

# SafeComp: Protocol For Certifying Cloud Computations Integrity

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## Abstract

We define a problem of certifying computation integrity performed by some remote party we do not necessarily trust. We present a multi-party interactive protocol called SafeComp that solves this problem under specified constraints. Comparing to the nearest related work, our protocol reduces a proof construction complexity from  $O(n \log n)$  to  $O(n)$ , turning a communication complexity to exactly one round using a certificate of a comparable length.

**Keywords:** computation integrity; interactive proofs; computation certificates

## 1 Introduction

Suppose, a user needs to compute some complex heavyweight function  $C(x)$ . It might be necessary to possess a non-trivial computational resource to do that, for example, a cluster of computational nodes. Chances are, the user does not have it. To compute this function, the user asks some cloud service - a provider - to compute it for him. The user supplies the provider with the function  $C(x)$  and some initial data  $d$ , asking to compute  $C(d)$ , and some time later the provider gives the result  $r$ . How can the user be sure that the provided result  $r$  is correct?

There are many cases when a computation result can be verified easily. Consider a sorting function that maps an arbitrary list of integers to a sorted list of the same integers. We can check that the output list is sorted using a simple one-way scanning function that ensures the sameness of input/ output list elements and their expected order.

On the contrary, there are cases when it is hard or maybe even impossible to verify a computation result without redoing the whole computation from scratch. Take a problem of checking that *there are no* Hamiltonian cycles within a given graph. <sup>1</sup> If

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<sup>1</sup> Hamiltonian cycle is a cycle in a graph that traverse each vertex exactly once.

the provider gives you a founded Hamiltonian cycle, it is easy to check its correctness. But what if the provider tells you that there are *no such cycles*: how would you ensure that it tells you the truth?

Consider yet another problem: a model-checking of computer programs - the well-known computationally hard problem. Suppose, the user supplies the provider with a program and a list of properties it must respect, and asks if this really holds. Some time later, the provider tells the user that a checking procedure did not find any deviations from the specified properties. How would you check that: 1) the result is correct 2) the result was not simply guessed (i.e. the computation was not halted until the result was obtained)?

In modern practice, a user has to trust in provider's integrity, rely on its market authority. A computation result is signed with a public key of the provider linking that trust to the obtained result. In this article, we present SafeComp: a multi-party interactive protocol that solves the described problem technically, removing the necessity to trust the authority of a cloud provider.

We still rely on a so-called *trusted weak computing device*, say, a smart-contract residing on a public blockchain. This device plays a role of autonomous transparent arbiter that is able to decide who is right and who is wrong in case of a dispute regarding published computation results, and do that performing only a tiny fraction of the whole computation.

Comparing to the nearest related work - TrueBit protocol - our protocol reduces proof construction complexity from  $O(n \log n)$  to  $O(n)$  and turning communication complexity from  $O(\log n)$  to exactly one round. This result is achieved by using a different proof construction and verification procedure, while the proof length stays effectively the same:  $O(n)$ .

The article is structured in the following way: in Section 2 we define the problem that our protocol is aiming to solve. In Section 3, we give a high-level overview of SafeComp protocol. Section 4 describes a way of presenting user computation function in a form that is appropriate to be used in the protocol. Section 5 contains a formal description of SafeComp protocol. Section 6 is devoted to a security analysis of some aspects of the protocol under clearly stated threat model. In Section 7, we discuss some of the known attack vectors and features of the protocol. Our first experimental results are discussed in Section 8. Section 9 contains the overview of related works in the field, and comparison of SafeComp with other known protocols.

## 2 Problem Statement

We are looking for an efficient verification procedure that allows us to check computation integrity giving that the computation was performed by some not necessarily fully trusted computing provider (or, simply, provider). Along with a computation result, the provider constructs a certificate whose size reasonably correlates with computation complexity. Using the certificate, we should be able to unambiguously check correctness of the performed computation, and do that much faster than a full recomputation.

**Definition 2.1.** (Computation Certification Problem)

For an arbitrary function  $C : \mathbb{N} \rightarrow Result$  computable at a point  $d \in \mathbb{N}$ , define computable functions:

$$proof(C, d) \rightarrow Result \times Proofs$$

$$verify(C, d, r, prf) \rightarrow \{True, False\}$$

where  $r \in Result$  is a result of computation under question, that is  $r \stackrel{?}{=} C(d)$ ,  $prf \in Proofs$  - a proof of computation integrity, also called *certificate*.

The following must hold:

1.  $verify(C, d, r, prf) = True \iff C(d) = r$
2. Computation complexity of  $proof(C, d)$  depends on complexity of  $C$  linearly.
3. Computation complexity of  $verify$  is essentially smaller than complexity of  $proof$  for any given function  $C$ , that is

$$\lim_{n \rightarrow \infty} \frac{v_C(n)}{e_C(n)} = 0$$

where  $v_C(n), e_C(n)$  - asymptotic estimates of computation complexity of functions  $verify$  and  $proof$  for the function  $C$  with input size  $n$ .

4. As complexity  $e_C(n)$  increases, the size of  $prf$  grows linearly, that is

$$\exists k \forall n. |prf(n)| \leq k \cdot e_C(n)$$

## Probabilistic Certificate Verification

In our case, the requirement 1 turns out to be overly rigorous. We will use a weaker requirement instead.

