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THE EQUIVALENCE OF BUNDLING AND ADVANCE SALES

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Abstract

The equivalence of bundling and advance sales[+]

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We show that a monopolist's problem of optimal advance selling strategy can be mathematically transformed into a problem of optimal bundling strategy if four conditions hold: i. consumers and the firm agree on the probability of the states occurring, ii. the firm pre-commits to the spot prices to be charged in the advance selling stage, iii. consumers are risk-neutral, and iv. consumers and the firm do not have time preferences or when they do have time preferences, they discount future at the same rate. The result allows both researchers and practitioners to apply the insights from the well-developed vast literature on bundling to advance selling problems. In particular, we show that advance selling is more profitable than spot selling when consumer valuations across the states are independent or negatively dependent or positively dependent up to a point. We furthermore illustrate the effect of advance selling on the spot prices and consumer welfare: When the firm offers advance selling discounts, it sets higher spot prices, so consumers who do not buy in advance are worse off due to the firm offering advance selling discounts. We extend our analysis to the cases of more than two states and competition only in one of the states. We also show how advance selling can be used as an entry deterrence strategy.

Keywords: Advance selling, bundling, price discrimination

JEL Codes: L11; D42

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1 Introduction

The literature on advance selling postulates that it might be profitable to give discounts to consumers who purchase a product/service before the consumption date, and so before some uncertainty is resolved.[1] We show that under certain conditions the problem of finding the optimal advance selling strategy can be mathematically transformed into a bundling problem or, in other words, that these two problems are isomorphic. This mathematical equivalence provides a new way of analyzing and comparing these problems. Using the findings of the well-developed bundling literature (since Adams and Yellen, 1976)[2] it enables us to show the effects of the correlation of valuations across states on the profitability of advanced selling by a monopoly firm: advance selling is more profitable than spot selling when consumer valuations across the states are independent or negatively dependent or positively dependent up to a point.[3]

We furthermore illustrate the effect of advance selling on the spot prices and consumer welfare: When the firm offers an advance purchasing option, it sets higher spot prices, so consumers who do not buy in advance are worse off due to the firm using advance selling. In the special case of independently and uniformly distributed valuations across states and zero marginal costs we show that consumers are overall worse off with advance selling.

We show that the equivalence result and the implied findings are robust to several extensions: when consumers and the firm have the same time preferences (i.e., when they discount future payments/consumption at the same rate), when there are $N > 2$ possible states, and when the firm faces a competitor in only one of the states. In particular, the second extension shows that the firm does not have to set 2^N prices, but can instead approximate the optimal pricing mechanism by setting N different prices for tickets valid in $k \leq N$ states. In the special case where consumer valuations across states are independently distributed and products have zero marginal cost, when there is a steady increase in the number of spot states, we show that the firm captures nearly all consumer surplus by advance selling. In the third extension, we also illustrate when it is profitable (for an incumbent) to use advance selling to deter potential entry to the market.

On the methodological front, going beyond the existing literature, we allow for almost

[1] See, e.g., Shugan and Xie (2000) and Xie and Shugan (2001).

[2] Chu, Leslie, and Sorensen (2011) and Eckalbar (2010) both provide new methods and references to the main old ones in empirical estimation and computational methods. Stremersch and Tellis (2002), Kobayashi (2005), Armstrong (2006), and Stole (2007) provide surveys on bundling and in general on non-linear pricing. These reviews have well over 200 citations to older literature and are a good place to start exploring the bundling literature after reading the newer research articles cited in this paper. See Belleflamme and Peitz (2010), p. 259-280 and p.417-420, for more influential papers in this literature.

[3] Dependence is essentially a nonlinear copula-based version of correlation. See Chen and Riordan (2013) and Clemen and Reilly (1999) for more.

arbitrary ex-ante (at the advance selling stage) consumer heterogeneity and ex-post (at the spot selling stage) aggregate demands which vary across states. Finally, we take a first step at joining the two literatures (bundling and advance selling), and show both researchers and practitioners that one can use the well-developed bundling intuition for advance-selling problems and vice-versa.

There is a simple insight behind the mathematical equivalence of an advance selling problem to a bundling problem. Suppose that the new basketball season is about to start and a firm is selling tickets to one of the last games in the regular season. Everyone agrees on the probability with which that game will be important (the team will or will not be able to achieve a higher overall position in the league and/or a playoff position), however this uncertainty is not going to be resolved soon. For easier exposition, suppose there are just two future states of the world: in state 1 the game is important and in state 2 it is not, and everyone agrees that the probability of state 1 occurring is a. The ticket to the last game can be viewed as a package (a bundle) of two single-state tickets: one of them is valid only in state 1 and the other ticket is valid only in state 2. The consumer's valuation of the single-state ticket is the value from going to the game in that state multiplied by the probability of that state occurring. The value of an advanced-sale ticket is then the sum of the values of the two single-state tickets. Selling a ticket to the game before the uncertainty is resolved (advance selling) is therefore analogous to selling the two single-state tickets as a bundle.[4]

The equivalence result is intuitive, but is not straightforward: There are important implementation differences between an advance selling strategy and a bundling strategy. If a firm sells a bundle, it sells one unit of each good included in the bundle and consumers decide whether to consume the bundle or the individual products separately, with the decisions being made at the same time. On the other hand, if the firm sells a good in advance, the good can actually only be consumed in one of the future states. Consumers make a sequential consumption decision: First, they decide whether to purchase the good in advance and then (after the uncertainty is resolved) the ones who did not buy in advance decide whether to buy the good at the spot price. Unlike a standard bundling problem, an advance selling problem has a time dimension. Some of the problems that we are interested in in the advance selling literature were of no particular interest in the contexts of bundling and so the researchers in bundling did not analyze these situations. For instance, the impact of uncertainty (risk aversion), commitment to future prices, time preferences (of consumers

[4]Another way to think about this is that the advance sale ticket is a bundle of several Debreu securities, with each security valid only in its state of the world. The states have to be mutually exclusive, but do not have to be collectively exhaustive, more on that follows in the discussion.

and the firm), different arrival times of consumers, and the impact on production costs (i.e., advance selling enables longer lead times). Due to these differences, we have identified four conditions to be satisfied for the mathematical equivalence of these problems to hold: consumers and the firm agree on the probability of the states occurring, the firm pre-commits to the spot prices to be charged in the advance selling stage, consumers are risk-neutral, and consumers and the firm do not have time preferences or when they do have time preferences, they discount future at the same rate.[5]

The assumption of time preferences is critical only if consumers and the firm discount future differently. In this case, we show (in section 5.1) that it is not possible to find the exact bundling equivalent to the advance selling model, but we can still specify a bundling model to understand the profitability of advance selling. In particular, we show that when consumers are more impatient than the firm, for all parameter spaces where bundling is not profitable in the bundling model, advance selling is not profitable in the advance selling model, either. Also, we derive a version of equivalence for consumers who value securing the product earlier (or later).[6]

The commitment assumption is in line with the most of the previous advance selling literature.[7] More importantly, in many real-world advance selling scenarios we believe that the firm can commit to spot prices, in particular due to reputational concerns.[8] In section 5.2 we provide examples of situations where firms can commit to one spot price (e.g., "advance price" and "gate price" for concert tickets, sport games, amusement parks, zoo tickets, fair tickets, exhibitions, early bird registration fees for conferences, recreation activities, and professional training classes), and situations where firms can commit to different spot prices depending on the state (e.g., electricity prices with smart-meters or energy supply prices based on an index).

In section 5.1 we discuss the potential effects of risk aversion on the demand for advance

[5]We do not model the supply side reasons of offering advance selling discounts. See below for more on these.

[6]The problem can be transformed into a bundling problem where products are complements or substitutes, see Venkatesh and Kamakura (2003). A less straightforward example is a reduced way of modeling a particular form of risk aversion. Suppose the consumer wants to secure the product in advance rather than risk anything unforeseen that might happen in the future, above and beyond the common-known state probabilities. In that case, the consumer values the advance purchase more than her expected valuation from purchasing the good at the spot, even if the expected spot price is the same as the advance sale price.

[7]From Fay and Xie (2010): "The advance selling literature (e.g., Xie and Shugan 2001, Shugan and Xie 2005) suggests that a seller is capable of committing in advance to announced spot prices if the latter are observable at the time when customers are making advance purchases."

[8]A firm usually promises to charge a particular price at a particular point of time either explicitly (conferences announcing early and late registration fees) or implicitly (the price that an airline charges one day before the flight depends on whether the flight is close to full, however, in either case frequent travelers have an expectation of the price). In either scenario, if the firm changes its pricing policy on-the-spot, it is likely to suffer a backlash in the future that will not be worth the one-time gain resulting from the deviation.

selling, explain why the case with risk-averse consumers would be particularly difficult to analyze within our advance selling model and provide some empirical evidence to support our argument that the advance selling model with risk-neutral consumers is a reasonable approximation of many real-world advance selling situations. We also provide a Taylor expansion in the Appendix to show that if the consumer utility functions are close to being linear around the consumers' current wealth levels (have low absolute magnitude derivatives, or, in other words, are close to being locally risk-neutral), then we also have an approximation.

We contribute to the advance selling literature that explains the profitability of advance selling solely by demand-related factors (See Shugan and Xie, 2000, Xie and Shugan, 2001, Fay and Xie, 2010, among others).[9] This literature mostly assumes ex-ante homogeneous and ex-post heterogeneous consumers.[10] While that is a good starting point for research, it has some undesirable qualities: for example, without any exogenously imposed restrictions, either all consumers purchase in advance or none of them do.

In our setup, however, consumer valuations for the good in a given state are drawn from a probability distribution function such that ex-ante consumers know their valuations and the probability of each state. Thus, some consumers purchase the good in advance and some wait and purchase (or not) when the state of the world is realized. Allowing for ex-ante, as well as ex-post, consumer heterogeneity enables us to illustrate that advance selling might be profitable even when the seller cannot fully homogenize the consumer types. The ex-ante heterogeneity allows us to illustrate when a monopolist wants to use advance selling as a price discrimination tool to exploit consumer heterogeneity in valuations across different states of world. The previous literature on yield management (Dana 1998; Gale and Holmes, 1992, 1993) also showed advance selling as a profitable price discrimination tool. Different from that literature we do not have costly capacity, so we do not link the profitability of advance selling to allocating capacity efficiently across the periods. We explain below how price discrimination works in our setup.

