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A Note on Modified Degenerate Gamma and Laplace Transformation

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Abstract: Kim-Kim studied some properties of the degenerate gamma and degenerate Laplace transformation and obtained their properties. In this paper, we define modified degenerate gamma and modified degenerate Laplace transformation and investigate some properties and formulas related to them.

Keywords: the degenerate gamma function; the modified degenerate gamma function; the degenerate Laplace transform; the modified degenerate Laplace transform

1. Introduction

It is well known that gamma function is defied by

$$\Gamma(s) = \int_0^\infty e^{-t} t^{s-1} dt, \text{ where } s \in \mathbf{C} \text{ with } Re(s) > 0, \tag{1}$$

(see [1,2]). From (1), we note that

$$\Gamma(s+1) = s\Gamma(s), \text{ and } \Gamma(n+1) = n!, \text{ where } n \in \mathbf{N}. \tag{2}$$

Let $f(t)$ be a function defined for $t \geq 0$. Then, the integral

$$L(f(t)) = \int_0^\infty e^{-st} f(t) dt, \tag{3}$$

(see [1–4]), is said to be the Laplace transform of f, provided that the integral converges. For $\lambda \in (0, \infty)$. Kim-Kim [2] introduced the degenerate gamma function for the complex variable $s$with $0 < Re(s) < \frac{1}{\lambda}$ as follows:

$$\Gamma_\lambda(s) = \int_0^\infty (1+\lambda t)^{-\frac{1}{\lambda}} t^{s-1} dt, \tag{4}$$

(see [2]) and degenerate Laplace transformation which was defined by

$$L_\lambda(f(t)) = \int_0^\infty (1+\lambda t)^{-\frac{s}{\lambda}} f(t) dt, \tag{5}$$

(see [2,5]), if the integral converges. The authors obtained some properties and interesting formulas related to the degenerate gamma function. For examples, For $\lambda \in (0,1)$ and $0 < Re(s) < \frac{1-\lambda}{\lambda}$,

$$\Gamma_\lambda(s+1) = \frac{s}{(1-\lambda)^{s-1}}\Gamma_{\frac{\lambda}{1-\lambda}}(s), \tag{6}$$

and $\lambda \in (0, \frac{1}{k+s})$ with $k \in \mathbf{N}$ and $0 < Re(s) < \frac{1-\lambda}{\lambda}$,

$$\Gamma_\lambda(s+1) = \frac{s(s-1)\cdots(s-(k+1)+1)}{(1-\lambda)(1-2\lambda)\cdots(1-k\lambda)(1-(k+1)\lambda)}\Gamma_{\frac{\lambda}{1-(k+1)\lambda}}(s-k), \tag{7}$$

and for $k \in \mathbf{N}$ and $\lambda \in (0, \frac{1}{k})$,

$$\Gamma_\lambda(k) = \frac{(k-1)!}{(1-\lambda)(1-2\lambda)\cdots(1-k\lambda)}. \tag{8}$$

The authors obtained some formulas related to the degenerate Laplace transformation. For examples,

$$L_\lambda(1) = \frac{1}{s-\lambda}, \text{ if } s > \lambda, \tag{9}$$

and

$$L_\lambda((1+\lambda t)^{-\frac{a}{\lambda}}) = \frac{1}{s+a-\lambda}, \text{ if } s > -a+\lambda, \tag{10}$$

and

$$L_\lambda(\cos_\lambda(at)) = \frac{s-\lambda}{(s-\lambda)^2+a^2}, \tag{11}$$

and

$$L_\lambda(\sin_\lambda(at)) = \frac{a}{(s-\lambda)^2+a^2}, \tag{12}$$

where $\cos_\lambda(t) = \frac{1}{2}\left((1+\lambda t)^{\frac{it}{\lambda}} + (1+\lambda t)^{-\frac{it}{\lambda}}\right)$ and $\sin_\lambda(t) = \frac{1}{2i}\left((1+\lambda t)^{\frac{it}{\lambda}} - (1+\lambda t)^{-\frac{it}{\lambda}}\right)$.