**Definition 2.2.** (Probabilistic Certificate Verification Criteria)  
Consider the following events:

$$E_0(\lambda) : proof_\lambda(C, d) = \langle r, prf_\lambda \rangle$$

$$E_1(\lambda) : verify_\lambda(C, d, r, prf_\lambda) = True$$

$$E_2 : C(d) = r$$

If functions  $proof_\lambda, verify_\lambda$  satisfy:

$$\lim_{\lambda \rightarrow \infty} \Pr[E_0(\lambda) \cdot E_1(\lambda) \cdot E_2] = 1$$

where  $\lambda$  is a parameter that can affect computational complexity of functions and the size of a certificate, then we say that algorithms described above meet probabilistic certificate verification criteria.

In other words, we are depicting not deterministic, but probabilistic method of solving the defined problem, where some special parameter regulates the probability of the desired outcome.

## Interactivity

Some approaches aiming to solve the defined problem require a so-called *interactivity*, that is, a verification procedure is performed in several rounds of message passing between a computing provider and a user. An upper bound on the number of message-passing rounds is called *communication complexity* of a protocol. An interactive approach for solving the depicted problem is also called *protocol for certifying outsourced computations*, or, cloud computations.

## Trusted Weak Computing Device

Some certification protocols rely on a presence of some trusted party which is able to perform simple computations in a verifiable way, but definitely not able to perform the whole computation  $C(d)$ .

For instance, such device may not be able to compute a factorial of some big number, but still it could perform an addition of any two natural numbers, providing some kind of proof of the result. If, say, we divide the whole computation of a factorial function into smaller operations of addition, we could carry out the entire computation in a verifiable manner.

**Definition 2.1.** (Trusted Weak Computing Device)

A device that is capable of certifying computations 2.1 for computable functions  $f$  such that computational complexity of  $f(n)$  grows linearly with the size of input, i.e.  $f(n) \in O(n)$ , is called *trusted weak computing device*.

Secure enclaves [12] and blockchain smart-contracts [5] are examples of such devices.

## Multi-party Protocol

A classical interpretation of the computation certification problem assumes that a computing provider proves correctness of an obtained result to a user in some way, and the user examines this proof. An interaction between two parties happens directly.

Our model of interaction is different: a user and the set of computing providers (or, simply, providers) interact with each other using a Trusted Weak Computing Device (TWCD) as a trusted verifiable intermediary. Several providers can run a computation task in parallel on their own computing resources. A computation result and a proof are then published within TWCD by some provider who has found a solution. Other providers are able to re-check the result. If a disagreement arises, other providers would be able to convince other parties that the result written in TWCD is faulty. Otherwise, after a period of time, the published result is considered to be correct and a user receives the solution to the published computation task. Such interaction model is called *multi-party protocol*.

## Problem Statement

In this article, we present a multi-party protocol for certifying cloud computations with a probabilistic certificate verification criteria and a reliance on a trusted weak computing device that is able to authenticate its users.

For ease of presentation, *we consider a public blockchain smart-contract to play a role of a trusted weak computing device*. But one can imagine other devices being used.

## 3 Protocol Overview

In this section, we give a high-level overview of the protocol. Here we presume that a user would like to compute the function  $C(x)$  in the point  $d$ , but do not have an ability to do the whole computation.

1. A user transforms his computable function  $C$  into another function  $f$  such that applying  $f$  to  $d$   $n$  times, the result converges to  $C(d)$ .

2. User publishes the function  $f$  and the point  $d$  in a smart contract. We assume that the smart contract is accessible for multiple computing providers.
3. Once a provider receives the computation task  $f$  and the point  $d$ , he starts performing the computation  $r = f(f(\dots f(d)))$  until the result converges to a fixpoint. At the end of each iteration  $f(x)$ , he also computes a hash  $c_i := H(x \circ c_{i-1})$ . These  $c_i$  values form a sequence  $cert := \langle c_1, \dots, c_n \rangle$  This sequence is called a *certificate sequence* (or, simply, a certificate).

The provider computes a certificate fingerprint  $hc := H(H(cert))$  and a *certificate projection*  $cp := \langle \pi(c_1), \pi(c_2), \dots, \pi(c_n) \rangle$ . The main idea behind introducing the fingerprint is to give other providers ability to check their own certificates against the computed one without publishing a certificate sequence in pure.

A certificate projection is needed to let other providers find the very first divergent step of a computation in case of a fingerprint disagreement, and do that without disclosing the certificate sequence in pure.

A certificate sequence plays a role of a secret value used in a later stage of the protocol to establish a list of all fair providers that performed the verification of a published result and a certificate, this is why we do not want to publish it in pure.

4. After finding a solution, one of the providers - we call him *the solver* - publishes the solution  $r$ , the certificate fingerprint  $hc$  and the projection  $cp$  in a smart-contract.
5. Other providers that performed the computation, but found the solution later than the solver, are able to check the published solution and the computation work integrity by comparing certificate fingerprints. This group of providers is called *auditors*.
6. In case of a disagreement, an auditor sends the number of a certificate part  $i$  from which the divergence starts, the partial value  $r_{i-1}$  and certificate parts  $c_{i-1}, c_i$ .
7. Using the provided values  $i, r_{i-1}, c_{i-1}, c_i$  the smart-contract arbiter ensures that the published result or computation certificate is indeed wrong by performing only a single computation step (details are in Section 5).
8. After some period of time, if no proofs of computation disintegrity has been provided, the published solution  $r$  is considered correct.
9. Each auditor sends the value  $H(H(cert) \circ id)$  into the smart-contract. Here  $id$  is a unique identifier of an auditor. This value plays a role of a proof that the computation work has been done.
10. Finally, the solver sends the value  $s$  such that  $hc = H(s)$  into the smart-contract. The list of fair auditors is then evaluated.

