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A STUDY OF AM AND FM SIGNAL RECEPTION OF TIME MODULATED LINEAR ANTENNA ARRAYS

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Abstract—The amplitude modulation (AM) and frequency modulation (FM) signal transmission of time modulated linear arrays (TMLA) is studied in this paper. The signal with a certain bandwidth received by a TMLA is time modulated by RF switches, generating many sideband signals at multiples of the time modulation frequency and each of them has the same bandwidth as the original signal. The AM and FM signals received by the TMLA were analyzed, and the requirements of the time modulation frequency for the recovery of the original signal is presented. Simulation results show that the time modulation frequency of the TMLA should be equal to or greater than the bandwidth of the original signal and a band-pass filter (BPF) has to be used to recover the original signal.

1. INTRODUCTION

Power pattern synthesis is an extremely important issue in antenna arrays design and has received much attention over several years. Besides some classical synthesis methods such as Chebyshev, Taylor and Woodward-Lawson synthesis technique, various novel approaches have been proposed during recent years to solve the problem [1–3]. On the other hand, time modulated antenna arrays which introduce a fourth dimension -time- into the array design have been demonstrated to be attractive for synthesizing low/ultra-low sidelobe levels (SLLs) [4–8]. As compared to conventional antenna arrays, each element in the time modulated antenna arrays is controlled by a high speed RF switch. The time parameter which can be used to

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taper the aperture distribution can be easily, rapidly and accurately adjusted. Consequently, the time modulated antenna arrays have more flexibility for the design, and the excitation amplitude dynamic-range ratios are usually low or even can be uniform as compared to conventional antenna arrays. By properly switching on the array elements within each period, it is possible to produce the desired performance of the power pattern. The major problem of the time modulated antenna arrays is that the time modulation generates unwanted harmonics or sidebands at multiples of the time modulation frequency. Many studies on the time modulated antenna arrays were concentrated on minimizing these harmonics [9–13]. However, sidebands do not always be harmful. The first harmonic is utilized in [14] to synthesize difference patterns with active null scanning capabilities for a two-element time modulated antenna array. Shanks proposed a simultaneous scan operation based on time modulation technology in which the beams at different sidebands were used to point at different directions [15].

So far, most of the studies on time modulated antenna arrays are mainly on array pattern synthesis, the analysis of signals with a certain bandwidth received by the time modulated antenna arrays and how to recover the original signals are rarely seen. The studies of the AM signal transmission in the time modulated antenna array were presented in [16, 17], which showed that after time modulation by the time modulated antenna arrays the original modulation of the signal is also preserved. However, how to recover the original signal was not mentioned. In this paper, the theoretical background of the TMLA reviewed in Section 2 and then the AM and FM signals received by the TMLA are analyzed in Section 3. Based on the analysis, the requirements for the time modulation frequency of the high-speed RF switches and the passband of the BPF for the signal recovery are presented in Section 4. Simulation results of a Gaussian pulse AM signal and a linear FM signal received by a 16-element TMLA are given in Section 5. The results show that in order to exactly recover the original signal, the time modulation frequency of the high-speed RF switches should not be less than the bandwidth of the original signal and a BPF has to be used to filter out all sideband signals. Finally, Section 6 presents the conclusion of this study.

2. THEORETICAL BACKGROUND

Let us consider an N-element linear array of equally spaced isotropic elements, and the array factor is given by [10]

$$F(\theta, t) = e^{j2\pi f_0 t} \sum_{k=1}^{N} A_k e^{j\alpha_k} U_k(t) \cdot e^{j(k-1)\beta d \sin\theta} \tag{1}$$

where f_0 is the center frequency of the antenna array, A_k and α_k are the static excitation amplitude and phase of the kth element, d is the element spacing, $\beta = 2\pi f_0/c$, c is the velocity of light in free space, and $U_k(t)$ is the periodic switch-on time sequences. $U_k(t) = 1$ during the period of "on" times τ_k and $U_k(t) = 0$ for the rest of the period. θ is the angle measured from the broadside direction of the array. Suppose that the time modulation frequency of the high-speed RF switches is f_p, due to that $U_k(t)$ is a periodic function of time, the space and frequency response of (1) can be obtained by decomposing it into Fourier series, and each frequency component has a frequency of nf_p ($n = 0, \pm 1, \pm 2, \ldots, \pm\infty$). The nth order Fourier component can be written as

