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TOPICAL ISSUE

Stabilization of Differential Repetitive Processes

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Abstract—Differential repetitive processes are a subclass of 2D systems that arise in modeling physical processes with identical repetitions of the same task and in the analysis of other control problems such as the design of iterative learning control laws. These models have proved to be efficient within the framework of linear dynamics, where control laws designed in this setting have been verified experimentally, but there are few results for nonlinear dynamics. This paper develops new results on the stability, stabilization and disturbance attenuation, using an H_∞ norm measure, for nonlinear differential repetitive processes. These results are then applied to design iterative learning control algorithms under model uncertainty and sensor failures described by a homogeneous Markov chain with a finite set of states. The resulting design algorithms can be computed using linear matrix inequalities.

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1. INTRODUCTION

Many industrial processes make a series of sweeps, termed passes, through a set of dynamics defined over a fixed finite duration known as the pass length [1]. Once each pass is complete, the process resets to the starting location ready for the start of the next pass. The output on each pass is termed the pass profile, which acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile. An industrial example described in [1] with references to the original modeling work, is long-wall coal cutting, where the pass profile represents the height of the stone/coal interface above some datum line and the objective lies in extracting the maximum amount of coal without penetrating the stone/coal boundary. The cutting machine rests on this profile during the production of the next pass and therefore the previous pass profile explicitly contributes to the dynamics of the next pass profile, with the result that oscillations that increase in amplitude from pass-to-pass can occur.

If these oscillations occur in a particular mining operation, then productive work must halt in order to enable their manual removal. The alternative is to use control action to prevent their appearance; however, the stabilization problem for these processes cannot be solved via standard, or 1D, systems theory/algorithms, since this ignores their inherent 2D systems structure. In particular, information propagation takes place in two directions: from pass-to-pass and along a given pass. Stability analysis of these processes in the case of linear models proceeds from the rigorous theory [1] based on exploring the properties of a certain linear operator in an appropriate Banach space.

Repetitive processes belong to the class of 2D systems, which date back to the 1970s in control theory and circuit design. The most widespread models include the Roesser model [2], the

Fornasini–Marchesini model [3] and the repetitive process model [1]. The Roesser model originates from image processing problems, where the state vector is partitioned into two sub-vectors, termed horizontal and vertical, respectively. In the Fornasini–Marchesini model (a doubly indexed dynamical system in the initial terminology of [3]) deals with a single state vector. A repetitive process differs from the Roesser model in the finite duration of one of the independent variables.

A new application area for 2D systems research and, in particular, repetitive processes can be traced in [4], where iterative learning control (ILC) was introduced. According to the survey papers [5, 6], iterative learning control (ILC) has many applications and research in this general area continues to grow, both in terms of theory and applications. The generic application area for ILC is systems consisting of multiple repetition of homogeneous operations, e.g., a portal robot moving goods from one location point to another along a given path. The novelty of such control lies in using information from the previous execution, or pass, to design the control signal for the next one. Consequently, there is information propagation in two independent variables, from pass to pass and also over a finite time interval, i.e., the duration of the pass, termed the pass length. Hence, 2D systems and, in particular, repetitive process theory is applicable.

In [7] an ILC law designed using linear repetitive process theory was verified experimentally. Another interesting example is an autonomous surveillance system [8] composed of an unmanned aerial vehicle (UAV) and autonomous ground sensors. This system detects infiltrators, captures a required target and transmits information on its location to an operator. Here the pass profile is the closed surveillance path. During a surveillance pass, a UAV flies over each autonomous ground sensor and a major control objective is to reduce possible deviations from a given path, which may increase from pass-to-pass. In the presence of such deviations, it is necessary to correct the path in order to maintain the surveillance. In [8] this problem was considered in an ILC setting, where control at a current flight pass is corrected on the basis of information acquired on the previous pass.

A very large volume of 2D control systems research (including the case of uncertain parameter systems) deals with linear stationary dynamics. Although the linear theory gives the necessary and sufficient stability conditions, their reduction to computable expressions can be problematic. One approach is to construct a polynomial positive definite matrix satisfying a Lyapunov-like inequality in complex variables. The recent work [9] developed a solution of this problem via linear matrix inequalities; however, the authors noted that the resulting computational complexity is high and even increases in the case of stabilization based on the necessary and sufficient conditions. An alternative proceeds from sufficient conditions in the Lyapunov equation setting with constraints imposed on the solution structure (the so-called 2D Lyapunov equation [1]).

Recent years have seen the appearance of research focused on 2D nonlinear systems. For instance, the stability of nonlinear Fornasini–Marchesini systems was analyzed in [10] and the publications [11, 12] considered different types of stability in nonlinear discrete-time Roesser systems. In [13, 14], the stability of discrete and differential nonlinear repetitive processes was considered and there is a need to extend this work to allow control law design. Among recent possible applications for repetitive process control theory in the nonlinear model setting are metal deposition processes [15] and wind turbine control [16].

This paper starts from the results in [17] and establishes new results on the stabilization of nonlinear differential repetitive processes by a nonstandard application of vector Lyapunov functions. The analysis is then extended to stabilization and disturbance attenuation as measured by an H_∞ norm. The case with possible failures in operation is also considered, where the failures are modeled as random switching, i.e., by a state-space model with jumps in the parameter values and/or structure governed by a Markov chain with a finite set of states. Such models are termed Markovian jump systems or systems with random structure [18, 19]. Note that the problem was solved in [20] for discrete repetitive processes. Finally, the new theory is applied to the ILC problem for a linear system with model uncertainty and sensor failures.

