), and one tree species (Scotch Pine); (3) the geographical location of the stands does not restrict treatment of the stands; (4) if necessary, each stand can be divided into parts.

The data employed for description of the structure and development of the forest stands have been largely drawn from a paper by Nyyssönen (1958). He has given data that illustrate the volume of and distribution of timber in different assortments as a function of site and age. The data relate to even-aged Scotch Pine stands in Southern Finland. The volumes are expressed as solid measure, excluding bark.

For analytical purposes, the forest stands are divided into young and old stands. These are defined as follows:

Site	Age of Young stand (years)	Age of Old stand (years)
1	0 — 39	40 or more
2	0 — 49	50 or more
3	0 — 59	60 or more

In young stands, the development of growing stock volume is based upon Nyyssönen's data (Table 1). It is assumed that the development of young stands strictly follows

Table 1. Volume growth and thinning removal in young forest stands, cu.m./ha./5 years.

Age, years	Growth			Removal		
	Site					
	1	2	3	1	2	3
0	0	0	0	0	0	0
5	5	0	0	0	0	0
10	15	15	8	0	0	0
15	27	20	10	9	5	5
20	35	22	10	10	5	6
25	44	27	11	13	10	6
30	45	30	14	20	13	8
35	50	32	16	30	16	8
40		35	15		19	8
45		40	16		22	9
50			17			10
55			17			10

these figures. Thus, there is thinning removal every fifth year in young stands. In old stands, cuttings are more flexible, and consequently a function is employed for determination of the growth between two cutting operations. It is assumed that the volume growth in old stands follows a volume growth function calculated for Scotch Pine stands by Kuusela and Kilkki (1963, p. 27). This function has the following form:

$$I_v = 11.38\ t^{-1.230}\ 10^{-0.001309v}\ v \tag{3}$$

where I_v = current volume growth, cu.m./ha./year
t = age, years
v = volume, cu.m./ha.

The unit values of growing stock as a function of site and age were calculated (Table 2) by application of the unit prices given by Nyyssönen (1958). As stumpage prices

Table 2. Unit values of growing stock, Fmk/cu.m.

Age, years	Site			Age, years	Site		
	1	2	3		1	2	3
0	0.0	0.0	0.0	65	16.9	13.0	7.5
5	0.0	0.0	0.0	70	18.8	14.3	8.0
10	0.0	0.0	0.0	75	20.1	15.9	8.7
15	1.5	0.0	0.0	80	21.4	17.4	9.4
20	6.8	2.3	0.0	85	22.5	18.7	10.2
25	7.1	4.8	0.0	90	23.6	19.9	11.1
30	7.5	7.3	2.0	95	24.3	20.8	12.0
35	8.2	7.5	2.4	100	25.0	21.7	13.0
40	9.0	7.7	2.9	105	25.5	22.5	14.4
45	10.3	8.7	4.2	110	26.0	23.2	15.9
50	11.7	9.7	5.2	115	26.3	23.8	17.1
55	13.3	10.7	6.1	120	26.6	24.4	18.3
60	15.0	11.7	6.9	125	26.8	24.8	19.4

have risen to some extent in Finland since Nyyssönen's paper (1958) was published these figures no longer indicate the real price level. Nevertheless, no drastic changes have occured in the price ratios of the different assortments during the last decade (Heikinheimo, Kuusela, and Sivonen 1967, pp. 15—30). Thus the figures in Table 2 probably still hold good as relative unit value estimates. The relatively small difference in unit values of the total growing stock and of the thinning removal has been ignored. The unit values of the total growing stock are also used for the thinning removals.

Based upon the data reported by Nyyssönen (1958, p. 41), reforestation costs are assumed to be:

Site	Reforestation cost, Fmk/ha.
1	60
2	50
3	40

These figures apply to natural regeneration, and may today be too low in comparison with the stumpage prices.

To reduce the number of calculations in linear programming, it is assumed that management costs are zero. As the management costs were taken as constant every year (p.8), it is possible to include them by raising the level of the *NCR* function (1) by their amount.

To make the calculations more rapid in execution it is assumed that the cutting and reforestation activities take place only at 5 year intervals. All the cuts needed to satisfy the net cash receipts requirements over a 5 year period are made at the beginning of the period. Moreover, the reforestation costs are paid at the beginning of the 5 year period. As an old forest stand is thinned, the thinning removal is not allowed to fall below a given minimum figure, since very slight thinnings do not occur in practice.

13 PURPOSE OF THE STUDY

The purpose of this study is that of developing methods for the formulation of anticipations needed in the decision-making process of the forest owner. This corresponds to phase three in Figure 1. The anticipations are obtained by the preparation of cutting budgets.

The cutting budget models to be developed in this study are based upon a given *NCR* function (1) for the forest undertaking. For practical reasons, a limit must be imposed on the period for which the *NCR* function is specified. This limited time is termed the planning period. An estimate of the *NCR* function after the planning period is expressed by the net present-value of the future income from the forest undertaking at the end of the planning period. This entity, referred to later as the value of the forest undertaking, does not indicate the form of the *NCR* function, but the average level of the net cash receipts after the planning period.

The aim of the cutting budget is that of defining the cutting schedule that satisfies a given *NCR* function during the planning period, and maximizes the value of the forest undertaking at the end of the planning period. Another purpose in mind is that the cutting budget provides adequate information of the development of the forest needed to evaluate different *NCR* functions, and the information needed to implement the cutting plan chosen on the basis of the alternative cutting budgets.

Two different methods, simulation and linear programming, are employed in construction of the cutting budget models. As the models are ready, their relative efficiency is tested. Analysis of the duality is carried out at the end of the study. The purpose is then that of illustrating the information provided by the models to the forest owner, and analysis of the interrelationships between the two models.

The cutting budget models to be developed in this study are so called deterministic models. The model is deterministic if there is only one possible outcome once stated input information has been given (Andersin 1967, p. 1).

2 PLANNING METHODS

21 SIMULATION

211 Principles

This section contains a discussion of the aspects of numerical simulation that are of interest to this study, largely based upon the paper by HILLIER and LIEBERMAN (1966).

Simulation is the technique of performing experiments on a model of a system. The experiments are made with the model, since the real system would be too expensive, time-consuming, and in many cases quite unsuitable for experimentation.

The simulation model describes the operation of the system in terms of individual events of the individual components of the system, rather than its over-all behaviour. The system is divided into elements with behaviour which can be predicted, at least in terms of probability distributions. The interrelationships between the elements are built into the model.

After the model has been constructed, it is activated by generating input data to simulate the actual operation of the system. By repeating this for the various alternative design configurations, and comparing their performances, the most promising configurations are discoverable. Thus the optimum, or a satisfactory, near optimum configuration, is derived as a result of an iterative procedure which call for a number of simulation operations (ANDERSIN 1967, pp. 167—187). The main problem is often that of reducing the number of alternatives to be tested. This can be effected by developing decision-making rules built into the simulation model which indicate the policy that leads near the desirable optimum. In a favourable case, a satisfactory near optimum may be found with one simulation operation, even though more operations are generally required to demonstrate this optimum.

Simulation is a versatile, and often the easiest means of approach to the problem. However, several serious weaknesses exist in simulation. In most cases, library programs are not available, and computer programming requires a great deal of time. It is also difficult to explain to other people what the simulation model really does. Nevertheless, simulation introduces the temptation of stopping half-way in formulating the problem. Thus, if it is possible to construct a solvable mathematical model that

is a reasonable idealization of the problem, this analytical approach is generally superior to simulation.

Simulation has already been used in forest planning (p. 8). In fact, all former cutting budget methods can be regarded as simulation, even though the calculations were made manually. However, the use of electronic computers has completely changed the situation. Before the advent of computers, only very few and limited calculations were practicable. Today, a number of sophisticated cutting budgets are effected with ease.

212 Decision-making rules

2121 Analyzing investment decisions

One of the key problems in simulation is that of determining the policy to be followed as decisions are made during the simulation process. The aim of this decision-policy is to make the simulation model more automatic (TÖRNQVIST 1963, p. 8), and to reduce the number of simulation operations required to find a satisfactory near-optimum configuration (p. 14).

Simple rules need to be found which will indicate the optimum allocation of growing stock at a given point of time, in order to maximize the value of the forest undertaking. The decisions based upon these rules are so-called automatic decisions (TÖRNQVIST 1963, p. 8), since the decision is always the same when the decision-making situation is the same. Naturally, these rules apply only to the old forest stands, as there is only one cutting schedule for young stands on each site (p. 11).

If the land and growing stock are regarded as capital, methods used in capital budgeting are available to find the decision-making rules for optimum allocation of the growing stock. The methods applied in evaluating investments are founded upon the use of two basic concepts: the present-value and the internal rate of return (cf. HONKO 1963, pp. 80—88; BIERMAN and SMIDT 1966, pp. 25—31).

As the present-value method is used, it is assumed that an unlimited amount of capital is available at a given rate of interest. Investments are ranked in accordance with the net present-value of the investments calculated as a difference between the present value of the proceeds and the present value of the outlays. The investment is accepted if the net present-value is positive, and rejected if negative. If several mutually exclusive investments exist the one chosen is that giving the highest net present-value, provided its net present-value exceeds zero.

As the internal rate of return is used, it is assumed that the amount of capital is limited. The problem is that of finding the investment objects with which the limited capital gives the highest yield. To solve this problem, the internal rate of return must be calculated for each investment alternative. The internal rate of return is the rate of interest which makes the net present-value of the investment zero. Ranking of the investment alternatives is done in accordance with the internal rates of return.

The amount of capital available determines the limit between the accepted and rejected investments. If there are two or more mutually exclusive investments, then generally the one chosen is that which yields the highest internal rate of return. However, in some cases a more detailed analysis is needed (BIERMAN and SMIDT 1966, p. 48).

As the amount of land and growing stock is limited in accordance with the planning situation (p. 8), the internal rate of return seems a better indicator for evaluation of the different allocations of growing stock. Difficulties in the use of the internal rate of return described by BIERMAN and SMIDT (1966, p. 48) also arise in this study. However, it was decided to use the internal rate of return, since the merits appeared to outweigh the demerits. Moreover, the present-value method has been suggested in forest planning by several authors (cf. JÖRGENSEN and SEIP 1954; POSO 1965; JOHNSTON, GRAYSON, and BRADLEY 1967, pp. 108—132).

The allocation of growing stock among forest stands can be changed by cuts. The problem is consequently how to determine the part of growing stock to be cut in one year to obtain given net cash receipts, and to maximize the value of the forest undertaking after cutting. As this is done on each occasion cuts are made during the planning period, the hoped for result is that the value of the forest undertaking will be maximized at the end of the planning period. Naturally, it may happen that an action which seems advisable at a certain moment is not the best one when the whole planning period is considered. However, it is part of the nature of simulation that the over-all behaviour of the model cannot be kept under control (p. 14). It is therefore not very important theoretically which method of investment evaluation is applied with simulation, as neither of them is really valid in a complex situation (WEINGARTNER 1963, p. 27).

The limit between the part of growing stock to be cut, and that to be left for future growth, is determined by means of the internal rate of return. The internal rate of return that divides the growing stock into parts to be cut and those to be left, so that the required net cash receipts are obtained exactly, is referred to in this study as the »Internal Rate of Return». Since cuts consist of both thinnings and clear cuttings, use of the internal rate of return is somewhat complicated. There are internal rates of return which correspond to both thinning and clear cutting activities. In the case of thinning, only the growing stock is considered as invested capital, but for clear cutting, both land and growing stock constitute the invested capital (KILKKI 1968).