## 4 Iterative Computation

The protocol requires the computation to be prepared in an iterative form. An example is shown on Fig.1: factorial function is implemented in two different ways: the standard recursive form 1a and iterative form 1b.

If you consecutively compute the function  $factFP$ , starting at the point  $\{N, 1\}$  and passing the evaluated result as a next point iteratively, then, after  $N$  iterations, the function will converge to the value  $\{0, N!\}$ .

<pre>fact(0) -&gt;   1; fact(N) <b>when</b> N &gt; 0 -&gt;   N * fact(N-1).</pre>	<pre>factFP({0, Acc}) -&gt;   {0, Acc}; factFP({N, Acc}) <b>when</b> N &gt; 0 -&gt;   {N - 1, Acc * N}.</pre>
(a) Recursive implementation	(b) Iterative implementation

Fig. 1: Recursive vs. Iterative factorial implementation (Erlang)

The main difference between two implementations is that the first one (Fig.1a) computes the result at once, while the second (Fig.1b) computes only a part of the whole computation and returns that partial result. The computation can be continued by passing the partial result into the function again, and so on, until a fixpoint - the solution - is found.

One can start feeling doubt whether it is always possible to transform an arbitrary computable function  $C(x)$  into such form. The next theorem shows that not only it is always possible, but can be done in several ways.

**Theorem 1.** For all functions  $C : \mathbb{N} \rightarrow \mathbb{N}$  computable at some point  $d$  there exist computable functions

$$inj : \mathbb{N} \rightarrow T, proj : T \rightarrow \mathbb{N}, F : T \rightarrow T$$

such that

$$proj(\mathbf{fix} F inj(d)) = C(d)$$

where  $\mathbf{fix} : (T \rightarrow T) \times \mathbb{N} \rightarrow T$  - an operator that calculates fixed point of a function starting at a given point. That is,

$$\mathbf{fix} F p = F(\mathbf{fix} F p)$$

$T$  - a finite set.

*Proof.* The proof is moved into Appendix 10. ■

## 5 Protocol Specification

SafeComp protocols rely on availability of a trusted weak computing device. We take a smart-contract as a practically viable form of such device. Parties - a user and providers - communicate with each other by sending *transactions* into the smart-contract.

Lets us define a smart-contract state to be a set of its internal variables of some type. Any transaction into a smart-contract can potentially change its state. Within our specification effort, we use a concept of a current state and a next state of a variable denoting the variable value before and after transaction processing respectively.

### Notation

$\mathbb{N}_p = \{0 \dots 2^p - 1\}$  - a set of natural numbers with a given upper bound  
 $X, X'$  - a value of variable  $X$  before and after transaction processing

### Variables Denotation

$Id \in 2^{\mathbb{N}} \cup \{0\}$  - a set of user identifiers. We use 0 to denote *an undefined* identifier.

$r \in \mathbb{N}$  - a computation result

$s \in \mathbb{N}$  - a certificate fingerprint

$solver \in Id$  - an identifier of a provider who published a solution for the user's task

$V \in 2^{Id}$  - a set of provider identifiers that managed to prove their verification work.

$L \in 2^{Id}$  - a set of participant identifiers that tried to compromise the protocol in any way

$P \subseteq Id \times \mathbb{N}$  - A set of pairs - identifier  $\times$  proof - sent by parties pretending to be considered as verifiers.

**Input:** User's task  $f : \mathbb{N} \rightarrow \mathbb{N}$  and an input data  $d \in \mathbb{N}$ .

**Output:**  $\langle r, s, solver, V, L \rangle$

### Preliminary Setup

1. The User defines the function  $f$  and the initial point  $d$ . We discussed requirements for  $C$  and  $f$  and their relation earlier.
2. Parties agree on some cryptographically secure hash function  $H : \mathbb{N} \rightarrow \mathbb{N}_q$ , where parameter  $q$  is taken according to recommendations for the hash-function.
3. The User constructs a computable function

$$F(\langle x, c \rangle) := \mathbf{IF} \ x == f(x) \ \mathbf{THEN} \ \langle x, c \rangle \ \mathbf{ELSE} \ \langle f(x), H(x \circ c) \rangle$$

Here  $\circ : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$  - some non-constant binary operation. Computation must start at the point  $\langle d, H(d) \rangle$

4. Parties agree on a function family  $\pi_k : \mathbb{N}^k \rightarrow \mathbb{N}_p^k$ , for  $k \geq 1$ , such that

$$\pi_1 : \mathbb{N} \rightarrow \mathbb{N}_p - \text{some arbitrary hash-function (not required to be equal to } H(x))$$

$$\pi_k(\langle c_1, \dots, c_k \rangle) := \langle \pi_1(c_1), \dots, \pi_1(c_k) \rangle, \text{ for } k > 1$$

Henceforth, we omit  $k$  index because its value is clear from a context: it is a length of a tuple argument.