2. SYSTEM DESCRIPTION AND PROBLEM STATEMENT

Consider a linear repetitive process with a pass length T described over $0 \leqslant t \leqslant T$ by the state-space model

$$\begin{aligned} \dot{x}_{k+1}(t) &= f_1(x_{k+1}(t), y_k(t), u_{k+1}(t), w_k(t)), \\ y_{k+1}(t) &= f_2(x_{k+1}(t), y_k(t), u_{k+1}(t), w_k(t)), \end{aligned} \tag{2.1}$$

where on pass k $x_k(t) \in \mathbb{R}^{n_x}$ is the state vector, $y_k(t) \in \mathbb{R}^{n_y}$ denotes the pass profile vector, $u_k(t) \in \mathbb{R}^{n_u}$ stands for the input vector, $w_k(t) \in \mathbb{R}^{n_w}$ means a disturbance vector; f_1 and f_2 are nonlinear functions such that $f_1(0,0,0,0) = 0$ and $f_2(0,0,0,0) = 0$. By assumption, the boundary conditions, i.e., the pass state initial vector sequence and the initial pass profile, are of the form

$$\begin{aligned} x_{k+1}(0) &= d_{k+1}, \quad k \geqslant 0, \\ y_0(t) &= f(t), \quad 0 \leqslant t \leqslant T, \end{aligned} \tag{2.2}$$

where the vector $d_{k+1} \in \mathbb{R}^{n_x}$ has known constant entries for each k, $f(t) \in \mathbb{R}^{n_y}$ is a vector whose entries represent known functions of t, $0 \leqslant t \leqslant T$. Moreover, it is assumed that $f(t)$ and d_{k+1} satisfy the inequalities

$$\begin{aligned} |f(t)|^2 &\leqslant M_f, \\ |d_{k+1}|^2 &\leqslant \kappa_d z_d^k, \quad k = 0, 1, \ldots, \end{aligned} \tag{2.3}$$

where M_f and κ_d are positive real scalars and $0 < z_d < 1$ determines the rate of convergence of the pass state initial vector sequence. Throughout the paper, the boundary conditions are supposed to satisfy (2.3).

In the systems theory developed for linear repetitive processes, the stability along the pass is the basic property in control law design and experimental verification [1, 7]. This property proceeds from linear operator theory in a Banach space setting. Hence, it cannot be directly transferred to the nonlinear case. The definitions introduced below form the basis of a stability theory for nonlinear repetitive processes.

Definition 1. A nonlinear differential repetitive process described by (2.1) with the boundary conditions (2.2) is said to be exponentially stable if, with $w_k(t) = 0$,

$$|x_k(t)|^2 + |y_k(t)|^2 \leqslant \kappa \exp(-\lambda t)\zeta^k, \quad \lambda > 0, \quad 0 < \zeta < 1, \tag{2.4}$$

where ζ and λ do not depend on T.

Assume that $w_k(t) \in L_2([0,\infty),[0,\infty))$, and consider

$$||w||_2 = \sqrt{\sum_{k=0}^{\infty} \int_0^{\infty} |w_k(t)|^2 dt} < \infty.$$

Definition 2. A nonlinear differential repetitive process described by (2.1) with the boundary conditions (2.2) is said to be exponentially stable with a prescribed H_∞ disturbance attenuation level γ if it is exponentially stable and for $f(t) \equiv 0$ and $d_k \equiv 0$:

$$||y||_2 < \gamma ||w||_2. \tag{2.5}$$

Let $\bar{x}_{k+1}(t) = [x_{k+1}(t)^{\mathrm{T}}\ y_k(t)^{\mathrm{T}}]^{\mathrm{T}}$ and write $u \in \Phi$ if $u = \varphi(\bar{x})$, where φ is a nonlinear function such that $\varphi(0) = 0$. The stabilization problem lies in constructing a control law $u \in \Phi$ so that the process (2.1) is exponentially stable. Similarly, the H_∞-stabilization problem is to design a control law $u \in \Phi$ such that the process (2.1) is exponentially stable with a prescribed H_∞ disturbance attenuation level γ.

3. STABILIZATION AND H_∞-STABILIZATION

3.1. General Stabilization Conditions

To obtain stabilization conditions for a process (2.1) with the boundary conditions (2.2), we employ the divergence approach developed in the papers [11, 13, 14, 20] and consider the candidate vector Lyapunov function

$$V(x, y) = \begin{bmatrix} V_1(x_{k+1}(t)) \\ V_2(y_k(t)) \end{bmatrix}, \tag{3.1}$$

$V_1(x) > 0$, $x \neq 0$, $V_2(y) > 0$, $y \neq 0$, $V_1(0) = 0$, $V_2(0) = 0$. The divergence operator of this function along the paths of the process (2.1) is defined as

$$\mathrm{div}V(x_{u,k+1}(t), y_{u,k}(t)) = \frac{dV_1(x_{u,k+1}(t))}{dt} + \Delta_k V_2(y_{u,k}(t)), \tag{3.2}$$

where $\Delta_k V_2(y_{u,k}(t)) = V_2(y_{u,k+1}(t)) - V_2(y_{u,k}(t))$ and subscript u indicates that the repetitive process (2.1) and (2.2) is considered for a given control $u_{k+1}(t)$. For brevity, this subscript will be omitted whenever no confusion occurs. Let $L(x, u)$ be a nonlinear function such that

$$L(\bar{x}, u) \geqslant c(|\bar{x}|^2 + |u|^2) \tag{3.3}$$

for some $c > 0$.

Theorem 1. *Assume that for some $u = \varphi(\bar{x}) \in \Phi$ the inequality*

$$\mathrm{div}V(x_{\varphi,k+1}(t), y_{\varphi,k}(t)) + L(\bar{x}_{\varphi,k+1}, \varphi(\bar{x}_{\varphi,k+1})) \leqslant 0 \tag{3.4}$$

has a solution $V(x, y) = [V_1(x)\ V_2(y)]^{\mathrm{T}}$ satisfying the conditions

$$c_1|x|^2 \leqslant V_1(x) \leqslant c_2|x|^2, \tag{3.5}$$

$$c_1|y|^2 \leqslant V_2(y) \leqslant c_2|y|^2, \tag{3.6}$$

where $c_1 > 0$, $c_2 > 0$. Then the controlled nonlinear differential repetitive process obtained by applying $u = \varphi(\bar{x})$ to (2.1) and (2.2) is exponentially stable.

The proof of this result is given in the Appendix.

