axioms

MDPI

Article

Nonrelativistic Approximation in the Theory of a Spin-2 Particle with Anomalous Magnetic Moment

Alina Ivashkevich [1], Viktor Red'kov [1] and Artur Ishkhanyan [2,*]

[1] B.I. Stepanov Institute of Physics of NAS of Belarus, 220072 Minsk, Belarus; ivashkevich.alina@yandex.by (A.I.); v.redkov@ifanbel.bas-net.by (V.R.)
[2] Institute for Physical Research of NAS of Armenia, Ashtarak 0204, Armenia
* Correspondence: aishkhanyan@gmail.com

Abstract: We start with the 50-component relativistic matrix equation for a hypothetical spin-2 particle in the presence of external electromagnetic fields. This equation is hypothesized to describe a particle with an anomalous magnetic moment. The complete wave function consists of a two-rank symmetric tensor and a three-rank tensor that is symmetric in two indices. We apply the general method for performing the nonrelativistic approximation, which is based on the structure of the 50×50 matrix Γ^0 of the main equation. Using the 7th-order minimal equation for the matrix Γ^0, we introduce three projective operators. These operators permit us to decompose the complete wave function into the sum of three parts: one large part and two smaller parts in the nonrelativistic approximation. We have found five independent large variables and 45 small ones. To simplify the task, by eliminating the variables related to the 3-rank tensor, we have derived a relativistic system of second-order equations for the 10 components related to the symmetric tensor. We then take into account the decomposition of these 10 variables into linear combinations of large and small ones. In accordance with the general method, we separate the rest energy in the wave function and specify the orders of smallness for different terms in the arising equations. Further, after performing the necessary calculations, we derive a system of five linked equations for the five large variables. This system is presented in matrix form, which has a nonrelativistic structure, where the term representing additional interaction with the external magnetic field through three spin projections is included. The multiplier before this interaction contains the basic magnetic moment and an additional term due to the anomalous magnetic moment. The latter characteristic is treated as a free parameter within the hypothesis.

Keywords: spin-2 particle; interaction with electromagnetic field; nonrelativistic approximation; projective operators; five-dimensional Pauli-like equation; anomalous magnetic moment

MSC: 81V10; 83C50; 81Q05

check for updates

Academic Editors: Elias Camouzis and Alexandre Landry

Received: 24 October 2024
Revised: 29 December 2024
Accepted: 31 December 2024
Published: 3 Januray 2025

Citation: Ivashkevich, A.; Red'kov, V.; Ishkhanyan, A. Nonrelativistic Approximation in the Theory of a Spin-2 Particle with Anomalous Magnetic Moment. *Axioms* **2025**, *14*, 35. https://doi.org/10.3390/axioms14010035

1. Introduction

The theory of massive and massless fields of spin-2, following the foundational work of Pauli and Fierz [1–3], has long attracted significant attention [4–36]. Several key aspects and challenges of this theoretical framework have been explored over the years.

Most studies have been conducted within the framework of second-order differential equations, with a particular focus on the additional constraints required to preserve the five independent degrees of freedom for a spin-2 particle. This problem becomes even more intricate when extending the theory to curved Riemannian space-times.

An additional complication arises when studying the massless spin-2 field. The well-known Pauli–Fierz solution for flat Minkowski space does not carry over to curved space-time. Extending the Pauli–Fierz prescription to a generally covariant form leads to unexpected constraints on space-time geometry: the Ricci tensor $R_{\alpha\beta}$ and the Riemann tensor $R_{\alpha\beta\rho\sigma}$ must vanish identically. To address this, a non-minimal interaction term involving the Riemann tensor can be introduced into the basic equations [30], allowing the constraints to be reduced to $R_{\alpha\beta} = 0$.

Another area of interest has been the problem of anomalous solutions in spin-2 theory. A technical alternative for studying spin-2 fields, both massive and massless, involves formulating first-order systems. This approach, based on the Gel'fand–Yaglom formalism [6], was first explored by Fedorov [7] and Regge [8]. Their work demonstrated that a spin-2 particle requires a 39-component set of tensors for its description.

This formalism allows for the exploration of new physical questions related to degrees of freedom. For instance, for the massless case, the 39-component matrix equation was solved in Minkowski space-time in [37] using cylindrical coordinates t, r, ϕ, z, and a tetrad. Six linearly independent solutions were found. By applying the Pauli–Fierz approach, adjusted to the tetrad formalism, the gauge solutions were constructed using exact solutions for the massless spin-1 field. This yielded four independent gauge solutions and two gauge-free solutions for the spin-2 field, as expected from physical reasoning.

Additionally, F.I. Fedorov introduced a more general theory for a spin-2 particle based on a 50-component set of tensors. This theory, in the presence of external electromagnetic fields, describes a spin-2 particle with a proposed anomalous magnetic moment [36,38]. In Riemannian space-time, the reduced theory automatically incorporates non-minimal interaction terms involving the Ricci and Riemann tensors.

One notable aspect of this framework is its allowance for a new massless limit for the spin-2 field [39]. This is particularly significant because the minimal Pauli–Fierz theory does not possess gauge symmetry in curved space-times with $R_{\alpha\beta} = 0$. However, the generalized framework exhibits gauge symmetry under these conditions, as demonstrated in [39].

In the present study, we focus on a specific problem within the 50-component framework: the nonrelativistic approximation for a hypothetical massive spin-2 particle. This task is closely tied to the physical interpretation of the generalized framework. A similar problem was previously addressed for the simpler 39-component theory in [40], where a Pauli-like equation was derived.

Section 2 introduces the basic definitions and notation, including the structure of the 50-component matrix equation and the role of the free parameter in the model. Explicit expressions for the four key matrices Γ^a of the equation, derived in [38], are assumed to be known. The system is formulated in Cartesian coordinates in the presence of external electromagnetic fields.

In Section 3, the nonrelativistic approximation is performed by distinguishing between large and small components of the wave function using three projective operators derived from the seventh-order minimal polynomial for the 50×50 matrix Γ^0. The explicit structure of these components is determined, revealing five independent large components and 45 independent small components.

In Section 4, the nonrelativistic equation for the five-component wave function is derived. The interaction term describing the magnetic moment of the spin-2 particle with the external magnetic field is isolated. This term includes contributions from the basic magnetic moment and an additional term corresponding to the proposed anomalous magnetic moment, governed by a free parameter in the framework. By physical reasoning, this parameter is expected to be small compared to the basic magnetic moment. In the absence of this additional term, the equation reduces to the Pauli-like equation for a hypothetical spin-2 particle, as derived in [40].

2. Initial System of Equations

The basic system of equations for a massive spin-2 field, comprising a set of a symmetric second-rank tensor and a third-rank tensor symmetric with respect to two indices, is known. (Initially, in the 50-component theory, the following set of tensors was used: a scalar, two vectors, a symmetric second-rank tensor, a symmetric third-rank tensor, and a 3-rank tensor antisymmetric in two indices. In the present paper, we apply a different but equivalent representation of the Lorentz group, where new variables are used: a symmetric second-rank tensor and a third-rank tensor symmetric in two indices. The symmetry properties of these new variables resemble the symmetry of the metric tensor and Christoffel symbols in General Relativity. This approach simplifies the elimination of additional components and the derivation of second-order equations for the basic symmetric tensor. Moreover, the relationships between nonlinear equations for gravitational fields (massive and massless) and linear Lorentz-invariant equations (massive and massless) emerge naturally in this framework. Notably, the 50-component approach can also describe the spin-2 particle without an anomalous magnetic moment, achieved by fixing the additional parameter of the model to $\sinh\alpha = 0$) [38]:

$$\begin{aligned} &-\tfrac{b_1}{2}\Big(D_\alpha\Psi_{\beta\sigma} + D_\beta\Psi_{\alpha\sigma}\Big) - b_2 D_\sigma\Psi_{\alpha\beta} - \tfrac{b_3}{2}\Big(g_{\alpha\sigma}(x)D_\beta\Psi_\mu{}^\mu + g_{\sigma\beta}(x)D_\alpha\Psi_\mu{}^\mu\Big) \\ &-b_4 g_{\alpha\beta}D_\sigma\Psi_\mu{}^\mu - \tfrac{b_5}{2}\Big(g_{\alpha\sigma}(x)D_\mu\Psi^\mu{}_\beta + g_{\sigma\beta}(x)D_\mu\Psi^\mu{}_\alpha\Big) - b_6 g_{\alpha\beta}(x)D_\mu\Psi^\mu{}_\sigma + M\Psi_{\sigma\alpha\beta} = 0, \end{aligned} \tag{1}$$

$$\begin{aligned} &\tfrac{a_1}{2}\Big(D_\alpha\Psi^\rho{}_{\rho\beta} + D_\beta\Psi^\rho{}_{\rho\alpha}\Big) - a_2 g_{\alpha\beta}(x)D_\mu\Psi_\rho{}^{\mu\rho} + \tfrac{a_3}{2}\Big(D_\alpha\Psi_{\beta\rho}{}^\rho + D_\beta\Psi_{\alpha\rho}{}^\rho\Big) \\ &-a_4 g_{\alpha\beta}(x)D_\rho\Psi^{\rho\mu}{}_\mu + a_5 D_\rho\Psi^\rho{}_{\alpha\beta} + \tfrac{a_6}{2}\Big(D_\rho\Psi_{\alpha\beta}{}^\rho + D_\rho\Psi_{\beta\alpha}{}^\rho\Big) + M\Psi_{\alpha\beta} = 0, \end{aligned} \tag{2}$$

where $D_a = \partial_a + ieA_a$. In Minkowski space, the metric tensor has the signature $(+,-,-,-)$.

The above system contains numerical coefficients a_i and b_i, determined by a free parameter α according to the relations [38]:

$$a_1 = \frac{1}{9}i\Big(2\sqrt{3}(\sqrt{2}-3)\sinh\alpha - 6\sqrt{2}\cosh\alpha - 3\sqrt{6}\Big),$$

$$a_2 = -\frac{1}{36}i\Big(2\sqrt{3}(3+2\sqrt{2})\sinh\alpha + 24\sqrt{2}\cosh\alpha + 3\sqrt{2}(\sqrt{3}-3)\Big),$$

$$a_3 = -\frac{1}{9}i\Big(5\sqrt{6}\sinh\alpha - 6\sqrt{2}\cosh\alpha + 3\sqrt{3}\Big),$$

$$a_4 = \frac{1}{36}i\Big(-8\sqrt{6}\sinh\alpha + 24\sqrt{2}\cosh\alpha - 3\sqrt{3} + 9\Big),$$

$$a_5 = -\frac{1}{3}i\sqrt{2}\Big(\sqrt{3}\sinh\alpha - 6\cosh\alpha\Big), \quad a_6 = -2i\Big(\sqrt{\frac{2}{3}}\sinh\alpha + \sqrt{2}\cosh\alpha\Big),$$

$$b_1 = \frac{i\Big(\cosh\alpha - 2\sqrt{3}\sinh\alpha\Big)}{3\sqrt{2}}, \quad b_2 = -\frac{i(\sqrt{3}\sinh\alpha + \cosh\alpha)}{3\sqrt{2}},$$

$$b_3 = \frac{1}{54}i\Big(\sqrt{3}(8+5\sqrt{2})\sinh\alpha - 3\sqrt{2}\cosh\alpha + (1+4\sqrt{2})(3+\sqrt{3})\Big),$$

$$b_4 = -\frac{1}{108}i\Big(\sqrt{3}(4+7\sqrt{2})\sinh\alpha - 6\sqrt{2}\cosh\alpha + (5+2\sqrt{2})(3+\sqrt{3})\Big),$$

$$b_5 = -\frac{1}{54}i\Big(2\sqrt{3}((16+\sqrt{2})\sinh\alpha + 8\sqrt{2}+2) - 3\sqrt{2}\cosh\alpha\Big),$$

$$b_6 = \frac{1}{54} i \Big(\sqrt{3}((8 + 23\sqrt{2}) \sinh \alpha + 4\sqrt{2} + 10) - 3\sqrt{2} \cosh \alpha \Big).$$