$$F_n(\theta, t) = e^{j2\pi(f_0 + nf_p)t} \sum_{k=1}^{N} a_{nk} \cdot e^{j[(k-1)\beta d \sin\theta + \alpha_k]} \tag{2}$$

where a_{nk} is the complex amplitude and is given by [11].

$$a_{nk} = A_k \tau_k f_p \cdot \frac{\sin[\pi n \tau_k f_p]}{\pi n \tau_k f_p} \cdot e^{-j\pi n \tau_k f_p} \tag{3}$$

At the center frequency ($n = 0$), (3) becomes

$$a_{0k} = A_k \tau_k f_p \tag{4}$$

Thus, various conventional excitation amplitude distributions, such as the Dolph-Chebyshev or Taylor distribution, can be applied to (4) to obtain a desired pattern at the center frequency. Alternatively, some optimization algorithms such as the differential evolution (DE) [6, 10, 11, 18], genetic algorithm (GA) [12] and simulated annealing (SA) [9, 13] can be used to optimize the time sequences of elements.

3. SIGNAL TRANSMISSION ANALYSIS

In this section, the AM and FM signals with a certain bandwidth (narrowband) received by the TMLA are analyzed and the noise is not considered in this paper.

3.1. Analysis of the Received AM Signal

The expression of an AM signal can be simply written as

$$z_{AM}\left(t\right)=s\left(t\right)e^{j2\pi f_0 t} \tag{5}$$

where $s(t)$ is typically referred to as the modulating signal. Let $z_{AM}(t)$ be a band-limited signal with $Z_{AM}(f)=0$ for $|f-f_0|>B/2$. After being received by the TMLA, the signal in time domain in the kth port of the array is

$$z_{rAMk}\left(t\right)=s\left(t\right)e^{j2\pi f_0 t}A_k e^{j\alpha_k}U_k\left(t\right)e^{j(k-1)\beta d\sin\theta} \tag{6}$$

after summing signals in all ports, the output signal can be expressed as

$$z_{rAM}\left(t\right)=\sum_{k=1}^{N}s\left(t\right)e^{j2\pi f_0 t}A_k e^{j\alpha_k}U_k\left(t\right)e^{j(k-1)\beta d\sin\theta} \tag{7}$$

Here we do not consider the antenna beam steering. In fact, beam steering is an important issue in TMLAs and some methods for beam steering have been proposed [14, 15]. The effect of beam steering on signal transmission will be our future studies. Without loss of generality, let $\alpha_k=0$ for all elements and the signal is from $\theta=0°$, then (7) becomes

$$z_{rAM}\left(t\right)=\sum_{k=1}^{N}s\left(t\right)e^{j2\pi f_0 t}A_k U_k\left(t\right) \tag{8}$$

The Fourier transform of (8) is

$$Z_{rAM}\left(f\right)=\sum_{n=-\infty}^{+\infty}\sum_{k=1}^{N}a_{nk}Z_{AM}\left(f-nf_p\right) \tag{9}$$

where a_{nk} has the same form as shown in (3), and $Z_{AM}(f)$ is the Fourier transform of $z_{AM}(t)$.