The internal rate of return expresses the marginal efficiency of the capital. Marginal analysis has already been used in the determination of optimum growing schedules for forests. NYYSSÖNEN (1958) and DUERR (1960, pp. 134—135) have shown how to determine the optimum rotation for the fully regulated forest given a guiding rate of interest. The same approach can be made for a forest stand: the optimum rotation is found at the point where the difference between the current net value growth and the rent of the growing stock falls below the average soil rent (KILKKI 1968).

Marginal analysis has also been used for determination of the optimum density of growing stock in selection forests (DUERR and BOND 1952). There, the optimum density corresponds to the level of growing stock where the cost of adding one value unit of timber equals the increase in the value growth.

2122 *Optimum density of growing stock*

In this study, the rough assumption is made that the optimum density of an even-aged stand can be determined by the same method as that applied in selection forests (DUERR and BOND 1952). It is thus assumed that the growth of an even-aged stand reflects only the present situation, and not the history of the stand (KILKKI 1968). With the data given in chapter 122, information is obtainable on the value growth of the stand. By interpolation, unit values of growing stock can be arrived at for each year from Table 2. This relationship can be denoted:

$$U = f(t, B) \tag{4}$$

where

U = unit value of growing stock, Fmk/ha.
t = age, years
B = site

By combining functions (3) and (4), a value growth function is obtained:

$$Z = (v + I_v)\, f(t + 1, B) - v\, f(t, B) \tag{5}$$

where

Z = current value growth, Fmk/ha./year
v = volume, cu.m./ha.
I_v = current volume growth, cu.m./ha./year

The optimum density of the growing stock is derivable from the value growth function (5). When the first derivative of this function is taken, and $\frac{dI_v}{dv}$ from function (3) is introduced, the derivative becomes:

$$\frac{dZ}{f(t, B)dv} = \frac{f(t+1, B)}{f(t, B)}\left(1 + \frac{I_v}{v} - 0.003014\, I_v\right) - 1 \tag{6}$$

which gives the internal rate of return from growing the stand for one more year, as only thinning is possible. When the internal rate of return under assumption of thinning equals the Internal Rate of Return, the density of the growing stock is at the optimum. However, no simple mathematical equation was derivable for calculation of the optimum growing stock, and accordingly the optimum growing stock levels were determined by computer iterations; some of the results of these are presented in Figure 2.

Some peculiarities are apparent in Figure 2. The optimum density at the same stand age is generally greater on poor sites than on good ones. This is partly attributable to the volume growth function (3), which does not exhibit any difference between the growth intensity on poor sites and that on good sites. The reasons for this phenomenon have been more widely discussed in the paper by KUUSELA and KILKKI (1963, pp.31—32).

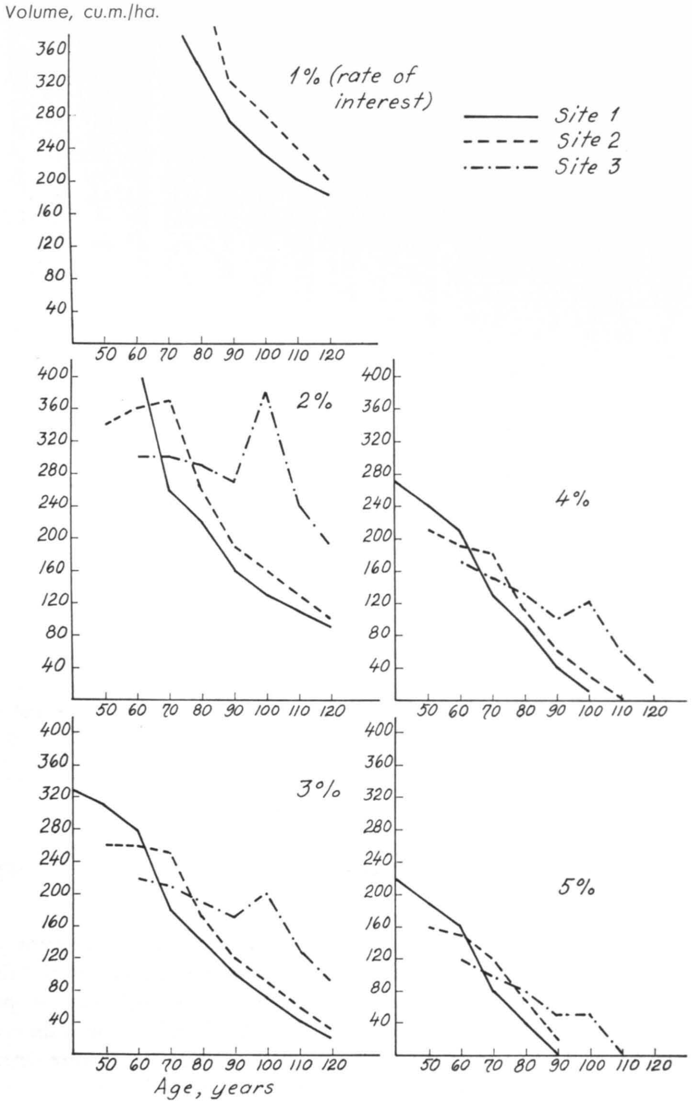


Figure 2. Optimum density of growing stock.

Another explanation may be that the influence of the density on the unit value function (4) has not been taken into account. Nevertheless, it is possible that as a consequence of different stages of development on different sites, the optimum density on poor sites is in fact higher than that on good sites of the same age.

Another peculiarity in Figure 2 is that in general the optimum growing stock decreases with increase in age. Mathematically, this is dependent upon the form of the volume growth function (3), as if it is assumed that the unit value of growing stock is independent of age, then the younger the forest the higher is the optimum growing stock figure derived by function (6). This is true for all rates of interest greater than zero. An increase in unit values according to age causes some exceptions to this rule. The question of whether the growth function is wrong, or the growth phenomenon really follows this kind of pattern is an interesting one; it was not studied more thoroughly in this work. However, some evidence is to be found of a possible diminution in the optimum growing stock as the age increases. CHAPPELLE and NELSON (1964) have noticed the same phenomenon in the use of marginal analysis for determination of the optimum growing stock. Moreover, by inference it is possible to say that the optimum growing stock will be zero in a forest stand which is old enough, as at a given age it is impossible to achieve even a value growth percentage which is as high as the marginal rate of return required. This will certainly happen even though the growing stock is reduced to the minimum to stimulate the growth rate. Another thing, of course, is that the forest stand may be clear cut before the density gets very low. Reasons that justify clear cutting are discussed in the following section.

As the cost of thinning per unit volume cut increases on reduction of the thinning removal, the growing densities indicated by the figures drawn from function (6) cannot be followed in practical forestry. Consequently, a minimum thinning removal is introduced (p. 13). Thus, it is assumed that as the density of the growing stock exceeds the optimum by the amount of the minimum thinning removal, the economic loss resulting from the reduced growth rate exceeds the extra cost involved in thinning. Of course, this method is no more than approximate, and more research would be required for determination of how the optimum stocking level does in fact depend upon the stand characteristics and logging costs.

2123 Financial maturity

In the use of marginal analysis for determination of the maturity of a forest stand, the test criterion (internal rate of return) is expressed by the following formula:

$$p = \frac{Z_t - C_t}{V_{t-1} + L} \, 100 \qquad \text{(KILKKI 1968)} \qquad (7)$$

where p = internal rate of return under assumption of clear cutting
Z_t = current value growth of a stand t years old, Fmk/ha./year
C_t = annual costs in a stand t years old, Fmk/ha./year
V_{t-1} = stumpage value of a stand t-1 years old, Fmk/ha./year
L = soil value, Fmk/ha.

When p equals the Internal Rate of Return, t expresses the age of financial maturity. This is true of stands beyond a critical age. In very young stands, p may be even negative, as the value growth is less than the costs. Moreover, if great variations exist in the value growth in old stands, formula (7) may not give correct answers. In such cases, p should be calculated as an internal rate of return for a number of years. Also, if the cutting interval is relatively long, p should be calculated for more than one year.

One of the key problems is concerned with determination of the soil value (L). If it is a matter of a fully regulated forest model, the soil value should be calculated by dividing the average soil rent of the fully regulated forest by the guiding rate of interest. For a single stand, the soil value should be determined by the Faustmann formula (cf. Endres 1923, p. 58; Keltikangas 1947, p. 37). In practice, however, financial maturities determined by means of either of these soil values are quite similar (Nyyssönen 1958, p. 21).

Theoretically, the appropriate level of the soil values employed in determination of the financial maturity should correspond to the level of the Internal Rate of Return, which again depends on the planning situation, and may vary over the planning period. It is consequently uncertain whether the soil values should also vary, or remain constant. This uncertainty is one of the difficulties in the use of the internal rate of return (p. 16). However, in some experiments with the simulation model it appeared that the use of either constant soil values, or exact following of the Internal Rate of Return in their determination did not induce any noteworthy change in the results. Nevertheless, it is important that as constant soil values are used, they should be at the correct level, as this determines the ratio between the thinnings and clear cuttings.

The soil value indicates the value ascribed by the forest owner to the income in the distant future. If the forest owner, for instance, wants simply to maximize the stumpage value of the growing stock at the end of the planning period, and the planning period is relatively short, the appropriate soil values are close to zero.

It is important that the decision whether to clear cut or not is made only after the stand has been thinned to the optimum density indicated by function (6). Otherwise, the internal rate of return under an assumption of clear cutting (formula 7) may show that the stand should be clear cut, although the low growth rate depends upon too great an amount of growing stock (Kilkki 1968).

To acquire information on the soil values on different sites, the Faustmann formula was applied, and the following procedure employed in calculating harvesting incomes.

In young stands, harvesting incomes were arrived at by direct application of the yield and unit value tables (Tables 1 and 2). Old stands were grown in accordance with volume growth function (3). When the volume of the growing stock was 50 cu.m./ha. higher than the optimum density (Figure 1), the stand was thinned to the optimum density. Rates of interest 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, and 5.0 per cent were used. The same rate of interest was employed both in discounting the incomes and costs, and in determination of the optimum densities.

These calculations were made for all three sites. The maximum soil values and the corresponding rotations are given in Table 3. Chappelle and Nelson (1964) employed a similar method for calculation of the soil values.

Table 3. Maximum soil values and the corresponding rotations.

Rate of interest, per cent	Soil value, Fmk/ha.			Rotation, years		
	Site					
	1	2	3	1	2	3
0.5	24330	17560	7910	100	120	120+
1.0	9740	6710	2850	95	110	120+
1.5	5240	3470	1340	90	105	120+
2.0	3190	2040	690	90	100	120+
2.5	2080	1290	380	85	95	120+
3.0	1430	850	210	80	90	120+
3.5	1010	580	120	80	90	120
4.0	730	410	70	75	85	115
4.5	540	290	40	70	80	110
5.0	400	210	10	70	80	95

2124 Combined use of thinning and clear cutting

If the previous concepts of the optimum density and financial maturity are combined, it is possible to derive automatic decision-making rules that indicate how forest stands should be treated in a given situation.

If consideration is given only to cutting and reforestation activities, only one of the following operations can be carried out in a forest stand at a given moment (Kilkki 1963):

1. Leave for future growth
2. Thin, leave for future growth
3. Harvest and regenerate

The following method can be applied for determination of which of these three treatments should take place. This method also indicates how much to thin in case 2.