We use the following notations for the 50 components [38], organizing them into five 10-dimensional columns:

$$\begin{array}{ccccc}
\varphi & \varphi_0 & \varphi_1 & \varphi_2 & \varphi_3 \\
\Psi_{11} = f_1 & \Psi_{0(11)} = f_{01} & \Psi_{1(11)} = f_{11} & \Psi_{2(11)} = f_{21} & \Psi_{3(11)} = f_{31} \\
\Psi_{22} = f_2 & \Psi_{0(22)} = f_{02} & \Psi_{1(22)} = f_{12} & \Psi_{2(22)} = f_{22} & \Psi_{3(22)} = f_{32} \\
\Psi_{33} = f_3 & \Psi_{0(33)} = f_{03} & \Psi_{1(33)} = f_{13} & \Psi_{2(33)} = f_{23} & \Psi_{3(33)} = f_{33} \\
\Psi_{23} = c_1 & \Psi_{0(23)} = c_{01} & \Psi_{1(23)} = c_{11} & \Psi_{2(23)} = c_{21} & \Psi_{3(23)} = c_{31} \\
\Psi_{31} = c_2 & \Psi_{0(31)} = c_{02} & \Psi_{1(31)} = c_{12} & \Psi_{2(31)} = c_{22} & \Psi_{3(31)} = c_{32} \\
\Psi_{12} = c_3 & \Psi_{0(12)} = c_{03} & \Psi_{1(12)} = c_{13} & \Psi_{2(12)} = c_{23} & \Psi_{3(12)} = c_{33} \\
\Psi_{01} = d_1 & \Psi_{0(01)} = d_{01} & \Psi_{1(01)} = d_{11} & \Psi_{2(01)} = d_{21} & \Psi_{3(01)} = d_{31} \\
\Psi_{02} = d_2 & \Psi_{0(02)} = d_{02} & \Psi_{1(02)} = d_{12} & \Psi_{2(02)} = d_{22} & \Psi_{3(02)} = d_{32} \\
\Psi_{03} = d_3 & \Psi_{0(03)} = d_{03} & \Psi_{1(03)} = d_{13} & \Psi_{2(03)} = d_{23} & \Psi_{3(03)} = d_{33} \\
\Psi_{00} = f_0 & \Psi_{0(00)} = f_{00} & \Psi_{1(00)} = f_{10} & \Psi_{2(00)} = f_{20} & \Psi_{3(00)} = f_{30}.
\end{array} \tag{3}$$

3. Projective Operators, Large and Small Components

It is convenient to apply the matrix form of the main 50-component system [38]:

$$(\partial_a \Gamma^a + M)\Psi = \partial_a \begin{vmatrix} 0 & K_0{}^a & K_1{}^a & K_2{}^a & K_3{}^a \\ L_0{}^a & 0 & 0 & 0 & 0 \\ L_1{}^a & 0 & 0 & 0 & 0 \\ L_2{}^a & 0 & 0 & 0 & 0 \\ L_3{}^a & 0 & 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} \varphi \\ \varphi_0 \\ \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{vmatrix} + M \begin{vmatrix} \varphi \\ \varphi_0 \\ \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{vmatrix} = 0. \tag{4}$$

The component φ refers to the second-rank symmetric tensor, while the components φ_a refer to the third-rank tensor symmetric in two indices. The explicit form of all 32 matrix blocks of dimension 10×10 was provided in [38]. These blocks depend on an arbitrary parameter that, as shown below, determines the value of the anomalous magnetic moment of the spin-2 particle.

In accordance with the general method for performing the nonrelativistic approximation, we specify the matrix Γ^0 of the main equation in its explicit form.

Using the explicit form of the 50-dimensional matrix Γ, we verified that its minimal equation is

$$(\Gamma^0)^7 - (\Gamma^0)^5 = 0.$$

This allows the introduction of three projective operators (let $\Gamma^0 = \Gamma$):

$$P_+ = P_1 = \frac{1}{2}\Gamma^5(\Gamma + 1), \quad P_- = P_2 = \frac{1}{2}\Gamma^5(\Gamma - 1), \quad P_0 = P_3 = I - \Gamma^6. \tag{5}$$

We derived explicit expressions for these 50×50 projective matrices, which depend on the free parameter α; their detailed forms are omitted for brevity. The complete wave function is decomposed as

$$\Psi = \Psi^+ + \Psi^- + \Psi^0,$$

where Ψ^+ represents the large component, and Ψ^- and Ψ^0 represent the small components. In the nonrelativistic limit, only the large components contribute to the final equations. The procedure involves dividing all components into two groups: large components (L) and small components (S), where $S \ll L$.

Among the elements of each component, independent variables are identified (detailed derivations are omitted). For the symmetric tensor φ, the projective constituents are

$$\Psi^+: \quad \varphi^+ = \begin{vmatrix} f_1^+ \\ f_2^+ \\ f_3^+ \\ c_1^+ \\ c_2^+ \\ c_3^+ \\ d_1^+ \\ d_2^+ \\ d_3^+ \\ f_0^+ \end{vmatrix} = \begin{vmatrix} f_1^+ \\ \frac{6i\sqrt{2}}{\cosh\alpha - 2\sqrt{3}\sinh\alpha} d_{33}^+ - f_1^+ \\ -\frac{6i\sqrt{2}}{\cosh\alpha - 2\sqrt{3}\sinh\alpha} d_{33}^+ \\ c_1^+ \\ c_2^+ \\ c_3^+ \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix},$$

$$\Psi^-: \quad \varphi^- = \begin{vmatrix} f_1^- \\ f_2^- \\ f_3^- \\ c_1^- \\ c_2^- \\ c_3^- \\ d_1^- \\ d_2^- \\ d_3^- \\ f_0^- \end{vmatrix} = \begin{vmatrix} f_1^- \\ -f_1^- - \frac{6i\sqrt{2}}{\cosh(\alpha) - 2\sqrt{3}\sinh(\alpha)} d_{33}^- \\ \frac{6i\sqrt{2}}{\cosh(\alpha) - 2\sqrt{3}\sinh(\alpha)} d_{33}^- \\ c_1^- \\ c_2^- \\ c_3^- \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix}, \quad \varphi^0 = \begin{vmatrix} f_1^0 \\ f_2^0 \\ f_3^0 \\ c_1^0 \\ c_2^0 \\ c_3^0 \\ d_1^0 \\ d_2^0 \\ d_3^0 \\ f_0^0 \end{vmatrix} = \begin{vmatrix} f_1^0 \\ f_1^0 \\ f_1^0 \\ 0 \\ 0 \\ 0 \\ d_1^0 \\ d_2^0 \\ d_3^0 \\ f_0^0 \end{vmatrix}.$$

To simplify, we introduce the following notation for the 15 independent variables (for convenience, large L and small S):

$$\begin{aligned} \varphi^+ &: \quad f_1^+ = L_1, \quad c_1^+ = L_2, \quad c_2^+ = L_3, \quad c_3^+ = L_4, \quad d_{33}^+ = L_5; \\ \varphi^- &: \quad f_1^- = S_1, \quad c_1^- = S_2, \quad c_2^- = S_3, \quad c_3^- = S_4, \quad d_{33}^- = S_5; \\ \varphi^0 &: \quad f_1^0 = S_6, \quad d_1^0 = S_7, \quad d_2^0 = S_8, \quad d_3^0 = S_9, \quad f_0^0 = S_{10}. \end{aligned} \tag{6}$$

4. Nonrelativistic Approximation

By eliminating the 40 variables associated with the three-rank tensor φ_a, we derive a system of ten second-order equations for the symmetric tensor φ. Using the following notations:

$$D_{(ab)} = \frac{1}{2}(D_{ab} + D_{ba}), \quad D_{[ab]} = \frac{1}{2}(D_{ab} - D_{ba}) = -\frac{ie}{2}F_{ab}, \quad A^2 = \sinh^2\alpha, \tag{7}$$

the system takes the form

1

$$\begin{gathered} 6ieA^2(F_{12}c_3 - F_{31}c_2 + F_{01}d_1) - M^2 f_1 \\ -\frac{1}{3}(3+\sqrt{3})D_{(12)}c_3 - \frac{1}{3}(-3+\sqrt{3})D_{(23)}c_1 - \frac{1}{3}(3+\sqrt{3})D_{(31)}c_2 \\ +\frac{1}{3}(3+\sqrt{3})D_{(01)}d_1 + \frac{1}{3}(-3+\sqrt{3})D_{(02)}d_2 + \frac{1}{3}(-3+\sqrt{3})D_{(03)}d_3 \\ +\frac{1}{12}D_{11}(-((3+2\sqrt{3})f_0) - 3f_1 + (3+2\sqrt{3})(f_2+f_3)) + \frac{1}{12}D_{22}(3f_0 + 9f_1 + (3-2\sqrt{3})f_2 - 3f_3) \\ +\frac{1}{12}D_{33}(3f_0 + 9f_1 - 3f_2 + (3-2\sqrt{3})f_3) + \frac{1}{12}D_{00}((3-2\sqrt{3})f_0 + 3(-3f_1 + f_2 + f_3)) = 0, \end{gathered}$$

2

$$
\begin{gathered}
6ieA^2(F_{23}c_1 - F_{12}c_3 + F_{02}d_2) - M^2 f_2 \\
-\frac{1}{3}(3+\sqrt{3})D_{(12)}c_3 - \frac{1}{3}(3+\sqrt{3})D_{(23)}c_1 - \frac{1}{3}(-3+\sqrt{3})D_{(31)}c_2 \\
+\frac{1}{3}(-3+\sqrt{3})D_{(01)}d_1 + \frac{1}{3}(3+\sqrt{3})D_{(02)}d_2 + \frac{1}{3}(-3+\sqrt{3})D_{(03)}d_3 \\
+\frac{1}{12}D_{11}(3f_0 + (3-2\sqrt{3})f_1 + 9f_2 - 3f_3) + \frac{1}{12}D_{22}(-((3+2\sqrt{3})f_0) \\
+(3+2\sqrt{3})f_1 - 3f_2 + (3+2\sqrt{3})f_3) \\
+\frac{1}{12}D_{33}(3f_0 - 3f_1 + 9f_2 + (3-2\sqrt{3})f_3) + \frac{1}{12}D_{00}((3-2\sqrt{3})f_0 + 3(f_1 - 3f_2 + f_3)) = 0,
\end{gathered}
$$

3

$$
\begin{gathered}
6ieA^2(F_{03}d_3 - F_{23}c_1 + F_{31}c_2) - M^2 f_3 \\
-\frac{1}{3}(-3+\sqrt{3})D_{(12)}c_3 - \frac{1}{3}(3+\sqrt{3})D_{(23)}c_1 - \frac{1}{3}(3+\sqrt{3})D_{(31)}c_2 \\
+\frac{1}{3}(-3+\sqrt{3})D_{(01)}d_1 + \frac{1}{3}(-3+\sqrt{3})D_{(02)}d_2 + \frac{1}{3}(3+\sqrt{3})D_{(03)}d_3 \\
+\frac{1}{12}D_{11}(3f_0 + (3-2\sqrt{3})f_1 - 3f_2 + 9f_3) \\
+\frac{1}{12}D_{22}(3f_0 - 3f_1 + (3-2\sqrt{3})f_2 + 9f_3) \\
+\frac{1}{12}D_{33}(-((3+2\sqrt{3})f_0) + (3+2\sqrt{3})f_1 + (3+2\sqrt{3})f_2 - 3f_3) \\
+\frac{1}{12}D_{00}((3-2\sqrt{3})f_0 + 3(f_1 + f_2 - 3f_3)) = 0,
\end{gathered}
$$