Furthermore, the authors obtained that

$$L_\lambda(t^n) = \frac{n!}{(s-\lambda)(s-2\lambda)\cdots(s-n\lambda)(s-(n+1)\lambda)}, \tag{13}$$

for $n \in \mathbf{N}$ and $s > (n+1)\lambda$, and

$$\begin{aligned} L_\lambda(f^{(n)}(t)) &= s(s+\lambda)(s+2\lambda)\cdots(s+(n-1)\lambda)L_\lambda((1+\lambda t)^{-n}f(t)) \\ &- \sum_{i=0}^{n-1} f^{(i)}(0)\left(\prod_{t=1}^{n-i-1} s+(l-1)\lambda\right). \end{aligned} \tag{14}$$

where $f, f^{(1)}, \cdots, f^{(n-1)}$ are continuous on $(0,\infty)$ and are of degenerate exponential order and $f^{(n)}(t)$ is piecewise continuous on $(0,\infty)$, and

$$L_\lambda((\log(1+\lambda t))^n f(t)) = (-1)^n\lambda^n\left(\frac{d}{ds}\right)^n L_\lambda(s), \tag{15}$$

for $n \in \mathbf{N}$.

At first, L. Carlitz introduced the degenerate special polynomials (see [6,7]). The recently works which can be cited in this and researchers have studied the degenerate special polynomials and numbers (see [2,8–19]). Recently, the concept of degenerate gamma function and degenerate Laplace transformation was introduced by Kim-Kim [2]. They studied some properties of the degenerate gamma and degenerate Laplace transformation and obtained their properties. We observe whether or not that holds. Thus, we consider the modified degenerate Laplace transform which are satisfied (16). The degenerate gamma and degenerate Laplace transformation applied to engineer's mathematical toolbox as they make solving linear ODEs and related initial value problems. This paper consists of two sections. The first section contains the modified degenerate gamma function and investigate the properties of the modified gamma function. The second part of the paper provide the modified degenerate Laplace transformation and investigate interesting results of the modified degenerate Laplace transformation.

$$L_\lambda(f*g) = L_\lambda(f)L_\lambda(g) \tag{16}$$

2. Modified Degenerate Gamma Function

In this section, we will define modified degenerate gamma functions which are different to degenerate gamma functions. For each $\lambda \in (0,\infty)$, we define modified degenerate gamma function for the complex variable s with $0 < Re(s)$ as follows:

$$\Gamma^*_\lambda(s) = \int_0^\infty (1+\lambda)^{-\frac{t}{\lambda}} t^{s-1}dt. \tag{17}$$

Let $\lambda \in (0,1)$. Then, for $0 < Re(s)$, we have

$$\begin{aligned}\Gamma^*_\lambda(s+1) &= \int_0^\infty (1+\lambda)^{-\frac{t}{\lambda}} t^{s}dt \\ &= \frac{1}{(\log(1+\lambda)^{-\frac{1}{\lambda}}}(1+\lambda)^{-\frac{t}{\lambda}}t^s\mid_0^\infty + \frac{\lambda}{\log(1+\lambda)}\int_0^\infty s(1+\lambda)^{-\frac{t}{\lambda}}t^{s-1}dt \\ &= \frac{\lambda}{\log(1+\lambda)}s\Gamma^*_\lambda(s).\end{aligned} \tag{18}$$

Therefore, by (18), we obtain the following theorem.