Fay and Xie (2010) draw a parallel between the profitability of advance selling and the profitability of pure bundling when consumer types are negatively correlated: both strategies work by aggregating consumers and thereby homogenizing consumer heterogeneity. We show that in our advance selling setup any kind of negative dependence between consumer valuations that are distributed by any joint p.d.f. (and c.d.f. satisfying the standard second

[9]Another line of literature relates the profitability of advance selling to supply side efficiency, such as using advance selling as a yield management tool, that is, to screen consumers by their uncertainty of demand while allocating capacity efficiently across periods when aggregate demand is uncertain (Dana, 1998, Gale and Holmes, 1992, 1993), improving capacity planning by using the information on advance sales (Boyaci and Özer, 2010), better supply chain coordination by allocating inventory risk between the partners (Cachon, 2004).

[10]An exception is an extension in the model of Shugan and Xie (2000), which we discuss below.

order conditions) implies that advance selling homogenizes at least some of the consumer heterogeneity, and thus improves profit compared to spot selling. Moreover, going beyond proving their conjecture, we show that a more profitable advance selling might arise even when consumer valuations are positively correlated.

Intuitively, advance purchasing discounts are profitable if there is a sufficiently high number of consumers who would purchase the good in only one state if there was no advance selling. This is the case if the correlation of the valuations across the states is not very positive, since then a consumer who has a high consumption value in state 1 is less likely to have a high value in state 2. In that case, advance selling discounts convince many consumers to buy the good in advance rather than buying in only one state.

Another big distinction from the previous advance selling literature is that our states are aggregate: all consumers are going to end up in the same state. The previous literature worked with individual states: each consumer independently drew the state that she was going to be in. The individual state assumption (coupled with a continuum of ex-ante homogeneous consumers) results in the same ex-post aggregate demand, regardless of the individual state realizations. Thus, once the uncertainty is resolved, the firm always faces the same demand curve, and so overall it does not face any uncertainty. The firm cannot charge different prices in different states because it cannot observe the individual state realizations of different consumers. The situation is different with our aggregate state assumption: the firm faces uncertainty; aggregate demand depends on the realized state, and thus the firm might want to charge different prices in different states.[11] In general, the two models (individual states versus aggregate state) are not nested and correspond to different real-world phenomena. Aggregate state assumption (our model) captures the cases where all consumers experience the same state: a basketball game at the end of the season (since it would be important to all the fans) or a flight that turned out to be in-demand (since it would be close to fully-booked for those trying to get a seat). On the other hand, the individual state assumption is more reasonable when different consumers face different state realizations. For example, when consumers decide whether to purchase gas in advance when renting a car: if they do not buy in advance, some consumers will end up in a good state where they find a gas station nearby, and some will end up in a bad state where they are late for their flight and do not have time to fill up the tank.

We are aware of only one paper analyzing competitive advance selling, Shugan and Xie

[11] In our advance selling model, we would get the same aggregate spot demand regardless of the state if the marginal density function of the valuations is the same for the two states (for example, normal with a mean of 5 and a standard deviation of 1, in both states). In this case, even with ex-ante heterogeneity, the firm charges the same spot price across two different states, and our model degenerates to the one where the firm chooses just two prices: the advance selling price and the spot price (same, regardless of the state).

(2005), which uses different demand specifications to show that the impact of competition on the profitability of advance selling is unclear. The literature on competitive bundling is much more developed,[12] so our equivalence result in the extension to competition contributes to the advance selling literature by illustrating that a firm which faces a competitor in one state will find advance selling profitable under similar conditions to a monopoly firm and an incumbent can use advance selling as a tool to deter potential entry to its market.

The equivalence that we prove is a technical one. It implies that the techniques for solving bundling problems can be used to solve advance selling problems and vice-versa. The intuition behind the two problems is very similar. This does not imply that in economic and marketing applications the two strategies are equivalent, or if a firm does one then there is no use in doing the other. Advance selling exploits consumer heterogeneity in the dimension of different valuations of the same product across different states of the world. Bundling exploits consumer heterogeneity in the dimension of different valuations for different products. The two might or might not be related, and the firm can use neither bundling nor advance selling, either one but not the other, or both at the same time.[13] While the logic behind the two price discrimination instruments is the same, and the techniques used for one are applicable to the other, there might still be value in using both at the same time since they exploit different dimensions of consumer heterogeneity.

We only consider the revenue benefits of advance selling and do not deal, for example, with the issues of capacity constraints, risk sharing between the suppliers and the retailers, or short sales seasons and replenishment.[14] We assume that the seller is not constrained on the production side and that production can occur instantaneously. We could build these and other supply features into our model; however, doing so would distract from the main purpose of this article.

Since the bundling literature is older and more developed than the advance selling literature, we mainly focus on which findings one can bring from the bundling literature to advance selling. This knowledge transfer will go the other way, too: research on advance can provide important insights about bundling since advance selling researchers have studied variations of the maximization problem that have not previously been considered in the bundling literature, for example, comparing advance selling to probabilistic selling, effects

[12]See e.g., Whinston 1990; Carlton and Waldman, 2002; Nalebuff, 2004; Chen and Riordan, 2013

[13]For example, some ski resorts start selling passes for the next season as early as April, taking advantage of the negative dependence of consumer valuations across several possible states of the world (winter with a lot of snow or winter with barely any snow). However, these ski resorts might also engage in the bundling of lodging and lift tickets, taking advantage of the negative dependence of consumer valuations between the values of lodging and lift tickets. We thank an anonymous referee for this example.

[14]Gale and Holmes (1993), Cachon (2004), and Tang, Rajaram, Alptekinoglu, and Ou (2004) show how advance purchase discounts improve profits under these scenarios.

of uncertainty and risk-aversion, and supply-side savings. Exploring these are left for future research. Hence, the current paper encourages, rather than discourages, future research on advance selling. Overall, we hope that this paper joins two currently separate and developed literatures, allowing researchers, practitioners, and instructors to leverage tools from one of the streams to another, and possibly combine these tools to create more.

2 The framework

We first describe a model of advance selling and then proceed by showing the mathematical equivalence of the optimal pricing decisions in this model to those in a bundling problem with appropriate parameter specifications.

2.1 A model of advance selling

We consider one seller and a continuum (mass 1) of consumers. The seller offers one product and each consumer has a unit demand for the product. The value from consumption depends on the state of the world. There are two states, and state 1 occurs with probability $a \in [0, 1]$. A consumer receives utility v_1 if she consumes the product in state 1 and v_2 if she consumes it in state 2. For now, we assume that consumers are risk-neutral.[15] Consumers differ in their valuations: (v_1, v_2) are drawn from a joint probability distribution function, $f(\bullet, \bullet)$ over region $[\underline{v_1}, \overline{v_1}]$ x $[\underline{v_2}, \overline{v_2}]$. We do not make any assumptions on the type of correlation between the valuations across the states.[16] Furthermore, we allow the marginal cost to the seller to be state dependent: in state j the marginal cost is c_j. We assume that at least some consumers have valuations above the respective marginal costs, and that the monopolist's second order condition is satisfied (there exists a global maximum).[17] We analyze the profitability of selling the product before the state is realized (advance selling) at price p^{AS} or after the state realization (spot selling) at price p_1^{SS} in state 1 and at price p_2^{SS} in state 2, or offering both an advance selling price and spot selling prices. The timing of the events is the following:

[15] We discuss how risk aversion of consumers would change the profitability of advance selling in section 5.1.

[16] More precisely, the valuations are independently distributed across the states if $Pr(v_i \geq x | v_j \geq y) = Pr(v_i \geq x)$ for $i \neq j$. Using the definition of Long (1984) the valuations are negatively (respectively, positively) correlated if $Pr(v_i \geq x | v_j \geq y)$ decreases (respectively, increases) in y.

[17] Defining straight-forward conditions for the S.O.C. is not a trivial task in this problem, or in the bundling problem in the next subsection. First, the maximum might not be unique – for example, if advance selling is not profitable, then there are many potential advance selling prices such that no advance sales are made, all resulting in the optimal profit. In general, the models (of advance selling and of bundling) that we present in this paper are variations of a multi-dimensional screening problem. A common assumption, made to ensure the convexity of such problems, is some version of the monotone hazard rate assumption on the probability distribution function. See McAfee and McMillan (1988) for more.

Period 1:

- Consumers arrive. Each consumer knows her valuation from consumption in the two states and the probability of each state occurring (v_1, v_2, and a), but she does not know which state is going to occur.

- The firm knows the p.d.f. from which consumer valuations are drawn and the state probabilities ($f(\bullet, \bullet)$ and a). The firm announces the advance sale price and two spot prices conditional on the state of the world (p^{AS}, p_1^{SS}, and p_2^{SS}).

- Consumers decide whether to buy now (and pay p^{AS}), or to wait until the uncertainty about the state is resolved.

Period 2:

- The state is realized (say, state 2).

- Consumers decide whether to purchase at the pre-announced spot price (p_2^{SS}).

- All buyers consume and receive their utility regardless of when they purchased.

Note that in this setup the firm faces unobserved consumer heterogeneity both before the state is realized (ex-ante) and after the state realization (ex-post), since the firm only knows the p.d.f. from which consumer valuations were drawn and the state probabilities. Ex-ante consumers know their value from consumption in a given state, and the probability of the state occurring. Thus, in particular, consumers know their expected value from consumption ex-ante. Ex-post (in period 2), they know the exact value of their utility since they know which state occurred.

We solve the model by backward induction. A consumer who did not purchase in advance purchases the good in the spot period (after the state realization) if her valuation is higher than the spot price. In the first period, consumers learn their valuation in each state, the advance selling price and the spot prices, and then decide whether or not to purchase in advance. In the decision problem of the first period, consumers anticipate the second period choice problem and the corresponding utility of each outcome. The expected utility from buying in advance is

$$U^{AS} = \alpha v_1 + (1 - \alpha) v_2 - p^{AS}.$$

If a consumer does not buy in advance and waits to buy at the spot price instead, then she buys at the spot only if her value in that state is higher than the spot price. So her expected utility from waiting until the spot period is the weighted average of the utility she gets in each state:

$$U^{SS} = \alpha(v_1 - p_1^{SS})\Gamma_1 + (1-\alpha)(v_2 - p_2^{SS})\Gamma_2$$

where $\Gamma_i = 1$ if $v_i \geq p_i^{SS}$ and $\Gamma_i = 0$ otherwise. She prefers to purchase in advance if and only if $U^{AS} \geq U^{SS}$. Let D^{AS} denote the advance selling demand and D_j^{SS} denote the spot selling demand in state j.[18] The monopolist's profit is the sum of the *expected* profits from advance selling and from spot selling:

$$\Pi^{AS}(p_1^{SS}, p_2^{SS}, p^{AS}) = (p^{AS} - ac_1 - (1-a)c_2)D^{AS} + a(p_1^{SS} - c_1)D_1^{SS} + (1-a)(p_2^{SS} - c_2)D_2^{SS}. \quad (1)$$

We assume that there exists a unique price vector $(p_1^{SS}, p_2^{SS}, p^{AS})$ which maximizes the seller's profit given in (1). The following lemma shows that when $p^{AS} > \alpha p_1^{SS} + (1-\alpha)p_2^{SS}$ every consumer is better off waiting until the spot period instead of buying in advance, and so the firm would make the same profit if it lowered its advance selling price to $p^{AS} = \alpha p_1^{SS} + (1-\alpha)p_2^{SS}$.