4

$$
\begin{gathered}
3ieA^2\Big(- F_{12}c_2 - F_{23}(f_2 - f_3) + F_{31}c_3 + F_{02}d_3 + F_{03}d_2\Big) - M^2 c_1 \\
+\frac{1}{6}D_{(23)}\Big(- ((3+\sqrt{3})f_0) + (3+\sqrt{3})f_1 + (-3+\sqrt{3})(f_2 + f_3)\Big) - D_{(12)}c_2 - D_{(31)}c_3 \\
+D_{11}c_1 + D_{(02)}d_3 + D_{(03)}d_2 - D_{00}c_1 = 0,
\end{gathered}
$$

5

$$
\begin{gathered}
3ieA^2\Big(F_{12}c_1 - F_{23}c_3 + F_{31}(f_1 - f_3) + F_{01}d_3 + F_{03}d_1\Big) - M^2 c_2 + D_{22}c_2 - D_{00}c_2 + D_{(01)}d_3 + D_{(03)}d_1 \\
+\frac{1}{6}D_{(31)}\Big(- (3+\sqrt{3})f_0 + (-3+\sqrt{3})f_1 + (3+\sqrt{3})f_2 + (-3+\sqrt{3})f_3\Big) - D_{(12)}c_1 - D_{(23)}c_3 = 0,
\end{gathered}
$$

6

$$
\begin{gathered}
3ieA^2\Big(- F_{12}(f_1 - f_2) + F_{23}c_2 - F_{31}c_1 + F_{01}d_2 + F_{02}d_1\Big) - M^2 c_3 + D_{33}c_3 - D_{00}c_3 + D_{(01)}d_2 + D_{(02)}d_1 \\
+\frac{1}{6}D_{(12)}\Big(- ((3+\sqrt{3})f_0) + (-3+\sqrt{3})f_1 + (-3+\sqrt{3})f_2 + (3+\sqrt{3})f_3\Big) - D_{(23)}c_2 - D_{(31)}c_1 = 0,
\end{gathered}
$$

7

$$
\begin{gathered}
3ieA^2\Big(F_{12}d_2 - F_{31}d_3 + (F_{01}(f_0 + f_1) + F_{02}c_3 + F_{03}c_2)\Big) - M^2 d_1 + D_{22}d_1 + D_{33}d_1 - D_{(12)}d_2 - D_{(31)}d_3 \\
+\frac{1}{6}D_{(01)}\Big(- (-3+\sqrt{3})f_0 + (-3+\sqrt{3})f_1 + (3+\sqrt{3})(f_2 + f_3)\Big) - D_{(02)}c_3 - D_{(03)}c_2 = 0,
\end{gathered}
$$

8

$$
\begin{gathered}
3ieA^2\Big(- F_{12}d_1 + F_{23}d_3 + F_{01}c_3 + F_{02}(f_0 + f_2) + F_{03}c_1\Big) - M^2 d_2 + D_{11}d_2 + D_{33}d_2 - D_{(12)}d_1 - D_{(23)}d_3 \\
-D_{(01)}c_3 + \frac{1}{6}D_{(02)}\Big(- ((-3+\sqrt{3})f_0) + (3+\sqrt{3})f_1 + (-3+\sqrt{3})f_2 + (3+\sqrt{3})f_3\Big) - D_{(03)}c_1 = 0,
\end{gathered}
$$

9

$$
3ieA^2\Big(- F_{23}d_2 + F_{31}d_1 + F_{01}c_2 + F_{02}c_1 + F_{03}(f_0 + f_3)\Big) - M^2 d_3 + D_{11}d_3 + D_{22}d_3 - D_{(230}d_2 - D_{(31)}d_1
$$

$$-D_{(01)}c_2 - D_{(02)}c_1 + \frac{1}{6}D_{(03)}\Big(- (-3+\sqrt{3})f_0 + (3+\sqrt{3})f_1 + (3+\sqrt{3})f_2 + (-3+\sqrt{3})f_3\Big) = 0,$$

10

$$6ieA^2(F_{01}d_1 + F_{02}d_2 + F_{03}d_3) - M^2 f_0 + \frac{1}{3}(-3+\sqrt{3})D_{(12)}c_3 + \frac{1}{3}(-3+\sqrt{3})D_{(23)}c_1$$
$$+ + \frac{1}{3}(-3+\sqrt{3})D_{(31)}c_2 - \frac{1}{3}(3+\sqrt{3})D_{(01)}d_1 - \frac{1}{3}(3+\sqrt{3})D_{(02)}d_2 - \frac{1}{3}(3+\sqrt{3})D_{(03)}d_3$$
$$+\frac{1}{12}D_{33}\Big(9f_0 + 3(f_1+f_2) + (-3+2\sqrt{3})f_3\Big) + \frac{1}{12}D_{22}\Big(9f_0 + (-3+2\sqrt{3})f_2 + 3(f_1+f_3)\Big)$$
$$+\frac{1}{12}D_{11}\Big(9f_0 + (-3+2\sqrt{3})f_1 + 3(f_2+f_3)\Big) + \frac{1}{12}D_{00}\Big(3f_0 + (3+2\sqrt{3})(f_1+f_2+f_3)\Big) = 0.$$

It may be noted that the parameter $A^2 = \sinh^2\alpha$ stays near the electromagnetic tensor F_{ab}.

Taking into account the decompositions of the 10 components through large and small variables (6), we obtain the following presentations for ten equations (for brevity, let $D_{(ab)} \to D_{ab}$):

1

$$6ie\Big(- F_{31}\Big(L_3+S_3\Big) + F_{12}\Big(L_4+S_4\Big) + F_{01}S_7\Big)A^2 + M^2\Big(- L_1 - S_1 - S_6\Big)$$
$$-\frac{1}{3}\Big(- 3+\sqrt{3}\Big)D_{23}\Big(L_2+S_2\Big) - \frac{1}{3}\Big(3+\sqrt{3}\Big)D_{31}\Big(L_3+S_3\Big) - \frac{1}{3}\Big(3+\sqrt{3}\Big)D_{12}\Big(L_4+S_4\Big)$$
$$+\frac{1}{12}D_{22}\Big(2\Big(3+\sqrt{3}\Big)L_1 + 2\Big(3+\sqrt{3}\Big)S_1 - \frac{12i\sqrt{2}\Big(- 3+\sqrt{3}\Big)\Big(L_5-S_5\Big)}{\cosh\alpha - 2\sqrt{3}\sin\alpha} + \Big(9-2\sqrt{3}\Big)S_6 + 3S_{10}\Big)$$
$$+\frac{1}{12}D_{11}\Big(- \Big(\Big(3+2\sqrt{3}\Big)\Big(L_1+S_1-2S_6\Big)\Big) - 3\Big(L_1+S_1+S_6\Big) - \Big(3+2\sqrt{3}\Big)S_{10}\Big)$$
$$+D_{33}\Big(L_1+S_1+\frac{i\sqrt{2}\Big(- 3+\sqrt{3}\Big)\Big(L_5-S_5\Big)}{\cosh\alpha - 2\sqrt{3}\sin\alpha} + \frac{1}{12}\Big(\Big(9-2\sqrt{3}\Big)S_6 + 3S_{10}\Big)\Big)$$
$$+\frac{1}{12}\Big(4\Big(3+\sqrt{3}\Big)D_{01}S_7 + 4\Big(- 3+\sqrt{3}\Big)\Big(D_{02}S_8 + D_{03}S_9\Big)$$
$$-D_{00}\Big(3\Big(4\Big(L_1+S_1\Big)+S_6\Big) + \Big(- 3+2\sqrt{3}\Big)S_{10}\Big)\Big) = 0,$$

2

$$6ie\Big(F_{23}\Big(L_2+S_2\Big) - F_{12}\Big(L_4+S_4\Big) + F_{02}S_8\Big)A^2 + M^2\Big(L_1+S_1+\frac{6i\sqrt{2}\Big(S_5-L_5\Big)}{\cosh\alpha - 2\sqrt{3}\sin\alpha} - S_6\Big)$$
$$-\frac{1}{3}\Big(3+\sqrt{3}\Big)D_{23}\Big(L_2+S_2\Big) - \frac{1}{3}\Big(- 3+\sqrt{3}\Big)D_{31}\Big(L_3+S_3\Big) - \frac{1}{3}\Big(3+\sqrt{3}\Big)D_{12}\Big(L_4+S_4\Big)$$
$$+\frac{1}{12}D_{22}\Big(2\Big(3+\sqrt{3}\Big)L_1 + 2\Big(3+\sqrt{3}\Big)S_1 - \frac{12i\sqrt{2}\Big(3+\sqrt{3}\Big)\Big(L_5-S_5\Big)}{\cosh\alpha - 2\sqrt{3}\sin\alpha}$$
$$+\Big(3+4\sqrt{3}\Big)S_6 - \Big(3+2\sqrt{3}\Big)S_{10}\Big)$$
$$+D_{11}\Big(\frac{6i\sqrt{2}\Big(L_5-S_5\Big)}{\cosh\alpha - 2\sqrt{3}\sin\alpha} + \frac{1}{12}\Big(- 2\Big(3+\sqrt{3}\Big)L_1 - 2\Big(3+\sqrt{3}\Big)S_1 + \Big(9-2\sqrt{3}\Big)S_6 + 3S_{10}\Big)\Big)$$
$$+\frac{1}{12}\Big(4\Big(\Big(- 3+\sqrt{3}\Big)D_{01}S_7 + \Big(3+\sqrt{3}\Big)D_{02}S_8 + \Big(- 3+\sqrt{3}\Big)D_{03}S_9\Big)$$
$$+D_{00}\Big(12L_1 + 12S_1 - \frac{72i\sqrt{2}\Big(L_5-S_5\Big)}{\cosh\alpha - 2\sqrt{3}\sin\alpha} - 3S_6 + \Big(3-2\sqrt{3}\Big)S_{10}\Big)\Big)$$
$$+D_{33}\Big(\frac{i\sqrt{2}\Big(3+\sqrt{3}\Big)\Big(L_5-S_5\Big)}{\cosh\alpha - 2\sqrt{3}\sin\alpha} + \frac{1}{12}\Big(\Big(9-2\sqrt{3}\Big)S_6 + 3\Big(S_{10} - 4\Big(L_1+S_1\Big)\Big)\Big)\Big) = 0,$$

3

$$-6ie\Big(F_{23}\Big(L_2+S_2\Big) - F_{31}\Big(L_3+S_3\Big) - F_{03}S_9\Big)A^2 + M^2\Big(\frac{6i\sqrt{2}\Big(L_5-S_5\Big)}{\cosh\alpha - 2\sqrt{3}\sin\alpha} - S_6\Big)$$

$$-\frac{1}{3}\Big(3+\sqrt{3}\Big)D_{23}\Big(L_2+S_2\Big)-\frac{1}{3}\Big(3+\sqrt{3}\Big)D_{31}\Big(L_3+S_3\Big)-\frac{1}{3}\Big(-3+\sqrt{3}\Big)D_{12}\Big(L_4+S_4\Big)$$
$$+\frac{1}{12}D_{22}\Big(2\Big(-3+\sqrt{3}\Big)L_1+2\Big(-3+\sqrt{3}\Big)S_1-\frac{12i\sqrt{2}\Big(3+\sqrt{3}\Big)\Big(L_5-S_5\Big)}{\cosh\alpha-2\sqrt{3}\sin\alpha}$$
$$+\Big(9-2\sqrt{3}\Big)S_6+3S_{10}\Big)+D_{11}\Big(\frac{1}{12}\Big(-2\Big(-3+\sqrt{3}\Big)L_1-2\Big(-3+\sqrt{3}\Big)S_1+\Big(9-2\sqrt{3}\Big)S_6+3S_{10}\Big)$$
$$-\frac{6i\sqrt{2}\Big(L_5-S_5\Big)}{\cosh\alpha-2\sqrt{3}\sin\alpha}\Big)+\frac{1}{12}\Big(4\Big(\Big(-3+\sqrt{3}\Big)D_{01}S_7$$
$$+\Big(-3+\sqrt{3}\Big)D_{02}S_8+\Big(3+\sqrt{3}\Big)D_{03}S_9\Big)+D_{00}\Big(\frac{72i\sqrt{2}\Big(L_5-S_5\Big)}{\cosh\alpha-2\sqrt{3}\sin\alpha}-3S_6+\Big(3-2\sqrt{3}\Big)S_{10}\Big)\Big)$$
$$+D_{33}\Big(\frac{i\sqrt{2}\Big(3+\sqrt{3}\Big)\Big(L_5-S_5\Big)}{\cosh\alpha-2\sqrt{3}\sin\alpha}+\frac{1}{12}\Big(\Big(3+4\sqrt{3}\Big)S_6-\Big(3+2\sqrt{3}\Big)S_{10}\Big)\Big)=0,$$