Theorem 1. *Let $\lambda \in (0,1)$. Then, for $0 < Re(s)$, we have*

$$\Gamma^*_\lambda(s+1) = \frac{\lambda s}{\log(1+\lambda)}\Gamma^*_\lambda(s). \tag{19}$$

Then, for $0 < Re(s)$ and $\lambda \in (0,1)$, repeatly we calculate

$$\Gamma^*_\lambda(s+1) = \frac{\lambda s}{\log(1+\lambda)}\Gamma^*_\lambda(s) = \frac{\lambda^2(s-1)}{(\log(1+\lambda))^2}\Gamma^*_\lambda(s-1). \tag{20}$$

Thus, continuing this process, for $0 < Re(s)$ and $\lambda \in (0,1)$, we have

$$\Gamma^*_\lambda(s+1) = \frac{\lambda^k(s-1)\cdots(s-k+1)}{(\log(1+\lambda))^k}\Gamma^*_\lambda(s-k). \tag{21}$$

Therefore, by (21), we obtain the following theorem.

Theorem 2. *Let $\lambda \in (0,1)$. Then, for $0 < Re(s)$, we have*

$$\Gamma^*_\lambda(s+1) = \tfrac{\lambda^k(s-1)\cdots(s-k+1)}{(\log(1+\lambda))^k}\Gamma^*_\lambda(s-k). \tag{22}$$

Let us take $s = k+1$. Then, by Theorem 2, we get

$$\begin{aligned}\Gamma^*_\lambda(k+2) &= \frac{\lambda^{k+1}k\cdots 2}{(\log(1+\lambda))^{k+1}}\Gamma^*_\lambda(1) \\ &= \frac{\lambda^{k+1}k!}{(\log(1+\lambda))^{k+1}}\Gamma^*_\lambda(1)\end{aligned} \tag{23}$$

and

$$\begin{aligned}\Gamma_\lambda^*(1) &= \int_0^\infty (1+\lambda)^{-\frac{t}{\lambda}} dt \\ &= -\frac{\lambda}{(\log(1+\lambda)}(1+\lambda)^{-\frac{t}{\lambda}} \mid_0^\infty \\ &= \frac{\lambda}{(\log(1+\lambda)}.\end{aligned} \tag{24}$$

Therefore, by (23) and (24), we obtain the following theorem.

Theorem 3. *For $k \in \mathbf{N}$ and $\lambda \in (0,1)$, we have*

$$\Gamma_\lambda^*(k+1) = \frac{\lambda^{k+1}k!}{(\log(1+\lambda))^{k+1}}. \tag{25}$$

3. Modified Degenerate Laplace Transformation

In this section, we will define modified Laplace transformation which are different to degenerate Laplace transformation. Let $\lambda \in (0,\infty)$ and let $f(t)$ be a function defined for $t \geq 0$. Then the integral

$$\mathcal{L}_\lambda^*(f(t)) = \int_0^\infty (1+\lambda s)^{-\frac{t}{\lambda}} f(t) dt. \tag{26}$$

is said to be the modified degenerate Laplace transformation of f if the integral converges which is also defined by $\mathcal{L}_\lambda^*(f(t)) = F_\lambda(s)$.

From (26), we get

$$\mathcal{L}_\lambda^*(\alpha f(t) + \beta g(t)) = \alpha \mathcal{L}_\lambda^*(f(t)) + \beta \mathcal{L}_\lambda^*(g(t)), \tag{27}$$

where α and β are constant real numbers.

First, we observe that for $n \in \mathbf{N}$,

$$\begin{aligned}\mathcal{L}_\lambda^*(t^n) &= \int_0^\infty (1+\lambda s)^{-\frac{t}{\lambda}} t^n dt \\ &= -\frac{\lambda}{\log(1+\lambda s)}(1+\lambda s)^{-\frac{t}{\lambda}} t^n \mid_0^\infty + \frac{\lambda n}{\log(1+\lambda s)} \int_0^\infty (1+\lambda s)^{-\frac{t}{\lambda}} t^{n-1} dt \\ &= \frac{\lambda n}{\log(1+\lambda s)} \mathcal{L}_\lambda^*(t^{n-1}) \\ &= \frac{\lambda n}{\log(1+\lambda s)} \left(-\frac{\lambda}{\log(1+\lambda s)} (1+\lambda s)^{-\frac{t}{\lambda}} t^{n-1} \mid_0^\infty + \frac{\lambda(n-1)}{\log(1+\lambda s)} \int_0^\infty (1+\lambda s)^{-\frac{t}{\lambda}} t^{n-2} dt \right) \\ &= \left(\frac{\lambda}{\log(1+\lambda s)}\right)^2 n(n-1)\mathcal{L}_\lambda^*(t^{n-2}) \\ &= \cdots \\ &= \left(\frac{\lambda}{\log(1+\lambda s)}\right)^n n!\mathcal{L}_\lambda^*(1) \\ &= \left(\frac{\lambda}{\log(1+\lambda s)}\right)^{n+1} n!.\end{aligned} \tag{28}$$