First, there is determined the Internal Rate of Return that results in the appropriate division between the part of the timber to be cut, and that to be grown forward on the forest area (p. 16). Thus, the situation in the forest before cutting and the *NCR* function (1) together determine the Internal Rate of Return. An iterative procedure which yields an estimate for this appropriate Internal Rate of Return is discussed in section 213. Following this, all the stands are examined. If the Internal Rate of Return is higher than the internal rate of return under the assumption of thinning (function 6), the stand will be thinned. If the internal rate of return under the assumption of clear cutting (formula 7) after possible thinning is higher than the Internal Rate of Return, the stand will be left for future growth; if lower, the stand will be regenerated.

An example of the limits between the three possible treatments of the stand is presented in Figure 3. The site is 2, and the Internal Rate of Return 3 per cent. The soil value employed in calculation of the internal rate of return under the assumption of clear cutting also corresponds to a rate of interest of 3 per cent (Table 3). If, for example, there is an 80-year-old stand 250 cu.m./ha. in volume, the stand should be thinned. The

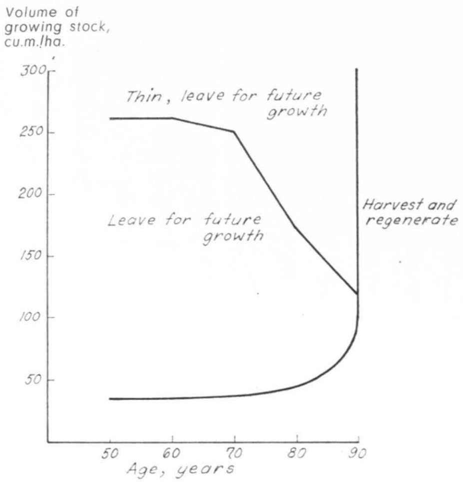


Figure 3. Determination of treatment of the forest stand. Site 2, rate of interest 3.0 per cent.

growing stock after thinning equals the optimum growing stock, i.e. 170 cu.m./ha. and the stand is left for future growth.

Probably, it would have been more realistic to choose another, more traditional growing pattern than that indicated by function (6). However, the growing densities obtained by function (6) were employed, because only then it is possible to make an objective comparison between the simulation and the linear models. If other growing densities than these had been used in the simulation model, this would have favoured the linear programming approach.

213 Simulation model

A simulation model for cutting budgets was prepared with FORTRAN IV programming language.

The input information comprises (1) data describing the original situation in the forest, (2) growth data and the necessary technical coefficients, (3) specifications for the decision-making rules, (4) *NCR* function (1) for the whole planning period, and (5) specifications for calculating the value of the forest undertaking at the end of the planning period.

At the beginning of the first 5 year period, there is calculated the value of the thinning removal from the young forest stands. The total net cash receipts requirement, minus this entity, gives the net cash receipts requirement from the old stands. The next step is that of finding the Internal Rate of Return which yields a combination of cutting activities (p. 16) such that the required net cash receipts are derived. The following iterative procedure is used to determine this Internal Rate of Return.

First, a rate of interest of 0.5 per cent is tested. All the old stands with a growing stock exceeding the optimum are thinned to the optimum density if the thinning removal exceeds the minimum removal. The forest stands for which the internal rate of return under the assumption of clear cutting after possible thinning is less than 0.5 per cent are clear cut and regenerated. The reforestation costs are subtracted from the timber selling incomes. If the difference covers the net cash receipts requirement from the old stands, this indicates that the appropriate Internal Rate of Return lies somewhere below 0.5 per cent, and this figure is employed as an estimate for it. If the net cash receipts requirement is unsatisfied, the rate of interest is increased in 0.5 per cent increments until the required net cash receipts are arrived at. The last rate of interest is considered as an estimate of the Internal Rate of Return.

If only the above procedure is applied, it is impossible to obtain the exact net cash receipts required to satisfy the *NCR* function; they always exceed the requirement. This occurs because the Internal Rate of Return is determined only to an accuracy of 0.5 per cent, and because the number of forest stands is limited. To arrive at an exact figure for the net cash receipts required the following procedure is applied.

After the Internal Rate of Return has been determined to an accuracy of 0.5 per cent, the old forest stands are so ranked that the stand with the lowest internal rate of return under the assumption of thinning comes first, and the stand with the highest internal rate of return last. Old stands are thinned to the optimum density in a given order if the possible removal exceeds the minimum removal. A check is made after the thinning of each stand to find whether the net cash receipts requirement is fulfilled. If not, the next stand on the list is thinned. This procedure is continued until all the stands have been thinned, or the net cash receipts requirement is fulfilled. If the net cash receipts requirement is satisfiable by thinnings, the harvest from the last stand to be thinned is reduced to meet the requirement exactly.

The unsatisfied net cash receipts requirement left after thinnings is covered by revenues from clear cuttings. Old forest stands are so ranked that the stand with the lowest internal rate of return under the assumption of clear cutting comes first, and the stand with the highest internal rate of return last. This ranking is applied until as many stands are clear cut as are necessary to meet the rest of the net cash receipts requirement. The last stand to be cut is always divided in two, and only one of these parts will be clear cut, in order to follow the *NCR* function exactly.

It should be mentioned that the average annual value growth during the following 5 year period is employed in calculation of the internal rates of return.

In the previous method, priority was given to thinnings, as financial maturity can be determined accurately by means of the internal rate of return only if the stands have first been thinned to the optimum density (p. 20).

The forest stands are grown for an additional 5 years. The foregoing procedure is repeated at the beginning of the new 5 year period, applying the net cash receipts requirement given by the *NCR* function for this period. This series of steps is repeated until the planning period is over.

After computations have been made for the whole planning period, each forest stand is grown for t more years, to determine the value of the forest undertaking at the end of the planning period. The growing procedure is quite similar to that described above. However, a constant quiding rate of interest is employed throughout the whole period of calculation in determination of the stand treatment. All the net cash receipts during these t years are discounted to the end of the planning period. Moreover, the stumpage value of the growing stock after $t+5$ years is discounted, and added to the sum of all the discounted net cash receipts. The total sum is considered as the net present-value of the future incomes from the forest undertaking at the end of the planning period or the value of the forest undertaking.

This method of calculating the value of the forest undertaking implies the assumption that the sum of the net present-values of all forest stands represents the total value of the forest undertaking at the end of the planning period. This is a strong assumption which does not hold in all circumstances (SAARI 1942, pp. 18—30). However, the value of the forest undertaking obtained by this method is very practicable, and is probably an adequate estimate if the planning period is comparatively long.

214 Example

To illustrate the simulation model, a cutting budget is made for a forest area which could be described in terms of the forest model presented in section 122. The total area of the forest is 45.92 hectares; it is divided into 11 stands. A cutting budget was constructed for a 20 year period, with the assumption that the net cash receipts requirement at the beginning of each 5 year period is 14,000 Fmk. Soil values corresponding to a rate of interest of 3 per cent were used in determination of the financial maturity. The minimum thinning removal was assumed to be 50 cu.m./ha. in old stands. The period over which the future incomes were discounted in order to determine the value of the forest undertaking at the end of the planning period was 100 years. A rate of interest of 3 per cent was applied in discounting.

The complete computer output is presented in the Appendix; this illustrates the detailed development of the forest area during the planning period.

22 LINEAR PROGRAMMING

221 Principles

»The general problem of linear programming is *the search for the optimum (minimum or maximum) of a linear function of variables constrained by linear relations (equations or inequalities) called constraints*» (SIMONNARD 1966, p. 6). The algebraic formulation of the preceding definition is:

minimize (or maximize)
$$z = \sum_{j=1}^{n} c_j x_j$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j \geqq d_i \qquad i = 1, \ldots, p$$

$$\sum_{j=1}^{n} a_{ij} x_j = d_i \qquad i = p+1, \ldots, m$$

$$x_j \geqq 0 \qquad j = 1, \ldots, n$$

The above condition of non-negativity of all variables (x_j) is imposed a priori in almost all economic problems.

A program, or a feasible solution, is a set of values of the variables which satisfies all the constraints of the problem, including those of non-negativity. An optimal program is the feasible solution that optimizes (minimizes or maximizes) the linear form to be optimized. The linear form to be optimized is called the objective function. The optimum value of the objective function is called the value of the linear program (DANTZIG 1963, p. 62).

In correspondence to the minimizing (maximizing) problem above, termed the primal problem, there exists the dual problem. The dual problem which corresponds to the primal minimizing problem has the following algebraic formulation:

maximize
$$w = \sum_{i=1}^{m} d_i u_i$$

subject to

$$\sum_{i=1}^{m} a_{ij} u_i \leqq c_j \qquad j = 1, \ldots, n$$

$$u_i \geqq 0 \qquad i = 1, \ldots, p$$

$$u_i \text{ arbitrary} \quad i = p+1, \ldots, m$$

If a finite optimum program exists for the primal problem, there also exists a finite optimum program for the dual problem where $z=w$. Mathematically, the primal and dual problems are reflexive, the dual of the dual is the primal. Variables x_j in the primal problem are often termed activities and variables u_i in the dual problem shadow prices.

Linear programming has been used in forest planning by several authors (p. 8). As a rule, it has not been very difficult to formulate linear programming problems, although difficulties have arisen by reason of the magnitude of many of them. The decomposition principle (DANTZIG 1963, p. 448), for instance, has been employed to cope with these difficulties (LIITTSCHWAGER and TCHENG 1967).

222 Linear model

This section presents a linear model usable in the preparation of cutting budgets. This model is amenable to linear programming techniques.

In the linear model, the activities consist of the different cutting schedules applicable to different forest stands. For each stand, there are one or more alternative activities. An activity (x_{ij}) expresses a cutting schedule j for a stand i over the whole planning period. An activity produces certain net cash receipts (a_{ijk}) at the beginning of the k'th 5 year period. The level of the activity indicates the area of the stand i treated by means of cutting schedule j.

The constraint vector comprises two types of constraints. First, there is the area constraint (b_i) for each forest stand. This means that the sum of the areas under different cutting schedules must equal the total area of the stand. Secondly, there are the constraints (d_k) expressed by the *NCR* function (1). They indicate the net cash receipts requirement at the beginning of each 5 year period.

The objective function to be maximized represents the value of forest undertaking at the end of the planning period. Each coefficient (c_{ij}) of the objective function represents the net present-value of a unit area of the forest stand i at the end of the planning period, as the stand has been treated by the application of cutting schedule j.

The algebraic formulation of the linear model is:

maximize $$z = \sum_{i=1}^{m} \sum_{j=1}^{n_i} c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^{n_i} x_{ij} = b_i \qquad i = 1, \ldots, m$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n_i} a_{ijk} x_{ij} = d_k \qquad k = 1, \ldots, p$$

$$x_{ij} \geqq 0$$

where

x_{ij} = the part of stand i treated by means of cutting schedule j, ha.
b_i = the area of stand i, ha.
d_k = the net cash receipts requirement during the k'th 5 year period, Fmk
a_{ijk} = net cash receipts during the k'th 5 year period from the part of stand i where cutting schedule j is applied, Fmk/ha.
c_{ij} = the net present-value of the part of stand i where cutting schedule j is applied at the end of the planning period, Fmk/ha.
z = the value of the forest undertaking at the end of the planning period, Fmk
n_i = the number of the cutting schedules for stand i

m = the number of the forest stands
p = the number of the 5 year periods

It should be remarked that this is not the only way of formulating the linear model appropriate to the cutting budget problem. An example of the alternative formulations is given by v. MALMBORG (1967, p. 69).