4

$$-3ie\Big(F_{12}\Big(L_3+S_3\Big)-F_{31}\Big(L_4+S_4\Big)+F_{23}\Big(-L_1-S_1+\frac{12i\sqrt{2}\Big(L_5-S_5\Big)}{\cosh\alpha-2\sqrt{3}\sin\alpha}\Big)$$
$$-F_{03}S_8-F_{02}S_9\Big)A^2+M^2\Big(-L_2-S_2\Big)+D_{11}\Big(L_2+S_2\Big)$$
$$-D_{00}\Big(L_2+S_2\Big)+D_{12}\Big(-L_3-S_3\Big)+D_{31}\Big(-L_4-S_4\Big)+D_{03}S_8+D_{02}S_9$$
$$+D_{23}\Big(L_1+S_1+\frac{1}{2}\Big(-1+\sqrt{3}\Big)S_6-\frac{1}{6}\Big(3+\sqrt{3}\Big)S_{10}\Big)=0,$$

5

$$3ie\Big(F_{12}\Big(L_2+S_2\Big)-F_{23}\Big(L_4+S_4\Big)+F_{31}\Big(L_1+S_1+\frac{6i\sqrt{2}\Big(L_5-S_5\Big)}{\cosh\alpha-2\sqrt{3}\sin\alpha}\Big)$$
$$+F_{03}S_7+F_{01}S_9\Big)A^2+M^2\Big(-L_3-S_3\Big)$$
$$+D_{12}\Big(-L_2-S_2\Big)+D_{22}\Big(L_3+S_3\Big)-D_{00}\Big(L_3+S_3\Big)+D_{(230}\Big(-L_4-S_4\Big)+D_{03}S_7+D_{01}S_9$$
$$+D_{31}\Big(-L_1-S_1+\frac{6i\sqrt{2}\Big(L_5-S_5\Big)}{\cosh\alpha-2\sqrt{3}\sin(\alpha)}+\frac{1}{2}\Big(-1+\sqrt{3}\Big)S_6-\frac{1}{6}\Big(3+\sqrt{3}\Big)S_{10}\Big)=0,$$

6

$$-3ie\Big(F_{31}\Big(L_2+S_2\Big)-F_{23}\Big(L_3+S_3\Big)+2F_{12}\Big(L_1+S_1+\frac{3i\sqrt{2}\Big(S_5-L_5\Big)}{\cosh\alpha-2\sqrt{3}\sin\alpha}\Big)$$
$$-F_{02}S_7-F_{01}S_8\Big)A^2+M^2\Big(-L_4-S_4\Big)$$
$$+D_{31}\Big(-L_2-S_2\Big)+D_{23}\Big(-L_3-S_3\Big)+D_{33}\Big(L_4+S_4\Big)-D_{00}\Big(L_4+S_4\Big)+D_{02}S_7+D_{01}S_8$$
$$+D_{12}\Big(-\frac{6i\sqrt{2}\Big(L_5-S_5\Big)}{\cosh\alpha-2\sqrt{3}\sin\alpha}+\frac{1}{2}\Big(-1+\sqrt{3}\Big)S_6-\frac{1}{6}\Big(3+\sqrt{3}\Big)S_{10}\Big)=0,$$

7

$$3ie\Big(F_{03}\Big(L_3+S_3\Big)+F_{02}\Big(L_4+S_4\Big)+F_{12}S_8-F_{31}S_9+F_{01}\Big(L_1+S_1+S_6+S_{10}\Big)\Big)A^2-M^2S_7$$
$$-D_{03}\Big(L_3+S_3\Big)-D_{02}\Big(L_4+S_4\Big)+D_{22}S_7+D_{33}S_7-D_{12}S_8-D_{31}S_9$$
$$-\frac{1}{6}D_{01}\Big(6L_1+6S_1-3\Big(1+\sqrt{3}\Big)S_6+\Big(-3+\sqrt{3}\Big)S_{10}\Big)=0,$$

8

$$-3ie\Big(-F_{03}\Big(L_2+S_2\Big)-F_{01}\Big(L_4+S_4\Big)+F_{12}S_7-F_{23}S_9$$
$$+F_{02}\Big(L_1+S_1-\frac{6i\sqrt{2}\Big(L_5-S_5\Big)}{\cosh\alpha-2\sqrt{3}\sin\alpha}-S_6-S_{10}\Big)\Big)A^2-M^2S_8$$

$$-D_{03}\Big(L_2+S_2\Big)-D_{01}\Big(L_4+S_4\Big)-D_{12}S_7+D_{11}S_8+D_{33}S_8$$

$$-D_{23}S_9+D_{02}\Big(L_1+S_1-\frac{6i\sqrt{2}\Big(L_5-S_5\Big)}{\cosh\alpha-2\sqrt{3}\sin\alpha}+\frac{1}{2}\Big(1+\sqrt{3}\Big)S_6-\frac{1}{6}\Big(-3+\sqrt{3}\Big)S_{10}\Big)=0,$$

9

$$3ie\Big(F_{02}\Big(L_2+S_2\Big)+F_{01}\Big(L_3+S_3\Big)+F_{31}S_7-F_{23}S_8+F_{03}\Big(\frac{6i\sqrt{2}\Big(S_5-L_5\Big)}{\cosh\alpha-2\sqrt{3}\sin\alpha}+S_6+S_{10}\Big)\Big)A^2-M^2S_9$$

$$-D_{02}\Big(L_2+S_2\Big)-D_{01}\Big(L_3+S_3\Big)-D_{31}S_7-D_{23}S_8+D_{11}S_9+D_{22}S_9$$

$$+\frac{1}{6}D_{03}\Big(\frac{36i\sqrt{2}\Big(L_5-S_5\Big)}{\cosh\alpha-2\sqrt{3}\sin\alpha}+3\Big(1+\sqrt{3}\Big)S_6-\Big(-3+\sqrt{3}\Big)S_{10}\Big)=0,$$

10

$$6ie\Big(F_{01}S_7+F_{02}S_8+F_{03}S_9\Big)A^2-M^2S_{10}+$$

$$+\frac{1}{3}\Big(-3+\sqrt{3}\Big)D_{23}\Big(L_2+S_2\Big)+\frac{1}{3}\Big(-3+\sqrt{3}\Big)D_{31}\Big(L_3+S_3\Big)$$

$$+\frac{1}{3}\Big(-3+\sqrt{3}\Big)D_{12}\Big(L_4+S_4\Big)-\frac{1}{3}\Big(3+\sqrt{3}\Big)\Big(D_{01}S_7+D_{02}S_8+D_{03}S_9\Big)+\frac{1}{4}D_{00}\Big(\Big(3+2\sqrt{3}\Big)S_6+S_{10}\Big)$$

$$+\frac{1}{12}D_{22}\Big(-2\Big(-3+\sqrt{3}\Big)L_1-2\Big(-3+\sqrt{3}\Big)S_1+\frac{12i\sqrt{2}\Big(-3+\sqrt{3}\Big)\Big(L_5-S_5\Big)}{\cosh\alpha-2\sqrt{3}\sin\alpha}$$

$$+\Big(3+2\sqrt{3}\Big)S_6+9S_{10}\Big)+\frac{1}{12}D_{11}\Big(-3\Big(L_1+S_1-2S_6\Big)+\Big(-3+2\sqrt{3}\Big)\Big(L_1+S_1+S_6\Big)+9S_{10}\Big)$$

$$+D_{33}\Big(\frac{1}{12}\Big(\Big(3+2\sqrt{3}\Big)S_6+9S_{10}\Big)-\frac{i\sqrt{2}\Big(-3+\sqrt{3}\Big)\Big(L_5-S_5\Big)}{\cosh\alpha-2\sqrt{3}\sin(\alpha)}\Big)=0.$$

According to the general method, the first step is to separate the rest energy, achieved via a formal transformation [40], where μ is a real-valued parameter related to the particle's mass:

$$D_0\Longrightarrow(D_0+i\mu),\quad D_{00}=D_0D_0\Longrightarrow(D_0+i\mu)(D_0+i\mu)=D_{00}+2i\mu D_0-\mu^2,$$
$$D_{0j}=\frac{1}{2}(D_0D_j+D_jD_0)\Longrightarrow D_{0j}+i\mu D_j,\quad D_{[kj]}=ieF_{kj},\quad D_{[0j]}=ieF_{0j}. \tag{8}$$

When performing the nonrelativistic approximation, we assume specific orders of smallness for different quantities [40]. These are derived from the analysis of plane wave scenarios:

$$\epsilon=M+E,\quad M\,\text{is the rest energy},\quad E\,\text{is the nonrelativistic energy},\quad E\ll M. \tag{9}$$

The operator counterpart is

$$iD_0\Longrightarrow M+iD_0,\quad E=\frac{P^2}{2M}\Longrightarrow\frac{E}{M}=\frac{P^2}{2M}, \tag{10}$$

with the following scaling rules:

$$\frac{P_j}{M}\sim x,\quad\frac{D_j}{M}\sim x,\quad\frac{E}{M}\sim x^2,\quad\frac{D_0}{M}\sim x^3. \tag{11}$$

In the presence of an external electromagnetic field, additional rules must be considered:

$$\frac{D_kD_l-D_lD_k}{M^2}\sim ie\frac{D_{kl}}{M^2}\sim x^2,\quad\frac{D_0D_j-D_jD_0}{M^2}\sim ie\frac{F_{0j}}{M^2}\sim x^3. \tag{12}$$

Applying these principles to the current system, we obtain:

$$
\begin{gathered}
L \sim 1, \quad S \sim x, \quad \frac{1}{\mu} D_i \sim x, \quad \frac{D_0}{\mu} \sim x^2, \\
\frac{D_{ij}}{\mu^2} \sim x^2, \quad \frac{D_{0j}}{\mu^2} \sim x^3, \quad \frac{D_{00}}{\mu^2} \sim x^4, \\
\frac{D_{[kj]}}{\mu^2} = \frac{ieF_{kj}}{\mu^2} \sim x^2, \quad \frac{D_{[0j]}}{\mu^2} = \frac{ieF_{0j}}{\mu^2} \sim x^3.
\end{gathered} \tag{13}
$$

These rules are fundamental for determining the procedure to derive nonrelativistic equations from relativistic ones. This methodology remains consistent regardless of the particle's spin.

To avoid imposing constraints on the independent large variables L_n, $n = 1, \ldots, 5$, we assume $\mu^2 = M^2$ and set $\mu = -M$. This ensures the formal transformation effectively separates the positive rest energy M:

$$
D_0 \Longrightarrow (D_0 + i\mu), \quad i(D_0 + i\mu) \Longrightarrow E + M, \quad \mu = -M. \tag{14}
$$

Using this transformation, the equations are reorganized to classify terms by their orders of smallness. Setting terms of orders x and x^2 to zero yields linear constraints for the small parameters, which are absent in other equations.

The remaining equations are grouped as follows: Group I relates the small variables S_k to terms $D_i L_j$, and Group II includes terms of the form $D_0 L_n$. Substituting the small variables from Group I into the equations of Group II and performing the necessary calculations, we derive a system of equations for the five independent large components, which exhibit a nonrelativistic structure.

The details of these calculations are provided in Appendix A.

The final nonrelativistic equation is

$$
iD_0 L = -\frac{1}{2M} \Delta L + \frac{e}{2M}(1 + 3A^2)(F_{23}S_1 + F_{31}S_2 + F_{12}S_3)L, \tag{15}
$$

where $A^2 = \sinh^2 \alpha$, and S_i are the 5×5 spin matrices. For $A = 0$, this equation corresponds to a spin-2 particle without an anomalous magnetic moment, with A^2 accounting for the contribution from the anomalous magnetic moment.