Therefore, by (28), we obtain the following theorem.

Theorem 4. *For $k \in \mathbf{N}$ and $\lambda \in (0,1)$, we have*

$$\mathcal{L}_\lambda^*(t^n) = \left(\frac{\lambda}{\log(1+\lambda s)}\right)^{n+1} n!. \tag{29}$$

Secondly, we note that if f is a periodic function with a period T.

$$\begin{aligned}\mathcal{L}_\lambda^*(f(t)) &= \int_0^\infty (1+\lambda s)^{-\frac{t}{\lambda}} f(t)dt \\ &= \int_0^T (1+\lambda s)^{-\frac{t}{\lambda}} f(t)dt + \int_T^\infty (1+\lambda s)^{-\frac{t}{\lambda}} f(t)dt \\ &= \int_0^T (1+\lambda s)^{-\frac{t}{\lambda}} f(t)dt + \int_0^\infty (1+\lambda s)^{-\frac{t+T}{\lambda}} f(t+T)dt \\ &= \int_0^T (1+\lambda s)^{-\frac{t}{\lambda}} f(t)dt + (1+\lambda s)^{-\frac{T}{\lambda}} \int_0^\infty (1+\lambda s)^{-\frac{t}{\lambda}} f(t)dt\end{aligned} \tag{30}$$

By (30), we get

$$\left(1-(1+\lambda s)^{-\frac{T}{\lambda}}\right)\mathcal{L}_\lambda^*(f(t)) = \int_0^T (1+\lambda s)^{-\frac{t}{\lambda}} f(t)dt. \tag{31}$$

Thus, by (31), we get

$$\mathcal{L}_\lambda^*(f(t)) = \frac{1}{\left(1-(1+\lambda s)^{-\frac{T}{\lambda}}\right)} \int_0^T (1+\lambda s)^{-\frac{t}{\lambda}} f(t)dt. \tag{32}$$

We recall that the degenerate Bernoulli numbers are introduced as

$$\frac{t}{(1+\lambda)^{-\frac{t}{\lambda}}} = \sum_{n=0}^\infty B_{n,\lambda}\frac{t^n}{n!}, \tag{33}$$

Thus, by (32) and (33), we have

$$\begin{aligned}\frac{1}{1-(1+\lambda S)^{-\frac{T}{\lambda}}} &= -\frac{1}{TS}\frac{ST}{(1+\lambda s)^{-\frac{TS}{\lambda S}}-1} \\ &= -\frac{1}{TS}\sum_{n=0}^\infty B_{n,\lambda S}(-1)^n S^n \frac{T^n}{n!}.\end{aligned} \tag{34}$$

Therefore, by (33) and (34), we obtain the following theorem.

