Max-min Fairness of K-user Cooperative Rate-Splitting in MISO Broadcast Channel with User Relaying

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Abstract—Cooperative Rate-Splitting (CRS) strategy, relying on linearly precoded rate-splitting at the transmitter and opportunistic transmission of the common message by the relaying user, has recently been shown to outperform typical Noncooperative Rate-Splitting (NRS), Cooperative Non-Orthogonal Multiple Access (C-NOMA) and Space Division Multiple Access (SDMA) in a two-user Multiple Input Single Output (MISO) Broadcast Channel (BC) with user relaying. In this work, the existing two-user CRS transmission strategy is generalized to the K-user case. We study the problem of jointly optimizing the precoders, message split, time slot allocation, and relaying user scheduling with the objective of maximizing the minimum rate among users subject to a transmit power constraint at the base station. As the user scheduling problem is discrete and the entire problem is non-convex, we propose a two-stage lowcomplexity algorithm to solve the problem. Both centralized and decentralized relaying protocols based on selecting K_1 ($K_1 < K$) strongest users are first proposed followed by a Successive Convex Approximation (SCA)-based algorithm to jointly optimize the time slot, precoders and message split. Numerical results show that by applying the proposed two-stage algorithm, the worstcase achievable rate achieved by CRS is significantly increased over that of NRS and SDMA in a wide range of network loads (underloaded and overloaded regimes) and user deployments (with a diversity of channel strengths). Importantly, the proposed SCA-based algorithm dramatically reduces the computational complexity without any rate loss compared with the conventional algorithm in the literature of CRS. Therefore, we conclude that the proposed K-user CRS combined with the two-stage algorithm is more powerful than the existing transmission schemes.

Index Terms—Cooperative Rate-Splitting (CRS), max-min fairness, Success Convex Approximation (SCA), relaying user, broadcast channel (BC), cooperative transmission

I. INTRODUCTION

RIVEN by the exponential rise in the volume of wireless traffic and increasing demands for higher data rates, considerable attention has been drawn to the design of innovative solutions for future wireless communication systems. Among numerous candidate technologies, Rate-Splitting Multiple Access (RSMA), a novel multiple access scheme

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proposed in the recent work [1], has been shown to be a promising and powerful strategy to boost the lower layers of next-generation communication systems. By using linearly precoded Rate-Splitting (RS) at the transmitter to split the user messages into common and private parts, and using Successive Interference Cancellation (SIC) at all receivers to sequentially decode the encoded common streams and the private streams, RSMA manages interference in a more versatile and robust manner than Space Division Multiple Access (SDMA) that fully treats residual interference as noise and power-domain Non-Orthogonal Multiple Access (NOMA) that fully decodes interference. Therefore, RSMA unifies and outperforms SDMA and NOMA from both spectral and energy efficiency perspectives [2]–[6].

The idea of RS was first introduced in a two-user Single-Input Single-Output (SISO) Interference Channel (IC) [7] and was further developed in recent works [6], [8]–[16] for modern Multiple Input Multiple Output (MIMO) Broadcast Channels (BCs). Among those works, 1-layer RS, the simplest building block of the framework of RSMA, has been studied and shown to be beneficial in the multi-antenna setup¹. RS splits the message of each user into one common and one private part. The common parts of all users are combined and encoded into one common stream to be decoded by all users while the privates parts are independently encoded and decoded by the corresponding users. RS superimposes the common stream on top of the private streams, and broadcasts to the users. In contrast to NOMA, RS does not require any user ordering or grouping at the transmitter and only one single layer of SIC is required at each user to decode and remove the common stream before decoding the intended private stream. The literature of RS in multi-antenna BC starts from an information theoretic perspective where RS is shown to achieve the optimal sum Degree of Freedom (DoF) [10] followed by the entire DoF region [13] of the K-user underloaded Multiple Input Multiple Out (MISO) BC with imperfect Channel State Information at the Transmitter (CSIT). Motivated by the DoF results showing the multiplexing gains of RS at high Signal-to-Noise Ratio (SNR), a number of recent works study the performance of RS in the finite SNR regime of MISO BC with both perfect CSIT [1], [3], [17] and imperfect CSIT [9]–[11], [14]–[16]. Besides the typical MISO BC, the benefits of RS has been further

¹ In the rest of the paper, "1-layer RS" will be referred to by "RS" for simplicity.

exploited in the finite SNR regime of massive MIMO [18], [19], millimeter wave systems [20], [21], multigroup multicasting [22], [23], multi-cell Coordinated Multipoint Joint Transmission (CoMP) [4], Simultaneous Wireless Information and Power Transfer (SWIPT) [24], joint unicast and multicast transmission [2], Cloud Radio Access Network (C-RAN) [25] as well as mutli-pair Decode-and-Forward (DF) full-duplex relay channel [26]. As the common stream in RS is required to be decoded by all users, its achievable rate is limited by the rate of the worst-case user. If the users experience heterogeneous channel strengths from BS, the rate of the common stream (which is also known as common rate) may drop. To overcome the limitation, one promising solution is to combine RS with cooperative transmission using user relaying.

Cooperative user relaying has been identified as one promising solution for effectively combating the shadowing effects to extend the radio coverage and improve the channel capacity simultaneously [27]. By appointing one or more relaying users to assist in receiving data from BS and forwarding it to other users, the spatial diversity is boosted without deploying dedicated relaying stations. BC has been shown to be well suited for cooperative user relaying in the literature of Relay BC (RBC) [28]–[30] from an information theoretical perspective due to its capability to simultaneously serve relaying users and other users in the same resource blocks. By incorporating superposition coding at the transmitter and Decoding-and-Forward (DF) at relaying users, RBC communication networks where the transmitter sends information to a number of users that cooperate by exchanging information, significantly improve the rate region of typical SISO BC achieved by Dirty Paper Coding (DPC) or superposition coding [30]. The benefits of RBC have been further explored in [31], [32] considering multiple transmit antennas at BS in order to embrace the spatial multiplexing gain of multi-antenna BC over singleantenna BC. In [31], the achievable rate region of two-user multi-antenna RBC with Zero-Forcing (ZF)-DPC is shown to be larger than that of ZF-DPC in the two-user MIMO BC without user relaying. A low-complex ZF beamforming with closed-form optimal power allocation is proposed in [32] for MIMO RBC.

Motivated by the recent findings about the performance benefits of RS over SDMA in multi-antenna BC, it is worth to explore the benefits of incorporating RS in multi-antenna RBC, which has only recently emerged. Indeed, [5] recently proposed a linearly precoded two-user Cooperative RS (CRS) transmission framework by allowing one of the two users to opportunistically forward its decoded common message to the other user. The "opportunistic" transmission comes from dynamic time slot allocation for the two transmission phases (namely, direct transmission phase from BS to the two users and cooperative transmission phase from the relaying user to the other user). It has been demonstrated in [5] that the proposed two-user CRS is more general than typical RS with only direct transmission, Cooperative-NOMA (C-NOMA), cooperative RS with equal time slot allocation as well as SDMA, and outperforms them all.

A. Motivations and Contributions

The objective of this work is to generalize the two-user CRS framework proposed in [5] to a K-user setup. This brings a number of issues to overcome:

- In the algorithm proposed in [5], the optimization of time slot allocation for the two transmission phases is separated from the optimization of precoders and message split. The time slot allocation is achieved via one dimensional exhaustive search while the precoder and the message split are jointly optimized for each given time slot allocation scheme. The computational complexity of the algorithm is relatively high. A first question arising is therefore "Is there any simpler algorithm that derives the optimal time slot allocation without requiring an exhaustive search?"
- In the direct transmission phase of CRS, the encoded common stream and the private streams for all users are superimposed and broadcast to the users simultaneously. Each user decodes the common stream and the intended private stream sequentially with the assistance of SIC. As all users first decode the common stream, all of them are potential candidates for forwarding the common stream in the cooperative transmission phase. One can therefore wonder how to smartly select one or more relaying users? If the relaying users are not properly selected, the cooperative transmission phase can be useless. For example, if the user with the worst transmission rate is selected as a relaying user, the cooperative transmission from the worst-case user to other users cannot improve the achievable common rate at all due to the fact that the achievable rate of the common stream is limited by the worst-case achievable rate. A second arising question is therefore "How do we design efficient relaying user scheduling protocols in the generalized K-user CRS?"
- Since only the two-user MISO BC is considered in [5], the impact of the number of users on the system performance remains unexplored. In the K-user case, the search space of the optimal solution is enlarged since the common rate is shared by more users and the relaying user scheduling becomes more complex as K increases. Hence, a third question arising is "How will the CRS design, optimization and performance change with the number of users K?"