5. Conclusions

We began with the well-established 50-component relativistic matrix equation for a hypothetical spin-2 particle interacting with external electromagnetic fields. This equation accounts for the particle's proposed anomalous magnetic moment. The complete wave function comprises a second-rank symmetric tensor and a third-rank tensor that is symmetric in two of its indices.

Through a nonrelativistic approximation, we decomposed the wave function into three parts: one dominant component and two smaller ones. This process revealed five independent large variables and 45 small ones.

To simplify the problem, we eliminated the variables associated with the third-rank tensor, leading to a relativistic system of second-order equations for the 10 components of the symmetric tensor. We then expressed these 10 variables as linear combinations of the large and small components, aligning with the nonrelativistic framework.

Following the general method, we separated the rest energy from the wave function and established the orders of smallness for various terms in the equations. By systematically performing the necessary calculations, we derived a system of five coupled equations for the large variables. These equations are presented in matrix form, exhibiting a nonrelativistic structure. Crucially, this system includes an interaction term representing the coupling with the external magnetic field through the three spin projections.

The interaction multiplier consists of two terms: the intrinsic magnetic moment and an additional contribution due to the proposed anomalous magnetic moment. This additional term is controlled by the framework's free parameter, $\sinh^2 \alpha$, which, on physical grounds, must be small relative to unity.

The resulting Pauli-like equation for the hypothetical spin-2 particle, derived within this framework, offers potential for generalization to curved space-time models.

It is worth noting that the presented framework applies to spin-2 particles with non-vanishing mass, a feature that distinguishes it from the massless Pauli–Fierz equations. While the Pauli–Fierz approach can be adapted to massive particles with known modifications that forgo gauge symmetry, our framework inherently incorporates mass. This distinction is of particular relevance in the context of covariant quantum gravity theories, where the necessity of a non-zero graviton mass has been emphasized as a prerequisite for theoretical consistency (see, for example, [41]). The inclusion of mass in our framework not only broadens its scope but also aligns with the physical requirements for the graviton proposed in these theories, providing a consistent and generalized equation for massive spin-2 particles.

Author Contributions: The authors equally contributed to this work in the conceptualization of the study, the analysis, and the writing of the manuscript. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the Armenian State Science Committee grant no. 21AG-1C064.

Data Availability Statement: This manuscript does not report data generation or analysis.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A. Technical Details

Group I of equations:

I

$$\frac{id_1 L_1}{M} + \frac{id_3 L_3}{M} + \frac{id_2 L_4}{M} - S_7 = 0,$$

$$\frac{d_2\left(-\frac{6\sqrt{2}L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)} - iL_1\right)}{M} + \frac{id_3 L_2}{M} + \frac{id_1 L_4}{M} - S_8 = 0,$$

$$\frac{6\sqrt{2}d_3 L_5}{M\left(\cosh(\alpha) - 2\sqrt{3}\sin(\alpha)\right)} + \frac{id_2 L_2}{M} + \frac{id_1 L_3}{M} - S_9 = 0,$$

Group II of equations:

$$1, \quad \frac{(6ieF_{12}L_4 - 6ieF_{31}L_3)A^2}{M^2}$$

$$+\frac{2iD_0 L_1}{M} + \frac{\left(-3-\sqrt{3}\right)D_{11}L_1}{6M^2} + \frac{\left(1-\frac{1}{\sqrt{3}}\right)D_{23}L_2}{M^2} + \frac{\left(-1-\frac{1}{\sqrt{3}}\right)D_{31}L_3}{M^2} + \frac{\left(-1-\frac{1}{\sqrt{3}}\right)D_{12}L_4}{M^2}$$

$$+\frac{D_{22}\left(\frac{1}{6}\left(3+\sqrt{3}\right)L_1 - \frac{i\sqrt{2}\left(-3+\sqrt{3}\right)L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M^2} + \frac{D_{33}\left(L_1 + \frac{i\sqrt{2}\left(-3+\sqrt{3}\right)L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M^2}$$

$$-\frac{i\left(3+\sqrt{3}\right)D_1 S_7}{3M} - \frac{i\left(-3+\sqrt{3}\right)D_2 S_8}{3M} - \frac{i\left(-3+\sqrt{3}\right)D_3 S_9}{3M} = 0,$$

$$2, \quad \frac{(6ieF_{23}L_2 - 6ieF_{12}L_4)A^2}{M^2} + \frac{\left(-1-\frac{1}{\sqrt{3}}\right)D_{23}L_2}{M^2} + \frac{\left(1-\frac{1}{\sqrt{3}}\right)D_{31}L_3}{M^2} + \frac{\left(-1-\frac{1}{\sqrt{3}}\right)D_{12}L_4}{M^2}$$

$$+\frac{D_0\left(-2iL_1-\frac{12\sqrt{2}L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M}+\frac{D_{11}\left(\frac{1}{6}\left(-3-\sqrt{3}\right)L_1+\frac{6i\sqrt{2}L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M^2}$$
$$+\frac{D_{22}\left(\frac{1}{6}\left(3+\sqrt{3}\right)L_1-\frac{i\sqrt{2}\left(3+\sqrt{3}\right)L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M^2}$$
$$+\frac{D_{33}\left(\frac{i\sqrt{2}\left(3+\sqrt{3}\right)L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}-L_1\right)}{M^2}-\frac{i\left(-3+\sqrt{3}\right)D_1S_7}{3M}-\frac{i\left(3+\sqrt{3}\right)D_2S_8}{3M}-\frac{i\left(-3+\sqrt{3}\right)D_3S_9}{3M}=0,$$

$$3,\qquad \frac{(6ieF_{31}L_3-6ieF_{23}L_2)A^2}{M^2}$$
$$+\frac{\left(-1-\frac{1}{\sqrt{3}}\right)D_{23}L_2}{M^2}+\frac{\left(-1-\frac{1}{\sqrt{3}}\right)D_{31}L_3}{M^2}+\frac{\left(1-\frac{1}{\sqrt{3}}\right)D_{12}L_4}{M^2}+\frac{12\sqrt{2}D_0L_5}{M\left(\cosh(\alpha)-2\sqrt{3}\sin(\alpha)\right)}$$
$$+\frac{i\sqrt{2}\left(3+\sqrt{3}\right)D_{33}L_5}{M^2\left(\cosh(\alpha)-2\sqrt{3}\sin(\alpha)\right)}+\frac{D_{11}\left(\frac{1}{6}\left(3-\sqrt{3}\right)L_1-\frac{6i\sqrt{2}L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M^2}$$
$$+\frac{D_{22}\left(\frac{1}{6}\left(-3+\sqrt{3}\right)L_1-\frac{i\sqrt{2}\left(3+\sqrt{3}\right)L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M^2}$$
$$-\frac{i\left(-3+\sqrt{3}\right)D_1S_7}{3M}-\frac{i\left(-3+\sqrt{3}\right)D_2S_8}{3M}-\frac{i\left(3+\sqrt{3}\right)D_3S_9}{3M}=0,$$

$$4,\qquad \frac{\left(3ieF_{23}L_1-3ieF_{12}L_3+3ieF_{31}L_4+\frac{36\sqrt{2}eF_{23}L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)A^2}{M^2}$$
$$+\frac{D_{23}L_1}{M^2}+\frac{2iD_0L_2}{M}+\frac{D_{11}L_2}{M^2}-\frac{D_{12}L_3}{M^2}-\frac{D_{31}L_4}{M^2}-\frac{iD_3S_8}{M}-\frac{iD_2S_9}{M}=0,$$

$$5,\qquad \frac{\left(3ieF_{31}L_1+3ieF_{12}L_2-3ieF_{23}L_4-\frac{18\sqrt{2}eF_{31}L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)A^2}{M^2}$$
$$-\frac{D_{12}L_2}{M^2}+\frac{2iD_0L_3}{M}+\frac{D_{22}L_3}{M^2}-\frac{D_{23}L_4}{M^2}+\frac{D_{31}\left(\frac{6i\sqrt{2}L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}-L_1\right)}{M^2}-\frac{iD_3S_7}{M}-\frac{iD_1S_9}{M}=0,$$

$$6,\qquad \frac{\left(-6ieF_{12}L_1-3ieF_{31}L_2+3ieF_{23}L_3-\frac{18\sqrt{2}eF_{12}L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)A^2}{M^2}$$
$$-\frac{D_{31}L_2}{M^2}-\frac{D_{23}L_3}{M^2}+\frac{2iD_0L_4}{M}+\frac{D_{33}L_4}{M^2}-\frac{6i\sqrt{2}D_{12}L_5}{M^2\left(\cosh(\alpha)-2\sqrt{3}\sin(\alpha)\right)}-\frac{iD_2S_7}{M}-\frac{iD_1S_8}{M}=0.$$

We solve for the variables S_7, S_8, and S_9 from the equations in group I and substitute these expressions into the equations of group II. We then obtain the following:

$$1,\qquad \frac{2iD_0L_1}{M}+\frac{(6ieF_{12}L_4-6ieF_{31}L_3)A^2}{M^2}$$
$$+\frac{\left(-3-\sqrt{3}\right)D_{11}L_1}{6M^2}+\frac{\left(1-\frac{1}{\sqrt{3}}\right)D_{23}L_2}{M^2}+\frac{\left(-1-\frac{1}{\sqrt{3}}\right)D_{31}L_3}{M^2}+\frac{\left(-1-\frac{1}{\sqrt{3}}\right)D_{12}L_4}{M^2}$$
$$+\frac{D_1\left(\left(1+\frac{1}{\sqrt{3}}\right)D_1L_1+\left(1+\frac{1}{\sqrt{3}}\right)D_3L_3+\left(1+\frac{1}{\sqrt{3}}\right)D_2L_4\right)}{M^2}$$
$$+\frac{D_{22}\left(\frac{1}{6}\left(3+\sqrt{3}\right)L_1-\frac{i\sqrt{2}\left(-3+\sqrt{3}\right)L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M^2}+\frac{D_{33}\left(L_1+\frac{i\sqrt{2}\left(-3+\sqrt{3}\right)L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M^2}$$
$$+\frac{D_2\left(\left(1-\frac{1}{\sqrt{3}}\right)D_2L_1+\left(-1+\frac{1}{\sqrt{3}}\right)D_3L_2+\left(-1+\frac{1}{\sqrt{3}}\right)D_1L_4+\frac{2i\sqrt{2}\left(-3+\sqrt{3}\right)D_2L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M^2}$$

$$+\frac{D_3\left(\left(-1+\frac{1}{\sqrt{3}}\right)D_2L_2+\left(-1+\frac{1}{\sqrt{3}}\right)D_1L_3-\frac{2i\sqrt{2}\left(-3+\sqrt{3}\right)D_3L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M^2}=0,$$

$$2,\qquad \frac{D_0\left(-2iL_1-\frac{12\sqrt{2}L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M}+\frac{(6ieF_{23}L_2-6ieF_{12}L_4)A^2}{M^2}$$
$$+\frac{\left(-1-\frac{1}{\sqrt{3}}\right)D_{23}L_2}{M^2}+\frac{\left(1-\frac{1}{\sqrt{3}}\right)D_{31}L_3}{M^2}+\frac{\left(-1-\frac{1}{\sqrt{3}}\right)D_{12}L_4}{M^2}$$
$$+\frac{D_1\left(\left(-1+\frac{1}{\sqrt{3}}\right)D_1L_1+\left(-1+\frac{1}{\sqrt{3}}\right)D_3L_3+\left(-1+\frac{1}{\sqrt{3}}\right)D_2L_4\right)}{M^2}+$$
$$+\frac{D_{11}\left(\frac{1}{6}\left(-3-\sqrt{3}\right)L_1+\frac{6i\sqrt{2}L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M^2}+\frac{D_{22}\left(\frac{1}{6}\left(3+\sqrt{3}\right)L_1-\frac{i\sqrt{2}\left(3+\sqrt{3}\right)L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M^2}$$
$$+\frac{D_{33}\left(\frac{i\sqrt{2}\left(3+\sqrt{3}\right)L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}-L_1\right)}{M^2}$$
$$+\frac{D_2\left(\left(-1-\frac{1}{\sqrt{3}}\right)D_2L_1+\left(1+\frac{1}{\sqrt{3}}\right)D_3L_2+\left(1+\frac{1}{\sqrt{3}}\right)D_1L_4+\frac{2i\sqrt{2}\left(3+\sqrt{3}\right)D_2L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M^2}$$
$$+\frac{D_3\left(\left(-1+\frac{1}{\sqrt{3}}\right)D_2L_2+\left(-1+\frac{1}{\sqrt{3}}\right)D_1L_3-\frac{2i\sqrt{2}\left(-3+\sqrt{3}\right)D_3L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M^2}=0,$$