3.2. Analysis of the Received FM Signal

The expression of a FM signal can be written as

$$z_{FM}\left(t\right)=s_c\left(t\right)\cdot e^{j2\pi f_0 t} \tag{10}$$

$$s_c\left(t\right)=e^{jK_f\int s(t)dt} \tag{11}$$

where $s_c(t)$ is the complex envelope of $z_{FM}(t)$, $s(t)$ is the original modulating signal and K_f is frequency modulation coefficient. After being received by the TMLA and summing signals in all ports of the array, the output signal can be expressed as

$$z_{rFM}\left(t\right) = z_{FM}\left(t\right) \sum_{k=1}^{N} A_k U_k\left(t\right) \tag{12}$$

Similar to the analysis of the received AM signal, the Fourier transform of $z_{rFM}(t)$ can be written as

$$Z_{rFM}\left(f\right) = \sum_{n=-\infty}^{+\infty} \sum_{k=1}^{N} a_{nk} Z_{FM}\left(f - nf_p\right) \tag{13}$$

where $Z_{FM}(f)$ is the Fourier transform of $z_{FM}(t)$. The bandwidth of FM is supposed to be B and we have $Z_{FM}(f) = 0$ for $|f - f_0| > B/2$.

According to (9) and (13), it is clear to see that both $Z_{rAM}(f)$ and $Z_{rFM}(f)$ are the periodic function of f consisting of a superposition of shifted replicas of $Z_{AM}(f)$ and $Z_{FM}(f)$, respectively. Due to the time modulation, the signals received by the TMLA produce many sideband signals at the frequency of $f_0 + nf_p$, ($n = \pm1, \pm2, \ldots, \pm\infty$). The amplitude of each sideband signal relies on $\sum_{k=1}^{N} a_{nk}$ and the bandwidth of each sideband signal is B. Based on (3), the power density of each sideband signal is lower than that of the signal at the center frequency.

4. REQUIREMENTS OF THE TIME MODULATED FREQUENCY

As shown in Section 3, the signal received by the TMLA generates many sideband signals and each of them has the bandwidth of B. In other words, each sideband signal contains all the information of the original signal. Similar to Nyquist sampling theorem, when

$$f_0 + \frac{B}{2} \leq f_0 + f_p - \frac{B}{2} \Rightarrow f_p \geq B, \tag{14}$$

there is no overlap between the sideband signals; whereas there is overlap for $f_p < B$. Details are illustrated in Figure 1. For the case of $f_p \geq B$, $Z_{AM}(f)$ and $Z_{FM}(f)$ are exactly reproduced at $f_0 + nf_p$, ($n = 0, \pm1, \pm2, \ldots, \pm\infty$). Consequently, if $f_p \geq B$, $Z_{AM}(f)$ and $Z_{FM}(f)$ can be recovered exactly from $Z_{rAM}(f)$ and $Z_{rFM}(f)$ respectively by means of a band-pass filter with a passband from $f_0 - B/2$ to $f_0 + B/2$ or from $f_0 - f_p + B/2$ to $f_0 + f_p - B/2$.

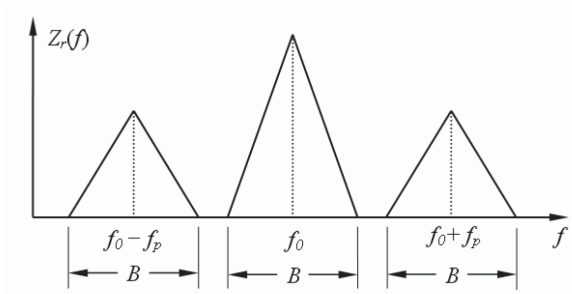

Figure 1. Frequency domain scheme of a signal with bandwidth B after being received by a time modulated antenna array.

5. SIMULATION RESULTS

In this section, a 16-element isotropic linear array with $\lambda/2$ equal spacing is considered. The center frequency of the antenna array is $f_0 = 1\,\text{GHz}$ and the array is excited with a static uniform amplitude and phase distribution. A $-40\,\text{dB}$ SLL power pattern is synthesized by the discrete Taylor distribution ($\bar{n} = 7$) and the time sequence of each antenna element is shown in Figure 2 where the shaded parts indicate that the RF switch is on.