In this work, we derive a generalized K-user CRS framework and provide solutions to overcome the above issues. The major contributions are summarized as follows:

- We formulate a max-min fairness problem of a K-user CRS network in order to optimize the precoders, message split and time slot allocation as well as relaying user scheduling. The objective is to maximize the minimum Quality of Service (QoS) rate among users subject to a transmit power constraint. This is the first work in the literature on the design of the generalized K-user CRS.
- We study both the centralized and decentralized user scheduling for the K-user CRS. Through analyzing the global optimal point of the max-min fairness problem, we prove that the worst common rate achieved by the

relaying users in the optimal relaying user group is always larger than that achieved by other users in the direct transmission phase. Motivated by the proposition, efficient centralized and decentralized relaying protocols are proposed and compared. Both protocols only require the information of channel strengths for relaying-user selection.

- We propose a Successive Convex Approximation (SCA)-based algorithm to jointly optimize the precoders, message split and time slot allocation. The qualitative complexities of the proposed algorithm and the algorithm proposed in [5] are analyzed. The proposed SCA-based algorithm dramatically reduces the computational complexity since only a single layer iterative procedure is required to yield a solution compared with the two-layer iterative procedures in [5].
- We show through numerical results that the proposed relaying protocols with one single relaying user selection is able to achieve a rate performance very close to that of the optimal relaying scheduling algorithm in a wide range of network loads (underloaded and overloaded regimes), user deployments (with a diversity of channel strengths) and power constraints at the relaying users.
- We further demonstrate that the proposed SCA-based algorithm achieves the same performance as the Weighted Minimum Mean Square Error (WMMSE) and exhaustive search-based algorithm proposed in [5] while the computational complexity of the system is much reduced. Moreover, we observe that the performance benefits of CRS over Non-cooperative RS (NRS) become more significant as the number of transmit antennas decreases or as the number of users/the channel strength disparity among user increases. The observations bring the conclusion that by letting the user with the strongest channel strength forward the common stream to other users with weaker channel strengths, the rate of decoding the common stream at all users can be further enhanced and therefore. the worst-case achievable rate of using CRS is further boosted even at low SNR.

B. Organization

The rest of the paper is organized as follows. In Section II, we introduce the system model of K-user CRS. The maxmin fairness optimization problem is formulated in Section III. The proposed low-complexity relaying protocol as well as the SCA-based joint optimization algorithm are specified in Section IV. Numerical results illustrating the benefits of the proposed algorithm are discussed in Section V, followed by the conclusions in Section VI.

C. Notations

The superscript $(\cdot)^T$ denotes transpose and $(\cdot)^H$ denotes conjugate-transpose. $\mathcal{CN}(\delta,\sigma^2)$ represents a complex Gaussian distribution with mean δ and variance σ^2 . The boldface uppercases represent matrices and lowercase letters represent vectors. $\operatorname{tr}(\cdot)$ is the trace. $|\cdot|$ is the absolute value and $||\cdot||$ is the Euclidean norm. $\mathbb C$ denotes the complex space. $|\mathcal A|$ is the cardinality of the set $\mathcal A$.

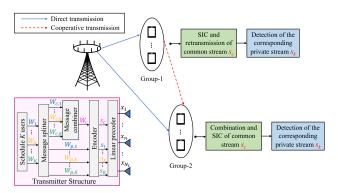


Fig. 1: The proposed K-user cooperative rate-splitting system.



Fig. 2: Time slot allocation for the two transmission phases.

II. SYSTEM MODEL

The system model is illustrated in Fig. 1. BS equipped with N_t transmit antennas simultaneously serves K single antenna users indexed by the set $\mathcal{K} = \{1, 2, \ldots, K\}$. The users are divided into two separated user groups, namely, group-1 indexed by the set \mathcal{K}_1 and group-2 indexed by the set \mathcal{K}_2 , where $\mathcal{K}_1 \cup \mathcal{K}_2 = \mathcal{K}$.

The signal transmission is accomplished in the direct transmission phase and cooperative transmission phase. As illustrated in Fig. 2, the time slot allocation for the two phases may not be equal. In the first time slot (also known as direct transmission phase), BS transmits signals to all users. θ is the fraction of time allocated to the direct transmission phase. In the second time slot (also known as the cooperative transmission phase), the users in group-1 cooperatively forward the signals to the users in group-2. The detailed transmission model is explained in the following subsections of the direct transmission phase and the cooperative transmission phase, respectively.

A. Direct transmission phase

In the direct transmission phase, BS serves K users as the typical multi-antenna RS transmission investigated in the recent literature [1], [6], [8]–[10]. Following the principle of RS, the message W_k intended for user-k is split into a common part $W_{c,k}$ and a private part $W_{p,k}$. The common parts of all users $W_{c,1},\ldots,W_{c,K}$ are combined into a common message W_c . It is encoded into the common stream s_c using a codebook shared by all users. s_c should be decoded by all users as it contains part of the messages of all users. The private part $W_{p,k}$ of user-k is independently encoded into the private stream s_k , which is only required to be decoded by user-k. The encoded streams are linearly precoded and the resulting transmit signal at BS in the first time slot is

$$\mathbf{x}^{[1]} = \mathbf{P}\mathbf{s} = \mathbf{p}_c s_c + \sum_{k \in \mathcal{K}} \mathbf{p}_k s_k. \tag{1}$$

Denoting $\mathbf{s} = [s_c, s_1, s_2, \dots, s_K]^T$ and assuming that $\mathbb{E}[\mathbf{s}\mathbf{s}^H] = \mathbf{I}$, the average transmit power constraint is given by $\operatorname{tr}(\mathbf{P}\mathbf{P}^H) \leq P_t$, where $\mathbf{P} = [\mathbf{p}_c, \mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_K]$ is the integrated precoder matrix and $\mathbf{p}_c, \mathbf{p}_k \in \mathbb{C}^{N_t \times 1}$.

The received signal at user-k in the first time slot is given by

$$y_k^{[1]} = \mathbf{h}_k^H \mathbf{x}^{[1]} + n_k, \forall k \in \mathcal{K}, \tag{2}$$

where $\mathbf{h}_k \in \mathbb{C}^{N_t \times 1}$ is the channel between BS and user-k. We assume that the Channel State Information (CSI) at the transmitter and receivers are perfect². $n_k \sim \mathcal{CN}(0,1)$ is the Additive White Gaussian Noise (AWGN). Therefore, the transmit SNR is equal to the total power consumption P_t .

Users of group-1 first decode the common stream s_c by fully treating the interference from all the private streams as noise. With the assist of SIC, the decoded common stream is removed from the received signal and then each user decodes the intended private stream by treating the remaining interference from other private streams as noise. Under the assumption of perfect SIC³, the rate of decoding the common stream at each user of group-1 in the first time slot is given by

$$R_{c,k}^{[1]} = \theta \log_2 \left(1 + \frac{\left| \mathbf{h}_k^H \mathbf{p}_c \right|^2}{\sum_{j \in \mathcal{K}} \left| \mathbf{h}_k^H \mathbf{p}_j \right|^2 + 1} \right), \forall k \in \mathcal{K}_1. \quad (3)$$

The rate of decoding the private stream at each user of group-1 in the first time slot is given by

$$R_k^{[1]} = \theta \log_2 \left(1 + \frac{\left| \mathbf{h}_k^H \mathbf{p}_k \right|^2}{\sum_{j \in \mathcal{K}, j \neq k} \left| \mathbf{h}_k^H \mathbf{p}_j \right|^2 + 1} \right), \forall k \in \mathcal{K}_1.$$
(4)

B. Cooperative transmission phase

In the cooperative transmission phase, users in group-1 employ the Non-regenerative Decode-and-Forward (NDF) protocol [33]–[35] to forward the common stream s_c to the users in group-2. Each user-j of group-1 re-encodes s_c with a codebook generated independently from that of BS and forwards s_c to the users of group-2 with the transmit power P_j . The transmit signal at user-j of group-1 in the second time slot is given by

$$x_j^{[2]} = \sqrt{P_j} s_c, \forall j \in \mathcal{K}_1. \tag{5}$$

The signal received at user-k of group-2 in the second time slot is

$$y_k^{[2]} = \sum_{j \in \mathcal{K}_1} h_{k,j} x_j^{[2]} + n_k, \forall k \in \mathcal{K}_2,$$
 (6)

where $h_{k,j}$ is the Single-Input Single-Output (SISO) channel between user-k and user-j.