223 Input data program

A computer program was prepared for calculation of the input data for the linear programming calculations. This program resembles the simulation model as far as growth and cut are concerned, although one fundamental difference exists between the simulation model and the linear programming input data program. In the simulation model, *NCR* function (1) is fixed, and the guiding rate of interest (the Internal Rate of Return) is the dependent variable. However, in the input data program, the guiding rate of interest is fixed for each 5 year period, and the *NCR* function is the dependent variable. This method of determining the cutting schedules for the stands is extremely flexible. Almost any cutting schedule can be obtained by giving the appropriate series of guiding rates of interest.

With exception of the *NCR* function, the input for the input data program contains the same information as the input for the simulation model (p. 22). Instead of the *NCR* function, series of guiding rates of interest are given. The output consists of the matrix of a_{ijk}, c_{ij}, b_i, and unit coefficients for the linear model. The program contains a system which automatically omits from the output all a_{ijk} coefficients that equal zero. Consequently, great importance was attached to the assumption of the management costs as zero, since otherwise almost all the a_{ijk} coefficients would have differed from zero (p. 13). The coefficients (c_{ij}) of the objective function are calculated by application of the same method as that for the simulation model (p. 23). If two or more interest rate series yield exactly the same cutting schedule, only one of them is given in the computer output. On addition of the net cash receipts constraints, the output is ready for the linear programming calculations.

224 Example

An example is presented to illustrate the linear model. The problem is that of finding the optimum cutting schedule for a 10 year period for a forest undertaking consisting of two stands. The situation at the beginning of the planning period is as follows:

Stand	Site	Age, years	Volume, cu.m./ha.	Area, ha.
1	1	65	170	1.0
2	3	95	60	1.0

The method described in section 223 was applied for determination of all possible cutting schedules for both of the stands, as guiding rates of interest 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, and 5.0 per cent are employed. Thus there were 100 (=10^2) potential alternative cutting schedules for each stand. Nevertheless, it appeared that only four schedules for the first stand, and three for the second, differed from each other. The reason for so many schedules appearing to be identical is primarily attributable to use of the lower boundary for the thinning removal. This was 50 cu.m./ha., the same as that taken in the example of application of the simulation model. The possible cutting schedules are indicated in Table 4.

Table 4. Cutting schedules for the linear model example.

Forest stand	Cutting schedule	Beginning of the first 5 year period			Beginning of the second 5 year period		
		Volume before cut	Removal	Volume after cut	Volume before cut	Removal	Volume after cut
		cu.m./ha.					
1	1	170	0	170	204	0	204
	2	170	0	170	204	54	150
	3	170	0	170	204	204	0
	4	170	170	0	0	0	0
2	1	60	0	60	71	0	71
	2	60	0	60	71	71	0
	3	60	60	0	0	0	0

As an example of how the figures in Table 4 are applied in determination of the a_{ijk} coefficients, a study can be made of cutting schedule 4 for stand 1 at the beginning of the first 5 year period. The volume to be cut is 170 cu.m./ha., and its unit value is 16.9 Fmk/cu.m. (Table 2). The reforestation cost is 60 Fmk/ha. (p. 12). Thus, if this cutting schedule is employed the net cash receipts are $170 \times 16.9 - 60 = 2{,}813$ Fmk/ha.

The coefficients (c_{ij}) of the objective function were calculated by exact application of the same procedure as that used with the simulation model. For example, if cutting schedule 3 for stand 1 is used, the net present-value of the 5-year-old stand at the end of the planning period is 1,595 Fmk/ha. The soil value obtained by means of the Faustmann formula (Table 3), multiplied by the 5 year and 3 per cent rate of interest compounding factor, results in 1,657 Fmk/ha., which is almost the same.

If it is assumed that the net cash receipts for both of the 5 year periods are 500 Fmk, the linear model is:

$$6146x_{11}+4983x_{12}+1595x_{13}+1862x_{14}+1253x_{21}+272x_{22}+325x_{23} = z \text{ (max)}$$

subject to

$$\begin{aligned}
x_{11}+x_{12}+x_{13}+x_{14} &= 1.0 \\
x_{21}+x_{22}+x_{23} &= 1.0 \\
2813x_{14}+680x_{23} &= 500 \\
1017x_{12}+3777x_{13}+881x_{22} &= 500
\end{aligned}$$

The optimum solution is:

$x_{11} = 0.738$ ha.
$x_{12} = 0.262$ «
$x_{22} = 0.265$ «
$x_{23} = 0.735$ «
$x_{13} = x_{14} = x_{21} = 0$ ha.
$z = 6152$ Fmk

The situation in the forest at the end of the planning period is:

Stand	Site	Age, years	Volume, cu.m./ha.	Area, ha.
1a	1	75	238	0.738
1b	1	75	179	0.262
2a	3	5	0	0.265
2b	3	10	0	0.735

3 EXPERIMENTATION WITH THE MODELS

As has been shown in the preceding sections, it is possible to prepare cutting budgets by the means of both the simulation model and the linear model. This section discusses the results of some experiments made to test the efficiency of these two models.

Ten cutting budgets, prepared with both the simulation model and the linear model, were constructed for the same model forest as that in the simulation example (p. 24). The net cash receipts for each 5 year period ranged from 11,500 to 16,000 Fmk in 500 Fmk increments. In each cutting budget, the net cash receipts requirement was assumed to remain unchanged from one 5 year period to another. The planning period was 50 years. The minimum thinning removal, discounting rate of interest, soil values, and the discounting period for calculating the value of the forest undertaking at the end of the planning period, were the same as those in the example in section 214.

Cutting budgets were first prepared with the simulation model. After these calculations, all the cutting schedules the simulation model had used were available. In terms of linear programming, these cutting schedules are feasible solutions (p. 25). Since there exists an infinite number of possible cutting schedules for the linear model, it is necessary to select by some means the cutting schedules taken along with the model. Accordingly, it was decided that the ten cutting schedules for each forest stand obtained from simulation, would be taken as the principal set of activities for the linear model. The simulation output contains ten series of Internal Rates of Return (see Appendix) that correspond approximately to these ten cutting schedules. By the addition of 26 other alternative series to these ten series, a group of altogether 36 series of guiding rates of interest was arrived at.

By employment of the 36 series of guiding rates of interest as an input for the input data program, a group of 149 alternative activities was obtained for the eleven forest stands. Thus, almost two thirds of the potential cutting schedules ($36 \times 11 = 396$) appeared to be identical with one or other of the 149 schedules. The set of 149 activities was employed on preparation of ten cutting budgets by means of the linear model.

The values of the forest undertaking at the end of the planning period, corresponding to the different *NCR* functions (1) and to both of the planning methods, are presented in Table 5. The main conclusion is that the two methods give results which are almost identical. The largest differences are 3.72 per cent in favour of the simulation model, and 0.96 per cent in favour of the linear model. In comparison with the

Table 5. Values of the forest undertaking at the end of the planning period, Fmk.

Net cash receipts requirement, Fmk/5 years	Method		Difference	
	Simulation	Linear programming	Fmk	Per cent
11500	240347	241950	+1603	+ .67
12000	230632	232857	+2225	+ .96
12500	222926	222993	+ 67	+ .03
13000	211544	212582	+1038	+ .49
13500	200965	201926	+ 961	+ .48
14000	190651	191103	+ 452	+ .24
14500	179614	179579	— 35	— .02
15000	167415	166889	— 526	— .31
15500	154512	152691	—1821	—1.18
16000	140883	135638	—5245	—3.72

linear model, the simulation model gives results which are slightly inferior, if the net cash receipts requirement is low, and results which are slightly superior, if the requirement is high. This phenomenon is easily explicable. When the net cash receipts requirement is high, it is difficult to find even a feasible solution for the linear programming problem, in view of the limited number of feasible solutions. If more alternative cutting schedules had been available when the net cash receipts requirement was high, linear programming would have provided better results. Theoretically, linear programming in this problem can always give at least as good results as simulation does. This could be accomplished, for instance, by the introduction of exactly the same cutting schedules as those used in the simulation model into the linear model.

The total computer time required for the simulation was 52 seconds (IBM 7094). The corresponding time required for calculation of the input data and the linear programs was 2 minutes and 20 seconds. On the other hand, the time required to construct the simulation model is much longer than the programming time for the linear model. When the linear model is used, it is necessary to prepare only the relatively simple input data program, as library programs are available for ordinary linear programming calculations.

If it is assumed that the computer programs are ready, the simulation model seems slightly preferable to the linear model in the previous example. However, some questions arise. First, is it possible to make any marked improvement in the results of either the linear model, by adding new activities, or the simulation model, by using other soil values? Secondly, is it possible to find a good set of activities for the linear model without use of the information provided by the simulation model?

The answer to the first question is probably »No». This is suggested since the optimum solutions of the linear model tended primarily to select the activities used by the simulation model, as the *NCR* functions were the same. It is thus evident that there does not exist an unused set of activities which would dramatically improve the linear programming results. Naturally, it is possible, as was mentioned above, to find the activities which yield results at least as good as those from the simulation model. Also, it might have been possible to improve the results of the linear programming a little by dividing the largest forest stands into smaller units, as then more alternative activities would have been available in the optimum solution.

The fact that the linear model could not in any event provide a value for the forest undertaking more than one per cent higher than that given by the simulation model, indicates moreover that few possibilities exist for improvement of the simulation results by changing the soil values on determination of the financial maturity. However, some improvement might be possible, as evidently the same soil values do not apply as efficiently to all levels of the *NCR* function.

The second question was explored, but for only one *NCR* function. The net cash receipts requirement was 14,000 Fmk in 5 years, and two sets of activities were employed. One set was based upon the cutting schedule used in the simulation model, as the *NCR* function was the same. Altogether, 77 activities were then available for the eleven stands. The other set of activities was obtained by the employment of ten series of constant guiding rates of interest in determination of the cutting schedules. In the latter case, the guiding rates of interest ranged from 0.5 to 5.0 per cent in 0.5 per cent increments. There were accordingly ten potential cutting schedules for each stand. However, only 96 schedules differed from each other.

Although the number of possible activities was slightly greater when the »artificial» set of activities was used (96 against 77) the set of activities based upon the simulation results gave appreciably better results. The value of the linear program in the latter case was 190,322 Fmk, or only slightly lower than the 191,103 Fmk obtained by use of the set of 149 activities. The value of the linear program when the »artificial» set of activities was used was only 170,850 Fmk, or more than ten per cent less. On comparison of this difference with the almost negligible differences of the results from the simulation model and the linear model in the case of ten cutting budgets (Table 5), it is evident that employment of the simulation results in the design of the linear programming activities had been of great assistance. This example indicates that the linear programming approach without a priori knowledge of the cutting schedules either requires a great number of activities, or leads to a result far from the optimum.

4 ANALYSIS OF DUALITY

Analysis of the dual linear programming problem corresponding to the original primal problem, or the linear model, provides an interesting insight into the cutting budget. There are two types of dual variables or shadow prices in this dual linear programming problem: (1) those corresponding to the forest stand area constraints and (2) those corresponding to the net cash receipts constraints.