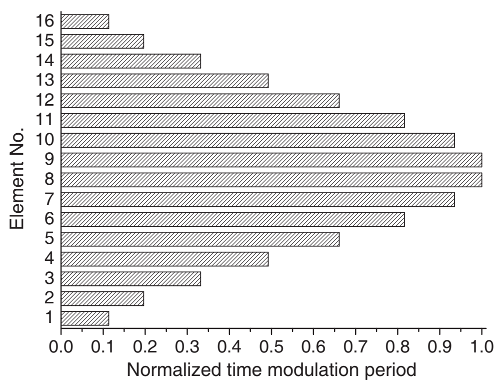

Figure 2. Time sequence of each antenna array element with a $-40\,\text{dB}$ SLL discrete Taylor distribution ($\bar{n} = 7$).

The first example is a Gaussian pulse AM signal. The modulating signal has the form

$$s(t) = \exp\left[-\left(\frac{t - t_0}{\sigma}\right)^2\right] \tag{15}$$

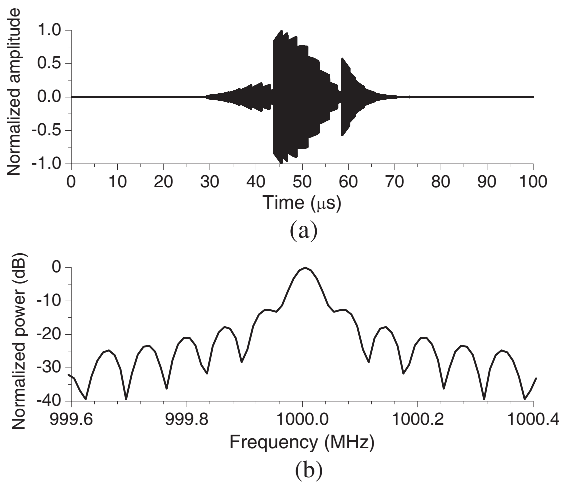

Figure 3. Gaussian pulse AM signal received by the TMLA. (a) Time domain. (b) Frequency domain.

Suppose that the duration of $s(t)$ is $T = 100\,\mu\text{s}$, the center of Gaussian pulse is at $t_0 = T/2$ and $\sigma = T/10$. The 10 dB bandwidth of the Gaussian pulse signal is approximately to be 68.3 kHz. After being received by the TMLA with the time modulation frequency $f_p = 68.3\,\text{kHz}$, the Gaussian pulse AM signal in time domain is shown in Figure 3(a) and the spectrum is shown in Figure 3(b). Obviously, after being received by the TMLA, the envelope of the signal no longer has the form of Gaussian pulse and the spectrum of the signal has many sideband signals. In order to recover the original modulating signal $s(t)$, f_p is set to be 68.3 kHz, 102.45 kHz and 136.6 kHz, respectively. After filtering out the sideband signals using a BPF with its passband from $f_0 - f_p/2$ to $f_0 + f_p/2$ and demodulation, the recovered signals are shown in Figure 4. The recovered signal closes to the original modulating signal as f_p increases. When $f_p = 136.6\,\text{kHz}$, the dotted line is nearly the same as the solid line in Figure 4.

The second example is a linear FM signal, which has the form of

$$z_{FM}(t) = s_c(t) \cdot \exp(j2\pi f_0 t) = \exp\left(j\pi \frac{B}{T} t^2\right) \cdot \exp(j2\pi f_0 t) \qquad (16)$$

Suppose that the duration of $z_{FM}(t)$ is also $T = 100\,\mu\text{s}$ and the bandwidth of the FM signal is 0.5 MHz. After being received by the TMLA with a time modulation frequency $f_p = 0.5\,\text{MHz}$, the linear FM signal in time domain is shown in Figure 5(a) and the spectrum is

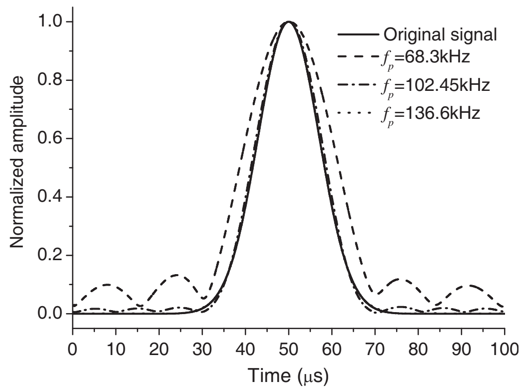

Figure 4. Results of a Gaussian pulse AM signal recovery with a time modulation frequency of 68.3 kHz, 102.45 kHz and 136.6 kHz, respectively.