Theorem 5. *If f is a function defined $t \geq 0$ and $\mathcal{L}_\lambda^*(f(t))$ exists, then we have*

$$\begin{aligned}\mathcal{L}_\lambda^*(f(t)) &= -\frac{1}{TS}\sum_{n=0}^\infty B_{n,\lambda S}(-1)^n S^n \int_0^T (1+\lambda s)^{-\frac{t}{\lambda}} f(t)dt \frac{T^n}{n!} \\ &= -\frac{1}{TS}\sum_{n=0}^\infty B_{n,\lambda S}(-1)^n S^n \mathcal{L}_\lambda^*(U(t-T)f(t)),\end{aligned} \tag{35}$$

where $U(t-a) = \begin{cases} 0, \text{for } 0 \geq t \geq a, \\ 1, \text{for } t \leq a. \end{cases}$ *is the Heviside function.*

Thirdly, we observe the modified degenerate Laplace transformation of $f(t-a)U(t-a)$ as follows:

$$\begin{aligned}\mathcal{L}_\lambda^*(f(t-a)U(t-a)) &= \int_0^\infty (1+\lambda s)^{-\frac{t}{\lambda}} f(t-a)U(t-a)dt \\ &= \int_a^\infty (1+\lambda s)^{-\frac{t}{\lambda}} f(t-a)dt \\ &= \int_0^\infty (1+\lambda s)^{-\frac{t+a}{\lambda}} f(t)dt \\ &= (1+\lambda s)^{-\frac{a}{\lambda}} \int_0^\infty (1+\lambda s)^{-\frac{t}{\lambda}} f(t)dt \\ &= (1+\lambda s)^{-\frac{a}{\lambda}} \mathcal{L}_\lambda^*(f(t)).\end{aligned} \tag{36}$$

Therefore, by (36), we obtain the following theorem.

Theorem 6. *For $\lambda \in (0,1)$ and $a \in (0,\infty)$ we have*

$$\mathcal{L}_\lambda^*(f(t-a)U(t-a)) = (1+\lambda s)^{-\frac{a}{\lambda}}\mathcal{L}_\lambda^*(f(t)), \tag{37}$$

where $U(t-a)$ is the Heviside function.

Fourthly, we observe the modified degenerate Laplace transformation of the convolution $f * g$ of two function f, g as follows:

$$\begin{aligned}
\mathcal{L}_\lambda^*(f)\mathcal{L}_\lambda^*(g) &= \left(\int_0^\infty (1+\lambda s)^{-\frac{t}{\lambda}} f(t)dt\right)\left(\int_0^\infty (1+\lambda s)^{-\frac{\tau}{\lambda}} g(\tau)d\tau\right) \\
&= \int_0^\infty \int_0^\infty (1+\lambda s)^{-\frac{t+\tau}{\lambda}} f(t)g(\tau)dtd\tau \\
&= \int_0^\infty f(t)\int_\tau^\infty (1+\lambda s)^{-\frac{\mu}{\lambda}} g(\mu-\tau)d\mu d\tau \\
&= \int_0^\infty \int_\tau^\infty f(t)(1+\lambda s)^{-\frac{\mu}{\lambda}} g(\mu-\tau)d\mu d\tau \\
&= \int_0^\infty (f*g)\,(1+\lambda s)^{-\frac{\mu}{\lambda}} d\mu \\
&= \mathcal{L}_\lambda(f*g).
\end{aligned} \tag{38}$$

Therefore, by (38), we obtain the following theorem.

Theorem 7. *For $\lambda \in (0,1]$, we have*

$$\mathcal{L}_\lambda^*(f*g) = \mathcal{L}_\lambda^*(f)\mathcal{L}_\lambda^*(g). \tag{39}$$

We note that

$$\begin{aligned}
\mathcal{L}_\lambda^*(1) &= \int_0^\infty (1+\lambda s)^{-\frac{t}{\lambda}} 1dt \\
&= -\frac{\lambda}{\log(1+\lambda s)}(1+\lambda s)^{-\frac{t}{\lambda}} \,|_0^\infty \\
&= \frac{\lambda}{\log(1+\lambda s)}.
\end{aligned} \tag{40}$$

By (40), we have

$$L_\lambda^*(f*1) = L_\lambda^*(f)L_\lambda^*(1) = L_\lambda^*(f)\frac{\lambda}{\log(1+\lambda s)}. \tag{41}$$

Therefore, by (41), we obtain the following theorem.