Economic interpretation of the shadow prices has been widely discussed, inter alia by Dorfman, Samuelson, and Solow (1958) and in a forestry application by v. Malmborg (1967, pp. 76—82). Here, a forest stand shadow price expresses the marginal increase in the value of the linear program if one hectare of the similar stand is added to the forest undertaking while all the other conditions remain unchanged. A net cash receipts shadow price expresses the marginal diminution in the value of the linear program if the net cash receipts requirement of the 5 year period is increased by one Fmk, while all the other conditions remain unchanged.

The following discussion is based upon the optimal solutions for the ten dual problems which correspond to the ten primal problems presented in section 3.

The forest stand shadow prices for all eleven stands and all ten *NCR* functions (1) are indicated in Figure 4. It is apparent that the better the site is, the higher is the shadow price. The 15-year-old stand on site 2 points to the shadow prices of the very young stands being lower than those containing marketable timber. As the net cash receipts requirement increases, so do the shadow prices. The older the stand, and the better the site, the greater is the increase. This illustrates that when good sites and old forests are available, the cuts can be increased without endangering the continuity of the forest undertaking (Kuusela 1959, pp. 14—15; Nersten 1965, p. 156).

The forest stand shadow prices also indicate that the value of the forest stand, from the forest owner's viewpoint largely depends upon the structure of the forest undertaking to which it belongs, and on the personal aims of the forest owner. Thus, the shadow prices offer a flexible tool for determination of the value of the single forest stand, and even the value of the bare area that is to be reforested and incorporated in a forest undertaking. This method might be applied in the acquisition of information on the values of small parts of a forest lot in the event of bargaining, or in determination of the profitability of different silvicultural measures to improve the yield. Nat-

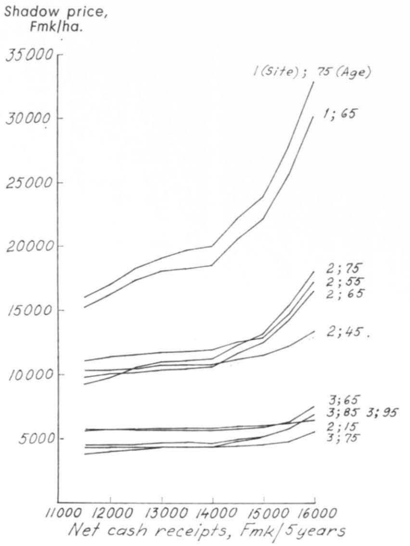


Figure 4. Forest stand shadow prices. ↑

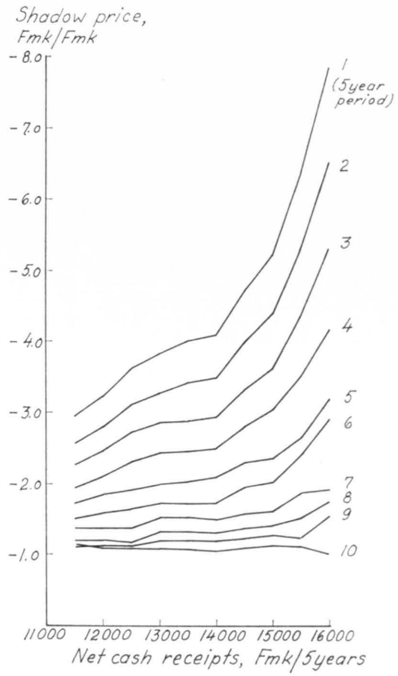


Figure 5. Net cash receipts shadow prices. →

urally, the values indicated by the forest stand shadow prices should be discounted to the beginning of the planning period to arrive at the net present-values.

The net cash receipts shadow prices for different 5 year periods and different *NCR* functions are presented in Figure 5. The earlier 5 year period is in question here, and the higher the net cash receipts requirement, the greater is the diminution in the value of the linear program induced by increase in the net cash receipts requirement. Some exceptions to these rules appear during the last 5 year period; these exceptions result from the limited number of activities. Then, the small weight of the last 5 year period makes its shadow prices vary in an irregular fashion.

The forecasts provided by the net cash receipts shadow prices for estimation of the effects of changing the *NCR* function could be tested by the real changes in the values of the linear programs. In Figure 6, a comparison is made between the real changes in the value of the linear program and the theoretical changes arrived at by means of the shadow prices. The theoretical changes were derived by multiplying each shadow price by 500 Fmk, and summing the products. Figure 6 illustrates that the theoretical changes have a close fit to the real changes. However, it seems rather difficult to make an accurate forecast of the effect of a major change in the *NCR* function by use of the shadow

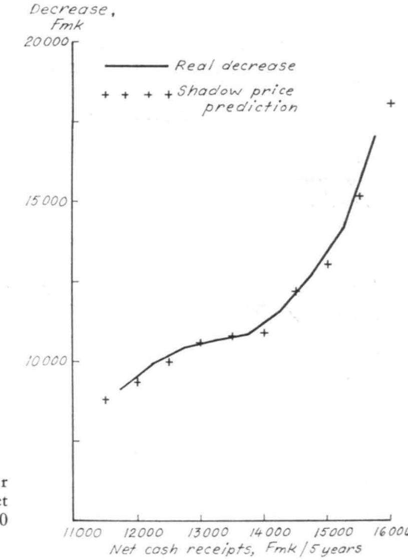


Figure 6. Decrease in the value of the linear program with increase in the net cash receipts requirement by 500 Fmk/5 years.

prices, since these also change rather rapidly with changes in the net cash receipts requirements.

An interesting relationship also exists between the net cash receipts shadow prices and the estimates of the Internal Rates of Return derived from the simulation model. A net cash receipts shadow price for a 5 year period can be regarded as a compounding factor for the rest of the planning period (Dorfman, Samuelson, and Solow 1958, p. 266). By calculating the ratio between the shadow prices of two successive 5 year periods, the compounding factor for the former 5 year period is obtained. This factor indicates the 5 year incremental rate of interest (Weingartner 1963, p. 144).

The 5 year compounding factors obtained by use of both the estimates of the Internal Rates of Return from the simulation model, and the ratios of the net cash receipts shadow prices, are presented in Figure 7. As the simulation model gives an overestimate for the Internal Rate of Return, this estimate is reduced by 0.25 per cent. Moreover, the fact that this estimate does not indicate the rate of compound interest (p. 23) has been taken into account in calculation of the compounding factors.

The correlation between the 5 year compounding factors obtained by the two methods is quite marked, as the net cash receipts requirement is below 14,500 Fmk in 5 years. As the *NCR* function exceeds this limit, the figures given by the shadow prices clearly exceed those based upon the estimates of the Internal Rates of Return. This indicates that there are insufficient alternative activities for the linear model, as the net cash receipts requirement is high (p. 31). It is consequently necessary to employ cutting schedules in which timber which would still grow quite well in the future

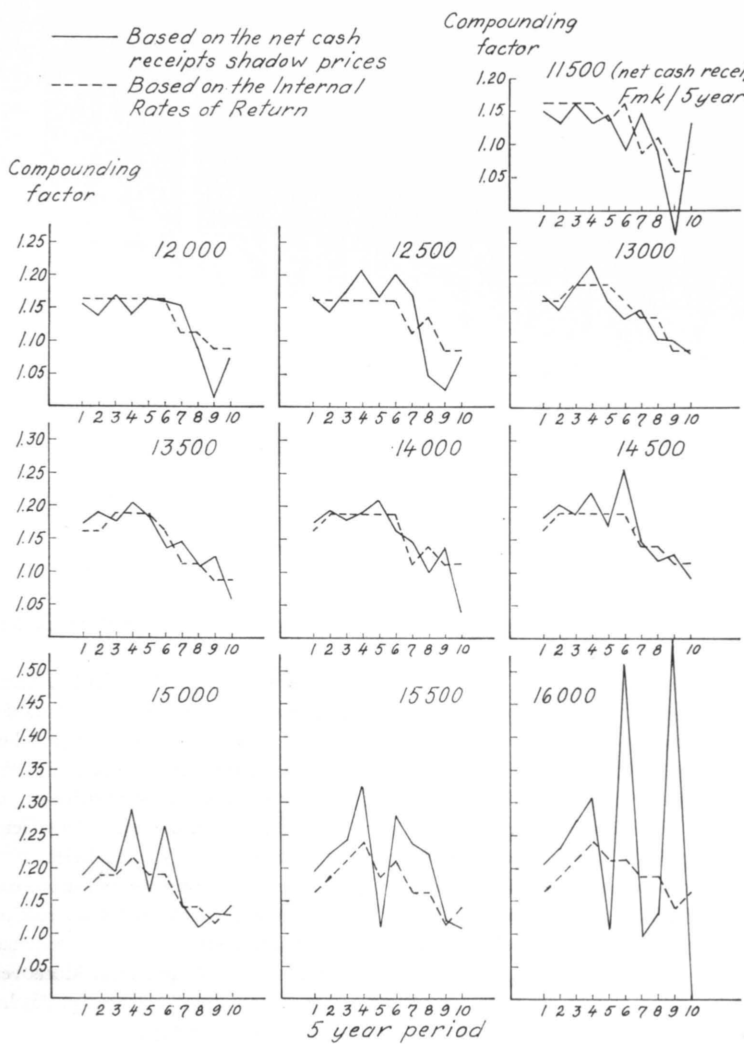


Figure 7. Compounding factors for 5 year periods.

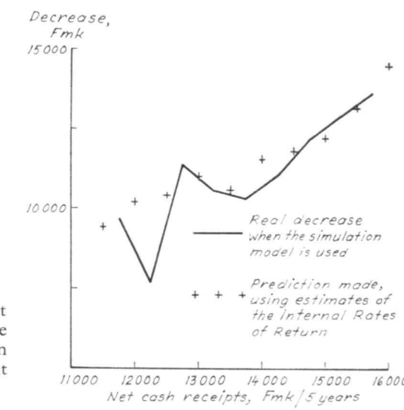


Figure 8. Decrease in the value of the forest undertaking at the end of the planning period with increase in the net cash receipts requirement by 500 Fmk/5 years.

has to be cut. Correspondingly, poorly-productive timber may remain stored, as the cutting cycle indicated by the activities does not coincide with the relative maturity of the timber. This observation is also confirmed by the marked variation in the 5 year compounding factors based upon the shadow prices, as the net cash receipts requirement is high.

The relationship between the net cash receipts shadow prices, from the linear model, and the estimates of the Internal Rates of Return, from the simulation model, also holds inversely. By use of the estimates of the Internal Rates of Return, it is possible to discover the corresponding »shadow prices». These new shadow prices are employed in estimation of the changes in the values of the objective function. Figure 8 indicates these theoretical changes, and the real changes based upon the simulation results. They are approximately on the same level, although the correlation between them is not so high as in the linear model (Figure 6). Figure 8 confirms that the method applied for estimation of the Internal Rates of Return in the simulation model gives results which are quite close to the real values.