The rates of decoding the common stream at user-k of group-2 in the first and the second time slots are respectively given by

$$R_{c,k}^{[1]} = \theta \log_2 \left(1 + \frac{\left| \mathbf{h}_k^H \mathbf{p}_c \right|^2}{\sum_{j \in \mathcal{K}} \left| \mathbf{h}_k^H \mathbf{p}_j \right|^2 + 1} \right), \forall k \in \mathcal{K}_2. \quad (7)$$

$$R_{c,k}^{[2]} = (1 - \theta) \log_2 \left(1 + \sum_{j \in \mathcal{K}_1} P_j \left| h_{k,j}^H \right|^2 \right), \forall k \in \mathcal{K}_2.$$
 (8)

Users in group-2 combine the decoded common stream in both time slots. The achievable rate of decoding the common stream at all users with the NDF [36]–[38] protocol⁴ is given as

$$R_{c} = \min \left(\{ R_{c,k}^{[1]} | k \in \mathcal{K}_{1} \}, \{ R_{c,k}^{[1]} + R_{c,k}^{[2]} | k \in \mathcal{K}_{2} \} \right)$$

$$= \min \{ R_{c,1}, R_{c,2} \},$$
(9)

where $R_{c,1} = \min_{k \in \mathcal{K}_1} \{R_{c,k}^{[1]}\}$ and $R_{c,2} = \min_{k \in \mathcal{K}_2} \{R_{c,k}^{[1]} + R_{c,k}^{[2]}\}$ are the achievable common rate of users in group-1 and group-2, respectively. R_c ensures that all users are able to decode the common stream successfully. As R_c is shared by all users for the transmission of the common stream s_c , it is equal to $\sum_{k \in \mathcal{K}} C_k = R_c$, where C_k is the portion of R_c transmitting $W_{c,k}$. Once s_c is decoded and removed from the received signal, user-k decodes the intended private stream. The rate of decoding the private stream at each user of group-2 follows equation (4). Denote the common rate vector as $\mathbf{c} = [C_1, \dots, C_K]$. \mathbf{c} is required to be optimized jointly with the precoder \mathbf{P} in order to maximize the worst-cast achievable rate. The total achievable rate of user-k is given by $R_{k,tot} = R_b^{[1]} + C_k, \forall k \in \mathcal{K}$.

III. PROBLEM FORMULATION

In this work, we emphasize the user fairness issue. The precoder P, the RS message split via common rate allocation c, the time slot allocation θ and the relaying user selection scheme \mathcal{K}_1 are optimized with the objective of maximizing the minimum user rate. The problem of maximizing the worst achievable rate among users has been studied in the literature of MISO BC [39], [40] as well as in the literature of RS without user-relaying [11], [41]. Motivated by those studies, we resort to investigate the max-min fairness in NDF CRS assisted transmission. The problem at hand differs from conventional max-min rate problems in two ways: 1) the time slot allocation and the relaying user selection in CRS are considered as optimization variables, 2) the rate constraint of the common stream is different from conventional RS in MISO BC without user-relaying due to the introduced NDF

⁴Note that there are two different types of DF relaying protocols, namely, NDF and Regenerative DF (RDF). The major difference between NDF and RDF is that the former considers independent codebooks while the latter uses the same codebook at BS and the relaying users. In this work, we focus on the application of RS in the NDF relaying protocol. Readers are referred to [33]–[35] for more detailed comparison of NDF and RDF protocols.

²The study of Non-cooperative RS (NRS) in imperfect CSIT has been widely studied in the literature [9]–[11], [14]–[16]. Due to the space limitations, the performance of CRS in imperfect CSIT is not investigated in this work. It is an interesting problem for future work.

³The impact of imperfect SIC on the system performance is beyond the scope of this work, which we will investigate in our future work.

CRS transmission model. The max-min rate problem for the K-user CRS is formulated as

$$\max_{\mathbf{P} \in \theta} \min_{K_{t}} R_{k,tot}$$
 (10a)

s.t.
$$\sum_{k \in \mathcal{K}} C_k \le R_c \tag{10b}$$

$$tr(\mathbf{PP}^H) < P_t \tag{10c}$$

$$C_k \ge 0, \forall k \in \mathcal{K}$$
 (10d)

$$\mathcal{K}_1 \subset \mathcal{K}, \mathcal{K}_2 = \mathcal{K} \setminus \mathcal{K}_1.$$
 (10e)

As \mathcal{K}_2 is determined once \mathcal{K}_1 is selected, i.e., $\mathcal{K}_2 = \mathcal{K} \setminus \mathcal{K}_1$, only \mathcal{K}_1 is considered as a variable. Constraint (10b) guarantees that the common stream is successfully decoded by all users. Constraint (10c) is the transmit power constraint at BS.

The formulated joint user selection and resource allocation problem is a mixed integer non-convex optimization problem. The optimal relaying user selection problem itself is of high computational complexity, particularly when the number of users is large. Therefore, in this paper, we propose a two-stage low-complexity algorithm to solve the problem. In summary, the algorithm has the following two stages:

Stage 1: Relaying user selection protocol: The two user groups \mathcal{K}_1 and \mathcal{K}_2 are determined by selecting the set of relaying users \mathcal{K}_1 that optimize the max-min fairness of the CRS transmission strategy. Both centralized and decentralized relaying user selection protocols are studied in this work. The former enables relaying user selection at BS while the latter allows users to accomplish the selection in a decentralized manner.

Stage 2: Joint precoder design and resource allocation: Based on the proposed relaying user selection protocols, we jointly optimize the precoders, message split and time slot allocation by solving the following max-min problem:

$$\max_{\mathbf{P}, \mathbf{c}, \theta} \min_{k \in \mathcal{K}} R_{k, tot}$$
s.t. (10b), (10c), (10d).

The proposed algorithm will be discussed thoroughly in the next section.

IV. PROPOSED LOW-COMPLEXITY ALGORITHM

In this section, we elaborate on the proposed two-stage low-complexity algorithm to solve problem (10).

A. Stage 1: Relaying user selection protocol

The transmission in the cooperative transmission phase helps to enhance the common rate R_c since only the common stream s_c is transmitted in this phase. As all users are required to decode s_c in the direct transmission phase, they all have the ability of re-transmitting the common stream. Two questions arises when designing the relaying user selection protocol: "how do we select each relaying user?" and "how many relaying users do we need?" To gain more insights, we further investigate the global optimal point $(\mathbf{P}^\star, \mathbf{c}^\star, \theta^\star, \mathcal{K}_1^\star)$ of problem (10) and we obtain Proposition 1:

Proposition 1. At the global optimal point $(\mathbf{P}^*, \mathbf{c}^*, \theta^*, \mathcal{K}_1^*)$ of problem (10), the common rates achieved by the users in

group-1 $R_{c,1}(\mathbf{P}^{\star}, \theta^{\star}, \mathcal{K}_{1}^{\star})$ and group-2 $R_{c,2}(\mathbf{P}^{\star}, \theta^{\star}, \mathcal{K}_{2}^{\star})$ are equal, which can be mathematically written as

$$\min_{k \in \mathcal{K}_{1}^{\star}} \{ R_{c,k}^{[1]}(\mathbf{P}^{\star}, \theta^{\star}) \}
= \min_{k \in \{\mathcal{K} \setminus \mathcal{K}_{1}^{\star}\}} \{ R_{c,k}^{[1]}(\mathbf{P}^{\star}, \theta^{\star}) + R_{c,k}^{[2]}(\theta^{\star}, \mathcal{K}_{1}^{\star}) \},$$
(12)

where

$$\begin{split} R_{c,1}(\mathbf{P}^{\star}, \boldsymbol{\theta}^{\star}, \mathcal{K}_{1}^{\star}) &= \min_{k \in \mathcal{K}_{1}^{\star}} \{R_{c,k}^{[1]}(\mathbf{P}^{\star}, \boldsymbol{\theta}^{\star})\}, \\ R_{c,2}(\mathbf{P}^{\star}, \boldsymbol{\theta}^{\star}, \mathcal{K}_{1}^{\star}) &= \min_{k \in \{\mathcal{K} \setminus \mathcal{K}_{1}^{\star}\}} \{R_{c,k}^{[1]}(\mathbf{P}^{\star}, \boldsymbol{\theta}^{\star}) + R_{c,k}^{[2]}(\boldsymbol{\theta}^{\star}, \mathcal{K}_{1}^{\star})\}, \end{split}$$

and the optimal relaying user grouping obeys the following rule when $0 < \theta^* < 1$:

$$\min_{k \in \mathcal{K}_{1}^{*}} \{ R_{c,k}^{[1]}(\mathbf{P}^{*}, \theta^{*}) \} > \min_{k \in \mathcal{K} \setminus \mathcal{K}_{1}^{*}} \{ R_{c,k}^{[1]}(\mathbf{P}^{*}, \theta^{*}) \}.$$
 (13)