3.2. Linear Process Stabilization

In this subsection, we demonstrate possible application of Theorem 1 to a special case of a process (2.1) described by the state-space equations

$$\begin{aligned} \dot{x}_{k+1}(t) &= A_{11}x_{k+1}(t) + A_{12}y_k(t) + B_1u_{k+1}(t), \\ y_{k+1}(t) &= A_{21}x_{k+1}(t) + A_{22}y_k(t) + B_2u_{k+1}(t), \end{aligned} \tag{3.7}$$

where A_{ij} $(i,j=1,2)$ and B_i $(i=1,2)$ are constant matrices of appropriate dimensions, the rest notation is that for (2.1) and the boundary conditions are again of the form (2.2). Choose the function L and the components of the vector Lyapunov function V as the quadratic forms

$$\begin{aligned} L(\bar{x}_{k+1}(t), u_{k+1}(t)) &= \bar{x}_{k+1}(t)^{\mathrm{T}} Q \bar{x}_{k+1}(t) + u_{k+1}(t)^{\mathrm{T}} R u_{k+1}(t), \\ V_1(x_{k+1}(t)) &= x_{k+1}(t)^{\mathrm{T}} P_1 x_{k+1}(t), \\ V_2(y_k(t)) &= y_k(t)^{\mathrm{T}} P_1 y_k(t), \end{aligned} \tag{3.8}$$

where $Q=Q^{\mathrm{T}}$ is a nonnegative definite matrix, $P_1=P_1^{\mathrm{T}},\ P_2=P_2^{\mathrm{T}},\ R=R^{\mathrm{T}}$ are positive definite matrices, all with compatible dimensions. Set

$$A=\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},\quad B=\begin{bmatrix} B_1 \\ B_2 \end{bmatrix},\quad P=\begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix},\quad I^{1,0}=\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix},\quad I^{0,1}=\begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}$$

and consider the problem of finding the control law $u=\varphi(\bar{x})$ from the condition

$$\min_{u\in\mathbb{R}^{n_u}} \{\mathrm{div} V(\bar{x}) + L(\bar{x}, u)\} < 0. \tag{3.9}$$

According to (3.9), this control satisfies (3.4) and, by application of Theorem 1, stabilizes the linear process. Moreover, minimization of the left-hand side of (3.9) gives

$$u_{k+1}(t) = -K\bar{x}_{k+1}(t),$$

where the gain matrix K is defined by the formula

$$K = [R + B^{\mathrm{T}} I^{0,1} P B]^{-1} B^{\mathrm{T}} [I^{1,0} P + I^{0,1} P A],$$

and the matrix P satisfies the inequality

$$\begin{aligned} A^{\mathrm{T}} I^{1,0} P + P I^{1,0} A - [I^{1,0} P B + A^{\mathrm{T}} I^{0,1} P B][R + B^{\mathrm{T}} I^{0,1} P B]^{-1} [B^{\mathrm{T}} I^{1,0} P + B^{\mathrm{T}} I^{0,1} P A] \\ + A^{\mathrm{T}} I^{0,1} P A - I^{0,1} P + Q < 0. \end{aligned} \tag{3.10}$$

The nonstandard square matrix inequality (3.10) can be solved by the progressive approximation method, but the construction of more efficient solution methods is an open question for future research together with the feasibility conditions of this inequality.

3.3. H_∞-stabilization

Consider the nonlinear repetitive process (2.1) with the boundary conditions (2.2) and an arbitrary function $w_k(t) \in L_2([0,\infty),[0,\infty))$. Then the following result can be established.

Theorem 2. *Assume that for some $u=\varphi(\bar{x}) \in \Phi$ the inequality*

$$\mathrm{div} V(x_{\varphi,k+1}(t), y_{\varphi,k}(t)) + \varepsilon(|x_{\varphi,k+1}(t)|^2 + |y_{\varphi,k}(t)|^2 - \gamma^2 |w_k(t)|^2) \leqslant 0, \tag{3.11}$$

where ε is a positive scalar, has a solution $V(x,y)$ satisfying (3.5) and (3.6). Then the nonlinear differential repetitive process obtained by applying $u=\varphi(\bar{x})$ to (2.1) and (2.2) is exponentially stable with the prescribed H_∞ disturbance attenuation level γ.

Proof. Let the pair $(V(x,y), \varphi(\bar{x}))$ be a solution of inequality (3.11). If $w \equiv 0$, it follows from (3.11) that

$$\mathrm{div} V(x_{\varphi,k+1}(t), y_{\varphi,k}(t)) \leqslant -\varepsilon(|x_{\varphi,k+1}(t)|^2 + |y_{\varphi,k}(t)|^2) \tag{3.12}$$

and the process described by (2.1) and (2.2) is exponentially stable by Theorem 1. Consider $w_k(t) \in L_2([0,\infty),[0,\infty))$ with $f(t) \equiv 0$ and $d_k \equiv 0$. In this case, inequality (3.12) can be rewritten as

$$\begin{aligned}\frac{dV_1(x_{\varphi,k+1}(t))}{dt} &+ V_2(y_{\varphi,k+1}(t)) - V_2(y_{\varphi,k}(t)) \\ &\leqslant -\varepsilon(|x_{\varphi,k+1}(t)|^2 + |y_{\varphi,k}(t)|^2 - \gamma^2|w_k(t)|^2) \\ &\leqslant -\varepsilon(|y_{\varphi,k}(t)|^2 - \gamma^2|w_k(t)|^2). \quad (3.13)\end{aligned}$$

Integrating and summing both sides of (3.13) and rearranging the summands (taking into account the zero boundary conditions) yields

$$\begin{aligned}\varepsilon\sum_{k=0}^{n}\int_0^t |y_{\varphi,k}(s)|^2 ds &\leqslant \varepsilon\sum_{k=0}^{n}\int_0^t \gamma^2|w_k(s)|^2 ds \\ &- \sum_{k=0}^{n} V_1(x_{\varphi,k+1}(t)) - \int_0^t V_2(y_{\varphi,k}(s))ds \leqslant \varepsilon\sum_{k=0}^{n}\int_0^t \gamma^2|w_k(s)|^2 ds. \quad (3.14)\end{aligned}$$

Finally, as $n \to \infty$ and $t \to \infty$ in (3.14), the inequality (2.5) is obtained. This concludes the proof.