By the agency of the differences in the values of the objective functions, the average Internal Rates of Return can be calculated during the 50 year planning period as different *NCR* functions are applied. As the fact that the net cash receipts are obtained at the beginning of each 5 year period is taken into account, the average Internal Rate of Return can be derived from the following formula:

$$D = 500 \frac{\left(1 + \frac{p}{100}\right)^{55} - \left(1 + \frac{p}{100}\right)^{5}}{\left(1 + \frac{p}{100}\right)^{5} - 1} \qquad (8)$$

where D = decrease in the value of the objective function on increase in the net cash receipts requirement by 500 Fmk/5 years, Fmk

p = average Internal Rate of Return during the 50 year planning period

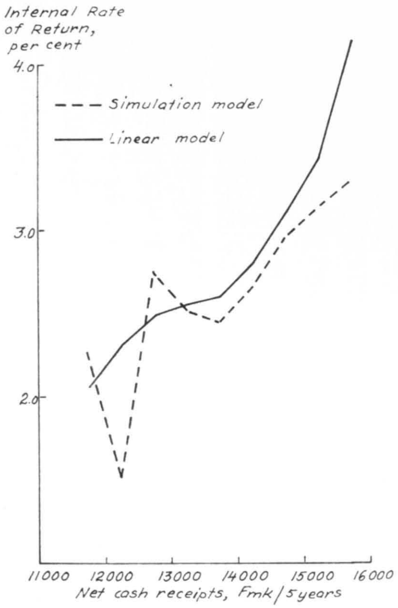


Figure 9. Average Internal Rate of Return during the 50 year planning period.

Figure 9 illustrates the average Internal Rate of Return arrived at by use of the *D*-values both from the simulation model and from the linear model. The results based upon the linear model exhibit a logical relationship between the *NCR* function and the average Internal Rate of Return. Contrary to this, the simulation results are not so logical. This indicates that the same decision-making rules do not apply as efficiently to all *NCR* functions. It might be recommendable to make new cutting budgets with the simulation model, using the soil values for each *NCR* function indicated by the average Internal Rate of Return.

The Internal Rates of Return for the 5 year periods can, of course, be employed for a more detailed analysis. Figure 7 indicates that a decrease in the net cash receipts during the third, fourth, and fifth 5 year periods would increase the value of the objective function quite rapidly from a relative standpoint. On the other hand, an increase in the net cash receipts at the end of the planning period might be motivated by the low Internal Rate of Return.

The value of the dual variables in the linear model, and similarly the value of the estimates of the Internal Rates of Return in the simulation model, lie in the information they provide to the forest owner on the profitability of different cutting schedules. They also give a dynamic aspect to the planning problem, although other tools such as parametric linear programming (cf. SIMONNARD 1966, pp. 138—158) are needed for more detailed analyses of the effects of changes in the planning situation.

5 DISCUSSION

For analysis of the applicability of the models prepared in this paper to practical cutting budget problems, the following questions need to be answered: (1) Is the planning situation described at the beginning of section 121 relevant to the situations that are generally involved? (2) Is it possible to describe the forest area adequately in terms of the variables presented at the beginning of section 122? (3) Is it possible to prepare cutting budgets of practical size by application of the models presented in this paper? (4) Is the information provided by the cutting budgets adequate for the evaluation of different *NCR* functions, and implementation of the cutting plan?

Several aspects other than those mentioned in section 121 (p. 8) could be taken into consideration in cutting budgets. In terms of linear programming, the problem is: what is the objective function, what are the activities, and what are the constraints? Clearly, the planning situation used in this study offers no more than a basis for more detailed analyses. A number of other activities could be added, such as fertilization and draining. Furthermore a number of other constraints could be employed, such as the maximum annual regeneration area, amount of manpower, and machines, and the regeneration material available. The important aspects of risk and uncertainty could also be included, to make the models more realistic.

Secondly, a study is made of the relevance of the forest model presented in section 122. Naturally, the model should be extended to many more sites and other tree species; this would not introduce any theoretical difficulties, since all the procedures would be analogous to those used in this study. The main difficulty is that of finding accurate data for all the different types of stands. Mixed forests in particular will give rise to difficulties in this respect. Moreover, difficulties will appear if the stands are not even-aged as has been assumed. The age of the stands does not then change chronologically, but may be affected by cuts (NYYSSÖNEN 1962). It should also be noticed that the cutting budget can at most be only as accurate as the data upon which it is based.

The assumption that a forest stand is divisible into parts (p. 11) does not hold good in a real world situation, but provides an adequate approximation if the original stands are very large, or represent classes of similar stands. Integer programming (cf. DANTZIG 1963, 514—550) might be used if the indivisibility of the stands is regarded as a necessity.

Another, perhaps more serious handicap is that the location of the stand does not restrict its treatment (p. 11). Thus, the cutting budget may result in a situation where a very small stand should be treated in a way totally different from the surrounding stands. To solve this problem, the study should be extended to analysis of the logging and silvicultural costs. If the stands are not assumed to be mutually independent, linear programming cannot be used, since then neither the assumption of additivity, nor the assumption of linear objective function holds (Dantzig 1963, p. 33).

If these practical aspects similar to the minimum thinning removal (p. 13) had been taken into account in formulation of the forest model, the value of the objective function would have diminished. Thus the value of the objective function in the models concerned in this study represents the theoretical maximum, and will decrease as practical constraints, such as were also mentioned in analysis of planning situation, are taken into account.

The third question was concerned with the magnitude of the problems solvable with the models presented in this work. As the simulation model is employed, even a small computer (IBM 1620) can handle a cutting budget for several decades in a relatively short time, as the forest area comprises less than 100 forest stands. Problems of the same size are solvable by linear programming if large computers are available. Application of the decomposition principle (p. 25) changes the situation, and linear programming may then be practicable even with small computers. It should be mentioned that in many cases it does not pay to include all single stands in the calculations, but it is practical to form relatively few classes by the combination of similar stands. This is often even necessary, since no reliable information exists on the single stands.

To answer the fourth question, it is necessary to analyze the output yielded by the cutting budget models. The Appendix indicates the output given by the simulation model; similar information is obtainable from the linear model. It is accordingly possible to follow the detailed development of the forest area in terms of volume, stumpage value, volume and value growth, and age class structure. If any of these factors appears to be critical at some stage of the planning period, the forest owner can notice it. Similarly, the cutting budget gives a detailed guide-line on how to allocate cuts among different stands during the planning period.

The Internal Rates of Return in the simulation model, and the net cash receipts shadow prices in the linear model, also offer information on the profitability of a cutting budget. Of course, if more detailed information is needed, several cutting budgets have to be prepared.

In conclusion, it can be said that no serious obstacles exist to application of the models developed in this study to practical forest planning. Of course, this presumes that the forest owner can at least approximately approve the assumptions upon which the models are based.

There still remains the question: which is the most advisable cutting budget method in practice? Theoretically, linear programming is superior to simulation. The decision-making rules used in the simulation model can take into account only the merits of the individual projects, but linear programming also includes their interrelationships (Weingartner 1963, p. 27). However, it seems to be difficult to get good results if linear programming alone is employed. This is due to the difficulty in finding an appropriate, but relatively limited set of activities. The capacity of the computer at hand is also of great importance. A small computer and expensive computer time evidently speak in favour of the simulation approach. Against this, linear programming is much more versatile if new constraints or other modifications need to be added to the model. Nevertheless, it is an open question whether the decision-making rules used in the simulation model are applicable to a situation in which the objective function is defined differently from that in the examples given in this paper. Probably, proper rules are discoverable even for very different situations, but a great deal of experimentation may be required.

The cutting budget method which might be most advisable consists of the combined use of simulation and linear programming. Simulation could be employed to find a rough cutting schedule, and linear programming to test how good this solution is. Nonetheless, the linear programming results can be applied for improvement of the decision-making rules used in the simulation model. The question at issue is concerned with the marginal benefits and costs which determine how far this feed-back system is allowed to run.

It should be noticed that only one linear model has been tested. As was mentioned above (p. 27), other alternatives exist for designing this model. Principally, the question is whether the number of variables, or the number of equations should be increased to improve the optimal solution at least cost. The answering of this question would require many more calculations than those made in this study.

The linear model might also be solved by the use of dynamic programming techniques. This approach would have the advantage that forest stands could be managed as indivisible units. It might also require less computer capacity than linear programming. Contrary to linear programming, no library programs probably exist for this special case of dynamic programming, and consequently more programming work would be required.

It might also be possible to turn the problem around. The objective function to be maximized would then correspond to the *NCR* function, and the value of the forest undertaking at the end of the planning period would just be one constraint. Probably, the models described in this study can be used in this new problem, since a clear correspondence is discernible between the level of the *NCR* function and the value of the objective function (Table 5).

Nevertheless, it is uncertain whether the assumption that the whole equals the sum of its parts holds good in determination of the value of the forest undertaking (p. 24). It might be worth-while, for instance, to try to apply quadratic programming. The possible effects of the age-class structure, for example, could be taken into account, at least to some extent. The objective function to be maximized might then represent the greatest permanent allowable cut after the planning period. Regression analysis, based upon a number of cutting budgets might be the method for calculation of this function (Nersten 1965; Kilkki 1966; Roiko-Jokela 1966). Furthermore, the age-class structure, or the structure of the growing stock, can be regulated by adding new constraints to the simulation and linear models. The calculated maximum value of the forest undertaking at the end of the planning period decreases, but the age-class structure or the structure of the growing stock is more desirable.

6 SUMMARY

The aim of this study was that of developing cutting budget methods for a forest undertaking. Cutting budgets are regarded as a part of the decision-making process the forest owner must pursue as he runs his undertaking. It provides information on the future income from the forest undertaking, and on the development of the forest. By the use of cutting budgets, the cutting plan can be derived which best corresponds to the expectations of the forest owner, and can also be effected in practice.

The cutting budget models developed in this study have been based on the assumption that the main interest of the forest owner lies in the income derived from his forest undertaking. This has been expressed by a net cash receipts function that expresses the net income the forest owner wishes to acquire from his forest undertaking in each year to come. For practical reasons, the net cash receipts requirements are specified only in 5 year periods. As it is neither possible nor necessary in practice to prepare cutting budgets for an infinitely long time in the future, the net cash receipts function is specified only for a limited time, termed the planning period. The length of the planning period is optional. The aim of the cutting budget is that of defining the cutting schedule which satisfies a given net cash receipts function during the planning period, and maximizes the value of the forest undertaking at its end. The value of the forest undertaking is calculated as the sum of the net present-values of the future income from the forest stands at the end of the planning period.

Two cutting budget models have been developed, by the application of simulation and linear programming. Both of these models are deterministic in nature, i.e. there is only one possible outcome once the stated input information has been given. To make the cutting budget models simpler, it has been assumed that thinning and clear cutting with reforestation are the only activities that can occur in the forest. A single forest stand, or a relatively homogeneous group of stands, constitutes the calculating unit. The cutting budget models are directly applicable only to forests consisting of even-aged Scotch Pine stands on three different sites. However, the models can easily be extended to cover forests comprising several tree species and many more sites.

Automatic decision-making rules have been developed for the simulation model; their purpose is that of attempting to allocate the cuts implied by the net cash receipts function, so that the value of the forest undertaking after cut is as high as possible. The decision-making rules are based upon the use of the internal rate of return in determination of both the optimum growing density and the financial maturity. In each 5 year period, two internal rates of return are determined for each forest stand. One of them indicates the thinning order and the other the clear cutting order. The rate of return which divides the growing stock into the part to be grown forward, and that to be cut to cover the net cash receipts requirement, is determined during each 5 year period by means of these internal rates of return. This rate of return can be regarded as the internal rate of return for the whole forest undertaking. Only one 5 year period is taken into account on division of the growing stock. As a consequence, the method is theoretically incorrect, even though quite convenient in practice. It may appear that a cutting schedule which seems best at a certain moment does not lead to the final goal, i.e. the highest possible value of the forest undertaking at the end of the planning period.