Proof: We first prove the equivalence of the common rate achieved by users in group-1 and group-2 at the global optimal point (12) by contradiction. By assuming \hat{k}_1 and \hat{k}_2 are the two users that respectively achieve the worst common rate in group-1 and group-2 at the global optimal point $(\mathbf{P}^*, \mathbf{c}^*, \theta^*, \mathcal{K}_1^*)$, we obtain that

$$R_{c,1}(\mathbf{P}^{\star}, \theta^{\star}, \mathcal{K}_{1}^{\star}) = \theta^{\star} f_{c,\hat{k}_{1}}^{[1]}(\mathbf{P}^{\star}),$$

$$R_{c,2}(\mathbf{P}^{\star}, \theta^{\star}, \mathcal{K}_{1}^{\star}) = \theta^{\star} f_{c,\hat{k}_{2}}^{[1]}(\mathbf{P}^{\star}) + (1 - \theta^{\star}) f_{c,\hat{k}_{2}}^{[2]}(\mathcal{K}_{1}^{\star}) \quad (14)$$

$$= \theta^{\star} \left(f_{c,\hat{k}_{2}}^{[1]}(\mathbf{P}^{\star}) - f_{c,\hat{k}_{2}}^{[2]}(\mathcal{K}_{1}^{\star}) \right) + f_{c,\hat{k}_{2}}^{[2]}(\mathcal{K}_{1}^{\star}),$$

where

$$f_{c,k}^{[1]}(\mathbf{P}^{\star}) = \log_2 \left(1 + \frac{\left| \mathbf{h}_k^H \mathbf{p}_c^{\star} \right|^2}{\sum_{j \in \mathcal{K}} \left| \mathbf{h}_k^H \mathbf{p}_j^{\star} \right|^2 + 1} \right)$$
$$f_{c,k}^{[2]}(\mathcal{K}_1^{\star}) = \log_2 \left(1 + \sum_{j \in \mathcal{K}_1^{\star}} P_j \left| h_{k,j}^H \right|^2 \right).$$

Note that $f_{c,\hat{k}_2}^{[1]}(\mathbf{P}^\star) < f_{c,\hat{k}_2}^{[2]}(\mathcal{K}_1^\star)$ should hold when $0 < \theta^\star < 1$. Otherwise, both $R_{c,1}(\mathbf{P}^\star,\theta,\mathcal{K}_1^\star)$ and $R_{c,2}(\mathbf{P}^\star,\theta,\mathcal{K}_1^\star)$ are increasing functions of θ for the optimal $\mathbf{P}^\star,\mathcal{K}_1^\star$ and the optimal θ^\star should be 1 in order to maximize the minimum rate among users. Hence, when $0 < \theta < 1$, $R_{c,1}(\mathbf{P}^\star,\theta,\mathcal{K}_1^\star)$ is a monotonic increasing function of θ while $R_{c,2}(\mathbf{P}^\star,\theta,\mathcal{K}_1^\star)$ is a monotonic decreasing function of θ .

If
$$R_{c,1}(\mathbf{P}^{\star}, \theta^{\star}, \mathcal{K}_{1}^{\star}) > R_{c,2}(\mathbf{P}^{\star}, \theta^{\star}, \mathcal{K}_{1}^{\star})$$
, we have
$$\theta^{\star} f_{c,\hat{k}_{1}}^{[1]}(\mathbf{P}^{\star}) > \theta^{\star} f_{c,\hat{k}_{2}}^{[1]}(\mathbf{P}^{\star}) + (1 - \theta^{\star}) f_{c,\hat{k}_{2}}^{[2]}(\mathcal{K}_{1}^{\star})$$

$$\Leftrightarrow \theta^{\star} > \frac{f_{c,\hat{k}_{1}}^{[2]}(\mathcal{K}_{1}^{\star})}{f_{c,\hat{k}_{1}}^{[1]}(\mathbf{P}^{\star}) - f_{c,\hat{k}_{2}}^{[1]}(\mathbf{P}^{\star}) + f_{c,\hat{k}_{2}}^{[2]}(\mathcal{K}_{1}^{\star})} \stackrel{\triangle}{=} \Gamma(\mathbf{P}^{\star}, \mathcal{K}_{1}^{\star}).$$

$$(15)$$

By decreasing θ from θ^* to $\theta' = \Gamma(\mathbf{P}^*, \mathcal{K}_1^*)$, we have $R_{c,2}(\mathbf{P}^*, \theta', \mathcal{K}_1^*) > R_{c,2}(\mathbf{P}^*, \theta^*, \mathcal{K}_1^*)$ and $R_{c,1}(\mathbf{P}^*, \theta', \mathcal{K}_1^*) = R_{c,2}(\mathbf{P}^*, \theta', \mathcal{K}_1^*)$. The achievable common rate $\sum_{k \in \mathcal{K}} C_k' = \min\{R_{c,1}(\mathbf{P}^*, \theta', \mathcal{K}_1^*) \ , R_{c,2}(\mathbf{P}^*, \theta', \mathcal{K}_1^*)\}$ increases. By allocating the improved common rate to the worst-case user, the value of the objective function achieved by using the new solution $(\mathbf{P}^*, \mathbf{c}', \theta', \mathcal{K}_1^*)$ is higher than that of $(\mathbf{P}^*, \mathbf{c}^*, \theta^*, \mathcal{K}_1^*)$, which contradicts the fact that $(\mathbf{P}^*, \mathbf{c}^*, \theta^*, \mathcal{K}_1^*)$ is the global optimal point. Similarly, we obtain that if $R_{c,1}(\mathbf{P}^*, \theta^*, \mathcal{K}_1^*) < \mathbf{e}^{\mathbf{r}}$

 $R_{c,2}(\mathbf{P}^{\star}, \theta^{\star}, \mathcal{K}_{1}^{\star}), \; \theta^{\star} < \Gamma(\mathbf{P}^{\star}, \mathcal{K}_{1}^{\star}) \; \text{holds. A better solution}$ is obtained by increasing θ from θ^* to $\theta' = \Gamma(\mathbf{P}^*, \mathcal{K}_1^*)$. The contradiction arises. Hence, we draw the conclusion that at the global optimal point $(\mathbf{P}^{\star}, \mathbf{c}^{\star}, \theta^{\star}, \mathcal{K}_{1}^{\star})$ of problem (10), $R_{c,1}(\mathbf{P}^{\star}, \theta^{\star}, \mathcal{K}_1^{\star}) = R_{c,2}(\mathbf{P}^{\star}, \theta^{\star}, \mathcal{K}_1^{\star}).$

As $\min_{k \in \{\mathcal{K} \setminus \mathcal{K}_1^*\}} \{R_{c,k}^{[1]}(\mathbf{P}^*, \theta^*) + R_{c,k}^{[2]}(\theta^*, \mathcal{K}_1^*)\} \ge \min_{k \in \{\mathcal{K} \setminus \mathcal{K}_1^*\}} \{R_{c,k}^{[1]}(\mathbf{P}^*, \theta^*)\} + \min_{k \in \{\mathcal{K} \setminus \mathcal{K}_1^*\}} \{R_{c,k}^{[2]}(\theta^*, \mathcal{K}_1^*)\}$ and $R_{c,k}^{[2]}(\theta^*, \mathcal{K}_1^*) > 0, \forall k \in \mathcal{K} \setminus \mathcal{K}_1^*$, we obtain that $\begin{array}{ll} R_{c,2}(\mathbf{P}^{\star},\theta^{\star},\mathcal{K}_{1}^{\star}) > \min_{k \in \{\mathcal{K} \setminus \mathcal{K}_{1}^{\star}\}} \{R_{c,k}^{[1]}(\mathbf{P}^{\star},\ \theta^{\star})\}. \text{ Based on (12), we have } R_{c,1}(\mathbf{P}^{\star},\theta^{\star},\mathcal{K}_{1}^{\star}) > \min_{k \in \{\mathcal{K} \setminus \mathcal{K}_{1}^{\star}\}} \{R_{c,k}^{[1]}(\mathbf{P}^{\star},\theta^{\star},\mathcal{K}_{1}^{\star})\}. \end{array}$ $R_{c,k}^{[1]}(\mathbf{P}^\star, \theta^\star)\}$ when $0 < \theta^\star < 1$. Proof of proposition 1 is

Proposition 1 provides insights into the design of relaying user selection protocol. To boost the common rate via CRS, users with large $R_{c,k}^{[1]}$ are suggested to be clustered in \mathcal{K}_1 while the users with low $R_{c,k}^{[1]}$ are suggested to be clustered in \mathcal{K}_2^{5} . As s_c is decoded by all users in the first time slot, the rate of $R_{c,k}^{[1]}$ is strongly influenced by users' channel strengths $\|\mathbf{h}_k\|^2$, $k \in \mathcal{K}$. Hence, an intuitive and simple selection algorithm is based on the channel strengths of users. In this work, we extend the Request-To-Send (RTS)/Clear-To-Send (CTS) collision avoidance mechanism proposed in [42] to our K-user dynamic CRS network. In principle, by exchanging RTS and CTS packets between BS and users, each user can not only estimate the channel strength in between but also get synchronized after handshake [42]. Both centralized and decentralized relaying protocols based on users' channel strengths are studied. Fig. 3 illustrates the two proposed protocols when a single "best" relaying user is selected $(K_1 = 1)$.