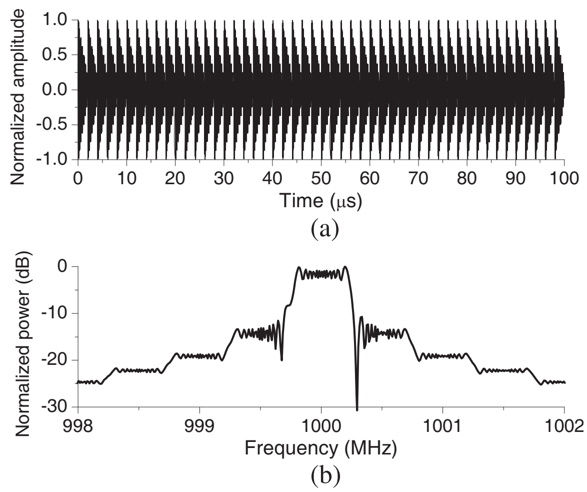

Figure 5. Linear FM signal with a bandwidth of 0.5 MHz received by the TMLA. (a) Time domain. (b) Frequency domain.

shown in Figure 5(b). It is observed that the envelope of the linear FM signal received by the TMLA is proportional to the number of "on" switches at a certain time instant. The spectrum consists of many sideband signals with bandwidth of 0.5 MHz at every $1000 + 0.5n$ MHz ($n = 0, \pm 1, \pm 2, \ldots, \pm\infty$). In order to recover the original signal, f_p is set to be 0.5 MHz and 0.75 MHz. Here we only consider the real part

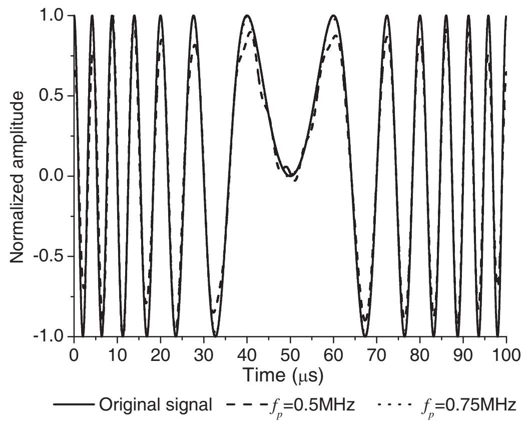

Figure 6. Results of a linear FM signal recovery with a time modulation frequency of 0.5 MHz and 0.75 MHz, respectively.

of complex envelope of the linear FM signal, which is given by

$$\mathrm{Re}\left\{s_c\left(t\right)\right\} = \cos\left(\pi\frac{B}{T}t^2\right) \tag{17}$$

After filtering out the sideband signals using a BPF with pass band from $f_0 - f_p/2$ to $f_0 + f_p/2$, the recovered signals are shown in Figure 6. Obviously, the dotted line almost coincides with the solid line when $f_p = 0.75$ MHz.

6. CONCLUSION

As compared to conventional antenna arrays, the time modulated antenna arrays have more flexibility for the synthesis of low/ultra-low SLLs patterns. However, the signal received by a time modulated antenna array is time modulated by the RF switches, which produce many sideband signals at multiples of the time modulation frequency and each of them has the same bandwidth as that of the original signal. The analysis of AM and FM signals shows that in order to exactly recover the original signals, the time modulation frequency should be equal to or greater than the bandwidth of original signals. Moreover, a BPF has to be used to filter out all sideband signals, which is similar to Nyquist sampling theorem. The simulation results show that the higher the time modulation frequency is the more exactly the signal recovers.

Finally, it should point out that the noise effect is not considered in this paper. In fact, the noise in an actual receiver is usually white.

Thus, as long as the time modulation frequency f_p is not lower than the bandwidth of the original signal and the signal to noise is high enough, the original signal can also be recovered according to the approach in this paper.

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