As uncertainty exists on how close to the optimum the simulation model results are, a linear model has also been developed for the preparation of cutting budgets. The objective function to be maximized represents the value of the forest undertaking at the end of the planning period, determined in exactly the same way as in the simulation model. The activities of the linear model consist of the different cutting schedules applicable to different forest stands over the whole planning period. The constraint vector comprises the areas of the forest stands and the net cash receipts requirements. It is attributable to the principles of linear programming that an optimum solution for the linear model is always discoverable, if one exists. The problems in use of the linear model arise from the limited number of variables acceptable by the computers. This involves that the calculated optimum solution may be far removed from the theoretical optimum attainable by a more successful choice of variables.

A limited number of cutting budgets have been prepared with both the simulation model and the linear model. They indicate that both models are applicable. The models gave approximately the same values for the forest undertaking at the end of the planning period if the cutting schedules for each stand obtained from the simulation model were used in selection of the variables for the linear model. If the variables for the linear model had to be selected without previous information of the appropriate cutting schedules in that planning situation, this model did not yield a value for the forest undertaking which was as high as that of the simulation model. A good result in this case would have implied a large number of variables which would again, have substantially raised computer costs.

The relationships between the simulation model and the linear model have also been studied by comparison of the internal rates of return for the whole forest undertaking in the simulation model with the shadow prices which correspond to the net cash receipts requirements in the linear model. It was then established that the ratio between the shadow prices of two successive 5 year periods bore a close correlation to the internal rate of return of the former period. This indicates that the decision-making rules employed in the simulation model approach closely to the maximization of the value of the forest undertaking at the end of the planning period. The fact that the optimal linear programs tended to use the same activities as the simulation model also confirms this observation.

In the light of this study, simulation seems today to be more appropriate than linear programming in the preparation of cutting budgets. However, the increasing capacity of electronic computers may even in the near future make linear programming quite competitive, especially as if it is borne in mind that the theoretical basis of linear programming is much firmer than that of simulation. The most advisable cutting budget method might consist of a combination of simulation and linear programming. Simulation could be employed to find a rough cutting schedule, and linear programming to test and improve this solution.

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ABBREVIATIONS

AFF = Acta Forestalia Fennica
MA = Metsätaloudellinen Aikakauslehti
MTJ = Metsäntutkimuslaitoksen julkaisuja — Communicationes Instituti Forestalis Fenniae
SF = Silva Fennica

SELOSTE:

TULOTAVOITTEESEEN PERUSTUVA HAKKUULASKELMA

Tämän tutkimuksen tarkoituksena on ollut metsälölle soveltuvien hakkuulaskelmamenetelmien kehittäminen. Hakkuulaskelma on osa siitä päätöksentekoprosessista, mikä metsänomistajan on läpikäytävä johtaessaan yritystensä toimintaa, ja se antaa kuvan metsälöstä tulevaisuudessa saatavista hakkuutuloista sekä metsälön kehityksestä. Hakkuulaskelmien avulla metsänomistaja voi määrittää hakkuusuunnitteen, joka parhaiten vastaa hänen toivomuksiaan ja joka on myös käytännössä toteutettavissa.

Nyt kehitetyissä hakkuulaskelmamenetelmissä on lähtökohtana ollut olettamus, että metsänomistajan päämielenkiinto kohdistuu metsälöstä saataviin tuloihin. Tämä on ilmaistu nettotulofunktiolla, joka ilmoittaa metsänomistajan metsälöstään vuosittain haluaman nettotulon nykyhetkestä eteenpäin. Käytännöllisistä syistä nettotulovaatimukset on määritetty vain 5-vuotiskausittain. Koska hakkuulaskelman laatiminen äärettömän pitkälle ajalle tulevaisuuteen ei ole mahdollista eikä käytännössä tarpeellistakaan, on nettotulofunktio määritetty ainoastaan rajoitetulle ajanjaksolle, jota kutsutaan suunnitelmakaudeksi. Suunnitelmakauden pituus on vapaasti valittavissa. Hakkuulaskelma pyritään laatimaan siten, että sen antamaa hakkuusuunnitetta noudatettaessa nettotulotavoitteet suunnitelmakauden aikana saavutetaan ja että metsälön arvo suunnitelmakauden lopussa on mahdollisimman korkea. Metsälön arvo lasketaan diskonttaamalla kustakin laskentayksikkönä käytetystä metsälön osasta suunnitelmakauden jälkeen tiettynä ajanjaksona saatavat hakkuutulot suunnitelmakauden loppuun ja summaamalla ne.

Hakkuulaskelmamallit on kehitetty simulointia ja viivallista ohjelmointia käyttäen. Molemmat näistä malleista ovat luonteeltaan deterministisiä, mikä merkitsee sitä, että mallin arvot tulevaisuudessa johtuvat yksikäsitteisesti sen nykyarvoista. Hakkuulaskelman yksinkertaistamiseksi on oletettu, että ainoat toiminnat, mitä metsälössä tapahtuu, ovat harvennus- ja paljaaksihakkuut sekä viimeksimainittuja seuraava metsänuudistaminen. Laskentayksikköinä käytetään yksityisiä metsiköitä tai verraten homogeenisia metsikköryhmiä. Metsälöt, joihin hakkuulaskelmamalleja sellaisenaan voidaan soveltaa, käsittävät ainoastaan tasaikäisiä MT-, VT- ja CT-männiköitä. Mallit voidaan kuitenkin helposti laajentaa koskemaan metsälöitä, joissa on useampia puulajeja ja kasvupaikkoja.

Simulointimallia varten on kehitetty automaattiset päätöksentekosäännöt, joiden avulla tulotavoitteen edellyttämät hakkuut pyritään sijoittamaan siten, että metsälön arvo hakkuun jälkeen on mahdollisimman korkea. Päätöksentekosäännöt perustuvat sisäisen korkokannan käyttöön sekä optimikasvatustiheyden että kiertoajan määrityksessä. Kunakin 5-vuotiskautena määritetään jokaiselle metsikölle kaksi sisäistä korkokantaa, joista toinen osoittaa harvennushakkuujärjestyksen ja toinen uudistusjärjestyksen. Näiden sisäisten korkokantojen avulla etsitään se korkokanta, joka jakaa metsälön puuston hakattavaan ja kasvatettavaan osaan siten, että hakattavaksi kuuluvasta osasta saatava tulo peittää täsmälleen nettotulotavoitteen. Tätä korkokantaa voidaan pitää koko metsälön sisäisenä korkokantana tuona 5-vuotiskautena. Koska puuston jaossa kasvatettavaan ja hakattavaan osaan on otettu huomioon puuston kehitys ja nettotulovaatimus ainoastaan kyseisellä 5-vuotiskaudella, ei käytetty menetelmä ole teoreettisesti moitteeton, vaikkakin verraten kätevä käytännössä. Saattaa nimittäin käydä siten, että hakkuutapa, joka tietyllä hetkellä näyttää parhaalta, ei tuotakaan lopullisesti haluttua tulosta eli mahdollisimman korkeaa metsälön arvoa suunnitelmakauden lopussa.

Koska simulointimallin tulosten hyvyydestä ei voida olla varmoja, on hakkuulaskelmia varten kehitetty myös viivallinen malli. Tavoitefunktio, jonka arvoa pyritään maksimoimaan, edustaa metsälön arvoa suunnitelmakauden lopussa. Metsälön arvo määritetään täsmälleen samoin kuin simulointimallissa. Kullekin metsikölle koko suunnitelmakaudeksi laaditut vaihtoehtoiset hakkuusuunnitelmat muodostavat viivallisen mallin muuttujajoukon. Rajoituksina ovat metsiköiden pinta-alat sekä nettotulofunktio. Viivallisen ohjelmoinnin perusominaisuuksista johtuu, että mallin optimiratkaisu löydetään aina, mikäli se on olemassa. Ongelmana viivallista mallia käytettäessä on muuttujien laskennallisista syistä rajoitettu lukumäärä. Tästä johtuu, että löydetty optimi ei saata olla lähelläkään teoreettista optimia, johon onnistuneemmalla muuttujien valinnalla olisi päästy.

Sekä simulointimallia että viivallista mallia käyttäen on tehty rajoitettu määrä hakkuulaskelmia. Ne osoittavat, että molemmat mallit ovat käyttökelpoisia. Jos viivallisen mallin muuttujien valinnassa on käytetty apuna simulointimallin yksityisille metsiköille antamia hakkuusuunnitelmia, ovat mallit antaneet likimain samoja arvoja metsälölle suunnitelmakauden lopussa. Jos viivallisen mallin muuttujat on jouduttu valitsemaan ilman ennakkotietoja tilanteeseen sopivista hakkuutavoista, ei metsälön arvo ole ollut yhtä korkea kuin simulointimallia käytettäessä. Hyvän tuloksen saavuttaminen olisi tällöin edellyttänyt erittäin monien muuttujien mukaanottamista, mikä taasen olisi lisännyt merkittävästi laskentakustannuksia.

Simulointimallin ja viivallisen mallin välisiä riippuvuussuhteita on pyritty selvittämään vertaamalla simulointimallin antamia metsälön sisäisiä korkokantoja nettotulotavoitetta vastaaviin varjohintoihin viivallisessa mallissa. Tällöin todettiin, että kahden peräkkäisen 5-vuotiskauden varjohintojen suhde korreloi voimakkaasti edellisen 5-vuotiskauden sisäisen korkokannan kanssa. Tämä ilmiö, samoin kuin se, että viivallinen malli pyrki käyttämään optimiratkaisuissa simulointimallin käyttämiä hakkuutapoja, vahvistaa käsitystä, että simulointimallissa käytetyt päätöksentekosäännöt johtavat lähelle metsälön loppuarvon maksimoitumista.

Tämän tutkimuksen valossa näyttää simulointi nykyisin viivallista ohjelmointia tarkoituksenmukaisemmalta menetelmältä hakkuulaskelman laadinnassa. Tietokoneiden lisääntyvä kapasiteetti saattaa kuitenkin jo lähiaikoina tehdä viivallisen ohjelmoinnin varsin kilpailukykyiseksi, kun vielä otetaan huomioon, että viivallisen ohjelmoinnin teoreettinen perusta on huomattavasti kestävämpi kuin simuloinnin. Suositeltavin menetelmä hakkuulaskelman laadinnassa saattaisi olla simuloinnin ja viivallisen ohjelmoinnin yhdistetty käyttö. Tällöin simuloinnilla voitaisiin löytää likimain optimaalinen hakkuusuunnite ja viivallisella ohjelmoinnilla voitaisiin tämän ratkaisun hyvyys tarkistaa ja ratkaisua parantaa.