$$3,\qquad \frac{12\sqrt{2}D_0L_5}{M\left(\cosh(\alpha)-2\sqrt{3}\sin(\alpha)\right)}+\frac{(6ieF_{31}L_3-6ieF_{23}L_2)A^2}{M^2}$$
$$+\frac{\left(-1-\frac{1}{\sqrt{3}}\right)D_{23}L_2}{M^2}+\frac{\left(-1-\frac{1}{\sqrt{3}}\right)D_{31}L_3}{M^2}+\frac{\left(1-\frac{1}{\sqrt{3}}\right)D_{12}L_4}{M^2}$$
$$+\frac{D_1\left(\left(-1+\frac{1}{\sqrt{3}}\right)D_1L_1+\left(-1+\frac{1}{\sqrt{3}}\right)D_3L_3+\left(-1+\frac{1}{\sqrt{3}}\right)D_2L_4\right)}{M^2}$$
$$+\frac{i\sqrt{2}\left(3+\sqrt{3}\right)D_{33}L_5}{M^2\left(\cosh(\alpha)-2\sqrt{3}\sin(\alpha)\right)}+\frac{D_{11}\left(\frac{1}{6}\left(3-\sqrt{3}\right)L_1-\frac{6i\sqrt{2}L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M^2}$$
$$+\frac{D_{22}\left(\frac{1}{6}\left(-3+\sqrt{3}\right)L_1-\frac{i\sqrt{2}\left(3+\sqrt{3}\right)L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M^2}$$
$$+\frac{D_2\left(\left(1-\frac{1}{\sqrt{3}}\right)D_2L_1+\left(-1+\frac{1}{\sqrt{3}}\right)D_3L_2+\left(-1+\frac{1}{\sqrt{3}}\right)D_1L_4+\frac{2i\sqrt{2}\left(-3+\sqrt{3}\right)D_2L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M^2}$$
$$+\frac{D_3\left(\left(1+\frac{1}{\sqrt{3}}\right)D_2L_2+\left(1+\frac{1}{\sqrt{3}}\right)D_1L_3-\frac{2i\sqrt{2}\left(3+\sqrt{3}\right)D_3L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M^2}=0,$$

$$4,\qquad \frac{2iD_0L_2}{M}+\frac{\left(3ieF_{23}L_1-3ieF_{12}L_3+3ieF_{31}L_4+\frac{36\sqrt{2}eF_{23}L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)A^2}{M^2}$$
$$+\frac{D_{23}L_1}{M^2}+\frac{D_{11}L_2}{M^2}-\frac{D_{12}L_3}{M^2}-\frac{D_{31}L_4}{M^2}$$
$$+\frac{D_3\left(-D_2L_1+D_3L_2+D_1L_4+\frac{6i\sqrt{2}D_2L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M^2}+\frac{D_2\left(D_2L_2+D_1L_3-\frac{6i\sqrt{2}D_3L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M^2}=0,$$

$$5,\qquad \frac{2iD_0L_3}{M}+\frac{\left(3ieF_{31}L_1+3ieF_{12}L_2-3ieF_{23}L_4-\frac{18\sqrt{2}eF_{31}L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)A^2}{M^2}-\frac{D_{12}L_2}{M^2}+\frac{D_{22}L_3}{M^2}-\frac{D_{23}L_4}{M^2}$$

$$+\frac{D_3(D_1L_1+D_3L_3+D_2L_4)}{M^2}+\frac{D_{31}\left(\frac{6i\sqrt{2}L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}-L_1\right)}{M^2}$$
$$+\frac{D_1\left(D_2L_2+D_1L_3-\frac{6i\sqrt{2}D_3L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M^2}=0,$$

$$6,\qquad \frac{2iD_0L_4}{M}+\frac{\left(-6ieF_{12}L_1-3ieF_{31}L_2+3ieF_{23}L_3-\frac{18\sqrt{2}eF_{12}L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)A^2}{M^2}$$
$$-\frac{D_{31}L_2}{M^2}-\frac{D_{23}L_3}{M^2}+\frac{D_{33}L_4}{M^2}+\frac{D_2(D_1L_1+D_3L_3+D_2L_4)}{M^2}$$
$$-\frac{6i\sqrt{2}D_{12}L_5}{M^2\left(\cosh(\alpha)-2\sqrt{3}\sin(\alpha)\right)}+\frac{D_1\left(-D_2L_1+D_3L_2+D_1L_4+\frac{6i\sqrt{2}D_2L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M^2}=0.$$

The structure of the equations can be concisely illustrated as follows:

$$1\quad D_0L_1;\quad 2\quad D_(L_1\oplus L_5);\quad 3\quad D_0L_5;\quad 4\quad D_0L_2;\quad 5\quad D_0L_3;\quad 6\quad D_0L_4.$$

In Equation (2), let us take into account the results of Equations (1) and (3):

$$2',\qquad \frac{D_2\left(\frac{6i\sqrt{2}\left(\sqrt{3}-1\right)D_2L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}+\left(1-\sqrt{3}\right)D_2L_1+\left(\sqrt{3}-1\right)D_3L_2+\left(\sqrt{3}-1\right)D_1L_4\right)}{M^2}$$
$$+\frac{D_3\left(-\frac{6i\sqrt{2}\left(\sqrt{3}-1\right)D_3L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}+\left(\sqrt{3}-1\right)D_2L_2+\left(\sqrt{3}-1\right)D_1L_3\right)}{M^2}$$
$$+\frac{D_1\left(\left(\sqrt{3}-1\right)D_1L_1+\left(\sqrt{3}-1\right)D_3L_3+\left(\sqrt{3}-1\right)D_2L_4\right)}{M^2}+\frac{6i\sqrt{2+\sqrt{3}}D_{33}L_5}{M^2\left(\cosh(\alpha)-2\sqrt{3}\sin(\alpha)\right)}$$
$$+\frac{D_{22}\left(\frac{1}{2}\left(1+\sqrt{3}\right)L_1-\frac{6i\sqrt{2+\sqrt{3}}L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M^2}$$
$$+\frac{\left(-1-\sqrt{3}\right)D_{11}L_1}{2M^2}+\frac{\left(-1-\sqrt{3}\right)D_{23}L_2}{M^2}+\frac{\left(-1-\sqrt{3}\right)D_{31}L_3}{M^2}+\frac{\left(-1-\sqrt{3}\right)D_{12}L_4}{M^2}=0.$$

In Equation (10), let us take into account the equations from Group I; this leads to the following:

$$10,\qquad \frac{D_2\left(-\frac{2i\sqrt{2}\left(3+\sqrt{3}\right)D_2L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}+\left(1+\frac{1}{\sqrt{3}}\right)D_2L_1+\left(-1-\frac{1}{\sqrt{3}}\right)D_3L_2+\left(-1-\frac{1}{\sqrt{3}}\right)D_1L_4\right)}{M^2}$$
$$+\frac{D_3\left(\frac{2i\sqrt{2}\left(3+\sqrt{3}\right)D_3L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}+\left(-1-\frac{1}{\sqrt{3}}\right)D_2L_2+\left(-1-\frac{1}{\sqrt{3}}\right)D_1L_3\right)}{M^2}$$
$$+\frac{D_1\left(\left(-1-\frac{1}{\sqrt{3}}\right)D_1L_1+\left(-1-\frac{1}{\sqrt{3}}\right)D_3L_3+\left(-1-\frac{1}{\sqrt{3}}\right)D_2L_4\right)}{M^2}$$
$$-\frac{i\sqrt{2}\left(\sqrt{3}-3\right)D_{33}L_5}{M^2\left(\cosh(\alpha)-2\sqrt{3}\sin(\alpha)\right)}+\frac{D_{22}\left(\frac{1}{6}\left(3-\sqrt{3}\right)L_1+\frac{i\sqrt{2}\left(\sqrt{3}-3\right)L_5}{\cosh(\alpha)-2\sqrt{3}\sin(\alpha)}\right)}{M^2}$$
$$+\frac{\left(\sqrt{3}-3\right)D_{11}L_1}{6M^2}+\frac{\left(\frac{1}{\sqrt{3}}-1\right)D_{23}L_2}{M^2}+\frac{\left(\frac{1}{\sqrt{3}}-1\right)D_{31}L_3}{M^2}+\frac{\left(\frac{1}{\sqrt{3}}-1\right)D_{12}L_4}{M^2}=0.$$

Again, taking in mind the identities,

$$D_a D_b = \frac{1}{2}(D_a D_b + D_b D_a) + \frac{1}{2}(D_a D_b - D_b D_a) = D_{ab} + ieF_{ab},$$

We transform all seven equations to the form where only the terms D_{ab} and ieF_{ab} are presented. In this way, we derive

$$1, \qquad 2iMD_0L_1 + (6ieF_{12}L_4 - 6ieF_{31}L_3)A^2 + 2ieF_{12}L_4 - 2ieF_{31}L_3$$

$$+\frac{1}{6}\Big(3+\sqrt{3}\Big)D_{11}L_1 + D_{22}\left(\frac{1}{6}\Big(9-\sqrt{3}\Big)L_1 + \frac{i\sqrt{2}\Big(-3+\sqrt{3}\Big)L_5}{\cosh(\alpha) - 2\sqrt{3}\sin(\alpha)}\right)$$

$$+D_{33}\left(L_1 - \frac{i\sqrt{2}\Big(-3+\sqrt{3}\Big)L_5}{\cosh(\alpha) - 2\sqrt{3}\sin(\alpha)}\right) + (-1 + \frac{1}{\sqrt{3}})\Big(D_{23}L_2 + D_{31}L_3 + D_{12}L_4\Big) = 0,$$

$$2, \qquad \frac{1}{2}\Big(-3+\sqrt{3}\Big)D_{11}L_1 + D_{22}\left(\frac{1}{2}\Big(3-\sqrt{3}\Big)L_1 + \frac{3i\sqrt{2}\Big(-3+\sqrt{3}\Big)L_5}{\cosh(\alpha) - 2\sqrt{3}\sin(\alpha)}\right)$$

$$-\frac{3i\sqrt{2}\Big(-3+\sqrt{3}\Big)D_{33}L_5}{\cosh(\alpha) - 2\sqrt{3}\sin(\alpha)} + (-3+\sqrt{3}\Big(D_{23}L_2 + D_{31}L_3 + D_{12}L_4) = 0,$$

$$3, \qquad +\frac{12\sqrt{2}MD_0L_5}{\cosh(\alpha) - 2\sqrt{3}\sin(\alpha)}$$

$$+(6ieF_{31}L_3 - 6ieF_{23}L_2)A^2 - 2ieF_{23}L_2 + 2ieF_{31}L_3 + (-1 + \frac{1}{\sqrt{3}})\Big(D_{12}L_4 + D_{23}L_2 + D_{31}L_3\Big)$$

$$+D_{11}\left(\frac{1}{6}\Big(-3+\sqrt{3}\Big)L_1 - \frac{6i\sqrt{2}L_5}{\cosh(\alpha) - 2\sqrt{3}\sin(\alpha)}\right)$$