Lemma 1 *There is no loss of generality from assuming that* $p^{AS} \leq \alpha p_1^{SS} + (1-\alpha)p_2^{SS}$.[19]

Proof. Suppose not, and $p^{AS} > \alpha p_1^{SS} + (1-\alpha)p_2^{SS}$. The expected price of purchasing at the spot, regardless of what state it is, is lower than the advance purchase price. Thus, a consumer is better off waiting for the spot market instead of purchasing in advance. Therefore, the firm receives the same profit if it lowers the advance selling price to $p^{AS} = \alpha p_1^{SS} + (1-\alpha)p_2^{SS}$: still no consumer purchases in advance, assuming that indifferent consumers wait until the spot market. ■

We could therefore write the consumers' choice problem as if they face four options in period 1: purchase in advance, purchase on-the-spot only in state 1, purchase on-the-spot only in state 2, and no purchase. Table 1 shows the utility a consumer gets from each of the four purchasing options.

[18] See Appendix A for the derivation of these demands.

[19] Note that this lemma might not be true if consumers were risk-averse and/or had some time preferences, or the firm cannot commit to future prices. We will discuss those cases later in the paper.

In particular, making comparisons with the earlier literature, this lemma is not true in the Xie and Shugan (2001) setup. In their case, this is due to the limited capacity assumption in that part of their model. This lemma is simply to cut down on the number of cases (rows in the table) discussed in the proof and is not crucial to the logic of the proof.

Moreover, wlog, the real value of the advance selling price is lower than the expected value of spot selling prices: if the consumer discount factor is δ, then $(1/\delta)p^{AS} \leq ap_1^{SS} + (1-a)p_2^{SS}$. See section 4 for more on discounting in our model.

Table 1: The expected utility of a consumer from different purchasing options in the advance selling model.

Purchasing options	**Expected utility**
Advance purchase	$av_1 + (1-a)v_2 - p^{AS}$
Spot purchase only in state 1	$a(v_1 - p_1^{SS})$
Spot purchase only in state 2	$(1-a)(v_2 - p_2^{SS})$
No purchase	0

2.2 The equivalence of the advance selling problem to a bundling problem

Now consider a static problem of monopoly bundling where the seller offers two products and consumers have a unit demand for each product. Suppose that a consumer receives utility av_1 from consuming product 1 and $(1-a)v_2$ from consuming product 2. As before, let $f(\bullet, \bullet)$ denote the joint distribution function of the valuations v_1, v_2 over region $[\underline{v_1}, \overline{v_1}]$ x $[\underline{v_2}, \overline{v_2}]$. There is unobserved consumer heterogeneity: consumers know their utilities (and a), but the seller only knows parameter $a \in [0,1]$ and the joint distribution function $f(\bullet, \bullet)$. Assume that the seller's marginal cost of product 1 is ac_1 and of product 2 is $(1-a)c_2$. Assume that the firm cannot prevent consumers from purchasing both products separately. The bundling problem involves analyzing the profitability of selling the products as a bundle at price p^B or selling only the individual products separately at prices p_1^I and p_2^I, or offering both a bundle price and individual product prices (mixed bundling).

First, we show that it is not feasible to sell the bundle at a price that is higher than the sum of the stand-alone prices, that is, implement a bundling premium, since the firm cannot prevent consumers from buying both of the items separately.[20]

Lemma 2 *There is no loss of generality from assuming that* $p^B \leq \alpha p_1^I + (1-\alpha)p_2^I$.

Proof. Suppose that $p^B > \alpha p_1^I + (1-\alpha)p_2^I$, but then every consumer is better off buying both goods separately and paying the sum of the stand-alone prices rather than buying the bundle and paying a premium, and so the firm would make the same profit if it lowered its bundle price to $p^B = \alpha p_1^I + (1-\alpha)p_2^I$. ■

Given the prices (p^B for the bundle, p_1^I, p_2^I for the individual products, and Lemma 1, consumers have four options: purchase both products as a bundle, purchase only product

[20]Indeed, the bundling literature shows that, if it is feasible to monitor purchases, the monopoly might want to charge a bundling premium rather than offering a bundling discount. Armstrong (2013) shows that this is the case if the demand for the bundle is less elastic than the demand for an individual item at the equilibrium prices without bundling. For example, when the valuations are independently distributed or negatively correlated this cannot be the case, so the monopolist prefers a bundling discount.

1, purchase only product 2, and no purchase. Table 2 shows the utility a consumer gets from each of the four available options. Consumers choose the option which gives them the

Table 2: The utility of a consumer from different purchasing options in the bundling model.

Purchasing options	**Utility**
Bundle of products 1 and 2	$av_1 + (1-a)v_2 - p^B$
Only product 1	$av_1 - p_1^I$
Only product 2	$(1-a)v_2 - p_2^I$
No purchase	0

highest utility. The monopolist's profit in the bundling model is:

$$\Pi^B(p_1^I, p_2^I, p^B) = (p^B - ac_1 - (1-a)c_2)D^B + (p_1^I - ac_1)D_1^I + (p_2^I - (1-a)c_2)D_2^I, \quad (2)$$

where D^B denotes the demand for the bundle and D_j^I refers to the demand for product j only.[21] Comparing consumer options in the advance selling model (Table 1) with those in the bundling model (Table 2), it is straightforward to see that if the price for the bundle is p^{AS} and the prices for product 1 and product 2 are, respectively, ap_1^{SS} and $(1-a)p_2^{SS}$, the purchasing options of the two models would coincide; advance selling corresponds to selling the bundle, spot selling only in state j corresponds to selling only product j, for $j = 1, 2$. Hence, at these prices, the demand for the bundle coincides with the demand for advance selling: $D^B \equiv D^{AS}$, and demand for only product j coincides with the demand for spot purchase only in state j, $D_j^I \equiv D_j^{SS}$.[22] As a result, at these prices the profit in the bundling model would be equivalent to the profit in the advance selling model:

$$\Pi^B(ap_1^{SS}, (1-a)p_2^{SS}, p^{AS}) = \Pi^{AS}(p_1^{SS}, p_2^{SS}, p^{AS}). \quad (3)$$

This proves our main result:

Proposition 1 *The problem of finding the optimal pricing strategy in the advance selling model described in Section 2.1 is mathematically equivalent to the problem of finding the optimal pricing strategy in the bundling problem described above. Moreover, if the optimal bundle price is p^*, and the optimal prices for the individual products are ap_1^* and $(1-a)p_2^*$, then the optimal advance selling price is p^* and the optimal spot selling prices are p_1^* and p_2^*.*[23]

[21] In Appendix B we drive these demand functions explicitly.

[22] Comparing the demands derived in Appendix A to those derived in Appendix B shows the equivalence of those demands when $p^B = p^{AS}$, $p_1^I = ap_1^{SS}$ and $p_2^I = (1-a)p_2^{SS}$.

[23] We extend the case of two products/states to $N > 2$ products/states in section 4.3 and discuss the implications of this result using the bundling literature on multi-product bundling.

The main idea behind this result is that an advance sale ticket is a bundle of two tickets, each valid only in a particular state in the future. The weights in the proposition adjust the valuations and the marginal costs of stand-alone products to conform to the fact that neither state occurs with certainty; and to make sure that a consumer's expected value of each item (and the cost of serving the consumer) in the advance selling model is the same as her value for each item (and the cost of serving her) in the bundling model.

The equivalence result seems to be straightforward. However, there are important implementation differences between the two problems. If the monopolist sells a bundle, it sells one unit of product 1 and one unit of product 2 with certainty. Consumers decide whether to consume the bundle or the individual products separately, with the decisions being made at the same time. In the advance selling model, however, if the monopolist sells a ticket in advance, this ticket can actually be consumed in only one of the two states. Consumers make a sequential consumption decision: first, they decide whether to purchase the good in advance and then (after the uncertainty is resolved) the ones who did not buy in advance decide whether to buy the good at the spot price. A priori, it is not clear whether the monopolist's problem is the same in these models.

We prove the mathematical equivalence of these models by rewriting consumer purchasing options in the advance selling model to reflect the fact that when consumers decide whether to advance purchase, they compare their expected utility from advance purchasing with their expected utility from spot purchasing in a given state (or to no purchase). Thus, even if advance purchasing and spot purchasing occur at different points in time, consumers decide at the same time whether to advance purchase or whether to delay their purchase to the spot period. Since consumers know spot prices from the beginning (and we assume that the firm can commit to spot prices), from the beginning consumers know whether they will purchase the good at the spot market in any given state.

□ Example

Suppose that consumer valuations for the two states are independently and uniformly distributed, with the valuations for State 1 in the range of $[0, V_1]$, and the valuations for State 2 in the range of $[0, V_2]$. Furthermore, suppose that the probability of State 1 is a, the marginal costs are zero regardless of the state, and $(1-a)V_2 \geq \frac{aV_1}{2}$ (see Eckalbar, 2010, for the case when this assumption is not satisfied).

Proposition 1 implies that this problem is equivalent to the following bundling problem: valuations for the two products are independently and uniformly distributed, with the valuations for Product 1 in the range of $[0, aV_1]$, and the valuations for Product 2 in the range of $[0, (1-a)V_2]$, and the marginal cost being 0 for either product. From Eckalbar

(2010), we know that the optimal mixed bundling solution for this problem is that optimal individual product prices are $p_1^I = \frac{2}{3}aV_1$ and $p_2^I = \frac{2}{3}(1-a)V_2$, the optimal bundle price is $p^B = \frac{1}{3}\left(2aV_1 + (1-a)V_2 - \sqrt{2a(1-a)V_1V_2}\right)$, and mixed bundling is more profitable than selling two independent products and not selling a bundle.

Then, using Proposition 1, in the advance selling model the optimal spot selling prices are $p_1^{SS} = \frac{2}{3}V_1$ and $p_2^{SS} = \frac{2}{3}V_2$, and the optimal advance selling price is

$p^{AS} = \frac{1}{3}\left(2aV_1 + (1-a)V_2 - \sqrt{2a(1-a)V_1V_2}\right)$, and advance selling along with spot selling is more profitable than spot selling and not offering advance selling, despite the fact that the correlation between the valuations in two states is zero.

3 Implications of the equivalence result

In this section we outline which findings from the monopoly bundling literature we can bring over to the advanced selling literature using our equivalence result (Proposition 1). First, we highlight an important insight from the bundling literature: the correlation of valuations across states is an important determinant of the profitability of offering advance sales. In particular, we show that, despite the intuition from the advance selling literature, advance selling can be profitable with positive correlation. We show in general when advance selling increases spot market prices and when it lowers consumer welfare.