- 1) Centralized relaying protocol: As BS requires the CSI to design the precoders, one natural method is to select relaying users centrally at BS. The proposed centralized relaying protocol is summarized in Algorithm 1. Once BS obtains the CSI of all users after handshake in Step 1, it selects the K_1 users with highest channel gain in Step 2. Once the K_1 users are selected, BS encapsulates the selection result into one "flag" packet and broadcasts it to all users. Each user decodes the received packet and gets the its own selection result.
- 2) Decentralized relaying protocol: As all users are capable of serving as relays, the cooperative transmission networks

⁵Though Proposition 1 only provides the selection policy for the worstcase users in K_1 and K_2 , we generalize it to all users in the two groups. There are two major reasons based on the analysis of KarushKuhnTucker (KKT) conditions as well as the computational complexity of problem (10). Mathematically, to reach the KKT point of problem (10), the complementary conditions should be met. Combining constraint (10b) and Proposition 1, we obtain the following constraint: $\mu_k(\sum_{k\in\mathcal{K}} C_k - R_{c,k}^{[1]}) = 0, \forall k\in\mathcal{K}_1$, where $\mu_k \geq 0$ is the Lagrangian dual variable. When all common rate constraints are active $(\mu_k > 0)$, we obtain that $\sum_{k \in \mathcal{K}} C_k - R_{c,k}^{[1]} = 0$ holds for all users in \mathcal{K}_1 . Similarly, we could derive the same conclusion for users in group-2. Hence, all users in group-1 achieve the same optimal common rate, which is larger than the common rate achieved by all users in group-2. When some of the common rate constraints are inactive, the problem becomes much difficult to handle. The computational complexity of optimizing relaying-user scheduling is NP-hard already. In total, $\sum_{k=1}^{K-1} {K \choose k}$ possible relaying-user groups should be searched. For each grouping method, the precoders, message split and time slot allocation are required to be optimized so as to obtain the achievable max-min rate for comparison. To gain insights in such circumstance is hard.

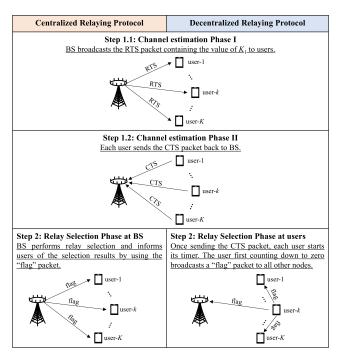


Fig. 3: The proposed centralized and decentralized relaying user selection protocols, $K_1 = 1$.

Algorithm 1: Proposed centralized relaying protocol

- 1 Initialize: $n \leftarrow 0, \mathcal{K}_1 = \emptyset, \mathcal{K}_2 = \mathcal{K};$
- 2 Step 1: BS broadcasts RTS packet to all users. Each user receives K_1 , replies BS with CTS. BS estimates the channel \mathbf{h}_k and calculates the channel strength $\|\mathbf{h}_k\|^2$;
- 3 Step 2: BS selects K_1 users with highest channel gain as
- 4 repeat
- $k^* = \max_k \{ \|\mathbf{h}_k\|^2 \};$ $\begin{array}{c|c} \mathbf{6} & \mathcal{K}_1 \leftarrow \mathcal{K}_1 \cup \{k^*\}; \\ \mathbf{7} & \mathcal{K}_2 \leftarrow \mathcal{K}_2 \setminus \{k^*\}; \\ \mathbf{8} & \mathbf{until} & |\mathcal{K}_1| = K_1; \end{array}$
- 9 Step 3: BS encapsulates the selection results into one "flag" packet and broadcasts it to all users.

can be designed adaptively based on the CSI conditions. However, due to the fact that the RTS packet is sent through a common pilot channel shared by all users while the CTS packets are fed back through uplink pilot channels dedicated to the corresponding users, it is easier to achieve perfect CSIR obtained from the common pilot compared with CSIT obtained from the uplink dedicated pilot [43]. Hence, decentralized selection approaches are preferred as they are not influenced by the CSIT inaccuracy.

The proposed decentralized relaying protocol is summarized in Algorithm 2. After handshake in step 1, each UE knows the number of relaying users K_1 to select as well as the channel strength $\|\mathbf{h}_k\|^2$. Perfect synchronization among users and BS is assumed after step 1. In the second step, each user starts the relaying user selection procedure. With the assistance of timer $T_k = \frac{\lambda}{\|\mathbf{h}_k\|^2}, \forall k \in \mathcal{K}_2$, where λ is a constant in time units [42], all users start counting down. The user first counting down to zero broadcasts a "flag" packet to all other users

Algorithm 2: Proposed decentralized relaying protocol

- 1 Initialize: $n \leftarrow 0, \mathcal{K}_1 = \emptyset, \mathcal{K}_2 = \mathcal{K};$
- 2 Step 1: BS broadcasts RTS packet that contains the value of K_1 to all users. Each user receives K_1 , replies BS with CTS, estimates the channel h_k and calculates the channel strength $\|\mathbf{h}_k\|^2$;
- 3 Step 2: Decentralized relaying user selection:
- 4 repeat
- At each user in K_2 , clear the existing timer if it has 5
- The timer is reset at each user in \mathcal{K}_2 as 6 $T_k = \frac{\lambda}{\|\mathbf{h}_k\|^2}, \forall k \in \mathcal{K}_2;$
- The user first counts down to zero, broadcasts a flag 7 packet to all users in \mathcal{K}_2 to announce itself as the best relaying user in \mathcal{K}_2 ;
- Once users in \mathcal{K}_2 receive the "flag" packet from the 8 best relaying user k^* , each of them update $\mathcal{K}_1 \leftarrow \mathcal{K}_1 \cup \{k^*\}, \, \mathcal{K}_2 \leftarrow \mathcal{K}_2 \setminus \{k^*\};$
- 9 **until** $|K_1| = K_1$;

to announce itself as the best relaying user. All other users add it to their relaying user list and reset the timer for the next round selection. The selection procedure repeats until K_1 relaying users are selected. As "how many relaying users do we need?" is a critical issue, the number of relaying users K_1 is considered to be a variable in the proposed algorithm. The influence of K_1 will be investigated in the numerical results section. The simplest selection is when $K_1 = 1$, which is a single "best" relaying user selection. In the numerical results, we will show that $K_1 = 1$ achieves a rate almost the same as the rate achieved by the optimal relaying user scheduling.

Remark 1: Compared with the decentralized relaying protocol without RS in [42], the protocol proposed for RS-assisted transmission is simplified as only the CSI between BS and the user is required at each user to perform decentralized relaying user selection. In contrast, without RS, each user should obtain CSI between every other user and itself as well as the CSI between BS and itself to perform decentralized relaying user selection [42]. Moreover, the protocol proposed in [42] can only select a single relaying user while our proposed protocol is generalized for multi-relaying user selection.

3) Comparison: As illustrated in Fig. 3, the major difference of the two proposed protocols stems from the relay selection phase where the nodes performing relay selection and broadcasting the "flag" packet are different. We further compare the proposed protocols in terms of overhead, synchronization and CSI requirement.