$$+D_{22}\left(\frac{1}{6}\Big(3-\sqrt{3}\Big)L_1 + \frac{i\sqrt{2}\Big(-9+\sqrt{3}\Big)L_5}{\cosh(\alpha) - 2\sqrt{3}\sin(\alpha)}\right) - \frac{i\sqrt{2}\Big(3+\sqrt{3}\Big)D_{33}L_5}{\cosh(\alpha) - 2\sqrt{3}\sin(\alpha)} = 0,$$

$$4, \qquad 2iMD_0L_2 + (D_{11} + D_{22} + D_{33})L_2$$

$$+\left(3ieF_{23}L_1 - 3ieF_{12}L_3 + 3ieF_{31}L_4 + \frac{36\sqrt{2}eF_{23}L_5}{\cosh(\alpha) - 2\sqrt{3}\sin(\alpha)}\right)A^2$$

$$+ieF_{23}L_1 - ieF_{12}L_3 + ieF_{31}L_4 + \frac{12\sqrt{2}eF_{23}L_5}{\cosh(\alpha) - 2\sqrt{3}\sin(\alpha)} = 0,$$

$$5, \qquad 2iMD_0L_3 + (D_{11} + D_{22} + D_{33})L_3$$

$$+\left(3ieF_{31}L_1 + 3ieF_{12}L_2 - 3ieF_{23}L_4 - \frac{18\sqrt{2}eF_{31}L_5}{\cosh(\alpha) - 2\sqrt{3}\sin(\alpha)}\right)A^2$$

$$+ieF_{31}L_1 + ieF_{12}L_2 - ieF_{23}L_4 - \frac{6\sqrt{2}eF_{31}L_5}{\cosh(\alpha) - 2\sqrt{3}\sin(\alpha)} = 0,$$

$$6, \qquad +2iMD_0L_4 + (D_{11}L_4 + D_{22} + D_{33})L_4$$

$$+\left(-6ieF_{12}L_1 - 3ieF_{31}L_2 + 3ieF_{23}L_3 - \frac{18\sqrt{2}eF_{12}L_5}{\cosh(\alpha) - 2\sqrt{3}\sin(\alpha)}\right)A^2$$

$$-2ieF_{12}L_1 - ieF_{31}L_2 + ieF_{23}L_3 - \frac{6\sqrt{2}eF_{12}L_5}{\cosh(\alpha) - 2\sqrt{3}\sin(\alpha)} = 0,$$

$$10, \qquad \frac{i\sqrt{2}\Big(9+\sqrt{3}\Big)D_{33}L_5}{\cosh(\alpha) - 2\sqrt{3}\sin(\alpha)} + D_{22}\Big(\frac{1}{6}\Big(9+\sqrt{3}\Big)L_1 - \frac{i\sqrt{2}\Big(9+\sqrt{3}\Big)L_5}{\cosh(\alpha) - 2\sqrt{3}\sin(\alpha)}\Big)$$

$$+\frac{1}{6}\Big(-9-\sqrt{3}\Big)D_{11}L_1 + \Big(-3 - \frac{1}{\sqrt{3}}\Big)D_{23}L_2 + \Big(-3 - \frac{1}{\sqrt{3}}\Big)D_{31}L_3 + \Big(-3 - \frac{1}{\sqrt{3}}\Big)D_{12}L_4 = 0.$$

We can observe that in all equations, the variable L_5 consistently appears in the form:

$$\frac{L_5}{\cosh\alpha - 2\sqrt{3}\sinh\alpha}.$$

Thus, it is reasonable to redefine the variables as follows:

$$L_1, \quad L_2, \quad L_3, \quad L_4, \quad \frac{L_5}{\cosh\alpha - 2\sqrt{3}\sinh\alpha} \Longrightarrow L_5. \tag{A1}$$

With this substitution, all equations adopt a simpler form:

1

$$\begin{gathered}2iMD_0L_1 + (6ieF_{12}L_4 - 6ieF_{31}L_3)A^2 + 2ieF_{12}L_4 - 2ieF_{31}L_3 \\ +\frac{1}{6}\left(3+\sqrt{3}\right)D_{11}L_1 + D_{22}\left(\frac{1}{6}\left(9-\sqrt{3}\right)L_1 + i\sqrt{2}\left(-3+\sqrt{3}\right)L_5\right) \\ +D_{33}\left(L_1 - i\sqrt{2}\left(-3+\sqrt{3}\right)L_5\right) + (-1+\frac{1}{\sqrt{3}})\left(D_{23}L_2 + D_{31}L_3 + D_{12}L_4\right) = 0,\end{gathered}$$

3

$$\begin{gathered}+12\sqrt{2}MD_0L_5 + (6ieF_{31}L_3 - 6ieF_{23}L_2)A^2 - 2ieF_{23}L_2 + 2ieF_{31}L_3 \\ +D_{11}\left(\frac{1}{6}\left(-3+\sqrt{3}\right)L_1 - 6i\sqrt{2}L_5\right) + D_{22}\left(\frac{1}{6}\left(3-\sqrt{3}\right)L_1 + i\sqrt{2}\left(-9+\sqrt{3}\right)L_5\right) \\ -i\sqrt{2}\left(3+\sqrt{3}\right)D_{33}L_5 + (-1+\frac{1}{\sqrt{3}})\left(D_{23}L_2 + D_{31}L_3 + D_{12}L_4\right) = 0,\end{gathered}$$

2

$$\begin{gathered}\frac{1}{2}\left(-3+\sqrt{3}\right)D_{11}L_1 + D_{22}\left(\frac{1}{2}\left(3-\sqrt{3}\right)L_1 + 3i\sqrt{2}\left(-3+\sqrt{3}\right)L_5\right) - 3i\sqrt{2}\left(-3+\sqrt{3}\right)D_{33}L_5 \\ +3(-1+\frac{1}{\sqrt{3}})\left(D_{23}L_2 + D_{31}L_3 + D_{12}L_4\right) = 0.\end{gathered}$$

From Equation (2), let us express the term

$$(-1+1/\sqrt{3})(D_{23}L_2 + D_{31}L_3 + D_{12}L_4)$$

and substitute it in Equations (1) and (3), obtaining

1′

$$A^2(6ieF_{12}L_4 - 6ieF_{31}L_3) + 2iD_0L_1M + D_{11}L_1 + D_{22}L_1 + D_{33}L_1 - 2ieF_{31}L_3 + 2ieF_{12}L_4 = 0;$$

3′

$$A^2\left(\frac{eF_{23}L_2}{\sqrt{2}} - \frac{eF_{31}L_3}{\sqrt{2}}\right) + 2iD_0L_5M + D_{11}L_5 + D_{22}L_5 + D_{33}L_5 + \frac{eF_{23}L_2}{3\sqrt{2}} - \frac{eF_{31}L_3}{3\sqrt{2}} = 0;$$

4

$$\begin{gathered}2iMD_0L_2 + (D_{11}+D_{22}+D_{33})L_2 \\ +\left(3ieF_{23}L_1 - 3ieF_{12}L_3 + 3ieF_{31}L_4 + 36\sqrt{2}eF_{23}L_5\right)A^2 \\ +ieF_{23}L_1 - ieF_{12}L_3 + ieF_{31}L_4 + 12\sqrt{2}eF_{23}L_5 = 0;\end{gathered}$$

5

$$\begin{gathered}2iMD_0L_3 + (D_{11}+D_{22}+D_{33})L_3 \\ +\left(3ieF_{31}L_1 + 3ieF_{12}L_2 - 3ieF_{23}L_4 - 18\sqrt{2}eF_{31}L_5\right)A^2 \\ +ieF_{31}L_1 + ieF_{12}L_2 - ieF_{23}L_4 - 6\sqrt{2}eF_{31}L_5 = 0;\end{gathered}$$

6

$$+2iMD_0L_4+(D_{11}L_4+D_{22}+D_{33})L_4$$
$$+\Big(-6ieF_{12}L_1-3ieF_{31}L_2+3ieF_{23}L_3-18\sqrt{2}eF_{12}L_5\Big)A^2$$
$$-2ieF_{12}L_1-ieF_{31}L_2+ieF_{23}L_3-6\sqrt{2}eF_{12}L_5=0;$$

10

$$2(D_{23}L_2+D_{31}L_3+D_{12}L_4)=-D_{11}L_1+6i\sqrt{2}D_{33}L_5+D_{22}\Big(L_1-6i\sqrt{2}L_5\Big);$$

The last is an identity incorporated into the above equations.

Thus, we have derived 5 equations with the needed structure.

$$2iMD_0L_1+\Big(D_{11}+D_{22}+D_{33}\Big)L_1+2ie\Big(F_{12}L_4-F_{31}L_3\Big)+6ieA^2\Big(F_{12}L_4-F_{31}L_3\Big)=0,$$

$$2iMD_0L_2+\Big(D_{11}+D_{22}+D_{33}\Big)L_2++ie\Big(F_{23}L_1-12i\sqrt{2}F_{23}L_5+F_{31}L_4-F_{12}L_3\Big)$$
$$+3ieA^2\Big(F_{23}L_1-12i\sqrt{2}F_{23}L_5+F_{31}L_4-F_{12}L_3\Big)=0,$$

$$2iMD_0L_3+\Big(D_{11}+D_{22}+D_{33}\Big)L_3++ie\Big(F_{31}L_1+6i\sqrt{2}F_{31}L_5-F_{23}L_4+F_{12}L_2\Big)$$
$$+3ieA^2\Big(F_{31}L_1+6i\sqrt{2}F_{31}L_5-F_{23}L_4+F_{12}L_2\Big)=0,$$
$$+2iMD_0L_4+\Big(D_{11}L_4+D_{22}+D_{33}\Big)L_4+ie\Big(F_{23}L_3-F_{31}L_2-2F_{12}L_1+6i\sqrt{2}F_{12}L_5\Big)$$
$$+3A^2ie\Big(F_{23}L_3-F_{31}L_2-2F_{12}L_1+6i\sqrt{2}F_{12}L_5\Big)=0.$$
$$2iMD_0L_5+\Big(D_{11}+D_{22}+D_{33}\Big)L_5+\frac{e}{3\sqrt{2}}\Big(F_{23}L_2-F_{31}L_3\Big)+eA^2\frac{1}{\sqrt{2}}\Big(F_{23}L_2-F_{31}L_3\Big)=0.$$

This system can be rewritten differently (let $\Delta=D_{11}+D_{22}+D_{33}$):

$$\begin{aligned}2iMD_0L_1+\Delta L_1+ie(1+3A^2)\Big(2F_{12}L_4-2F_{31}L_3\Big)&=0,\\ 2iMD_0L_2+\Delta L_2+ie(1+3A^2)\Big(F_{23}L_1-12i\sqrt{2}F_{23}L_5+F_{31}L_4-F_{12}L_3\Big)&=0,\\ 2iMD_0L_3+\Delta L_3+ie(1+3A^2)\Big(F_{31}L_1+6i\sqrt{2}F_{31}L_5-F_{23}L_4+F_{12}L_2\Big)&=0,\\ +2iMD_0L_4+\Delta L_4+ie(1+3A^2)\Big(F_{23}L_3-F_{31}L_2-2F_{12}L_1+6i\sqrt{2}F_{12}L_5\Big)&=0,\\ 2iMD_0L_5+\Delta L_5+ie(1+3A^2)\Big(-i\frac{1}{3\sqrt{2}}F_{23}L_2+i\frac{1}{3\sqrt{2}}F_{31}L_3\Big)&=0.\end{aligned} \tag{A2}$$

This system can be presented in the matrix form:

$$2iMD_0L+\Delta L+ie(1+3A^2)(F_{23}T_1+F_{31}T_2+F_{12}T_3)L=0,\quad L=\begin{vmatrix}L_1\\L_2\\L_3\\L_4\\L_5\end{vmatrix},$$

where

$$T_1=\begin{vmatrix}0&0&0&0&0\\1&0&0&0&-12i\sqrt{2}\\0&0&0&-1&0\\0&0&1&0&0\\0&-i\frac{1}{3\sqrt{2}}&0&0&0\end{vmatrix},T_2=\begin{vmatrix}0&0&-2&0&0\\0&0&0&1&0\\1&0&0&0&6i\sqrt{2}\\0&-1&0&0&0\\0&0&i\frac{1}{3\sqrt{2}}&0&0\end{vmatrix},$$

$$T_3 = \begin{vmatrix} 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 & 6i\sqrt{2} \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix}.$$

We can verify that the following commutation relations hold:

$$T_1 T_2 - T_2 T_1 = -T_3, \quad \text{and so on.}$$

Introducing the new matrices,

$$S_1 = -iT_1, \quad S_2 = -iT_2, \quad S_3 = -iT_3,$$

we arrive at the commutation relations of the rotation group:

$$S_1 S_2 - S_2 S_1 = iS_3, \quad S_2 S_3 - S_3 S_2 = iS_1, \quad S_3 S_1 - S_1 S_3 = iS_2.$$

This implies that S_j represents the spin matrices for a particle of spin 2.