5. A smart-contract must be published on a blockchain. The smart-contract should give its users ability to:
  - Publish a computational task  $F$ , initial data point  $d$ ; the user request status is set to "published". For every new user request, the smart-contract allocates a new set of state variables:  $\langle r, s, solver, V, L, P \rangle$ , initially empty.
  - For any published user request, receive a user request status and a computational task  $\langle F, d \rangle$ . If a request status is set to "completed", then also receive a certificate projection  $cp$  and a solution  $r$ . If a request status is set to "verified", then also receive values  $\langle r, s, solver, V, L \rangle$
  - For requests with status "published", provide the solution  $r_n, c_n, cp, hc$ , such that

$$F(\langle r_n, c_n \rangle) = \langle r_n, c_n \rangle$$

$$cp := \pi(\langle c_1, \dots, c_n \rangle)$$

$$hc := H(H(\langle c_1, \dots, c_n \rangle))$$

Let  $u$  be an identifier of a provider that sent a solution - the solver. The smart-contract checks that the pair  $\langle r_n, c_n \rangle$  is indeed a fixpoint of  $F$ , in this case the user request status is changed to “completed” and

$$solver' = u \wedge r' = r_n$$

If the point  $\langle r_n, c_n \rangle$  is not a fixpoint of  $F$ , the solution is declined, the user request status does not change. In this case,

$$L' = L \cup \{u\}$$

- For user requests with status “completed”, refute the published solution by providing the refutation data  $i, c_{i-1}, c_i, r_{i-1}$ , such that

$$\pi(c_{i-1}) = cp[i-1] \wedge F(\langle r_{i-1}, c_{i-1} \rangle) = \langle r_i, c_i \rangle \wedge \pi(c_i) = cp[i] \quad (1)$$

$$c_{i+1} := H(r_i \circ c_i) \quad (2)$$

$$\pi(c_{i+1}) \neq cp[i+1] \quad (3)$$

Here  $cp[i]$  denotes an element of a tuple  $cp$ , residing on the position  $i$ .

Let  $u$  be an identifier of a provider that has sent a refutation. If the smart-contract establishes the fact of refutation correctness, the user request status is changed back to “published”. In this case,

$$L' = L \cup \{solver\} \wedge V' = V \cup \{u\} \wedge solver' = 0$$

- For user requests with status “completed”, provide a proof of computation  $prf$ , such that:

$$prf := H(H(\langle c_1, \dots, c_n \rangle) \circ id)$$

where  $id$  - a unique identifier of a provider. In this case,

$$P' = P \cup \{(id, prf)\}$$

- For user requests with status “completed”, after verification period  $T$  has elapsed, provide a secret  $s$  from a user with identifier  $solver$ , such that:

$$hc = H(s)$$

If the above does not hold, then the status is changed to “published” and

$$L' = L \cup \{solver\}$$

$$solver' = 0$$

Otherwise, the status is changed to “verified”. In this case,

$$V' = V \cup \{x : (x, p) \in P \wedge H(hc \circ x) = p\}$$

$$L' = L \cup \{x : (x, p) \in P \wedge H(hc \circ x) \neq p\}$$

## Protocol Steps

1. A user publishes a computation request, providing  $F, d$



2. Potential computing providers read computation requests with the status “published” from the smart-contract and possibly start to compute  $F$  from the point  $\langle d, H(d) \rangle$
3. A provider publishes a computed solution  $r_n, c_n, cp, hc$  in the smart-contract. We call this provider *the solver*.
4. Other providers that were also performing computation, verify the published result by comparing the value  $cp$  to their own.
  - If values are different, then send a refutation data into the smart-contract.
  - If values are the same, after verification period, send the proof of computation into the smart-contract.
5. Last of all, the solver sends  $s$ , the hash of a certificate, into the smart-contract, making it possible to find out all fair providers.
  - In case of successful secret validation, the smart-contract constructs an output:  $\langle r, s, solver, V, L \rangle$
  - Otherwise, the user request status is changed back to “published” and the protocol goes back to step 2.

## 6 Protocol Security Analysis

Any participant of the protocol trying to break its functional properties is called *intruder*. We measure protocol security degree as a probability value of any undesirable event - that is when functional properties of the protocol gets broken - within a given intruder model.

**Definition 6.1.** (Intruder model  $M_1$ )

- Finding a pre-image of a chosen hash-function  $H(x)$  is considered a computationally difficult task for an intruder.
- Intruder’s knowledge regarding user’s function  $f(x)$  implementation details is considered to be the same as of a user itself.

**Definition 6.2.** (Protocol Security Threats)

- Event  $E_1$  : Refutation of a correct solution.
- Event  $E_2$  : False proof of a solution verification.

**Theorem 2.** A refutation of a correct published solution (Event  $E_1$ ) is a computationally difficult task for an intruder from  $M_1$ .

*Proof.* To refute a published solution, the protocol requires providing such  $i, c_{i-1}, c_i, r_{i-1}$  that

$$\pi(c_{i-1}) = cp[i - 1] \wedge \pi(c_i) = cp[i] \wedge H(r_{i-1} \circ c_{i-1}) = c_i \quad (4)$$

$$F(\langle r_i, c_i \rangle) = (r_{i+1}, c_{i+1}) \wedge \pi(c_{i+1}) \neq cp[i + 1] \quad (5)$$

Suppose,  $F$  is a total function  $\mathbb{N} \times \mathbb{N}_q \rightarrow \mathbb{N}$  - in practice, this is almost always not the case, but let us take this assumption to simplify work for an intruder in our analysis.