4. NONLINEAR DIFFERENTIAL REPETITIVE PROCESSES WITH FAILURES

This section extends the results of the previous section to nonlinear differential repetitive processes in the presence of failures. The failures are modeled by a state-space model with jumps in the parameter values and/or structure governed by a Markov chain with a finite set of states, often termed Markovian jump systems or systems with random structure [18, 19].

The differential nonlinear repetitive processes under consideration are described by the state-space model

$$\begin{aligned}\dot{x}_{k+1}(t) &= g_1(x_{k+1}(t), y_k(t), u_{k+1}(t), w_k(t), r(t)), \\ y_{k+1}(t) &= g_2(x_{k+1}(t), y_k(t), u_{k+1}(t), w_k(t), r(t)),\end{aligned} \quad (4.1)$$

where $r(t)$ $(t \geqslant 0)$ denotes a Markov chain with a finite set of states $\mathbb{N} = \{1,\ldots,\nu\}$ and transition probabilities given by

$$\mathrm{P}(r(t+\tau) = j \mid r(t) = i) = \begin{cases} \pi_{ij}\tau + o(\tau), & \text{if } j \neq i \\ 1 + \pi_{ii}\tau + o(\tau), & \text{if } j = i, \end{cases} \quad (4.2)$$

$i,j = 1,\ldots,\nu$, $\pi_{ij} > 0$, $\pi_{ii} = -\sum_{i\neq j}^{\nu}\pi_{ij}$; g_1 and g_2 represent nonlinear functions such that for all $r \in \mathbb{N}$: $g_1(0,0,0,0,r) = 0$, $g_2(0,0,0,0,r) = 0$. The rest of the notation is the same as in (2.1) and the boundary conditions again have the form (2.2).

The following are definitions of exponential stability and H_∞ disturbance attenuation for the repetitive processes considered in this section.

Definition 3. A nonlinear differential repetitive process described by (4.1), (2.2) with $w_k(t) = 0$ is said to be exponentially stable in the mean square if

$$\mathrm{E}[|x_k(t)|^2 + |y_k(t)|^2] \leqslant \kappa\exp(-\lambda t)\zeta^k, \quad \lambda > 0, \quad 0 < \zeta < 1, \quad (4.3)$$

where E denotes the expectation operator and the constants ζ, λ do not depend on T.

Assume that $w_k(t) \in L_2([0,\infty),[0,\infty))$ and define

$$||w||_{\mathrm{E}} = \sqrt{\mathrm{E}\left[\sum_{k=0}^{\infty}\int\limits_0^{\infty}|w_k(t)|^2dt\right]} < \infty.$$

Definition 4. A differential nonlinear repetitive process described by (4.1) and (2.2) is said to be exponentially stable in the mean square with a prescribed H_∞ disturbance attenuation level γ if it is exponentially stable and for all $w_k(t) \in L_2([0,\infty),[0,\infty)) \neq 0$ with $f(t) \equiv 0$ and $d_k \equiv 0$:

$$||y||_{\mathrm{E}} < \gamma||w||_{\mathrm{E}}. \tag{4.4}$$

Suppose that $u = \varphi(\bar{x}) \in \Phi$. To derive the conditions for exponential stability in the mean square for a process (4.1) and (2.2), consider the candidate vector Lyapunov function

$$V(x_{k+1}(t), y_k(t), r(t)) = \begin{bmatrix} V_1(x_{k+1}(t), r(t)) \\ V_2(y_k(t), r(t)) \end{bmatrix}, \tag{4.5}$$

where $V_1(x,r) > 0,\ x \neq 0,\ V_2(y,r) > 0,\ y \neq 0,\ V_1(0,r) = 0,\ V_2(0,r) = 0.$

Also introduce the operators $\mathcal{D}_1$ and $\mathcal{D}_2$ defined along the paths of (4.1):

$$\mathcal{D}_1V(\xi,\eta,i) = \lim_{\Delta t\to 0}\frac{1}{\Delta t}\mathrm{E}[V_1(x_{k+1}(t+\Delta t), r(t+\Delta t) - V_1(x_{k+1}(t), r(t)) \mid x_{k+1}(t) = \xi, y_k(t) = \eta, r(t) = i],$$

$$\mathcal{D}_2V(\xi,\eta,i) = \mathrm{E}[V_2(y_{k+1}(t), r(t)) - V_2(y_k(t), r(t)) \mid x_{k+1}(t) = \xi, y_k(t) = \eta, r(t) = i].$$

Let $V_1(\xi, i)$ be differentiable in ξ for each $i \in \mathbb{N}$. Hence, using (4.1) and (4.2), it follows immediately that

$$\mathcal{D}_1V(\xi,\eta,i) = g_1^{\mathrm{T}}(\xi,\eta,\varphi(\bar{\xi}),w,i)\frac{\partial V_1(\xi,i)}{\partial\xi} + \sum_{j=1}^{\nu}\pi_{i,j}V_1(\xi,j), \tag{4.6}$$

where $\bar{\xi} = [\xi^{\mathrm{T}}\ \eta^{\mathrm{T}}]^{\mathrm{T}}$. Introduce the operator $\mathcal{D}$ as the stochastic counterpart of the divergence operator from the previous section:

$$\mathcal{D}V(\xi,\eta,i) = \mathcal{D}_1V(\xi,\eta,i) + \mathcal{D}_2V(\xi,\eta,i). \tag{4.7}$$

Theorem 3. *Consider a nonlinear differential repetitive process described by* (4.1) *and* (2.2) *with* $u = \varphi(\bar{x}) \in \Phi$. *Assume that the inequality*

$$\mathcal{D}V(\xi,\eta,i) + L(\bar{\xi},\varphi(\bar{\xi})) \leqslant 0, \quad i \in \mathbb{N} \tag{4.8}$$

has a solution $V(\xi,\eta,i) = [V_1(\xi,i)\ V_2(\eta,i)]^{\mathrm{T}}$ *satisfying*

$$c_1|\xi|^2 \leqslant V_1(\xi,i) \leqslant c_2|\xi|^2, \tag{4.9}$$

$$c_1|\eta|^2 \leqslant V_2(\eta,i) \leqslant c_2|\eta|^2, \tag{4.10}$$

$c_1 > 0,\ c_2 > 0.$ *Then this process is exponentially stable in the mean square.*

The proof of this result is given in the Appendix.