APPENDIX

Example of the simulation model output

PERIOD 1

FOREST STANDS BEFORE CUTTING

						5 YEAR GROWTH		INT.R. OF RET.	
NO	SITE	AGE	VOLUME	VALUE	AREA	VOLUME	VALUE	CLEAR C.	THIN.
1	1	65	170.	2873.	0.99	34.09	963.9	4.48	4.25
2	2	45	100.	870.	17.26	40.00	488.0		
3	2	15	10.	0.	4.45	20.00	69.0		
4	2	75	110.	1749.	4.88	22.62	558.5	4.30	4.64
5	3	85	70.	714.	4.82	14.09	219.4	4.75	4.90
6	3	95	60.	720.	1.10	10.85	201.1	4.32	4.59
7	1	75	170.	3417.	5.93	28.58	832.7	3.44	2.96
8	2	55	130.	1391.	2.04	36.80	560.5	5.00	5.32
9	3	75	60.	522.	0.31	14.67	179.9	4.92	5.52
10	2	65	110.	1430.	3.53	27.07	530.2	4.65	5.28
11	3	65	90.	675.	0.61	23.76	235.1	5.31	5.11

```
SITE1    15.07   SITE2    70.03   SITE3    14.90   TOTAL AREA   45.92
AGE 1    74.     AGE 2    54.     AGE 3    84.     MEAN AGE     62.
VOLUME     4614. GROWTH    1363. STUMP. VALUE     59351. VALUE GR.  21840.
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FOREST STANDS AFTER CUTTING

						5 YEAR GROWTH		INT.R. OF RET.	
NO	SITE	AGE	VOLUME	VALUE	AREA	VOLUME	VALUE	CLEAR C.	THIN.
1	1	65	170.	2873.	0.99	34.09	963.9	4.48	4.25
2	2	45	78.	679.	17.26	40.00	466.0		
3	2	15	5.	0.	4.45	20.00	57.5		
4	2	75	110.	1749.	4.88	22.62	558.5	4.30	4.64
5	3	85	70.	714.	4.82	14.09	219.4	4.75	4.90
6	3	95	60.	720.	1.10	10.85	201.1	4.32	4.59
7	1	75	110.	2211.	4.28	22.62	627.0	3.44	4.03
8	2	55	130.	1391.	2.04	36.80	560.5	5.00	5.32
9	3	75	60.	522.	0.31	14.67	179.9	4.92	5.52
10	2	65	110.	1430.	3.53	27.07	530.2	4.65	5.28
11	3	65	90.	675.	0.61	23.76	235.1	5.31	5.11
12	1	0	0.	0.	1.65	0.00	0.0		

```
INTERNAL RATE OF RETURN   3.5    NCR     14000.   TOTAL CUT   939.
THIN. AREA     4.28   REF. AREA     1.65   REF. COST   99. AD. COST     0.
VOLUME     3675. GROWTH    1290. STUMP. VALUE     45252. VALUE GR.  19156.
```

PERIOD 2

FOREST STANDS BEFORE CUTTING

						5 YEAR GROWTH		INT.R. OF RET.	
NO	SITE	AGE	VOLUME	VALUE	AREA	VOLUME	VALUE	CLEAR C.	THIN.
1	1	70	204.	3837.	0.99	33.38	936.3	3.56	2.67
2	2	50	118.	1145.	17.26	39.30	538.5	5.40	6.33
3	2	20	25.	58.	4.45	22.00	168.1		
4	2	80	133.	2308.	4.88	23.31	608.2	3.85	3.62
5	3	90	84.	933.	4.82	15.01	255.8	4.47	4.27
6	3	100	71.	921.	1.10	11.58	265.9	4.70	4.72
7	1	80	133.	2838.	4.28	23.31	670.3	3.14	3.14
8	2	60	167.	1952.	2.04	37.31	701.9	5.01	4.50
9	3	80	75.	702.	0.31	15.97	222.7	4.88	4.97
10	2	70	137.	1960.	3.53	28.02	664.8	4.73	4.67
11	3	70	114.	910.	0.61	25.18	298.7	5.33	4.66
12	1	5	0.	0.	1.65	5.00	0.0		

SITE1 15.07 SITE2 70.03 SITE3 14.90 TOTAL AREA 45.92
AGE 1 77. AGE 2 58. AGE 3 89. MEAN AGE 65.
VOLUME 4965. GROWTH 1311. STUMP. VALUE 64408. VALUE GR. 22364.

FOREST STANDS AFTER CUTTING

						5 YEAR GROWTH		INT.R. OF RET.	
NO	SITE	AGE	VOLUME	VALUE	AREA	VOLUME	VALUE	CLEAR C.	THIN.
1	1	70	130.	2444.	0.71	27.22	716.1	3.70	3.94
2	2	50	118.	1145.	17.26	39.30	538.5	5.40	6.33
3	2	20	20.	46.	4.45	22.00	155.6		
4	2	80	133.	2308.	4.88	23.31	608.2	3.85	3.62
5	3	90	84.	933.	4.82	15.01	255.8	4.47	4.27
6	3	100	71.	921.	1.10	11.58	265.9	4.70	4.72
7	1	0	0.	0.	4.28	0.00	0.0		
8	2	60	167.	1952.	2.04	37.31	701.9	5.01	4.50
9	3	80	75.	702.	0.31	15.97	222.7	4.88	4.97
10	2	70	137.	1960.	3.53	28.02	664.8	4.73	4.67
11	3	70	114.	910.	0.61	25.18	298.7	5.33	4.66
12	1	5	0.	0.	1.65	5.00	0.0		
13	1	0	0.	0.	0.28	0.00	0.0		

INTERNAL RATE OF RETURN 4.0 NCR 14000. TOTAL CUT 700.
THIN. AREA 0.71 REF. AREA 4.56 REF. COST 274. AD. COST 0.
VOLUME 4265. GROWTH 1198. STUMP. VALUE 50135. VALUE GR. 19017.

PERIOD 3

FOREST STANDS BEFORE CUTTING

						5 YEAR GROWTH		INT.R. OF RET.	
NO	SITE	AGE	VOLUME	VALUE	AREA	VOLUME	VALUE	CLEAR C.	THIN.
1	1	75	157.	3160.	0.71	27.58	794.5	3.46	3.16
2	2	55	157.	1683.	17.26	40.48	630.9	4.98	4.62
3	2	25	42.	202.	4.45	27.00	302.1		
4	2	85	156.	2916.	4.88	23.53	655.4	3.48	2.90
5	3	95	99.	1189.	4.82	15.71	303.4	4.34	3.89
6	3	105	82.	1187.	1.10	12.18	317.4	4.54	4.31
7	1	5	0.	0.	4.28	5.00	0.0		
8	2	65	204.	2653.	2.04	36.54	787.9	4.50	3.42
9	3	85	91.	925.	0.31	17.01	270.3	4.77	4.48
10	2	75	165.	2625.	3.53	28.21	738.5	4.25	3.64
11	3	75	139.	1209.	0.61	25.90	340.8	4.80	3.80
12	1	10	5.	0.	1.65	15.00	30.0		
13	1	5	0.	0.	0.28	5.00	0.0		

SITE1 15.07 SITE2 70.03 SITE3 14.90 TOTAL AREA 45.92
AGE 1 71. AGE 2 62. AGE 3 93. MEAN AGE 66.
VOLUME 5463. GROWTH 1285. STUMP. VALUE 69152. VALUE GR. 22360.

FOREST STANDS AFTER CUTTING

						5 YEAR GROWTH		INT.R. OF RET.	
NO	SITE	AGE	VOLUME	VALUE	AREA	VOLUME	VALUE	CLEAR C.	THIN.
1	1	75	90.	1809.	0.71	19.82	541.1	3.34	4.46
2	2	55	157.	1683.	17.26	40.48	630.9	4.98	4.62
3	2	25	32.	154.	4.45	27.00	277.1		
4	2	85	60.	1122.	3.96	12.50	320.8	3.25	4.62
5	3	95	99.	1189.	4.82	15.71	303.4	4.34	3.89
6	3	105	82.	1187.	1.10	12.18	317.4	4.54	4.31
7	1	5	0.	0.	4.28	5.00	0.0		
8	2	65	204.	2653.	2.04	36.54	787.9	4.50	3.42
9	3	85	91.	925.	0.31	17.01	270.3	4.77	4.48
10	2	75	110.	1749.	3.53	22.62	558.5	4.30	4.64
11	3	75	139.	1209.	0.61	25.90	340.8	4.80	3.80
12	1	10	5.	0.	1.65	15.00	30.0		
13	1	5	0.	0.	0.28	5.00	0.0		
14	2	0	0.	0.	0.92	0.00	0.0		

INTERNAL RATE OF RETURN 4.0 NCR 14000. TOTAL CUT 810.
THIN. AREA 8.20 REF. AREA 0.92 REF. COST 46. AD. COST 0.
VOLUME 4653. GROWTH 1194. STUMP. VALUE 55106. VALUE GR. 19507.

PERIOD 4

FOREST STANDS BEFORE CUTTING

						5 YEAR GROWTH		INT.R. OF RET.	
NO	SITE	AGE	VOLUME	VALUE	AREA	VOLUME	VALUE	CLEAR C.	THIN.
1	1	80	110.	2350.	0.71	20.84	589.6	3.12	3.55
2	2	60	198.	2314.	17.26	39.87	775.4	4.90	3.91
3	2	30	59.	431.	4.45	30.00	236.8		
4	2	90	73.	1443.	3.96	13.46	345.2	3.01	3.75
5	3	100	115.	1493.	4.82	16.21	394.2	4.63	4.02
6	3	110	95.	1504.	1.10	12.67	330.3	3.85	3.43
7	1	10	5.	0.	4.28	15.00	30.0		
8	2	70	241.	3441.	2.04	34.96	940.9	4.38	3.07
9	3	90	108.	1195.	0.31	17.75	309.9	4.41	3.85
10	2	80	133.	2308.	3.53	23.31	608.2	3.85	3.62
11	3	80	165.	1550.	0.61	26.04	397.4	4.52	3.32
12	1	15	20.	30.	1.65	27.00	289.6		
13	1	10	5.	0.	0.28	15.00	30.0		
14	2	5	0.	0.	0.92	0.00	0.0		

SITE1 15.07 SITE2 70.03 SITE3 14.90 TOTAL AREA 45.92
AGE 1 52. AGE 2 63. AGE 3 98. MEAN AGE 68.
VOLUME 5847. GROWTH 1270. STUMP. VALUE 74612. VALUE GR. 23504.

FOREST STANDS AFTER CUTTING

						5 YEAR GROWTH		INT.R. OF RET.	
NO	SITE	AGE	VOLUME	VALUE	AREA	VOLUME	VALUE	CLEAR C.	THIN.
1	1	0	0.	0.	0.71	0.00	0.0		
2	2	60	198.	2314.	17.26	39.87	775.4	4.90	3.91
3	2	30	46.	336.	4.45	30.00	234.2		
4	2	0	0.	0.	3.96	0.00	0.0		
5	3	100	115.	1493.	4.82	16.21	394.2	4.63	4.02
6	3	110	95.	1504.	1.10	12.67	330.3	3.85	3.43
7	1	10	5.	0.	4.28	15.00	30.0		
8	2	70	180.	2574.	2.04	31.87	794.8	4.64	3.90
9	3	90	108.	1195.	0.31	17.75	309.9	4.41	3.85
10	2	80	133.	2308.	1.47	23.31	608.2	3.85	3.62
11	3	80	165.	1550.	0.61	26.04	397.4	4.52	3.32
12	1	15	11.	17.	1.65	27.00	241.9		
13	1	10	5.	0.	0.28	15.00	30.0		
14	2	5	0.	0.	0.92	0.00	0.0		
15	2	0	0.	0.	2.06	0.00	0.0		

INTERNAL RATE OF RETURN 4.0 NCR 14000. TOTAL CUT 834.
THIN. AREA 2.04 REF. AREA 6.73 REF. COST 343. AD. COST 0.
VOLUME 5013. GROWTH 1147. STUMP. VALUE 60269. VALUE GR. 20078.

PERIOD 20 YRS RATE OF INTEREST 3.0
THE VALUE OF THE FOREST UNDERTAKING
AT THE BEGINNING OF THE PLANNING PERIOD 121311.
AT THE END OF THE PLANNING PERIOD 136960.

ACTA FORESTALIA FENNICA

EDELLISIÄ NITEITÄ — PREVIOUS VOLUMES