Since Stigler's (1968) examples of profitable bundling of movies, the bundling literature has developed significantly (see footnote 2 for references to review articles). A large proportion of the literature studied whether and when a multi-product monopoly firm wants to sell its products as a bundle rather than separately or do mixed bundling (combining the two options): a price for the bundle and individual product prices. This literature emphasizes the role of bundling as a price discrimination tool. The main finding is that the profitability of bundling depends on the correlation between valuations for the two products.

Chen and Riordan (2013) differ from the previous work by allowing for (non-continuous) general distribution functions of valuations (subject to the standard second order condition) and deriving intuitive sufficient conditions using *copulas*[24] under which mixed bundling strictly dominates separate selling. They also examine the effect of bundling on stand-alone prices. We use their findings (unless otherwise specified) to derive the implications of our equivalence result. The following corollaries are direct equivalents of corresponding results from the bundling literature.

[24]To define the dependence between the distributions of valuations for the two products, they use a copula, which is a function that couples marginal distributions of random variables to form a joint distribution.

Corollary 1 *If the valuations for consumption in the two possible states of nature are negatively dependent, independent, or not too positively dependent, then optimal advance selling discounts strictly increase profit.*[25]

Offering an advance sale discount lowers profits from consumers who purchase the good in both states at spot prices, but increases profits by convincing some of the consumers, who would purchase only in one state of the world, to buy in advance. The gains from advance sale discounts are likely to be higher than the losses when there are many consumers who switch from buying the good only in one state to buying it in advance. This is the case when consumer valuations across states are not too positively dependent: if a consumer's valuation from the good is high in one state, it is unlikely to also be high in the other state.

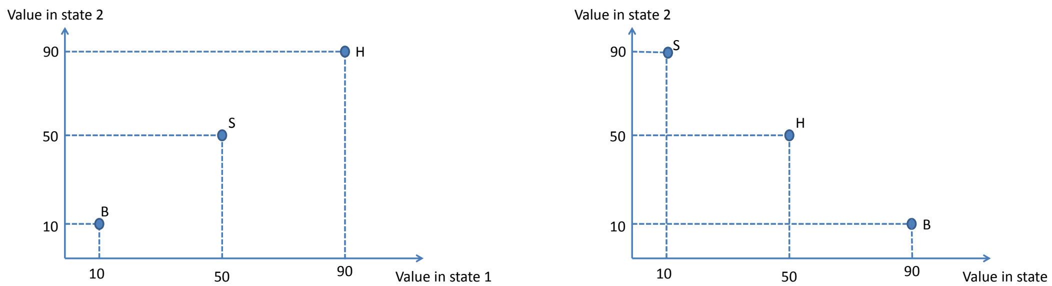
□ **Example**

While we are used to interpreting correlation in bundling settings, understanding correlation of consumer valuations for consumption across different states of the world is not as straightforward, at least at first. This example aims to illustrate this issue. Let us get back to the example of selling tickets to a future basketball game that might or might not be important. Say there are three types of fans: hardcore, bandwagon, and snobs.

If consumers receive the same value from the game regardless of the state, then consumer values are perfectly and positively correlated. For instance, if the hardcore fans value the game at 90 dollars both in the case when it is going to be important and when it is not, snobs value it at 50 in either case, and bandwagon fans value it at 10 in either case. Figure 1a plots the consumer types of this example where the x-axis refers to valuations for the game in the good state and the y-axis refers to valuations in the bad state. Our equivalence result implies that, in this case, offering advance sale tickets is not profitable (correlation is 1, and so it is above any threshold where a positive correlation still results in increased profits).

When a consumer receiving a higher value from the game in one state gets a lower value in the other state, then consumer valuations across states are negatively correlated. For instance, if the hardcore fans value the game at 50 dollars in either case, the bandwagon fans value it at 90 when the game matters and at 10 if it does not, and the snobs value it at 10 when it matters and at 90 when it does not, then there is negative correlation, and advance selling improves profit. Figure 1b shows the negatively correlated consumer types of this example.

[25]See Chen and Riordan (2013) for the exact bounds.



(a) Positively correlated values (b) Negatively correlated values

Figure 1: Correlation of values across states

Corollary 2 *If the valuations for consumption in the two possible states of nature are negatively dependent, independent, or not too positively dependent, then advance selling discounts strictly increase spot prices.*

This corollary shows that consumers who purchase in one of the spot markets are worse off due to the availability of the advance selling discounts. The firm engages in price discrimination, and effectively discriminates against the consumers who purchase in one of the states. These consumers did not buy in advance because their expected valuation for the other state was not that high, but they are going to buy in the spot market, and thus their valuations for this state must be higher than those of the whole consumer group before the advance selling period.

Corollary 2 shows the use of advance selling increases prices for those consumers who delay their purchase. However, by Lemma 1, consumers buying in advance get a better deal than a consumer who would purchase in either state at the spot price. Hence, the impact of advance selling on consumer welfare is not straightforward and clear implications can be drawn only by making some functional assumptions about the distribution function of the valuations.[26] In particular, using Eckalbar (2010), our equivalence result implies that

Corollary 3 *When marginal costs are zero, and consumer valuations are independently and uniformly distributed, advance selling together with spot selling results in a lower consumer surplus than spot selling.*

[26]Note that in the standard price discrimination model, the price discrimination between two segments with different demand function results in an increase in both the price to one set of consumers and the firm's profit (like in the previous two corollaries). However, the effect on consumer welfare is ambiguous and depends on the exact shapes of demand and cost functions.

4 Extensions

4.1 Time preferences

□ Base model

We extend our setup to account for some ways that consumers' and the firm's time preferences can display themselves. By purchasing early consumers spend the money upfront, but get the actual product later, at the same time as those consumers who wait until the spot sale. If we normalize the value of unit utility/money to 1 at the consumption stage, the price paid upfront should have a unit value higher than one, more precisely, multiplied by the inverse of the discount factor, $1/\delta$ for $\delta \in (0, 1]$. This implies that the utility from buying in advance is now:

$$av_1 + (1-a)v_2 - (1/\delta)p^{AS},$$

Apart from this, the rest of Table 1 remains the same. If $\delta = 1$, there is no discounting, so we have the benchmark model. For $0 < \delta < 1$ consumers discount future utility/payments, so advance purchasing becomes less valuable to consumers. Furthermore, suppose that the firm values a unit profit at 1 at the spot sale stage and at $1/\delta$ at the advance selling stage. Also, assume that the firm's production takes place at the advance selling stage. So the firm's profit from advance selling and spot selling becomes

$$\begin{aligned}\Pi^{AS}(p_1^{SS}, p_2^{SS}, p^{AS}) = (1/\delta)(p^{AS} - ac_1 - (1-a)c_2)D^{AS} + a(p_1^{SS} - (1/\delta)c_1)D_1^{SS} \\ +(1-a)(p_2^{SS} - (1/\delta)c_2)D_2^{SS}.\end{aligned}$$

So with $0 < \delta < 1$ advance selling becomes more profitable for the firm (keeping the demand and price constant).

The bundling model remains the same except that we need to scale the firm's marginal costs by $1/\delta$ to account for the firm's time preferences. To do so, assume that in the bundling model the marginal cost of product 1 is ac_1/δ and the marginal cost of product 2 is $(1-a)c_2/\delta$. With this parametrization we obtain a result, analogous to Proposition 1.

Proposition 2 *Suppose that in the advance selling model consumers and the firm discount future (consumption and payments at the spot selling period) by $0 < \delta \le 1$ (compared to the advance selling period). The problem of finding the optimal pricing strategy in the advance selling model with time preferences is mathematically equivalent to the problem of finding the optimal pricing strategy in the bundling problem where the marginal cost of product 1 is*

ac_1/δ and the marginal cost of product 2 is $(1-a)c_2/\delta$. Moreover, if the optimal bundle price is $(1/\delta)p^*$, and the optimal prices for the individual products are ap_1^* and $(1-a)p_2^*$, then the optimal advance selling price is p^* and the optimal spot selling prices are p_1^* and p_2^*.

Recall the example (from Eckalbar, 2010) of section 2 where consumer valuations are uniformly and independently distributed and the firm's marginal costs are zero. Using the closed form solution to this bundling problem and Proposition 2 implies that in the advance selling problem where consumers and the firm discount future at the same rate, δ, the optimal spot selling prices are the same as the ones without discounting: $p_1^{SS} = \frac{2}{3}V_1$ and $p_2^{SS} = \frac{2}{3}V_2$, but the optimal advance selling price is lower reflecting the discount factor: $p^{AS} = \frac{\delta}{3}\left(2aV_1 + (1-a)V_2 - \sqrt{2a(1-a)V_1V_2}\right)$. In this example introducing time preferences does not affect spot selling prices, but lowers the advance selling price by accounting for the discount factor.

□ Different discount factors

If consumers have time preferences as above, but the firm does not, then the equivalence result of the previous corollary breaks down (however, see the special case in the next subsection). In this case, we do not need to scale the costs in the bundling model by $1/\delta$ and so we keep the cost parametrization of the bundling model the same as the benchmark. If the bundle price is $(1/\delta)p^{AS}$, product 1's price is ap_1^{SS}, and product 2's price is $(1-a)p_2^{SS}$, then the demand for the bundle, the demand for only product 1, and the demand for only product 2 correspond respectively to the advance selling demand, demand for spot sales in state 1, and the demand for spot sales in state 2. However, at those prices, the firm's profit is higher in the bundling model than it is in the advance selling model since in the former model specification it receives a higher price, $(1/\delta)p^{AS}$ rather than p^{AS}, from the same number of consumers. Hence, it is not possible to obtain the exact equivalence of the advance selling model to the bundling model in this case. More generally, the equivalence breaks down if the firm discounts future at a different rate from the discount rate of consumers, since then it is not possible to obtain the equivalence of demands as well as of profits with a unique parametrization of prices. In this case we obtain the following result from the comparison of the two problems.

Proposition 3 *Suppose that in the advance selling model consumers discount future (consumption and payments at the spot selling period) by $0 < \delta_c \leq 1$ (compared to the advance selling period), and the firm discounts future (profits at the spot selling period) by $0 < \delta_f \leq 1$ such that $\delta_f \neq \delta_c$. Suppose that in the bundling problem the marginal cost of product 1 is ac_1/δ_f and the marginal cost of product 2 is $(1-a)c_2/\delta_f$. If the firm discounts future less* *than consumers, i.e., $\delta_f > \delta_c$, the region where the advance selling is profitable is smaller than the region where bundling is profitable, and the latter includes the former. The opposite is true otherwise.*

This implies that if consumers are more impatient than the firm, which is more likely to be true as the firm should have easier access to capital market than individuals, for all parameter values where bundling is not profitable, advance selling is not profitable either. More interestingly, this result shows that even if one cannot obtain the exact equivalence between the bundling problem and the advance selling problem, the comparison of the two models enables us to obtain insights on the profitability of advance selling if we know whether the firm or consumers are more impatient and have a characterization of the profitability of the bundling strategy.