Table I illustrates the overhead comparison in terms of signaling and time consumption between the two proposed relaying protocols as well as the strategies without user relaying. N_{RTS} and N_{CTS} represent the number of symbols in each RTS and CTS packet, respectively. T_S denotes the symbol duration. N_{flag}^C and N_{flag}^D are the number of symbols in each flag packet of the centralized and decentralized relaying protocols, respectively. Note that N_{flag}^C is larger than N_{flag}^D (i.e., $N_{\mathit{flag}}^C > N_{\mathit{flag}}^D$) since the "flag" packet sent by BS in the

centralized relaying protocol contains the selection information of all users while the one sent by the selected user in the decentralized relaying protocol only contains the selection result of one single user. The handshake based on RTS and CTS is required for all strategies so as to obtain CSI at BS. Therefore, the signaling overhead of RTS N_{RTS} and CTS for K users $N_{CTS}K$ (assuming TDD system) is required for all strategies. Comparing the signaling and time overhead of all strategies, the signaling overhead of the centralized relaying protocol is relatively higher since $N^C_{\mathit{flag}} = K N^D_{\mathit{flag}}$ when BS simultaneously sends K "flag" packets to all users (via spatial multiplexing). In such case, the size of each "flag" packet for the centralized protocol is the same as the one for the decentralized protocol. However, the time consumption of the decentralized relaying protocol is higher since additional timer is required for the selection of each relaying user. Apparently, the strategies without user relaying has the lowest overhead.

Synchronization is required to be more accurate in the two strategies with user relaying compared with those without user relaying. In the relay selection phase, no explicit synchronization protocol is required among the users since the timing process is triggered by the CTS or flag packet and the collision due to the inaccurate synchronization can be controlled by λ [42]. A larger λ will reduce the probability of collision⁶. Readers are referred to [42] for more details. In the information transmission phases (as illustrated in Fig. 1 and Fig. 2), the synchronization is assumed to be perfect. This assumption is commonly made in the literature of multi-relay cooperative transmission [44]-[46]. In the numerical results, we have shown that a single relaying user selection is enough to achieve the near optimal result. Hence, synchronization among relaying users is not necessary.

As mentioned previously, the centralized relaying protocol requires BS to select relaying users based on the obtained CSIT while the decentralized relaying protocol allows users to select relaying users themselves based on CSIR. Hence, the major motivation of adopting the decentralized protocol is that its performance is not sensitive to the CSIT inaccuracy at all. In contrast, the performance of centralized relaying selection drops as CSIT becomes worse.

B. Stage 2: Joint precoder design and resource allocation

The two user groups K_1 and K_2 are determined after stage 1. In the second stage, a SCA-based algorithm is proposed to jointly optimize the precoders, message split and time resource allocation for problem (11). We first equivalently transform (11) by introducing an auxiliary variable t:

$$\max_{\mathbf{P.c.}\theta.t} t \tag{16a}$$

$$\max_{\mathbf{P}, \mathbf{c}, \theta, t} t$$
 (16a)
s.t. $R_k^{[1]} + C_k \ge t, \forall k \in \mathcal{K}$ (16b)
(10b), (10c), (10d).

The equivalence between (16) and (11) is guaranteed by noting that constraint (16b) is the same as $\min_{k \in \mathcal{K}} (R_k^{[1]} + C_k) \ge t$

⁶ If the transmission still fails for a large λ , one solution is to switch from the decentralized relaying protocol to the centralized one. The study on the failure of transmission is beyond the research scope in this paper, which will be considered as a future research direction.

TABLE I: Overhead comparison of different strategies

Schemes	Signaling overhead	Time overhead
Centralized relaying	$N_{RTS} + N_{CTS}K + N_{flag}^C$	$N_{RTS}T_S + N_{CTS}T_SK + N_{flag}^CT_S$
Distributed relaying	$N_{RTS} + N_{CTS}K + N_{flag}^D K_1$	$N_{RTS}T_S + N_{CTS}T_SK + N_{flag}^DT_SK_1 + \sum_{k \in \mathcal{K}_1} T_k$
Without user relaying	$N_{RTS} + N_{CTS}K$	$N_{RTS}T_S + N_{CTS}T_SK$

and it must hold with equality at optimum. The problem is still non-convex due to the rate expressions $R_k^{[1]}$ and $R_{c,k}^{[1]}$ in (16b) and (10b). Next, we introduce slack variable vectors $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_K], \ \boldsymbol{\alpha}_c = [\alpha_{c,1}, \ldots, \alpha_{c,K}], \ \boldsymbol{\rho} = [\rho_1, \ldots, \rho_K], \ \boldsymbol{\rho}_c = [\rho_{c,1}, \ldots, \rho_{c,K}]. \ \boldsymbol{\alpha}$ and $\boldsymbol{\alpha}_c$ are adopted to respectively represent the rate vectors of the private streams and the common streams at all users. $\boldsymbol{\rho}$ and $\boldsymbol{\rho}_c$ respectively denote the Signal-to-Interference-plus-Noise Ratio (SINR) vectors of the private streams and the common streams. With the assistance of the new variables, the optimization problem (16) is equivalently rewritten into

$$\max_{\mathbf{P}, \mathbf{c}, \theta, t} t \tag{17a}$$

s.t.
$$\theta \alpha_k + C_k \ge t, \forall k \in \mathcal{K}$$
 (17b)

$$\theta \alpha_{c,k} \ge \sum_{j \in \mathcal{K}} C_j, \forall k \in \mathcal{K}_1$$
 (17c)

$$\theta \alpha_{c,k} + R_{c,k}^{[2]} \ge \sum_{j \in \mathcal{K}} C_j, \forall k \in \mathcal{K}_2$$
 (17d)

$$1 + \rho_k - 2^{\alpha_k} \ge 0, \forall k \in \mathcal{K} \tag{17e}$$

$$1 + \rho_{c,k} - 2^{\alpha_{c,k}} \ge 0, \forall k \in \mathcal{K}$$
 (17f)

$$\frac{\left|\mathbf{h}_{k}^{H}\mathbf{p}_{k}\right|^{2}}{\sum_{j\in\mathcal{K},j\neq k}\left|\mathbf{h}_{k}^{H}\mathbf{p}_{j}\right|^{2}+1} \geq \rho_{k}, \forall k \in \mathcal{K} \quad (17g)$$

$$\frac{\left|\mathbf{h}_{k}^{H}\mathbf{p}_{c}\right|^{2}}{\sum_{j\in\mathcal{K}}\left|\mathbf{h}_{k}^{H}\mathbf{p}_{j}\right|^{2}+1} \geq \rho_{c,k}, \forall k \in \mathcal{K}$$
(17h)
(10c), (10d).

To deal with all the non-convex constraints (17b)–(17d), (17g), (17h), we adopt the following SCA method. For constraints (17b)–(17d), the bilinear function $\theta\alpha_k$ can be equivalently written as $\theta\alpha_k = \frac{1}{4}(\theta + \alpha_k)^2 - \frac{1}{4}(\theta - \alpha_k)^2$. Hence, $\theta\alpha_k$ is approximated at the point $(\theta^{[n]}, \alpha_k^{[n]})$ by the first-order Taylor approximation of $(\theta + \alpha_k)^2$, which is given by

$$\theta \alpha_k \ge \frac{1}{2} \left(\theta^{[n]} + \alpha_k^{[n]} \right) (\theta + \alpha_k) - \frac{1}{4} \left(\theta^{[n]} + \alpha_k^{[n]} \right)^2 - \frac{1}{4} (\theta - \alpha_k)^2 \triangleq \Phi^{[n]}(\theta, \alpha_k)$$

$$(18)$$

Constraints (17b)–(17d) are approximated around the point $(\theta^{[n]}, \alpha^{[n]}, \alpha^{[n]})$ at iteration n as

$$\Phi^{[n]}(\theta, \alpha_k) + C_k \ge t, \forall k \in \mathcal{K},
\Phi^{[n]}(\theta, \alpha_{c,k}) \ge \sum_{j \in \mathcal{K}} C_j, \forall k \in \mathcal{K}_1,
\Phi^{[n]}(\theta, \alpha_{c,k}) + R_{c,k}^{[2]} \ge \sum_{j \in \mathcal{K}} C_j, \forall k \in \mathcal{K}_2.$$
(19)

Constraints (17g), (17h) are equivalently written into Difference-of-Convex (DC) forms, which are given by

$$\sum_{j \in \mathcal{K}, j \neq k} |\mathbf{h}_k^H \mathbf{p}_j|^2 + 1 - \frac{|\mathbf{h}_k^H \mathbf{p}_k|^2}{\rho_k} \le 0, \forall k \in \mathcal{K},$$

$$\sum_{j \in \mathcal{K}} |\mathbf{h}_k^H \mathbf{p}_j|^2 + 1 - \frac{|\mathbf{h}_k^H \mathbf{p}_c|^2}{\rho_{c,k}} \le 0, \forall k \in \mathcal{K}.$$
(20)