Theorem 8. *For $\lambda \in (0,1]$, we have*

$$\mathcal{L}_\lambda^{*-1}(L_\lambda^*(f)\frac{\lambda}{\log(1+\lambda s)}) = f*1(t) = \int_0^t f(t)dt. \tag{42}$$

Fifthly, we observe that the modified degenerate Laplace transformation of derivative of f which is $f(t) = 0((1+\lambda s)^{-\frac{t}{\lambda}})$, where $f(t) = 0(u(t))$ means

$$\begin{aligned}\mathcal{L}^*_\lambda(f') &= \int_0^\infty (1+\lambda s)^{-\frac{t}{\lambda}} f' dt \\ &= (1+\lambda s)^{-\frac{t}{\lambda}} f(t)\mid_0^\infty + \int_0^\infty \frac{\log(1+\lambda s)}{\lambda}(1+\lambda s)^{-\frac{t}{\lambda}} f(t)dt \\ &= -f(0) + \frac{\log(1+\lambda s)}{\lambda}\mathcal{L}^*_\lambda(f).\end{aligned} \tag{43}$$

and

$$\begin{aligned}\mathcal{L}^*_\lambda(f^{(2)}) &= \int_0^\infty (1+\lambda s)^{-\frac{t}{\lambda}} f^{(2)} dt \\ &= (1+\lambda s)^{-\frac{t}{\lambda}} f'(t)\mid_0^\infty + \frac{\log(1+\lambda s)}{\lambda}\int_0^\infty (1+\lambda s)^{-\frac{t}{\lambda}} f'(t)dt \\ &= -f(0) + \frac{\log(1+\lambda s)}{\lambda}\left(-f(0) + \frac{\log(1+\lambda s)}{\lambda}\mathcal{L}^*_\lambda(f)\right) \\ &= \left(\frac{\log(1+\lambda s)}{\lambda}\right)^2 \mathcal{L}^*_\lambda(f) - \frac{\log(1+\lambda s)}{\lambda} f(0) - f'(0).\end{aligned} \tag{44}$$

By using mathematical induction, we obtain the following theorem.

Theorem 9. *For $\lambda \in (0,1]$, we have*

$$\mathcal{L}^*_\lambda(f^{(n)}) = \left(\frac{\log(1+\lambda s)}{\lambda}\right)^n \mathcal{L}^*_\lambda(f) - \sum_{i=0}^{n-1}\left(\frac{\log(1+\lambda s)}{\lambda}\right)^{n-1-i} f^{(i)}(0). \tag{45}$$

Finally, we observe

$$\begin{aligned}\frac{dF^*_\lambda}{ds} &= \int_0^\infty \frac{\lambda}{1+\lambda s}(-\frac{t}{\lambda})(1+\lambda s)^{-\frac{t}{\lambda}} f(t)dt \\ &= -\frac{1}{1+\lambda s}\int_0^\infty (1+\lambda s)^{-\frac{t}{\lambda}} t f(t) dt \\ &= -\frac{1}{1+\lambda s}\mathcal{L}^*_\lambda(tf(t)).\end{aligned} \tag{46}$$

By (46), we obtain the following theorem.

Theorem 10. *For $\lambda \in (0,1]$ and $0 < Re(s)$, we have*

$$\frac{dF^*_\lambda}{ds} = -\frac{1}{1+\lambda s}\mathcal{L}^*_\lambda(tf(t)). \tag{47}$$

4. Conclusions

Kim-Kim ([9]) defined a degenerate gamma function and a degenerate Laplace transformation. The motivation of this paper is to define modified degenerate gamma functions and modified degenerate Laplace transformations which are different to degenerate gamma function and degenerate Laplace transformation and to obtain more useful results which are Theorems 7 and 8 for the modified degenerate Laplace transformation. We do not obtain these result from the degenerate Laplace transformation. Also, we investigated some results which are Theorems 1 and 3 for modified degenerate gamma functions. Furthermore, Theorems 6 and 9 are some interesting properties which are applied to differential equations in engineering mathematics.

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