□ Consumers placing a premium on getting the product earlier

The value of a product (or a service) purchased in advance could be larger than the expected value of the same product purchased in all the possible future states. One example is tweaking the model to allow consumers who purchased the good in advance to enjoy it in Period 1 as well. The utility in that period is certain, since there is no uncertainty to resolve. Then, in Period 2, we return to the model described earlier. In general, scenarios where a consumer gets an extra benefit from securing the transaction earlier fit this model.[27]

There are several papers in the bundling literature that allow nonadditive valuations: consumers' valuation from purchasing a bundle is not simply the sum of the valuations for the individual products included in the bundle.[28] To model the situation where consumers get an extra utility from purchasing in advance, we use the advance selling equivalent of Venkatesh and Kamakura's (2003) bundling complements model.[29] Hence, we assume that the extra benefit from advance purchasing is likely dependent on the expected value from spot purchasing. Thus, the utility of the advance purchase in this scenario is: $(1+\theta)(av_1 + (1-a)v_2) - p^{AS}$, where $\theta > 0$ implies that consumers prefer to secure their product early.

Proposition 4 *The problem of finding the optimal pricing strategy in the advance selling model with time preferences as described above is mathematically equivalent to the problem of finding the optimal pricing strategy in the bundling problem, where goods are complements,*

[27] In particular, this could be a reduced form way of modeling risk aversion.

[28] This assumption allows for cases where the products are complements so that consumers get an extra utility from buying two products together or where the products are substitutes so that consumers' marginal utility from consuming the second product is lower if they have already bought the first one.

[29] Venkatesh and Kamakura analyze optimal bundle pricing when the utility of a bundle of goods 1 and 2 is more than the sum of the values from independent consumption, more precisely, $(1+\theta)(u_1 + u_2)$.

as described above. Moreover, if the optimal bundle price is p^*, and the optimal prices for the individual products are ap_1^* and $(1-a)p_2^*$, then the optimal advance selling price is p^* and the optimal spot selling prices are p_1^* and p_2^*.

This proposition and Proposition 2 both generalize Proposition 1, and show that all of our results and intuition still apply in these cases with time preferences.

Using Venkatesh and Kamakura's (2003) findings, this result implies the following:[30]

Corollary 4 *Assume consumer valuations from consumption in different states are uniformly distributed over a compact interval, $[0, a]$, and the firm's marginal cost is the same in both states, $c_1 = c_2 = c$.*

- *When c/a is low, pure advance selling is more profitable than pure spot selling. Otherwise, pure spot selling is more profitable than pure advance selling except when the time preferences are very strong, in which case the two strategies are equally profitable.*
- *Simulations show that when c/a is low, advance selling together with spot selling is more profitable when time preferences are weak, whereas pure advance selling is more profitable when the time preferences are very strong. When c/a is high, pure spot selling is more profitable when the time preferences are weak and pure advance selling is more profitable otherwise. When c/a is moderate, pure spot selling is more profitable when the time preferences are moderate, advance selling together with spot selling is more profitable when time preferences are weak, and pure advance selling is more profitable when time preferences are strong.*

4.2 More than two states of the world

Chen and Riordan (2013) also analyze the bundling problem of a monopolist selling more than two goods. To apply their findings of that case to the advance selling problem, we extend the model of advance selling by allowing for $N > 2$ states of nature where state i occurs with probability $a_i \in [0, 1]$ such that $\sum_i a_i = 1$ for $i = 1, .., N$. A consumer receives utility v_i if she consumes the product in state i. Let $f(\bullet, ..., \bullet)$ denote the joint probability distribution function of valuations across the states over region $[\underline{v_1}, \overline{v_1}]$ x...x $[\underline{v_N}, \overline{v_N}]$. As before, we do not make any assumption on the type of correlation between the valuations across the states. Furthermore, the marginal cost of the firm is c_i in state i. We assume that

[30] Also see Armstrong (2013) for a more general model covering substitute products ($\theta < 0$), or consumers preferring not to secure the transaction in advance. This could occur if consumers are cash-constrained and do not want to commit any of their cash to this future purchase or simply do not have the money to pay now.

at least some consumers have valuations above the respective marginal costs, and that the monopolist's second order condition is satisfied (there exists a global maximum). Assuming the same timing as before, we analyze the profitability of selling the product before the state is realized (advance selling) at price p^{AS} or after the state realization (spot selling) at price p_i^{SS} in state i, or offering both an advance selling price and spot selling prices.

Similarly, we extend the bundling model to more than two products: suppose that the seller offers $N > 2$ products and consumers have unit demand for each product. Suppose that a consumer receives utility $a_i v_i$ from consuming product i and let $f(\bullet, ..., \bullet)$ denote the joint distribution function of the valuations $v_1, ..., v_N$ over region $[\underline{v_1}, \overline{v_1}]$x... x $[\underline{v_N}, \overline{v_N}]$. Assume also that the seller's marginal cost of product i is $a_i c_i$ and that the firm cannot prevent consumers from purchasing any two (or more) products separately. The bundling problem involves analyzing the profitability of selling the products as a bundle at price p^B or selling only the individual products separately at prices $p_1^I, .., p_N^I$, or offering both a bundle price and individual product prices (mixed bundling).

Following a similar argument as the one in Proposition 1, one can show the mathematical equivalence of these two problems.

Proposition 5 *The problem of finding the optimal pricing strategy in the advance selling model when there are $N > 2$ possible states of the world (described above) is mathematically equivalent to the problem of finding the optimal pricing strategy in the bundling problem when the firm sells $N > 2$ products (described above). Moreover, if the optimal bundle price is p^*, and the stand-alone optimal price for product i is $a_i p_i^*$, then the optimal advance selling price is p^* and the optimal spot selling price in state i is p_i^*.*

Using the previous bundling literature, this equivalence result implies that advance selling with more than two possible future states is profitable under similar conditions as the ones for two states.

Corollary 5 *Consider the advance selling problem where there are $N > 2$ possible states of the world. If the valuations across any two states are either negatively dependent, independent, or not too positively dependent, then advance selling increases profits.*

Another potential issue arising from the bundling literature is sometimes firms bundle only a subset of the products, if only for the practical purposes of having a manageable size menu. For example, with mixed bundling setting prices to 41 products results in $2^{41} - 1$ possible prices, and no firm will realistically subject its consumers to this number of choices. Chu, Leslie, and Sorensen (2011) show numerically (and confirm empirically) that, for particular distributions, bundle-size pricing (setting prices that depend only on the size of the bundle purchased) tends to closely approximate the profits from mixed bundling. This result significantly simplifies the analysis of bundling with many products.

Similarly, in the problem of advance selling, we can think of selling different types of advance tickets such that each ticket incorporates a different number of possible states. For example, a consumer can have a ticket for the basketball game at the end of the season no matter what the standings are, or she can buy a ticket that lets her attend only if the team won more than x out of the preceding 81 games.[31]

Using Chu *et. al.* (2011), our equivalence result (Proposition 2) implies the following:

Corollary 6 *Consider the problem of advance selling with $N > 2$ states of the world where the firm can offer advance purchase tickets for each of the possible combinations of states. The firm's profit under this problem is closely approximated by offering advance purchase tickets such that their prices depend only on the number of states in which they are valid.*

Bakos and Brynjolfsson (1999) consider the problem of monopoly bundling when the number of products is very large (goes to infinity). In the case where consumer valuations across states are independent and products have zero marginal cost (digital products, tickets to events where almost all costs are fixed, and so on), our equivalence result brings out the following finding.

Corollary 7 *Consider the problem of advance selling with $N > 2$ states of the world. Assume that consumer valuations are distributed independently across the states, uniformly bounded with continuous density functions and nonnegative support, and marginal costs are zero. Then, as N increases, the consumer surplus from purchasing the bundle decreases to zero, and the firm extracts nearly all consumer surplus.*

The intuition comes from the law of large numbers. More and more possible future states result in more and more independent draws for each consumer. As long as consumer valuations are drawn from the same distribution (within the same state, not necessarily the same distribution across the states), their expected values of the advance selling ticket covering all the states become more and more homogeneous, and in the limit the monopolist simply charges that price in advance, sells to everyone in advance, and captures all consumer surplus.

[31] The alternative results in the advance selling menu of the size of the cardinality of the power set with 82 elements (2^{82}): an advance sale ticket that is valid only if the team won 0 or 1 games, an advance sale ticket that is valid only if the team won 1 or 2 games... an advance ticket that is valid only if the team won 0, 1, or 2 games.... culminating with an advance ticket that is valid in any state.

4.3 Competition

Chen and Riordan (2013) also analyzed the profitability of bundling when one firm offers both products and a competitor can offer only a differentiated version of one product and show that bundling is profitable under similar conditions to the case without competition. To analyze a similar competitive scenario in the advance selling model, we extend it to the case where the firm faces a competitor which offers a differentiated alternative to the firm's product only in one state, say state 1.

For example, if the basketball game is not important, consumers might consider going to see a hockey game instead. Another example is the industrial transportation market between Detroit and Chicago. There are two ways of getting goods cheaply from Detroit to Chicago. If the weather is really cold, then there is only a train. If the weather is warm and there is no ice on the lakes, then there is also a boat. The question is, when does the train company have an incentive to sell in advance, knowing that consumers are going to get stuck with only one choice if the weather is cold. To analyze this question we modify the benchmark setup. The full analysis, including the Proposition describing the equivalence, is in Appendix C. Below, we briefly describe the timing of the game, and discuss the main conclusions.

The timing is as follows:

1. The firm sets an advance selling price (p^{AS}) and a spot selling price for each state (p_1^{SS}, p_2^{SS}). Simultaneously, the competitor sets its price, $\tilde{p}_1^{SS}$.
2. Consumers decide whether to purchase in advance or to wait until the uncertainty about the state is resolved.
3. The state is realized. In state 1, the two firms compete for consumers who did not buy in advance. In state 2, the firm is the monopoly seller and consumers who did not buy in advance decide whether to buy the product.