By reconstructing the concave parts of the DC constraints with the first-order Taylor approximations, the constraints (17g), (17h) are respectively approximated around the point $(\mathbf{P}^{[n]}, \boldsymbol{\rho}_c^{[n]}, \boldsymbol{\rho}_c^{[n]})$ at iteration n by

$$\sum_{j \in \mathcal{K}, j \neq k} |\mathbf{h}_{k}^{H} \mathbf{p}_{j}|^{2} + 1 - \frac{2\Re\{(\mathbf{p}_{k}^{[n]})^{H} \mathbf{h}_{k} \mathbf{h}_{k}^{H} \mathbf{p}_{k}\}}{\rho_{k}^{[n]}} + \frac{|\mathbf{h}_{k}^{H} \mathbf{p}_{k}^{[n]}|^{2} \rho_{k}}{(\rho_{k}^{[n]})^{2}} \leq 0, \forall k \in \mathcal{K},$$

$$\sum_{j \in \mathcal{K}} |\mathbf{h}_{k}^{H} \mathbf{p}_{j}|^{2} + 1 - \frac{2\Re\{(\mathbf{p}_{c}^{[n]})^{H} \mathbf{h}_{k} \mathbf{h}_{k}^{H} \mathbf{p}_{c}\}}{\rho_{c,k}^{[n]}} + \frac{|\mathbf{h}_{k}^{H} \mathbf{p}_{c}^{[n]}|^{2} \rho_{c,k}}{(\rho_{c,k}^{[n]})^{2}} \leq 0, \forall k \in \mathcal{K}.$$

$$(21)$$

Based on the above approximation methods, the original optimization problem can be solved using the SCA method. The main idea of SCA is to successively solve a sequence of convex subproblems. At iteration n, based on the optimal solution $(\mathbf{P}^{[n]}, \theta^{[n]}, \boldsymbol{\alpha}^{[n]}, \boldsymbol{\alpha}^{[n]}, \boldsymbol{\rho}^{[n]}, \boldsymbol{\rho}^{[n]})$ obtained from the previous iteration n-1, we solve the following subproblem:

$$\max_{\substack{\mathbf{P}, \mathbf{c}, \theta, t, \\ \alpha, \alpha_c, \rho, \rho_c}} t \\
\text{s.t.} \quad (10c), (10d), (17e), (17f), (19), (21).$$

Problem (22) is a convex Quadratically Constrained Quadratic Program (QCQP), which can be solved using interior-point methods [47]. The proposed SCA-based algorithm is summarized in Algorithm 3.

C. Convergence and Complexity

1) Convergence: The proposed SCA-based algorithm iteratively solves the approximated problem (22) until convergence, where ϵ is the tolerance of convergence.

Proposition 2. For any feasible initial point, the proposed SCA-based algorithm is guaranteed to converge to a stationary point of problem (11).

Proof: SCA ensures monotonic improvement of t, i.e., $t^{[n]} \geq t^{[n-1]}$. This is due to the fact that the solution generated by solving problem (22) at iteration n-1 is a feasible point of problem (22) at iteration n. Due to the transmit

Algorithm 3: Proposed SCA-based algorithm

```
 \begin{array}{ll} \textbf{Initialize:} \ n \leftarrow 0, t^{[n]} \leftarrow 0, \\ \mathbf{P}^{[n]}, \theta^{[n]}, \boldsymbol{\alpha}^{[n]}, \boldsymbol{\alpha}^{[n]}, \boldsymbol{\rho}^{[n]}, \boldsymbol{\rho}^{[n]}; \\ \textbf{2 repeat} \\ \textbf{3} & n \leftarrow n+1; \\ \textbf{4} & \text{Solve problem (22) using} \\ \mathbf{P}^{[n-1]}, \theta^{[n-1]}, \boldsymbol{\alpha}^{[n-1]}, \boldsymbol{\alpha}^{[n-1]}, \boldsymbol{\rho}^{[n-1]}, \boldsymbol{\rho}^{[n-1]} \text{ and denote the optimal value of the objective function as} \\ t^{\star} \ \text{and the optimal solutions as} \\ \mathbf{P}^{\star}, \theta^{\star}, \boldsymbol{\alpha}^{\star}, \boldsymbol{\alpha}^{\star}_{c}, \boldsymbol{\rho}^{\star}, \boldsymbol{\rho}^{\star}_{c}; \\ \textbf{5} & \text{Update } t^{[n]} \leftarrow t^{\star}, \mathbf{P}^{[n]} \leftarrow \mathbf{P}^{\star}, \ \theta^{[n]} \leftarrow \theta^{\star}, \ \boldsymbol{\alpha}^{[n]} \leftarrow \boldsymbol{\alpha}^{\star}, \\ \boldsymbol{\alpha}^{[n]}_{c} \leftarrow \boldsymbol{\alpha}^{\star}_{c}, \ \boldsymbol{\rho}^{[n]} \leftarrow \boldsymbol{\rho}^{\star}, \ \boldsymbol{\rho}^{[n]}_{c} \leftarrow \boldsymbol{\rho}^{\star}_{c}; \\ \textbf{6 until} \ |t^{[n]} - t^{[n-1]}| < \epsilon; \\ \end{array}
```

power constraint (10c), the sequence $\{t^{[n]}\}_{n=1}^{n=\infty}$ is bounded above, which implies that the convergence of the proposed SCA-based algorithm is guaranteed. Next, we show that the sequence of $(\mathbf{P}^{[n]}, \theta^{[n]}, \boldsymbol{\alpha}^{[n]}, \boldsymbol{\alpha}^{[n]}, \boldsymbol{\rho}^{[n]}, \boldsymbol{\rho}^{[n]}, \boldsymbol{\rho}^{[n]})$ converges to the set of stationary points of problem (11). The proposed SCA-based algorithm is in fact an inner approximation algorithm in the nonconvex optimization literature [48], [49]. This is proved by showing the equivalence of the KKT conditions of problem (11) and problem (22) when the solution $(\mathbf{P}, \theta, \alpha, \alpha_c, \rho, \rho_c)$ is equal to $(\mathbf{P}^{[n]}, \theta^{[n]}, \alpha^{[n]}, \alpha^{[n]}, \rho^{[n]}, \rho^{[n]})$. Combining with the fact that the approximations made in (19), (21) are asymptotically tight as $n \to \infty$ [49], we can obtain that the solution of the proposed SCA-based algorithm converges to the set of KKT points (which is also known as the stationary points) of problem (11).

2) Complexity: At each iteration of the proposed SCA-based algorithm, the convex subproblem (22) is solved. Due to the exponential cone constraints (17e) and (17f), problem (22) is a generalized nonlinear convex program. The nonlinear solvers such as "fmincon" in the Matlab optimization toolbox can be adopted to solve the problem. An alternative efficient method is to approximate (17e) and (17f) by a sequence of Second Order Cone (SOC) constraints via the successive approximation method [50], [51]. The resulting SOC Programming (SOCP) can be solved by using interior-point methods with computational complexity $\mathcal{O}([KN_t]^{3.5})$. The total number of iterations required for the convergence is approximated as $\mathcal{O}(\log(\epsilon^{-1}))$. Hence, the worst-case computational complexity is $\mathcal{O}(\log(\epsilon^{-1})[KN_t]^{3.5})$.

Remark 2: The Weighted Minimum Mean Square Error (WMMSE) algorithm with one dimensional exhaustive search proposed in [5] can also be adopted to solve problem (11) for a given relaying user scheduling scheme, where θ is exhaustively searched over the range (0,1] and $\mathbf{P}(\theta), \mathbf{c}(\theta)$ are updated by using the WMMSE-based Alternative Optimization (AO) algorithm for each θ . Compared with the algorithm proposed in [5], our proposed SCA-based algorithm is more efficient as $\theta, \mathbf{P}, \mathbf{c}$ are jointly optimized without one-dimensional exhaustive search. The computational complexity of the WMMSE and exhaustive search-based algorithm proposed in [5] is $\mathcal{O}(\delta^{-1}\log(\epsilon^{-1})[KN_t]^{3.5})$ where $\delta \in (0,1)$ is

the increment between two adjacent candidates of θ . The set of candidate θ is assumed to be regularly-spaced over the range (0,1]. ϵ is the convergence tolerance of the AO algorithm to update $\mathbf{P}(\theta)$, $\mathbf{c}(\theta)$ for each θ . It is assumed to be the same as in the proposed SCA-based algorithm. Therefore, the worst-case computational complexity of our proposed SCA-based algorithm is δ^{-1} folds lower than the algorithm proposed in [5].

V. NUMERICAL RESULTS

In this section, we evaluate the performance of the generalized *K*-user CRS using the proposed low-complexity algorithm.