In this case, it is easy to satisfy 5. The requirement could be fulfilled for almost any point  $\langle r, c \rangle$ . Therefore, the main burden falls on the predicate 4.

Suppose that hash-functions have the following signatures:  $H : \mathbb{N} \rightarrow \mathbb{N}_q$ ,  $\pi : \mathbb{N}_q \rightarrow \mathbb{N}_p$ . We also require  $H$  to be cryptographically secure. Then, a problem of finding values satisfying the predicate 4 is reduced to the known problem of finding a pre-image by a given image of a cryptographically secure hash-function, and is considered to be hard for an intruder in  $M_1$  with appropriately chosen  $p, q$  values. ■

**Theorem 3.** Submitting of false proof of verification for a published solution (event  $E_2$ ) is computationally difficult task for any intruder from  $M_1$ .

*Proof.* To prove a published solution verification work, the protocol requires a participant to provide the following value:

$$prf = H(s \circ id)$$

Here,  $id$  is a unique identifier of a participant. Every unique  $id$  is allowed to publish only a single  $prf$  as a proof. Additionally, we consider an intruder to know the following values:

$$hc = H(s) \tag{6}$$

$$prf_i = H(s \circ id_i), \text{ for } i = 1, 2, 3, \dots \tag{7}$$

Identifiers  $id_i$  are also known to the intruder. Here,  $s = H(\langle c_1, c_2, \dots, c_n \rangle)$  is a fingerprint of the certificate. A probability of finding the desired pre-image of  $s$  is:

$$P_s = \frac{1}{2^{q \cdot n}}, \text{ где } , q \geq 64, n \gg 10^3$$

Here, values for  $q$  - a hash-function image size, and  $n$  - a number of computation steps, are specified according to practical considerations. So, in this case, an event  $E_2$  can be considered to be highly improbable.

An intruder could make an attempt to find a collision in 6: find  $s'$  such that  $H(s') = hc$ . But, in this case, with carefully selected operation  $\circ$ , a probability of finding satisfying values for 7 with the founded collision  $s'$  is also insignificant. ■

## Economic Incentives

Earlier, we have discussed some aspects of the protocol security relying on an assumption that the protocol's cryptographic components is secure enough. It is possible to enhance reliability of the protocol even further by exploring the immanent feature of smart-contracts: *economic incentives* for its users - a powerful tool for stimulating rational parties strictly follow protocol rules.

In this paradigm, a stability of a protocol depends not only on security of cryptographic mechanisms, but also on an incentive scheme consisting of penalties and premiums for participants. We are not going to discuss this in deep, but give a high level overview of such scheme in form of an extension to protocol steps.

1. User publishes a computation request in a smart-contract, supplying the request with  $F, d$  and a cryptocurrency deposit  $D_r$ .

2. Providers read the request and start seeking a solution.
3. A provider who computed the solution first, publish his result in the smart-contract, supplying the transaction with a cryptocurrency deposit  $D_s$
4. Other providers verify the published result.
  - If a divergence is found, one publishes a refutation in the smart-contract, supplying the transaction with a cryptocurrency deposit  $D_p$
  - If no divergence is found, one sends a proof of computation in the smart-contract, supplying the transaction with a cryptocurrency deposit  $D_w$

At the end, when a solution is found and verified, a smart-contract has an amount of cryptocurrency on its account equal to  $s = D_r + D_s + m \cdot D_p + n \cdot D_w$  where  $n$  - number of participants successfully proved the computational work,  $m$  - number of refuted solutions. The protocol output is a tuple:  $\langle r, s, solver, V, L \rangle$ , so the smart-contract is able to redistribute funds  $s$  between a *solver* and fair auditors  $V$ , penalizing bad actors from  $L$ .

## 7 Protocol Features

Some aspects of SafeComp protocol may cause difficulties in a practical implementation, or even present attack vectors. Here, we discuss some of those issues and ways to overcome them.

**Transforming computation into iterative form.** In theorem 1, it was shown that a computation could be always presented in an iterative form and there are multiple ways to do the transformation.

For example, in the work [19] the following approach is used: a user's program is compiled from high-level language into a byte-code of some virtual machine. Then,  $n$  elementary steps of interpreter is taken as a single computation step. A state of the interpreter - registers and memory - is then taken as a computation state. The next step is executed starting from that interpreter state, and so on, until the solution is computed.

We use a different approach: the user's program is written in a so-called *Continuation Passing Style (CPS)* [14], but instead of a tail call, the argument together with a tail call function tag is returned as an intermediary result. Such representation achieves the same technical result while may sometimes lead to more compact state representations and does not rely on a special interpreter. A program's source code can be automatically transformed into CPS-style form [1], but we did not investigate this question in deep.

**Divergent function  $F$ .** Suppose someone places a divergent computation task in a smart-contract. A computational provider will spend its computing resource, but will never find a solution: an obvious loss for a provider.

There are some ways to overcome this difficulty:

- Implement a computational algorithm in a programming language that *guarantees* termination. For example, any *primitive recursive language* gives such guarantee by construction, and is expressive enough to develop nearly all practically interesting algorithms [17].

In this case, a user sends not a byte-code, but a program source code  $P$ , written in one of those languages. A provider then compiles  $P$  into  $f$ . If compilation is successful, the provider is guaranteed that the computation is finite and can be taken into work.