Corollary 8 *Suppose that the firm will face a competitor only in one state of the world. Then advance selling discounts are strictly more profitable if the valuations are negatively dependent, independent, or not too positively dependent.*[32]

The corollary shows that the firm facing a competitor only in one state has similar incentives to advance sell as the monopoly firm, even if the level of prices is going to change. As Chen and Riordan (2013) show, the bounds with competition require a stronger negative dependence property than the monopoly case. Intuitively, when the firm faces a competitor, consumers have one more option to choose from (spot purchase only from the competitor in

[32] See Chen and Riordan (2013) for the proof in the bundling case.

state 1). Since competition lowers the spot selling price in state 1 and raises the expected utility from delaying to purchase, there will be less demand for advance purchasing and therefore the optimal advance selling price would be lower than in the monopoly case. This implies that advance selling margins would be lower and so the profitability of advance selling discounts would require more people to switch from purchasing the good only in one state to buying the good in advance, which would be case if there was stronger negative dependence between the valuations across the states.

Most of the literature on competitive bundling has focused on anti-competitive bundling and analyzed when a multi-product incumbent facing potential entry to a single market wants to bundle its monopoly good with the competitive good to deter entry (for example, Whinston, 1990). To analyze whether advance selling can be used as a tool to deter potential entry in the spot period, we use the equivalent of Nalebuff's (2004) bundling model and modify our advance selling model as follows: consumer valuations across the states are uniformly and independently distributed over a unit square.[33] With some probability the challenger who offers a perfect substitute to the incumbent's product enters the market either in state 1 or in state 2. The timing of the events is: 1. The incumbent decides whether to sell its product in advance, and if so, it sets its advance selling price. Simultaneously, it sets its spot prices. 2. The challenger decides whether to enter, in either of the two states, and if it does, then it sets its spot price. 3. Consumers decide which firm to patronize and whether to purchase in advance or wait for the spot markets. Regardless of whether the challenger enters, the incumbent cannot change its prices.[34]

Corollary 9 *In the advance selling model described above, advance selling makes entry less profitable without lowering the profits of the incumbent (if entry is deterred).*

Intuitively, if entry occurs, advance selling reduces the market share of the entrant. Without advance selling, if the entrant offers an ϵ lower spot price than the incumbent, it gains the entire market. With advance selling, if the entrant offers an ϵ lower spot price than the incumbent, it steals less than the entire market. Offering more advance purchasing discounts than an uncontested monopoly is profitable since it only has a second order effect on the incumbent profit, but a first order effect on the entrant's profit.

[33] Nalebuff argues that uniform density is for the sake of simplicity and qualitative arguments on how bundling deters entry would apply for the class of quasi-concave density functions, such as multi-variate normal.

[34] Fixing prices of the incumbent pre-entry is favorable to the entrant and so conservative for the entry deterrence role of advance selling. See Nalebuff (2004) for more on this.

5 Limitations

There are important implementation differences between an advance selling strategy and a bundling strategy. Some of the problems that we are interested in in the advance selling literature were not of particular interest in the context of bundling and so the researchers in bundling did not analyze these situations. Unlike a standard bundling problem, an advance selling problem has a time dimension and incorporates uncertainty. For instance, the impact of uncertainty (risk aversion), commitment to future prices, and different arrival times of consumers were not analyzed by the bundling literature.[35] Hence, our paper does not directly contribute to understanding these problems in the advance selling context. However, even in these cases it is often possible to draw relatively straightforward parallels (see our discussion of time preferences, for example) or, at least, to point out which assumptions are not satisfied and to conjecture the effect of these issues on the model. We discuss some of these issues below.

5.1 Risk aversion

Advance purchasing decisions involve uncertainty and so risk aversion might be an important factor driving consumer decisions of whether to buy a product in advance. However, the existing models of bundling make restrictive assumptions (the linearity of the utility from the bundle in the individual product valuations) that are isomorphic to assuming risk neutrality. Moreover, unlike the few other cases covered in this paper, risk aversion does not affect profitability and consumer decision-making in a way that is convenient to analyze analytically. This was illustrated by Shugan and Xie (2001) who presented some examples of risk averse utility functions where consumers are assumed to be homogeneous. The analysis of risk aversion is even more complicated in our setup since we allow for ex-ante consumer heterogeneity and ex-post different aggregate demands depending on the state. As a simple example can indicate, the impact of risk aversion on the profitability of advance selling depends on the distribution of consumer valuations across the states.[36] Modeling this goes

[35]For different arrival times, some consumers might not have been there in the advance sale period, but might come in later and be able to purchase on-the-spot after the uncertainty is resolved (see Shugan and Xie, 2001). Allowing sequential arrival of consumers would change the distribution of consumer types at the spot selling stage. In the bundling setting, this would be equivalent to a segment of consumers that, for exogenous reasons, cannot buy the bundle, but can only buy individual products, for instance, because the bundle is physically not available in some geographic locations.

[36]Whether buying in advance is riskier depends on consumer valuations relative to prices. For example, suppose there are two equally likely states, the spot prices are 10 and 5, and the advance sale price is 7.40. If a consumer's valuations are 11 and 6, then waiting gives a certain payoff of 1, while advance purchase is a lottery between 3.60 and -1.40. If the consumer is sufficiently risk-averse, he waits, even though the expected value of advance purchase is higher. On the other hand, if a consumer's valuations are 8.40 for both states,

beyond the scope of our paper.

In appendix D we illustrate how one can incorporate risk aversion into our benchmark models and apply Taylor expansions to the versions of tables 1 and 2 to show that the bundling problem provides an approximation to the advance selling problem even in the risk aversion case, with the market of risk-neutral consumers resulting in the already stated equivalence. One can then use empirical estimates of the derivatives of consumers' utility functions to at least bound our risk-neutral (linear) approximation. An interesting empirical question is how close the risk-neutral approximation actually is. The answer depends on the ratio of the net utility differences of purchasing in advance or waiting until the spot period in different states (the risk involved) to the overall wealth of the consumer (the starting point in the concave utility function calculations). We conjecture that with most everyday purchases, like concert tickets for consumers, or even most of the airplane tickets, with median U.S. household wealth, assuming standard functional utility forms like Constant Absolute Risk Aversion (CARA) or Constant Relative Risk Aversion (CRRA), the risk-neutral approximation is reasonably close.[37] This raises many other empirical questions, such as whether the standard functional forms like CARA and CRRA do a good job approximating everyday purchases; however, we leave these questions and a potential generalization of our model of risk-averse consumers to future researchers.

5.2 Commitment to spot prices

For the purpose of our analysis we assume that the firm can pre-commit to spot prices in the advance selling model. Indeed, we observe many real-world situations where firms announce and commit to their spot prices at the advance selling stage, for example, "advance price" and "gate price" for concert tickets, sport games, amusement parks, zoo tickets, fair tickets,

then the advance purchase gives a certain payoff of 1, while waiting is a lottery between 3.40 and 0. In this case, if the consumer is sufficiently risk-averse, he purchases in advance, even though the expected value of waiting for the spot market is higher. On one hand, advance purchasing leads to a risk of paying a higher price than the actual valuation at the spot (if the state is bad), on the other hand, it removes the risk of paying a higher price at the spot (if the state is good).

[37] We use the results of an empirical estimation of risk aversion, Fullenkamp, Tenorio, and Battalio (2003). They use the data from a game show to analyze the contestants' risk aversion patterns. They show that, for example, using the CARA and CRRA functional forms, and estimating the models using several possible bounded rationality scenarios, the authors find that a certainty equivalent of a 50/50 gamble between \$0 and \$1,000 ranges from \$491.26 to \$499.19, depending on the exact function and a possible bounded rationality scenario that the authors use. In comparison with other studies, the authors point out that 'for a gamble offering a 50-50 chance of winning nothing and \$1,000, the certainty equivalent ranges from \$461.40 (Gertner, 1993) to \$496.73 (Hersch and McDougall, 1997), with most estimates implying a certainty equivalent above \$490.' While the amount of risk aversion is not trivial, it shows that our approximation should be not too far off even for a gamble of this size: 50/50 between \$0 and \$1,000. However, we believe that most real-world advance sales products do not present as much of a gamble for a potential consumer, and thus our bundling approximation should be reasonably accurate.

exhibitions, early bird registration fees for conferences, recreation activities, and professional training classes.[38]

Another example is Smart-Meters. Smart-Meters show consumers the current price of the electricity that they are consuming, depending on whether this is a peak time for consumption or not. Consumers know the peak and non-peak prices in advance, but do not know exactly when the peak time will occur in the future. Letting your electricity company install a Smart-Meter is akin to spot pricing. Not letting it install a Smart-Meter is like buying in advance and securing a price regardless of what state of the world it is (peak or off-peak).[39]

Similarly, some companies offer a guaranteed lock-in energy supply price to their consumers (including other businesses) for the following time period (for example, a year), instead of fluctuating prices based on a given index. The expectations of the index fluctuations should be the same for all the market participants. Some of the examples are gasoline and electricity.[40]

A less recent, but worth mentioning example, is a leaked internal discussion of the Coca-Cola Company in 1999, regarding the idea of charging different prices in their vending machines, depending on the temperature outside. The idea fell through due, reportedly, to the consumer backlash, and it illustrates that as with any price discrimination device, setting different prices for different demand or cost states should be done carefully, in particular, without forgetting to manage consumer expectations.[41]

As noted in the introduction, the assumption of exogenous credibility is a common assumption in the advance selling literature and is usually justified in real-world situations by reputation concerns (see, e.g., Shugan and Xie, 2000, 2005; Fay and Xie, 2010). One of the few exceptions in the literature is Xie and Shugan (2001) who analyze an extension the seller cannot pre-commit to spot prices and show that the seller's credibility is critical for the profitability of advance selling. With no exogenous credibility, they show that large

[38] See, e.g., All Good Music Festival and Campout, www.allgoodfestival.com, Minnesota State Fair, www.mnstatefair.org/tickets_discounts/admission.html, conference fee announcements of European Association for Research in Industrial Economics, www.earie2013.org/index.php?/earie/Registration-and-Submission/Conference-Fees.

[39] Some Smart-Meters have predefined peak periods (say, 9am-5pm on workdays), thus allowing consumers to strategically switch, and so this situation would not fit our model. Others (critical peak pricing) have no predefined peak periods and charge consumers high (and known in advance) prices when utilities observe high wholesale prices or power-system emergency conditions, 'such as 3p.m. to 6p.m. on a hot summer weekday,' see Wall Street Journal (2013).

[40] Consumers cannot take advantage of the seller and resell the product if they get particularly lucky with their locked-in rate: in the United States, the seller is protected by the given State's Uniform Commercial Code provisions, or similar statutes (see U.C.C. §§2-103(1)(b) and 2-306, and Alexandrov, 2013, for more).

[41] See New York Times (2005). The advance purchase in this case would be buying a Coke in a grocery store.

marginal costs and/or capacity constraints might enable the seller to commit to high spot prices. They also find that limiting advance sales can be profitable when three conditions hold: (1) selling to all early arrivals would leave insufficient capacity in the spot period to sell to all second period arrivals with high valuations, (2) the optimal spot price is high, and (3) marginal costs are sufficiently small to make advance selling profitable.