A nonlinear differential repetitive process described by (4.1) and (2.2) with $w_k(t) \in L_2([0,\infty),[0,\infty))$ possesses the following property.

Theorem 4. *Assume that for some $u = \varphi(\bar{x}) \in \Phi$ the inequality*

$$\mathcal{D}V(\xi,\eta,i) + \varepsilon(|\xi|^2 + |\eta|^2 - \gamma^2|w|^2) \leqslant 0, \tag{4.11}$$

where ε is a positive scalar, has a solution $V(\xi,\eta,r)$ meeting the conditions (4.9) *and* (4.10). *Then the nonlinear differential repetitive process obtained by applying $u = \varphi(\bar{x})$ to* (4.1) *and* (2.2) *is exponentially stable in the mean square with the prescribed H_∞ disturbance attenuation level γ.*

Proof. Let the pair $(V(\xi,\eta,r),\varphi(\bar{x}))$ be a solution of (4.11). If $w \equiv 0$, inequality (4.11) gives that

$$\mathcal{D}V(\xi,\eta,i) \leqslant -\varepsilon(|\xi|^2 + |\eta|^2) \tag{4.12}$$

and by Theorem 3 a repetitive process described by (4.1) and (2.2) is exponentially stable in the mean square. Suppose that $w_k(t) \in L_2([0,\infty),[0,\infty))$, $f(t) \equiv 0$ and $d_k \equiv 0$. Then (4.12) leads to

$$\begin{aligned}&\mathrm{E}[\mathcal{D}_1 V(x_{k+1}(t), r(t))] + \mathrm{E}[V_2(y_{k+1}(t), r(t)) - V_2(y_k(t), r(t))] \\ &\quad\leqslant -\varepsilon\mathrm{E}[(|x_{k+1}(t)|^2 + |y_k(t)|^2] - \gamma^2\mathrm{E}[|w_k(t)|^2)] \leqslant -\varepsilon\mathrm{E}[|y_{\varphi,k}(t)|^2 - \gamma^2|w_k(t)|^2].\end{aligned} \tag{4.13}$$

Integrating and summing both sides of (4.13) and rearranging the summands gives

$$\begin{aligned}&\varepsilon\mathrm{E}\left[\sum_{k=0}^{n}\int_0^t |y_{\varphi,k}(s)|^2 ds\right] \leqslant \varepsilon\mathrm{E}\left[\sum_{k=0}^{n}\int_0^t \gamma^2|w_k(s)|^2 ds\right] \\ &\quad - \mathrm{E}\left[\sum_{k=0}^{n} V_1(x_{k+1}(t), r(t))\right] - \mathrm{E}\left[\int_0^t V_2(y_k(s), r(s))ds\right] \leqslant \varepsilon\mathrm{E}\left[\sum_{k=0}^{n}\int_0^t \gamma^2|w_k(s)|^2 ds\right].\end{aligned} \tag{4.14}$$

As $n \to \infty$ and $t \to \infty$ in inequality (4.14), we obtain (4.4). This completes the proof.

5. ITERATIVE LEARNING CONTROL UNDER UNCERTAINTY AND FAILURES

In this section the stability results of the previous section are applied to ILC design under parameter uncertainty and possible sensors faults for linear systems described by the state-space model

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= C(\delta(t), r(t))x(t),\end{aligned} \tag{5.1}$$

where $x \in \mathbb{R}^n$ denotes the state vector, $u \in \mathbb{R}^m$ is the input vector, $y \in \mathbb{R}^p$ is the output vector, $\delta \in \mathbb{R}^N$ is the vector of uncertain parameters in the sensors, $r(t)$ represents a Markov chain with a finite set of states $\mathbb{N} = \{1,\ldots,\nu\}$ corresponding to the number of possible failures with transition probabilities given by (4.2).

The uncertainty associated with the system dynamics has the affine parallelotopic model

$$C(\delta(t), r) = C + \sum_{j=1}^{N} \delta_j(t) C_j(r), \quad r \in \mathbb{N}, \tag{5.2}$$

where $\delta_j(t)$, $j = 1,\ldots,N$, are the components of the uncertainty vector $\delta(t)$, N denotes its dimension and C, $C_j(r)$, $j = 1,\ldots,N$, $r \in \mathbb{N}$ indicate matrices of appropriate dimensions. Each component $\delta_j(t)$ of the uncertainty vector in (5.2) is assumed to be bounded in an interval

$$\underline{\delta}_j(t) \leqslant \delta_j(t) \leqslant \overline{\delta}_j(t). \tag{5.3}$$

Designate by $\mathbf{\Delta}$ the set of uncertainties $\delta(t)$:

$$\mathbf{\Delta} = \{\ \delta(t) = [\ \delta_1(t)\ \ \ldots\ \ \delta_N(t)\ ]^{\mathrm{T}} : \delta_j(t) \in [\underline{\delta}_j, \overline{\delta}_j],\ j = 1, \ldots, N\}.$$

And the finite set of extremal values (vertices) of the set $\mathbf{\Delta}$ is defined by

$$\mathbf{\Delta}_v = \{\ \delta(t) = [\ \delta_1(t)\ \ \ldots\ \ \delta_N(t)\ ]^{\mathrm{T}} : \delta_j(t) \in \{\underline{\delta}_j, \overline{\delta}_j\},\ j = 1, \ldots, N\ \}. \tag{5.4}$$

The process (5.1) evolves in the repetitive mode with a pass length T with resetting to the initial state after each pass is complete. Moreover, within the time interval $0 \leqslant t \leqslant T$, the output signal $y(t)$ must follow a reference signal $y_{ref}(t)$ with a given accuracy ϵ. An illustrative example is a portal robot with multiple repetition of homogeneous operations in a production conveyor. In such an operational mode, it seems natural to design control laws using information not only from a current pass, but also from one or several previous passes. The ILC problem lies in constructing feedback control correction algorithms based on the above information to achieve the required accuracy. To formulate the ILC problem, let the integer k denote the pass (also termed trial in some literature) and $u_k(t), x_k(t)$ and $y_k(t)$ stand for the input, state and output vectors, respectively, on this pass and have the same dimensions as their counterparts in (5.1). Then the dynamics of the uncontrolled process are described by

$$\begin{aligned} \dot{x}_k(t) &= Ax_k(t) + Bu_k(t), \\ y_k(t) &= C(\delta(t), r(t))x_k(t) \end{aligned} \tag{5.5}$$

with the boundary conditions

$$y_0(t) = 0, \quad 0 \leqslant t \leqslant T, \quad x_k(0) = x_0, \quad k = 0, 1, \ldots, \tag{5.6}$$

where T is the pass length.