The final form of the nonrelativistic equation is

$$iD_0 L = -\frac{1}{2M}\Delta L + \frac{e}{2M}(1 + 3A^2)(F_{23}S_1 + F_{31}S_2 + F_{12}S_3)L. \tag{A3}$$

At $A = 0$, this equation describes a spin-2 particle without an anomalous magnetic moment. Recall that $A^2 = \sinh^2 \alpha$, where α is an arbitrary parameter.

To simplify further, let us find a basis in which the matrix S_3 becomes diagonal:

$$S_3 \Psi = \sigma \Psi, \quad \bar{\Psi} = S\Psi, \quad S^{-1}\bar{\Psi} = \Psi, \quad S S_3 S^{-1} = \begin{vmatrix} \sigma_1 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3 & 0 & 0 \\ 0 & 0 & 0 & \sigma_4 & 0 \\ 0 & 0 & 0 & 0 & \sigma_5 \end{vmatrix} = \bar{S}_3.$$

The required transformation is

$$S = \begin{vmatrix} -i & 0 & 0 & 1 & -3\sqrt{2} \\ 0 & 1 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & i & 0 & 0 \\ i & 0 & 0 & 1 & 3\sqrt{2} \end{vmatrix}, \quad S^{-1} = \begin{vmatrix} \frac{i}{2} & 0 & 3i\sqrt{2} & 0 & -\frac{i}{2} \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{i}{2} & 0 & -\frac{i}{2} & 0 \\ \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 0 \end{vmatrix}. \tag{A4}$$

This transformation leads to the new spin matrices:

$$\bar{S}_1 = \begin{vmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & -9\sqrt{2} & 0 & 0 \\ 0 & -\frac{1}{6\sqrt{2}} & 0 & -\frac{1}{6\sqrt{2}} & 0 \\ 0 & 0 & -9\sqrt{2} & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 \end{vmatrix}, \quad \bar{S}_2 = \begin{vmatrix} 0 & i & 0 & 0 & 0 \\ -i & 0 & -9i\sqrt{2} & 0 & 0 \\ 0 & \frac{i}{6\sqrt{2}} & 0 & -\frac{i}{6\sqrt{2}} & 0 \\ 0 & 0 & 9i\sqrt{2} & 0 & -i \\ 0 & 0 & 0 & i & 0 \end{vmatrix},$$

$$\bar{S}_3 = \begin{vmatrix} -2 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{vmatrix}.$$

References

1. Fierz, M. Über die relativistische theorie Kraftefreier Teilchen mit beliebigem Spin. *Helv. Phys. Acta*. **1939**, *12*, 3–37.
2. Pauli, W.; Fierz, M. Über relativistische Feldleichungen von Teilchen mit beliebigem Spin im elektromagnetishen Feld. *Helv. Phys. Acta*. **1939**, *12*, 297–300.
3. Fierz, M.; Pauli, W. On relativistic wave equations for particles of arbitrary spin in an electromagnetic field. *Proc. R. Soc. Lond.* **1939**, *A173*, 211–232.
4. de Broglie, L. Sur l'interprétation de certaines équations dans la théorie des particules de spin 2. *CR Acad. Sci. Paris* **1941**, *212*, 657–659.
5. Wild, E. On first order wave equations for elementary particles without subsidiary conditions. *Proc. R. Soc. Lond.* **1947**, *A191*, 253–268.
6. Gelfand, I.M.; Yaglom, A.M. General relativistically invariant equations and infinitedimensional representations of the Lorentz group. *J. Exp. Theor. Phys.* **1948**, *18*, 703–733. (In Russian)
7. Fedorov, F.I. On the theory of the spin 2 particle. Proceedings of Belorussian State University. *Ser.-Phys.-Math.* **1951**, *12*, 156–173.
8. Regge, T. On properties of the particle with spin 2. *Nuovo C* **1957**, *5*, 325–326. [CrossRef]
9. Buchdahl, H.A. On the compatibility of relativistic wave equations for particles of higher spin in the presence of a gravitational field. *Nuovo C* **1958**, *10*, 96–103. [CrossRef]
10. Buchdahl, H.A. On the compatibility of relativistic wave equations in Riemann spaces. *Nuovo C* **1962**, *25*, 486–496. [CrossRef]
11. Shelepin, L.A. Covariant theory of relativictic wave equations. *Nucl. Phys.* **1962**, *33*, 580–593. [CrossRef]
12. Velo, G.; Zwanziger, D. Noncausality and other defects of interaction Lagrangians for particles with spin one and higher. *Phys. Rev.* **1969**, *188*, 2218–2222. [CrossRef]
13. Aragone, C.; Deser, S. Constraints on gravitationally coupled tensor fields. *Nuovo C* **1971**, *A3*, 709–720. [CrossRef]
14. Velo, G. Anomalous behavior of a massive spin two charged particle in an external electromagnetic field. *Nucl. Phys.* **1972**, *B43*, 389–401. [CrossRef]
15. Hagen, C.R. Minimal electromagnetic coupling of spin-two fields. *Phys. Rev.* **1972**, *D6*, 984–987. [CrossRef]
16. Reilly, J.F. Minimal coupling in spin-2 field. *Nucl. Phys.* **1974**, *B1*, 356–364. [CrossRef]
17. Mathews, P.M.; Seetharaman, M.; Prabhakaran, J. Inconsistencies in the symmetric tensor field description of spin-2 particles in an external homogeneous magnetic field. *Phys. Rev.* **1976**, *D14*, 1021–1031. [CrossRef]
18. Kobayashi, M.; Shamaly, A. Minimal electromagnetic coupling for massive spin-2 fields. *Phys. Rev.* **1978**, *D17*, 2179. [CrossRef]
19. Aragone, C.; Deser, S. Consistency problems of spin-2 gravity coupling. *Nuovo C* **1980**, *B57*, 33–49. [CrossRef]
20. Mathews, P.M.; Govindarajan, T.R.; Seetharaman, M.; Prabhakaran, J. Relativistic wave equations coupled to external fields: An algebraic study of the problem of constraints. *J. Math. Phys.* **1980**, *21*, 1495–1505. [CrossRef]
21. Cox, W. First-order formulation of massive spin-2 field theories. *J. Phys. A* **1982**, *15*, 253–268. [CrossRef]
22. Wald, R.M. Spin-two fields and general covariance. *Phys. Rev.* **1986**, *D33*, 3613–3625. [CrossRef] [PubMed]
23. Loide, R.K. On conformally covariant spin-3/2 and spin-2 equations. *J. Phys. A* **1986**, *19*, 827–829. [CrossRef]
24. Higuchi, A. Forbidden mass range for spin-2 field theory in de Sitter space-time. *Nucl. Phys.* **1987**, *B282*, 397. [CrossRef]
25. Bogush, A.A.; Kisel, V.V. On description of anomalous magnetic moment of a massive particle with spin 2 in the theory of relativistic wave equations. *Izvestia Vuzov. SSSR Fizika.* **1988**, *31*, 11–16. (In Russian) [CrossRef]
26. Vasiliev, M.A. More on equations of motion for interacting massless fields of all spins in (3+1)-dimensions. *Phys. Lett.* **1992**, *B285*, 225–234. [CrossRef]
27. Buchbinder, I.L.; Krykhtin, V.A.; Pershin, V.D. On consistent equations for massive spin-2 field coupled to gravity in string theory. *Phys. Lett.* **1999**, *B466*, 216–226. [CrossRef]
28. Buchbinder, I.L.; Gitman, D.M.; Krykhtin, V.A.; Pershin, V.D. Equations of motion for massive spin 2 field coupled to gravity. *Nucl. Phys.* **2000**, *B584*, 615–640. [CrossRef]
29. Casini, H.; Montemayor, R.; Urrutia, L.F. Duality for symmetric second rank tensors. The linearized gravitational field. *Phys. Rev.* **2003**, *D68*, 065011. [CrossRef]
30. Red'kov, V.M.; Tokarevskaya, N.G.; Kisel, V.V. Graviton in a curved space-time background and gauge symmetry. *Nonlinear Phenom. Complex Syst.* **2003**, *6*, 772–778.
31. Schmidt, A.M. Classically Consistent Theories of Interacting Spin-2 Fields. Doctoral Thesis, Theoretical Physics, Stockholm, Sweden, 2013.
32. Bernard, L.; Deffayet, C.; Schmidt-May, A.; von Strauss, M. Linear spin-2 fields in most general backgrounds. *Phys. Rev.* **2016**, *D93*, 084020. [CrossRef]
33. Fukuma, M.; Kawai, H.; Sakai, K.; Yamamoto, J. Massive higher spin fields in curved spacetime and necessity of non-minimal couplings. *Prog. Theor. Exp. Phys.* **2016**, *2016*, 073B02. [CrossRef]
34. Koenigstein; Giacosa, F.; Rischke, D.H. Classical and quantum theory of the massive spin-two field. *Ann. Phys.* **2016**, *368*, 16. [CrossRef]

35. Mazuet, C.; Volkov, M.S. Massive spin-2 field in arbitrary spacetimes, the detailed derivation. *Phys. Rev.* **2017**, *D96*, 124023. [CrossRef]
36. Kisel, V.V.; Ovsiyuk, E.M.; Beko, O.V.; Voynova, Y.A.; Red'kov, V.M.; Balan, V. *Red'kov. Elementary Particles with Internal Structure in External Fields. I. General Theory; II. Physical Problems*; Nova Science Publishers Inc.: New York, NY, USA, 2018.
37. Ivashkevich, A.V.; Red'kov, V.M.; Ishkhanyan, A.M. Massless spin 2 field in 50-component approach, exact solutions with cylindrical symmetry, eliminating the gauge degrees of freedom. *Proc. Natl. Acad. Sci. USA* **2024**, *60*, 132–145.
38. Ivashkevich, A.V.; Bury, A.V.; Red'kov, V.M.; Kisel, V.V. On new form of the 50-component theory for spin 2 particle with anomalous magnetic moment in the basis of tensors of 2-nd and 3-rd ranks. *Nonlinear Dyn. Appl.* **2023**, *29*, 289–330.
39. Dudko, I.G.; Semenyuk, O.A.; Kisel, V.V.; Red'kov, V.M. Spin 2 particle with anomalous magnetic moment in Riemann space-time, restriction to massless case, gauge symmetry. *Nonlinear Phenom. Complex Syst.* **2022**, *25*, 286–296. [CrossRef]
40. Ivashkevich, A.V.; Bury, A.V.; Ovsiyuk, E.M.; Kisel, V.V.; Red'kov, V.M. Nonrelativistic approximation in 39-component theory for a spin 2 particle. Proceedings of the Komi Science Centre of the Ural Branch of the Russian Academy of Sciences. *Phys. Math. Sci.* **2024**, *5*, 46–57.
41. Cremaschini, C.; Tessarotto, M. Hamiltonian approach to GR—Part 2: Covariant theory of quantum gravity. *Eur. Phys. J. C* **2017**, *77*, 330. [CrossRef]