Unfortunately, analyzing the no commitment case is particularly hard in our model since we allow consumers to be heterogeneous at the advance selling stage. In Xie and Shugan (2001) consumers are ex-ante homogeneous.[42] In our model, however, consumers are heterogeneous ex-ante. And depending on the price charged in advance, a distinct, non-random, and self-selected subset of consumers waits for the spot period. Consumers and the firm play a Bayesian Nash game, where consumers form expectations over who exactly purchases in the first period based on the advance selling price, and thus what the monopolist will charge in the second. An equilibrium advance purchase price depends on the consumers' beliefs, and the spot price depends on who exactly purchases in advance and who does not. There are, potentially, many possible equilibria, including tipping and self-fulfilling expectations, when consumer beliefs about the identities of the advance purchasers might result in the spot markets shutting down. While this topic is important, it is arguably less relevant in many real world examples due to reputation concerns – as we discussed above and in the introduction – and is beyond the scope of this paper.

6 Conclusion

We have shown that the problem of optimal advance selling is technically equivalent to the problem of optimal bundle pricing after picking the appropriate comparison costs and valuations. Our main assumptions are that consumers and the seller agree on the probability of each state occurring, consumers are risk-neutral, the seller can commit to the pre-announced spot prices, and that consumers and the firm have the same time preferences.

This equivalence allows us to apply numerous insights from the bundling literature to the study of advance selling problems and provides researchers and practitioners with a new way of analyzing advance selling and bundling problems. In particular, it shows that the correlation of consumer valuations across states plays a crucial role in determining whether

[42]Thus, regardless of whether the firm can commit to spot prices, all consumers are indifferent between purchasing in advance and waiting for the spot period. This implies that the consumers who do not purchase in advance are a random sample of all the consumers, and the prices chosen by the firm and consumers' beliefs will not change the distribution of consumers who do not purchase in advance. Therefore, the firm always picks the same price once it gets to the spot purchasing stage, and in particular this price does not depend on the advance sale price.

advance selling is profitable, including the cases of more than two states, the case where consumers and the firm have the same time preferences, and the case of potential competition in one of the states. We also illustrate a profitable use of advance selling as an entry deterrence tool.

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Appendix

A Spot selling and advance selling demands

Consumers who would purchase on-the-spot in either state (types $v_1 \geq p_1^{SS}$ and $v_2 \geq p_2^{SS}$) purchase in advance since the advance selling price is lower than the expected spot selling price (by Lemma 1). Consumers who would spot purchase only in state 1 (types $v_1 \geq p_1^{SS}$ and $v_2 < p_2^{SS}$) prefer to delay their purchase and purchase on-the-spot (only when state 1 occurs) if

$$a(v_1 - p_1^{SS}) \geq av_1 + (1-a)v_2 - p^{AS}$$

or equivalently their state 2 valuation is very low ($v_2 < (p^{AS} - ap_1^{SS})/(1-a)$). Aggregation of these consumer types gives us the demand for spot purchase only in state 1:

$$D_1^{SS}(p_1^{SS}, p^{AS}) = \int_{p_1^{SS}}^{\overline{v_1}} \int_{\underline{v_2}}^{\frac{p^{AS} - ap_1^{SS}}{1-a}} f(v_1, v_2) dv_2 dv_1. \tag{4}$$

Consumers who would spot purchase only in state 2 (types $v_1 < p_1^{SS}$ and $v_2 \geq p_2^{SS}$) prefer to delay their purchase and purchase on-the-spot (only when state 2 occurs) if

$$(1-a)(v_2 - p_2^{SS}) \geq av_1 + (1-a)v_2 - p^{AS}$$

or equivalently their state 1 valuation is very low ($v_1 < (p^{AS} - (1-a)p_2^{SS})/a$). An aggregation of these consumer types gives us the demand for spot purchase only in state 2:

$$D_2^{SS}(p_2^{SS}, p^{AS}) = \int_{\underline{v_1}}^{\frac{p^{AS} - (1-a)p_2^{SS}}{a}} \int_{p_2^{SS}}^{\overline{v_2}} f(v_1, v_2) dv_2 dv_1. \tag{5}$$

Consumers who would not purchase on-the-spot (types $v_1 < p_1^{SS}$ and $v_2 < p_2^{SS}$) purchase in advance if the advance selling price is sufficiently low ($av_1 + (1-a)v_2 \geq p^{AS}$) and do not purchase at all otherwise. Hence, the proportion of consumers who do not make any purchase is given by

$$D^{NoPurchase}(p_1^{SS}, p_2^{SS}, p^{AS}) = \int_{\underline{v_2}}^{p_2^{SS}} \int_{\underline{v_1}}^{\frac{p^{AS} - (1-a)v_2}{a}} f(v_1, v_2) dv_1 dv_2. \tag{6}$$

and the demand for advance selling is equal to

$$D^{AS}(p_1^{SS}, p_2^{SS}, p^{AS}) = 1 - D^{NoPurchase}(p_1^{SS}, p_2^{SS}, p^{AS}) - D_1^{SS}(p_1^{SS}, p^{AS}) - D_2^{SS}(p_2^{SS}, p^{AS}). \tag{7}$$

B Individual product and bundling demands

Consumers who would purchase both products (types $v_1 \geq \frac{p_1^I}{a}$ and $v_2 \geq \frac{p_2^I}{1-a}$) buy the bundle since it costs less than purchasing both products separately (by Lemma 2). Consumers who would purchase only product 1 (types $v_1 \geq \frac{p_1^I}{a}$ and $v_2 < \frac{p_2^I}{1-a}$) prefer to buy only product 1 rather than the bundle if

$$av_1 - p_1^I \geq av_1 + (1-a)v_2 - p^B$$

or equivalently if their product 2 valuation is very low: $v_2 < (p^B - p_1^I)/(1-a)$. Aggregation of these consumer types gives us the demand for product 1 only:

$$D_1^I(p_1^I, p^B) = \int_{\frac{p_1^I}{a}}^{\overline{v_1}} \int_{\underline{v_2}}^{\frac{p^B - p_1^I}{1-a}} f(v_1, v_2) dv_2 dv_1. \tag{8}$$

Consumers who would purchase only product 2 (types $v_1 < \frac{p_1^I}{a}$ and $v_2 \geq \frac{p_2^I}{1-a}$) prefer to buy only product 2 rather than the bundle if

$$(1-a)v_2 - p_2^I \geq av_1 + (1-a)v_2 - p^B$$

or equivalently if their product 1 valuation is very low ($v_1 < (p^{AS} - p_2^I)/a$). Aggregation of these consumer types gives us the demand for product 2 only:

$$D_2^I(p_2^I, p^B) = \int_{\underline{v_1}}^{\frac{p^B - p_2^I}{a}} \int_{\frac{p_2^I}{1-a}}^{\overline{v_2}} f(v_1, v_2) dv_2 dv_1. \tag{9}$$

Consumers who would not purchase the products separately (types $v_1 < \frac{p_1^I}{a}$ and $v_2 < \frac{p_2^I}{1-a}$) purchase the bundle if the bundle price is sufficiently low ($av_1 + (1-a)v_2 \geq p^B$) and do not purchase at all otherwise. Hence, the proportion of consumers who do not make any purchase is given by

$$D^{NoPurchase}(p_1^I, p_2^I, p^B) = \int_{\underline{v_2}}^{\frac{p_2^I}{1-a}} \int_{\underline{v_1}}^{\frac{p^B - (1-a)v_2}{a}} f(v_1, v_2) dv_1 dv_2. \tag{10}$$

and the demand for the bundle is equal to

$$D^B(p_1^I, p_2^I, p^B) = 1 - D^{NoPurchase}(p_1^I, p_2^I, p^B) - D_1^I(p_1^I, p^B) - D_2^I(p_2^I, p^B). \tag{11}$$

C Extensions

C.1 Time preferences

□ Proof of Proposition 3

The utility from buying in advance is now:

$$av_1 + (1-a)v_2 - (1/\delta_c)p^{AS},$$

Apart from this, the rest of Table 1 remains the same. But now we need to scale the costs in the bundling model by $1/\delta_f$, so the marginal cost of product 1 becomes ac_1/δ_f and the marginal cost of product 2 is $(1-a)c_2/\delta_f$. If the bundle price is $(1/\delta_c)p^{AS}$, product 1's price is ap_1^{SS}, and product 2's price is $(1-a)p_2^{SS}$, then the demand for the bundle, the demand for only product 1, and the demand for only product 2 correspond respectively to the advance selling demand, demand for spot sales in state 1, and the demand for spot sales in state 2. At those prices in the advance selling model the firm gets

$$\Pi^{AS}(p_1^{SS}, p_2^{SS}, p^{AS}) = (1/\delta_f)(p^{AS} - ac_1 - (1-a)c_2)D^{AS} + a(p_1^{SS} - (1/\delta_f)c_1)D_1^{SS} + (1-a)(p_2^{SS} - (1/\delta_f)c_2)D_2^{SS}.$$

On the other hand, at those prices in the bundling model the firm gets

$$\begin{aligned}\Pi^{B}(ap_1^{SS}, (1-a)p_2^{SS}, (1/\delta_c)p^{AS}) &= ((1/\delta_c)p^{AS} - (1/\delta_f)(ac_1 + (1-a)c_2))D^{AS} \\ &\quad + a(p_1^{SS} - (1/\delta_f)c_1)D_1^{SS} + (1-a)(p_2^{SS} - (1/\delta_f)c_2)D_2^{SS}.\end{aligned}$$

Obviously, when $\delta_f < \delta_c$, $\Pi^{AS}(p_1^{SS}, p_2^{SS}, p^{AS}) > \Pi^{B}(ap_1^{SS}, (1-a)p_2^{SS}, (1/\delta_c)p^{AS})$, and so for parameter values where the bundling is profitable, it must be the case that advance selling is profitable.

C.2 Competition

Assume that consumer value from the dominant firm's product is v_i in state i, for $i = 1, 2$, and their value from the rival product is $\tilde{v}_1$ in state 1, and state 1 occurs with probability a. Let $f(\bullet, \bullet)$ denote the joint distribution of (v_1, v_2) over region $[\underline{v_1}, \overline{v_1}]$ x $[\underline{v_2}, \overline{v_2}]$ (as in the benchmark) and also let $\tilde{f}(\bullet, \bullet)$ denote the joint distribution of $(\tilde{v}_1, v_2)$ over region $[\underline{\tilde{v}_1}, \overline{\tilde{v}_1}]$ x $[\underline{v_2}, \overline{v_2}]$. Assume that the dominant seller's marginal cost of product 1 is c_1 and of product 2 is c_2, and the entrant's marginal cost is $\tilde{c}_1$. The timing of the events is the following:
The timing of the game is as described in the body of the text.