- Extend the protocol to support certification of partial computation results, i.e. results that have not converged to a fixpoint, but is correct from a certificate point of view. In this case, the longest result respecting the certificate integrity check is considered to be a correct solution.

**Bytecode size of  $F$ .** It might be the case that the size of a function  $F$  or a point  $d$  is greater than a maximum size of data unit of chosen blockchain platform.

Unfortunately, it is a fundamental limitation of the protocol. The following inequation must hold:  $|F| + |d| \leq T_{max}$ , where  $T_{max}$  - maximum size of data unit for a chosen TWCD. In case of smart-contracts, if this condition does not hold, such transaction will be declined.

**Size of refutation.** If a dispute against a published solution arises, a provider must send a refutation data  $c_{i-1}, c_i, r_{i-1}$  into the smart-contract.

The point  $r_{i-1}$  represents an intermediary computing state. It might be the case that the size of  $r_{i-1}$  is too big to be sent into the smart-contract. Unfortunately, it is a fundamental limitation of the protocol. The following must hold:  $\forall r, |f(r)| \leq T_{max}$ , where  $T_{max}$  - maximum size of data unit for a chosen TWCD.

**Certificate projection size.** It might be the case that a certificate projection is too big to be placed directly into a smart-contract or other chosen TWCD. At least in case of smart-contracts, it is possible to use an external immutable storage to store the projection there. For example, IPFS distributed file system may be used [3]. A smart-contract could use an oracle technology to access the corresponding data [10].

## 8 Experimental Evaluation

In order to evaluate SafeComp protocol, we have implemented its logic and one user computation task.

Due to time constraints, we have chosen not implement the protocol logic on a real blockchain. We implemented the protocol using Erlang programming language and its BEAM virtual machine instead. <sup>2</sup>

As for the user task, we took UNSAT problem: the problem of determining that there is no satisfying assignment for a Boolean formula. UNSAT problem is conjugated to a famous NP-complete problem SAT [6], but unlike SAT, its solution can not be checked in polynomial time in general case. Our choice is justified by the fact that an algorithm for solving UNSAT problem has a direct practical application - a symbolic model-checking of computer algorithms.

There are many algorithms for solving SAT/UNSAT problem, and it is known that no one of them guarantees a working time better, than  $O(2^n)$ , where  $n$  - the number of boolean variables in Boolean formula. Still, they are quite good in practice.

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<sup>2</sup> Our first attempts to implement the user task in Solidity (Ethereum blockchain) turned out to be highly cumbersome activity due to limited expressive power of the language and limitations of a virtual machine.

We have chosen to implement one of the simplest such algorithms - DPLL [7]. There are many DPLL implementations available, but in our case, we had to implement it in an iterative form discussed in Section 4.

As for the input, passed into DPLL algorithm, we took an program equivalence checking problem. Particulary, we check equivalence of programs where each program implement a FIFO queue, but in different way (the problem is taken from [4]).

The problem is encoded in a Conjunctive Normal Form (CNF) formula  $d$ , such that if  $DPLL(d) = Unsat$ , it means an equivalence of programs. Otherwise the algorithm outputs a counter-example.

In this experiment, we were interested in the following:

1. How much the computation time of the function  $f$  will change between the usual recursive form ( $T_1$ ) and its iterative form ( $T_2$ ).
2. How much a bytecode size of the function  $f$  will change between the usual recursive ( $S_1$ ) and iterative ( $S_2$ ) forms.
3. What will be a number of iterations needed to get from the initial point to a fixpoint of  $f$  (denoted as  $n$ ); what will be a certificate size ( $C_f$ )?
4. What will be the greatest size of an intermediary function result  $f^i(d)$  (denoted as  $d_{max}$ ), and the size of initial point  $d_0$ ?

Experiments were carried out on a computing node Intel Xeon Gold 6254 CPU 3.10GHz x 4 Cores, 16GB RAM; Execution environment: Erlang 20, ERTS 9.3.3.11, running under Linux Fedora 29 OS.

	$d_0$	$S_1$	$S_2$	$T_1$	$T_2$	$d_{max}$	$C_f$	$n$
<i>QueueInvar<sub>2</sub></i>	5703	2064	2272	0.1	0.4	42028	52992	1656
<i>QueueInvar<sub>4</sub></i>	13707	2064	2272	193	423	170758	13145664	410802

Here  $T_1, T_2$  is measured in seconds,  $d_0, d_{max}, S_1, S_2, C_f$  - in bytes,  $n$  - number of iterations.

Besides, we implemented the solution refutation step scenario consisting of the following steps:

1. The user task DPLL(x) presented in an iterative form is published together with the initial data  $d$ .  
The size of a user task  $S_2$  and its data  $d_0$  permits to send it into TWCD - the smart-contract.
2. Some participant sends intentionally incorrect solution, consisting of values  $r_n, cp, hc$ . It might be the case that the size of  $cp$  (measured as a fraction of  $C_f$ ) is larger than smart-contract is able to process. In this case,  $cp$  is published on an external immutable storage, for example IPFS. Instead of  $cp$ , it sends a link to the data object residing on IPFS. It is a responsibility of the solution provider to make this link alive and available.
3. Some other participant, after checking the published solution and detecting the error, constructs a refutation. The refutation  $i, c_{i-1}, c_i, r_{i-1}$  is sent into the smart-contract.