Suppose that the components of the reference signal $y_{ref}(t)$ are differentiable on the interval $[0, T]$. Then $e_k(t) = y_{ref}(t) - y_k(t)$ is the error on pass k and the aim of ILC is to construct a sequence of inputs such that the error decreases with each pass. In the absence of failures, this can be expressed as the convergence condition on the input and error:

$$\lim_{k\to\infty} |e_k(t)| = 0, \quad \lim_{k\to\infty} |u_k(t) - u_\infty(t)| = 0, \tag{5.7}$$

where u_∞ is termed the learned control.

A commonly used ILC law is to select the input on the current pass as that used on the previous pass plus a correction, i.e., the ILC law on pass $k+1$ is of the form

$$u_{k+1}(t) = u_k(t) + \Delta u_{k+1}(t), \tag{5.8}$$

where $\Delta u_{k+1}(t)$ is the correction term whose design can involve information generated over the complete previous pass, in contrast to standard feedback laws.

Returning to the case of failures, the stochastic nature of $r(t)$ requires the following modified definition of ILC convergence.

Definition 5. A system described by (5.5) is said to be ILC convergent if for all $0 \leqslant t \leqslant T$:

$$\mathrm{E}[|e_k(t)|^2] = \mathrm{E}[|y_{ref}(t) - y_k(t)|^2], \quad \mathrm{E}[|u_k(t) - u_\infty(t)|^2] \to 0, \quad k \to \infty. \tag{5.9}$$

To write the ILC dynamics as a linear differential repetitive process, introduce the auxiliary vector $\upsilon_k(t)$ defined by

$$\dot{\upsilon}_{k+1}(t) = x_{k+1}(t) - x_k(t). \tag{5.10}$$

Then, given (5.5),

$$e_{k+1}(t) - e_k(t) = -C(\delta(t), r(t))A \int_0^t (x_{k+1}(\tau) - x_k(\tau))d\tau - C(\delta(t), r(t))B \int_0^t (u_{k+1}(\tau) - u_k(\tau))d\tau. \quad (5.11)$$

Also by (5.10), (5.11), the ILC dynamics can be described by a linear differential repetitive process with uncertainty of the form

$$\dot{\upsilon}_{k+1}(t) = A\upsilon_{k+1}(t) + B \int_0^t \Delta u_{k+1}(\tau)d\tau, \quad (5.12)$$

$$e_{k+1}(t) = e_k(t) - C(\delta(t), r(t))A\upsilon_{k+1}(t) - C(\delta(t), r(t))B \int_0^t \Delta u_{k+1}(\tau)d\tau e_k(t).$$

Consider also the case when

$$\Delta u_{k+1}(t) = F_1(i)\dot{\upsilon}_{k+1}(t) + F_2(i)\dot{e}_k(t), \quad \text{if} \quad r(t) = i. \quad (5.13)$$

Then if (5.13) guarantees exponential stability in the mean square of (5.12), then by Definition 5 the ILC law (5.8) is convergent.

To construct the stabilizing control law matrices $F_1(i)$ and $F_2(i)$, $i \in \mathbb{N}$, the stability conditions of Theorem 3 can be applied. Choose the candidate vector Lyapunov function as (4.5) with $V_1(\upsilon_{k+1}(t), r(t)) = \upsilon_{k+1}^{\mathrm{T}}(t)P_1(r(t))\upsilon_{k+1}(t)$, $V_2(e_k(t), r(t)) = e_k^{\mathrm{T}}(t)P_2(r(t))e_k(t)$, $P_1 > 0$, $P_2 > 0$. In this case, the stochastic divergence operator $\mathcal{D}$ of the function (4.5) must satisfy the condition (4.8). By calculating this operator along the paths of the system described by (5.12), (5.13), the following are sufficient conditions for the exponential stability in the mean square:

$$\begin{gathered} P(i) = \operatorname{diag}\{P_1(i)\ P_2(i)\} > 0, \\ A_{c1}^{\mathrm{T}}(\delta, i)P(i) + P(i)A_{c1}(\delta, i) + \sum_{j=1}^{\nu} \pi_{ij} I^{1,0} P(j) \\ -I^{0,1}P(i) + A_{c2}^{\mathrm{T}}(\delta, i)P(i)A_{c2}(\delta, i) < 0, \ \ i \in \mathbb{N}, \ \ \delta \in \mathbf{\Delta}, \end{gathered} \quad (5.14)$$

where

$$I^{1,0} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \ \ I^{0,1} = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix},$$

$$A_{c2}(\delta, i) = \begin{bmatrix} 0 & 0 \\ -C(\delta, i)A - C(\delta, i)BF_1(i) & I - C(\delta, i)BF_2(i) \end{bmatrix},$$

$$A_{c1}(i) = \begin{bmatrix} A + BF_1(i) & BF_2(i) \\ 0 & 0 \end{bmatrix}.$$

Introduce the new variables $X(i) = P^{-1}(i)$, $Y_1(i) = F_1(i)X_1(i)$, $Y_2(i) = F_2(i)X_2(i)$. Then using the Schur complement lemma, routine calculations and convexity properties, the inequalities (5.14) are reduced to the following coupled set of linear matrix inequalities (LMIs) with respect to these variables:

$$\begin{bmatrix} S_{11}(\delta,i) & S_{12}(\delta,i) & S_{13}(i) \\ S_{12}^{\mathrm{T}}(\delta,i) & -X(i) & 0 \\ S_{13}^{\mathrm{T}}(i) & 0 & S_{33}(i) \end{bmatrix} < 0, \qquad X(i) > 0, \quad \delta \in \mathbf{\Delta}, \quad i \in \mathbb{N}, \tag{5.15}$$

where

$$S_{11}(\delta,i) = \begin{bmatrix} A_{c11}(i) & BY_2(i) \\ (BY_2(i))^{\mathrm{T}} & -X_2(i) \end{bmatrix}, \quad S_{12}(\delta,i) = \begin{bmatrix} 0 & 0 \\ A_{c12}(\delta,i) & A_{c22]}(\delta,i) \end{bmatrix}^{\mathrm{T}},$$

$$A_{c11}(i) = AX_1(i) + BY_1(i) + (AX_1(i) + BY_1(i))^{\mathrm{T}} + \pi_{ii}X_1(i),$$
$$A_{c12}(\delta,i) = -C(\delta,i)AX_1(i) - C(\delta,i)BY_1(i),$$
$$A_{c22}(\delta,i) = X_2(i) - C(\delta,i)BY_2(i),$$
$$S_{13}(i) = \left[\pi_{i1}^{\frac{1}{2}}X(i)I^{1,0} \ldots \pi_{i\,i-1}^{\frac{1}{2}}X(i)I^{1,0}\pi_{i\,i+1}^{\frac{1}{2}}X(i)I^{1,0} \ldots \pi_{i\nu}^{\frac{1}{2}}X(i)I^{1,0}\right],$$
$$S_{33}(i) = \mathrm{diag}[-X(1) \ldots - X(i-1) \ -X(i+1) \ldots - X(\nu)].$$

The inequalities (5.15) are convex and, by the well-known theorem of inequalities on convex hulls, it is necessary and sufficient that they hold true on the set $\mathbf{\Delta}_v$ and the following result has been established.

Theorem 5. *Consider the ILC dynamics described by* (5.12). *Suppose that the LMIs* (5.15) *with* $\delta \in \mathbf{\Delta}_v$ *and* $i \in \mathbb{N}$ *are feasible and set* $F_1(i) = Y_1(i)X_1^{-1}(i)$, $F_2(i) = Y_2(i)X_2^{-1}(i)$, $i \in \mathbb{N}$. *Then the ILC scheme defined by* (5.8) *and* (5.13) *is convergent.*

6. CONCLUSION

This paper has developed new results on the stability of nonlinear differential repetitive processes with potential applications. To demonstrate their role for the latter purpose, they have been applied to ILC design, including the case when failures in implementation may arise. These results provide a basis for further research to fully evaluate the potential of a systems theory for nonlinear repetitive processes.

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APPENDIX

Proof of Theorem 1. If the pair $(V(x,y), \varphi(\bar{x}))$ satisfies (3.4), the controlled nonlinear differential repetitive process (in the absence of the disturbance terms) can be rewritten as

$$\begin{aligned} \dot{x}_{k+1}(t) &= \bar{f}_1(x_{k+1}(t), y_k(t), \varphi(x_{k+1}(t), y_k(t))), \\ y_{k+1}(t) &= \bar{f}_2(x_{k+1}(t), y_k(t), \varphi(x_{k+1}(t), y_k(t))), \end{aligned} \tag{A.1}$$

where $\bar{f}_1$ and $\bar{f}_2$ represent nonlinear functions such that $\bar{f}_1(0,0,0) = 0$ and $\bar{f}_2(0,0,0) = 0$. Moreover, it follows from (3.3) and (3.4) that

$$\mathrm{div} V(x_{\varphi,k+1}(t), y_{\varphi,k}(t)) \leqslant -c(|x_{\varphi,k+1}|^2 + |y_{\varphi,k}(t)|^2). \tag{A.2}$$

Hence, it is required to show that the controlled process (A.1) is exponentially stable under (3.5), (3.6) and (A.2).

Given (A.2), there exists a number $c_3 < c$ such that $c_3 < c_2$ and $z_d^{\frac{1}{2}} < 1 - \frac{c_3}{c_2} < 1$. Next, the inequalities (3.5), (3.6) and (A.2) give

$$\frac{dV_1(x_{k+1}(t))}{dt} + \lambda V_1(x_{k+1}(t)) + V_2(y_{k+1}(t)) - \zeta V_2(y_k(t)) \leqslant 0, \tag{A.3}$$

where $\lambda = \frac{c_3}{c_2}$, $\zeta = 1 - \frac{c_3}{c_2} \in (0,1)$. Solving inequality (A.3) with respect to $V_1(x_{k+1}(t))$ yields

$$V_1(x_{k+1}(t)) \leqslant V_1(x_{k+1}(0))\mathrm{e}^{-\lambda t} - \int\limits_0^t \mathrm{e}^{-\lambda(t-s)}[V_2(y_{k+1}(s)) - \zeta V_2(y_k(s))]ds. \tag{A.4}$$

Introducing

$$W_{k+1}(t) = V_1(x_{k+1}(0))\mathrm{e}^{-\lambda t} - V_1(x_{k+1}(t)),$$
$$H_k(t) = \int\limits_0^t \mathrm{e}^{-\lambda(t-s)} V_2(y_k(s))ds,$$

enables (A.4) to be rewritten as

$$H_{k+1}(t) \leqslant \zeta H_k(t) + W_{k+1}(t). \tag{A.5}$$

Solving (A.5) with respect to H gives

$$H_n(t) \leqslant \zeta^n H_0(t) + \sum_{k=1}^n W_k(t)\zeta^{n-k}, \tag{A.6}$$

or

$$\sum_{k=1}^n V_1(x_k(t))\zeta^{n-k} + \int\limits_0^t \mathrm{e}^{-\lambda(t-s)} V_2(y_n(s))ds \leqslant \mathrm{e}^{-\lambda t}\sum_{k=1}^n V_1(x_k(0))\zeta^{n-k} + \zeta^n \int\limits_0^t \mathrm{e}^{-\lambda(t-s)} V_2(y_0(s))ds.$$