Similar to the monopoly model of advance selling (section 2.1), Table 3 shows consumers' purchasing options at the advance selling stage and their corresponding utilities.

Table 3: The expected utility of a consumer from different purchasing options in the advance selling model with competition.

Purchasing options	**Expected utility**
Advance purchase	$av_1 + (1-a)v_2 - p^{AS}$
Spot purchase only the dominant firm's product in state 1	$a(v_1 - p_1^{SS})$
Spot purchase only the competitor's product in state 1	$a(\tilde{v}_1 - \tilde{p}_1^{SS})$
Spot purchase only in state 2	$(1-a)(v_2 - p_2^{SS})$
No purchase	0

Let $D^{AS}, D_1^{SS}, \tilde{D}_1^{SS}, D_2^{SS}$ denote respectively the demand for advance purchasing, the demand for spot purchasing only the dominant firm's product in state 1, the demand for spot purchasing only the competitor's product in state 1, and the demand for spot purchasing only in state 2. Note that these demand functions are different from their counterparts of the benchmark (derived in appendix A) since now while deriving demands we need to account for one more option for consumers in state 1: Purchasing the product of the competitor. The dominant firm's profit is the sum of the *expected* profits from advance selling and from spot selling:

$$\Pi^{AS}(p_1^{SS}, p_2^{SS}, p^{AS}, \tilde{p}_1^{SS}) = (p^{AS} - ac_1 - (1-a)c_2)D^{AS} + a(p_1^{SS} - c_1)D_1^{SS} + (1-a)(p_2^{SS} - c_2)D_2^{SS}. \quad (12)$$

The competitor's profit is the expected profit from spot selling only:

$$\tilde{\Pi}^{AS}(p_1^{SS}, p_2^{SS}, p^{AS}, \tilde{p}_1^{SS}) = a(\tilde{p}_1^{SS} - \tilde{c}_1)\tilde{D}_1^{SS}. \quad (13)$$

To obtain a bundling equivalent of the advance selling model with competition, we modify the benchmark model of bundling (section 2.2) as the following: Consider a static problem of competitive bundling where a dominant seller offers two products, a competitor offers a differentiated version of product 1 and consumers have unit demand for each product. Suppose that a consumer receives utility av_1 from consuming product 1 of the dominant firm, $a\tilde{v_1}$ from consuming product 1 of the entrant, and $(1-a)v_2$ from consuming product 2. As before, let $f(\bullet, \bullet)$ denote the joint distribution function of the valuations (v_1, v_2) over region $[\underline{v_1}, \overline{v_1}]$ x $[\underline{v_2}, \overline{v_2}]$ and also let $\tilde{f}(\bullet, \bullet)$ denote the joint distribution of $(\tilde{v}_1, v_2)$ over region $[\underline{\tilde{v}_1}, \overline{\tilde{v}_1}]$ x $[\underline{v_2}, \overline{v_2}]$. Assume that the dominant seller's marginal cost of product 1 is ac_1 and of product 2 is $(1-a)c_2$, and the entrant's marginal cost is $a\tilde{c_1}$. Assume also that the dominant firm cannot prevent consumers from purchasing both products separately. The timing of the events is the following:

1. The dominant firm sets a bundle price (p^B) and individual product prices (p_1^I, p_2^I). Simultaneously, the competitor sets its price, $\tilde{p}_1^I$.

2. Consumers decide whether to purchase the bundle or individual products only. In the latter case, they decide from which firm to buy product 1.

Similar to the previous advance selling problem, we write consumer options and their corresponding utilities:

Table 4: The utility of a consumer from different purchasing options in the bundling model with competition.

Purchasing options	**Expected utility**
Bundle of products 1 and 2	$av_1 + (1-a)v_2 - p^B$
Only product 1 from the dominant firm	$av_1 - p_1^I$
Only product 1 from the competitor	$a\tilde{v}_1 - \tilde{p}_1^I$
Only product 2	$(1-a)v_2 - p_2^I$
No purchase	0

Consumers choose the option which gives them the highest utility. The dominant firm's profit in the bundling model is

$$\Pi^B(p_1^I, p_2^I, p^B, \tilde{p}_1^I) = (p^B - ac_1 - (1-a)c_2)D^B + (p_1^I - ac_1)D_1^I + (p_2^I - (1-a)c_2)D_2^I, \quad (14)$$

and the competitor's profit is

$$\tilde{\Pi}^B(p_1^I, p_2^I, p^B, \tilde{p}_1^I) = (\tilde{p}_1^I - a\tilde{c}_1)\tilde{D}_1^I, \quad (15)$$

Comparing table 3 and table 4 shows that at prices $p^B = p^{AS}$, $p_1^I = ap_1^{SS}$, $\tilde{p}_1^I = a\tilde{p}_1^{SS}$, $p_2^I = (1-a)p_2^{SS}$ consumers face exactly the same set of purchasing options and utilities in the advance selling model and in the bundling model. This implies that at these prices the demand for advance purchasing corresponds to the demand for the bundle, $D^{AS} = D^B$, the demand for spot purchasing the dominant firm's product in state 1 corresponds to the demand for only product 1 from the dominant firm, $D_1^{SS} = D_1^I$, the demand for spot purchasing the competitor's product in state 1 corresponds to the demand for only product 1 from the competitor, $\tilde{D}_1^{SS} = \tilde{D}_1^I$ and finally the demand for spot purchasing only in state 2 corresponds to the demand for only product 2, $D_1^{SS} = D_1^I$. But then at those prices the profits of both models coincide:

$$\Pi^{AS}(p_1^{SS}, p_2^{SS}, p^{AS}, \tilde{p}_1^{SS}) = \Pi^{B}(ap_1^{SS}, (1-a)p_2^{SS}, p^{AS}, a\tilde{p}_1^{SS})$$
$$\tilde{\Pi}^{AS}(p_1^{SS}, p_2^{SS}, p^{AS}, \tilde{p}_1^{SS}) = \tilde{\Pi}^{B}(ap_1^{SS}, (1-a)p_2^{SS}, p^{AS}, (1-a)\tilde{p}_1^{SS})$$

Hence, we obtain our equivalence result

Proposition 6 *The problem of finding the optimal pricing strategy in the competitive advance selling model (described above) is mathematically equivalent to the problem of finding the optimal pricing strategy in the competitive bundling model (described above). Moreover, if the dominant firm's optimal bundle price is* p^**, and optimal prices for the individual products are* ap_1^* *and* $(1-a)p_2^*$*, and the competitor's product price is* $a\tilde{p}_1^*$*, then in the advance selling model the dominant firm's optimal advance selling price is* p^* *and the optimal spot selling prices are* p_1^* *and* p_2^**, and the competitor's product price is* $\tilde{p}_1^*$*.*

D Consumers with risk preferences

In the advance selling model we assume that consumers are risk-neutral. In particular, this implies that a consumer's utility from consuming a unit of a good bringing utility v at a price p is $U(w+v-p) = u(w) + v - p$ where w refers to the consumer's endowment. In most microeconomic and marketing problems, the endowment is assumed to be fixed and thus the $u(w)$ term generally drops out of the calculations. In this section, we show how an advance selling problem can be approximated by a bundling problem even when consumers are not risk-neutral.

The only difference from the setup of Section 2 is that in both the advance selling model and the bundling model we assume that consumers have a utility function of $U(\bullet)$, which incorporates consumer risk preferences. In particular, a concave U implies risk-aversion, a convex U implies risk-lovingness, and a linear U degenerates to our benchmark models. We again assume that in both models there is a unique solution to the seller's optimal pricing problem (the second order conditions are satisfied). Keep the notation as before. Denote consumer i's current wealth level by w_i. Similar to Tables 1 and 2, Tables 5 and 6 show the purchasing options of a consumer in each model.

Each consumer is going to pick the purchasing option which gives her the maximum (expected) utility. Using Taylor Expansion, we transform Tables 5 and 6 into Table 7 and 8, respectively.

Comparing Tables 7 and 8 shows that if the price for the bundle is p^{AS} and the prices for product 1 and product 2 are, respectively, ap_1^{SS} and $(1-a)p_2^{SS}$, the purchasing options of the

Table 5: The utility of a consumer from different purchasing options in the advance selling model.

Purchasing options	**Expected utility**
Advance purchase	$aU(w_i + v_1 - p^{AS}) + (1-a)U(w_i + v_2 - p^{AS})$
Spot purchase only in state 1	$aU(w_i + v_1 - p_1^{SS}) + (1-a)U(w_i)$
Spot purchase only in state 2	$aU(w_i) + (1-a)U(w_i + v_2 - p_2^{SS})$
No purchase	$U(w_i)$

Table 6: The utility of a consumer from different purchasing options in the bundling model.

Purchasing options	**Utility**
Bundle of products 1 and 2	$U(w_i + av_1 + (1-a)v_2 - p^B)$
Only product 1	$U(w_i + av_1 - p_1^I)$
Only product 2	$U(w_i + (1-a)v_2 - p_2^I)$
No purchase	$U(w_i)$

bundling model would differ from the purchasing options of the advance selling model for Taylor expansion of orders two and more. And so how close the bundling model approximates the advance selling model depends on the magnitude of the higher order derivatives of the utility function.

Table 7: The (expected) utility of a consumer from different options in the advance selling model (Taylor Expansion).

Purchasing options	**Expected Utility**
Advance purchasing	$U(w_i) + U'(w_i)(av_1 + (1-a)v_2 - p^{AS}) + U''(w_i)(a(v_1 - p^{AS})^2 + (1-a)(v_2 - p^{AS})^2)/2 + ...$
only in spot 1	$U(w_i) + aU'(w_i)(v_1 - p_1^{SS}) + aU''(w_i)(v_1 - p_1^{SS})^2/2 + ...$
only in spot 2	$U(w_i) + (1-a)U'(w_i)(v_2 - p_2^{SS}) + (1-a)U''(w_i)(v_2 - p_2^{SS})^2/2 + ...$
No purchase	$U(w_i)$

Table 8: The utility of a consumer from different options in the bundling model (Taylor Expansion).

Purchasing options	**Utility**
Bundle	$U(w_i) + U'(w_i)(av_1 + (1-a)v_2 - p^B) + U''(w_i)(av_1 + (1-a)v_2 - p^B)^2/2...$
Product 1 only	$U(w_i) + U'(w_i)(av_1 - ap_1^I) + U''(w_i)(av_1 - ap_1^I)^2/2 + ...$
Product 2 only	$U(w_i) + U'(w_i)((1-a)v_2 - (1-a)p_2^I) + U''(w_i)((1-a)v_2 - (1-a)p_2^I)^2/2 + ...$
No purchase	$U(w_i)$

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