	$inter(n)$	$proof(n)$	$verify(n)$	$cert(n)$
Libra Non-ZKP <sup>3</sup>	$O(d(n) \cdot \log s(n))$	$O(s(n))$	$O(d(n) \cdot \log s(n))$	$O(d(n) \cdot \log s(n))$
TrueBit	$O(\log n)$	$O(n \log n)$	$O(1)$	$O(n)$
SafeComp	1	$O(n)$	$O(1)$	$O(n)$

Fig. 2: Asymptotic estimates for several certification algorithms. Denotation:  $proof(n)$  - complexity of proof construction,  $verify(n)$  - complexity of proof verification,  $inter(n)$  - number of interaction rounds,  $cert(n)$  - size of certificate,  $s(n)$  - size of functional element circuit as a function of task input size,  $d(n)$  - depth of a circuit, i.e. a maximum path length in the circuit.

- The smart-contract checks the provided refutation by applying rules 1, 2, 3 from Section 5. If the link to IPFS is published instead of pure  $cp$  value, then the smart-contract asks a trusted oracle to provide data locating at  $i - 1$  and  $i$  offset, receiving  $c_{i-1}$ ,  $c_i$ . If the IPFS link is unavailable, the solution is declined.

A Proof-of-Concept implementation of the protocol together with described scenario and the user task program is available at the repository [18].

All IPFS and Oracle-related functionality is modelled using a usual program code, without making any external service calls.

## 9 Related Works

Certification of computation integrity performed by some party we do not fully trust is considered a classic problem in modern cryptography. Several solutions to this problem have been proposed, each solution with its own set of compromises. One set of methods rely on a so-called *Probabilistically Checkable Proofs (PCP)* theorem for computations in class NP.

A reminder: NP class consists of decision problems such that there exists a verification procedure able to check its solutions in polynomial time.

PCP theorem states that any solution for an NP problem could be checked by a polynomial time algorithm using a fixed number of randomly chosen structural elements of a solution [2].

Methods belonging to this family could be characterized by the following pattern of interaction: 1) A user defines his computation task in a form of a boolean circuit (or functional elements circuit [11]); constructs a special polynomial - Algebraic Normal Form (ANF; amount of terms in ANF depends on the number of functional elements in the circuit [21]); sends the circuit and the input data to the provider 2) The provider derives corresponding boolean function; using this function, he gets the ANF and encodes the output of each of the circuit nodes in ANF; computes both the result of a computation and a certificate. The certificate can be stored on the provider's part, in the cloud or transmitted with the result to the user 3) To verify the certificate, the user utilizes one of approaches shown below. Generally, in order to verify a certificate, it is enough to randomly chose a limited number of circuit nodes for each approach, as PCP theorem implies.

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<sup>3</sup>Libra is a Zero-Knowledge (ZK) protocol: it hides the user function input from the prover, and this is

There are known approaches to certificate verification: 1) Interactive approach without commitments: Query a certificate (residing on a provider’s part or in a cloud) at a different nodes until the user is satisfied [8] [22] [20] 2) Interactive approach with commitments: form commitments with cryptographic protocols that verifies the integrity of a certificate. The key difference from the previous approach is a significant extension of the class of verification tasks [15] [16] [13]

PCP-inspired approaches impose considerable limitations on verifiable computations. For instance, accurate upper estimates for the number of loop iterations and the size of data structure ought to be known in advance.

Protocol Libra [22] belongs to those methods relying on PCP theorem, and is one of the latest methods known from the literature. It cannot be directly matched with our protocol because it has a number of limitations, for example, on the structure of user’s computations. Nevertheless, in some cases it can be applied to obtain the same technical result. Therefore, we also give bounds for it.

In the table 2, we compare the nearest known analogues in terms of their asymptotic behaviour.

## Comparison with TrueBit

With the advent of blockchains equipped with smart-contracts programming capability, another approach for solving the computation certification problem has emerged. In this setting, a smart-contract is used as a transparent autonomous arbiter able to resolve computation disputes among parties.

The first protocol to explore this idea was TrueBit [19]. We now give a brief comparison, highlighting the main differences between TrueBit and SafeComp.

**Solution refutation procedure.** If a dispute against a solution arises, TrueBit entails an interactive game between two computation providers with the aim of finding the very first incorrect computation step and convincing the smart-contract that this step is indeed wrong. The game is played in several rounds. The number of rounds is bounded by  $O(\log n)$ , where  $n$  is the number of computation steps. On each round, a solver and another provider compute several Merkle tree roots: the computation states are taken for the leafs. Those values are needed to find the source of disagreement. Complexity of computing one such root is bounded by  $O(n)$ . Total computation complexity for constructing a refutation, for all rounds, is bounded by  $O(n \log n)$ . The advantage of such approach is that it allows parties to optimize the data volume sent into a smart-contract, but with an extra cost of computing Merkle Tree roots, several times.

In SafeComp, we use another approach: a solver publishes both a solution to a task and a certificate projection - a partially disclosed certificate sequence (Section 3, p.3) - that allows other parties to find a divergent part of a certificate fast, in  $O(\log n)$  steps using binary search. Both the size and computation complexity for a certificate has a bound  $O(n)$ . If a certificate projection is large, we use an external immutable storage, IPFS for example, using an oracle technology to move data into a smart-contract in case it is needed. The solution refutation procedure in our case is, thereby, done in 1 round, lowering the complexity of the procedure from  $O(n \log n)$  to  $O(n + \log n) = O(n)$ , but at the expense of writing extra  $O(n)$  data, i.e. a certificate projection, into the network.