The last inequality is equivalent to

$$\mathrm{e}^{\lambda t}\sum_{k=1}^n V_1(x_k(t))\zeta^{-k} + \zeta^{-n}\int\limits_0^t \mathrm{e}^{\lambda s} V_2(y_n(s))ds \leqslant \zeta^{-n}\sum_{k=1}^n V_1(x_k(0))\zeta^{n-k} + \mathrm{e}^{\lambda t}\int\limits_0^t \mathrm{e}^{-\lambda(t-s)} V_2(y_0(s))ds. \tag{A.7}$$

Estimating the right-hand side of (A.7) in combination with the boundary conditions (2.2) gives

$$\zeta^{-n}\sum_{k=1}^{n} V_1(x_k(0))\zeta^{n-k} + \mathrm{e}^{\lambda t}\int\limits_0^t \mathrm{e}^{-\lambda(t-s)}V_2(y_0(s))ds$$
$$\leqslant \frac{c_2 M_f(\mathrm{e}^{\lambda T}-1)}{\lambda} + c_2\kappa_d\sum_{k=1}^{\infty}\zeta^k = \frac{c_2 M_f(\mathrm{e}^{\lambda T}-1)}{\lambda} + \frac{c_2\kappa_d}{1-\zeta} = C, \quad \text{(A.8)}$$

and it follows from (A.7) and (A.8) that

$$c_1|x_n(t)|^2\zeta^{-n}\mathrm{e}^{\lambda t} \leqslant C \quad \text{for all} \quad t\in[0,\infty], \quad n=0,1,\ldots, \quad \text{(A.9)}$$
$$\int\limits_0^t c_1\zeta^{-n}\mathrm{e}^{\lambda s}|y_n(s)|^2 ds \leqslant C \quad \text{for all} \quad t\in[0,\infty], \quad n=0,1,\ldots.$$

The last inequality means that the function $c_1\zeta^{-n}\mathrm{e}^{\lambda s}|y_n(s)|^2$ is integrable for all $s\in[0,\infty]$, $n=0,1,\ldots$, and necessarily bounded, i.e.,

$$c_1|y_n(s)|^2\zeta^{-n}\mathrm{e}^{\lambda s} \leqslant \bar{C} < \infty \quad \text{for all} \quad s\in[0,\infty],\ n=0,1,\ldots. \quad \text{(A.10)}$$

Finally, (A.9) and (A.10) directly imply (2.4) and the proof is complete.

Proof of Theorem 3. It follows from (3.3) and (4.8) that

$$\mathcal{D}V(\xi,\eta,i) \leqslant -c(|\xi|^2+|\eta|^2), \quad \text{(A.11)}$$

and by (A.11), there exists $c_3 < c$ such that $c_3 < c_2$ and $z_d^{\frac{1}{2}} < 1-\frac{c_3}{c_2} < 1$. Conversely, (4.9), (4.10) and (A.11) give that

$$\mathrm{E}[\mathcal{D}_1 V(x_{k+1}(t), y_k(t), r(t))] + \lambda\mathrm{E}[V_1(x_{k+1}(t), r(t))]$$
$$+\,\mathrm{E}[V_2(y_{k+1}(t), r(t))] - \zeta\mathrm{E}[V_2(y_k(t), r(t))] \leqslant 0, \quad \text{(A.12)}$$

where $\lambda = \frac{c_3}{c_2}$, $\zeta = 1-\frac{c_3}{c_2}\in(0,1)$. Solving (A.12) with respect to $V_1(x_{k+1}(t))$ gives

$$\mathrm{E}[V_1(x_{k+1}(t), r(t))] \leqslant \mathrm{E}[V_1(x_{k+1}(0), r(0))]\mathrm{e}^{-\lambda t}$$
$$-\int\limits_0^t \mathrm{e}^{-\lambda(t-s)}\mathrm{E}[V_2(y_{k+1}(s), r(s)) - \zeta V_2(y_k(s), r(s))]ds. \quad \text{(A.13)}$$

Introduce

$$W_{k+1}(t) = \mathrm{E}[V_1(x_{k+1}(0), r(0))\mathrm{e}^{-\lambda t} - V_1(x_{k+1}(t), r(t))],$$
$$H_k(t) = \int\limits_0^t \mathrm{e}^{-\lambda(t-s)}\mathrm{E}[V_2(y_k(s), r(s))]ds$$

and rewrite (A.13) as

$$H_{k+1}(t) \leqslant \zeta H_k(t) + W_{k+1}(t). \quad \text{(A.14)}$$

Solving (A.14) with respect to H yields

$$H_n(t) \leqslant \zeta^n H_0(t) + \sum_{k=1}^{n} W_k(t)\zeta^{n-k}, \quad \text{(A.15)}$$

or

$$\sum_{k=1}^{n} \mathrm{E}[V_1(x_k(t), r(t))]\zeta^{n-k} + \int_0^t \mathrm{e}^{-\lambda(t-s)}\mathrm{E}[V_2(y_n(s), r(s))]ds$$
$$\leqslant \mathrm{e}^{-\lambda t} \sum_{k=1}^{n} \mathrm{E}[V_1(x_k(0), r(0))]\zeta^{n-k} + \zeta^n \int_0^t \mathrm{e}^{-\lambda(t-s)}\mathrm{E}[V_2(y_0(s), r(s))]ds.$$

This last inequality is equivalent to

$$\mathrm{e}^{\lambda t} \sum_{k=1}^{n} \mathrm{E}[V_1(x_k(t), r(t))]\zeta^{-k} + \zeta^{-n} \int_0^t \mathrm{e}^{\lambda s}\mathrm{E}[V_2(y_n(s), r(s))]ds$$
$$\leqslant \zeta^{-n} \sum_{k=1}^{n} \mathrm{E}[V_1(x_k(0), r(0))]\zeta^{n-k} + \mathrm{e}^{\lambda t} \int_0^t \mathrm{e}^{-\lambda(t-s)}\mathrm{E}[V_2(y_0(s), r(s))]ds,$$

and the remainder of the proof follows identical steps to that of Theorem 1 with obvious modifications to the notation.

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