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not what we would like to compare with, because, both SafeComp and TrueBit do not hide user input. If we omit ZK property, Libra inherits communication complexity of GKP protocol [8] with some modifications.

**Form of Computational Task.** In case of TrueBit, a user task must be compiled into a bytecode of some virtual machine. One have to place an interpreter for the bytecode in the blockchain, so a smart-contract will be able to resolve disputes. In our case, one needs to present its computation in a special - iterative - form, meaning that it may entail program rewriting in a different style. This may cause inconvenience, but maybe softened by the fact that one does not need to place an interpreter on the blockchain. We are unsure if a suitable interpreter exists at the moment.

## 10 Conclusion and Future Work

In this article, we presented a protocol called SafeComp aiming to solve a cloud computation integrity certification problem in a specified context.

We have given a formal specification of the protocol, proved some of its security properties within a specified threat model.

Comparing to the nearest related work, SafeComp lowers the refutation procedure complexity considerably, and communication complexity becomes exactly one round.

We evaluated the protocol and made some measures that convinces us in viability of the protocol in practice. For the future work, we would like to do the following:

- 1) Propose a provably reliable economic incentives model for participants. As for now, we almost completely omitted the subject.
- 2) Implement SafeComp protocol within some public blockchain, and perform a computation certification for some practically interesting computations.
- 3) Investigate a relation between an iterative representation of a user program and its intermediary state size, comparing to the virtual machine byte-code representation.

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## Appendix 1

**Theorem.** For all functions  $C : \mathbb{N} \rightarrow \mathbb{N}$  computable at some point  $d$  there exist computable functions

$$inj : \mathbb{N} \rightarrow T, proj : T \rightarrow \mathbb{N}, F : T \rightarrow T$$

such that

$$proj(\mathbf{fix} F inj(d)) = C(d)$$

where  $\mathbf{fix} : (T \rightarrow T) \times \mathbb{N} \rightarrow T$  - an operator that calculates fixed point of a function starting from a given point. That is,

$$\mathbf{fix} F p = F(\mathbf{fix} F p)$$

$T$  - a finite set.

*Proof.* Let

$$\begin{aligned} inj(d) &= \langle d, 0 \rangle & proj(\langle x, y \rangle) &= x \\ F(\langle x, 0 \rangle) &= \langle C(x), 1 \rangle & F(\langle x, 1 \rangle) &= \langle x, 1 \rangle \end{aligned}$$

Selected functions satisfy conditions of the theorem. Such representation of  $F$  could be called *trivial*: it does not help to divide function  $C(x)$  into composite elementary units. But is it possible to represent  $F$  in any non-trivial way? Yes, we can. The following argument proofs this fact in a constructive way.

According to Turing thesis, every function  $C(x)$ , computable at a point, can be represented as a Turing machine. Let  $M$  be a machine corresponding to the function  $C(x)$ , that is

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, Q_f)$$

$Q$  – non-empty set of states,  $\Gamma$  – non-empty set of tape alphabet symbols

$\Sigma \subseteq \Gamma$  – set of input symbols,  $Q_f \subseteq Q$  – set of possible states

$q_0 \in Q$  – initial state

$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$  – transition function

Lets define the function  $F$  as:

$$F(\langle M, I, q, p \rangle) = \begin{cases} \langle M, I', q', p' \rangle, & \text{if } q \notin Q_f \\ \langle M, I, q, p \rangle, & \text{otherwise} \end{cases}$$

where

$$(q', X, p') = \delta(q, I_p), \text{ where } I_p \text{ is a content of the tape's cell at the position } p$$

$$I' = I[X/p]$$

Starting from the position  $p$  and the state  $q$ , the machine  $M$  changes state of the tape from  $I$  to  $I'$  (replacing content of  $p$  with  $X$ ) and the position of its head  $p'$ . If it reaches an accepting state (i.e. a member of  $Q_f$ ) it wouldn't do any steps further, so a fixpoint of  $F$  is obtained.

Theorem statement suggests that a function  $C$  is computable at a point  $d$ , so, after some number of iterations, the machine will necessarily reach an accepting state.

Notice that elements  $\langle M, I, q, p \rangle$  can be encoded, for instance, by a finite number of natural numbers. An example of such an encoding can be found in [9].

The function  $F$  is computable because it relies on a few computable functions: a transition function  $\delta(q, X)$ , a function of checking whether an element belongs to a set, i.e  $q \in Q_f$ , and a choice function *if – then – else*. All these functions are computable if sets  $Q$  and  $\Gamma$  are finite.

Let  $inj(d) := \langle M, I_0, q_0, 1 \rangle$ , where  $I_0$  is an initial state of the tape that contains  $d$  among other things. Let  $proj(\langle M, I, q, p \rangle)$  be a function that extracts the value  $C(d)$  from the tape  $I$ . Such functions could be constructed because it is always possible to allocate space for an initial value and for an answer. The answer  $C(d)$  is presented as a finite number of cells on the tape since computability of the function at the point guarantees a finite number of steps of the machine.

If a size of an answer can not be known in advance, we could always fix the initial cell and introduce a unique mark denoting an end of area reserved for an answer.

The described representation of functions  $F, inj, proj$  satisfies requirements of the theorem and is not trivial. ■