We compare the following relaying user scheduling protocols for the design of K_1 and K_2^7 :

- Optimal—the optimal relaying protocol where the relaying user selection is performed centrally at BS by enumerating all possible relaying user combinations. The scheduling scheme with the highest max-min rate is selected. It achieves the upper bound of the max-min rate of all relaying protocols but has the highest selection complexity.
- 1 best relay—the proposed relaying protocols when $K_1 = 1$.
- K/2 best relays—the proposed relaying protocols when $K_1 = \frac{K}{2}$. If K is odd, $\frac{K}{2}$ is rounded up to the closest integer.
- 1 random relay— BS randomly selects one user from

 K and informs the decision to all users via the "RTS"
 packet. It has the lowest selection complexity.

For a determined relaying user grouping \mathcal{K}_1 and \mathcal{K}_2 , we compare the following precoder, message split and time resource allocation algorithms:

- CRS: Proposed SCA—the cooperative RS model proposed in Section II and the proposed SCA-based algorithm (Algorithm 3) is adopted to solve problem (10). As discussed in Section IV, its worst-case computational complexity is $\mathcal{O}(\log(\epsilon^{-1})[KN_t]^{3.5})$.
- CRS: WMMSE—the cooperative RS model proposed in Section II, but the optimization problem (10) is solved using the WMMSE algorithm proposed in [5] with one-dimensional exhaustive search for θ . Its worst-case computational complexity is $\mathcal{O}(\lambda^{-1}\log(\epsilon^{-1})[KN_t]^{3.5})$.
- ERS: WMMSE—Equal RS is a special instance of "CRS: WMMSE" scheme when θ is fixed to 0.5. The time slot allocations for the direct and cooperative transmission phases are equal. Its worst-case computational complexity is $\mathcal{O}(\log(\epsilon^{-1})[KN_t]^{3.5})$.
- NRS: WMMSE—Non-cooperative RS is also a special instance of "CRS: WMMSE" scheme when θ is fixed to 1. This is the RS scheme that has been investigated in [1], [3], [9]–[11] for MISO BC without cooperative transmission. The transmission completes at the end of

 $^{^7}$ Under the assumption of perfect CSIT, the selection results of the proposed centralized and decentralized relaying protocols are the same for a given K_1 . The influence of CSIT inaccuracy to the two different relaying protocols is worth to be studied as a future work.

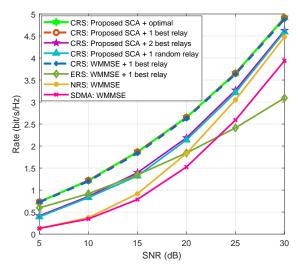
the direct transmission phase and the relaying transmission is blocked. Its worst-case computational complexity is $\mathcal{O}(\log(\epsilon^{-1})[KN_t]^{3.5})$.

• SDMA: WMMSE— the traditional multi-user linear precoding-based SDMA investigated in [52]. There is no RS and no cooperative transmission (i.e., $\|\mathbf{p}_c\|^2 = 0$ and $\theta = 1$). Its worst-case computational complexity is $\mathcal{O}(\log(\epsilon^{-1})[KN_t]^{3.5})$.

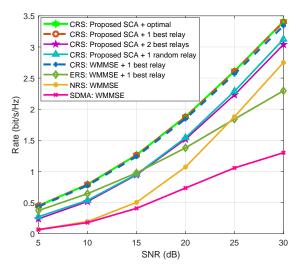
Different from [5] where only the specific channel realization is investigated for the two-user CRS, we consider the scenarios when each channel h_k has independent and identically distributed (i.i.d.) complex Gaussian entries with a certain variance, i.e., $\mathcal{CN}(0, \sigma_k^2)$. The channel between userk and user-j follows $h_{k,j} \sim \mathcal{CN}(0, \sigma_{k,j}^2)$. In the following simulation, we assume $\sigma_{k,j}^2 = 1, \forall k, j \in \mathcal{K}, k \neq j$. The tolerance of all the algorithms are set to $\epsilon = 10^{-3}$. Without loss of generality, we assume that the transmit power at BS and the relaying users in K_1 are equal, i.e., $P_t = P_j, \forall j \in$ \mathcal{K}_1 . Following the literature [1], [5], [10], the precoders of the proposed SCA-based algorithm are initialized by using Maximum Ratio Transmission (MRT) combined with Singular Maximum Ratio Transmission (MRT) combined with Singular Value Decomposition (SVD). The precoder $\mathbf{p}_k^{[0]}$ for the private stream s_k is initialized as $\mathbf{p}_k^{[0]} = \sqrt{p_k} \frac{\mathbf{h}_k}{\|\mathbf{h}_k\|}$, where $p_k = \frac{\beta P_t}{2}$ and $0 \le \beta \le 1$. The precoder $\mathbf{p}_c^{[0]}$ for the common message s_c is initialized as $\mathbf{p}_c^{[0]} = \sqrt{p_c}\mathbf{u}_c$, where $p_c = (1 - \beta)P_t$ and \mathbf{u}_c is the largest left singular vector of the channel matrix $\begin{aligned} \mathbf{H} &= [\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_K]. \text{ It is calculated by } \mathbf{u}_c = \mathbf{U}(:,1) \text{ where } \\ \mathbf{H} &= \mathbf{U}\mathbf{S}\mathbf{V}^H. \ \{\alpha_{c,k}^{[0]}, \alpha_k^{[0]}, \rho_{c,k}^{[0]}, \rho_k^{[0]} | k \in \mathcal{K}\} \text{ are initialized} \end{aligned}$ as $\rho_{c,k}^{[0]} = \frac{\left|\mathbf{h}_{k}^{H}\mathbf{p}_{c}^{[0]}\right|^{2}}{\sum_{j\in\mathcal{K}}\left|\mathbf{h}_{k}^{H}\mathbf{p}_{j}^{[0]}\right|^{2}+1}, \ \rho_{k}^{[0]} = \frac{\left|\mathbf{h}_{k}^{H}\mathbf{p}_{k}^{[0]}\right|^{2}}{\sum_{j\in\mathcal{K},j\neq k}\left|\mathbf{h}_{k}^{H}\mathbf{p}_{j}^{[0]}\right|^{2}+1}, \ \alpha_{c,k}^{[0]} = \log_{2}(1+\rho_{c,k}^{[0]}), \ \alpha_{k}^{[0]} = \log_{2}(1+\rho_{k}^{[0]}), \text{ respectively.}$ $\theta^{[0]}$ is initialized as 0.8. The precoder initialization of the WMMSE algorithm is the same as the proposed SCA-based algorithm. θ is searched with increment $\delta = 0.1$ in the "CRS: WMMSE" algorithm. Therefore, the precoders and message split are optimized by using the WMMSE algorithm 10 times for each value of θ selected from the set $[0.1, 0.2, \dots, 1]$. The MATLAB toolbox YALMIP [53] is used to solve the QCQP problem (22).

Fig. 4 shows the average max-min rates of different strategies used in two stages versus SNR at BS over 100 random channel realizations with varied transmit antennas and channel strength disparities among users (K=3). The variances of user channels are set to $\sigma_1^2=1,\ \sigma_2^2=0.3,\ \sigma_3^2=0.1$ in subfigure (a) and (b) while $\sigma_1^2=1,\ \sigma_2^2=1,\ \sigma_3^2=0.3$ in subfigure (c). The number of transmit antennas is 4 in subfigure (a) (which is an underloaded case) while there is 2 transmit antennas in subfigure (b) and (c) (which are overloaded cases).

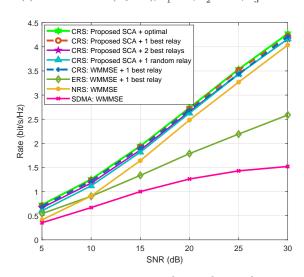
Comparing the relaying protocols for the given joint optimization algorithm "CRS: Proposed SCA" within each subfigure of Fig. 4, we observe that "1 best relay" achieves almost the same rate performance as the optimal relaying scheduling algorithm. In comparison, "K/2 best relays", which is equal to "2 best relays" when K=3, deteriorates the max-min rate and is closer to the performance of the baseline "1



(a) Underloaded case $(N_t = 4)$, $\sigma_1^2 = 1$, $\sigma_2^2 = 0.3$, $\sigma_3^2 = 0.1$



(b) Overloaded case $(N_t = 2)$, $\sigma_1^2 = 1$, $\sigma_2^2 = 0.3$, $\sigma_3^2 = 0.1$



(c) Overloaded case $(N_t = 2)$, $\sigma_1^2 = 1$, $\sigma_2^2 = 1$, $\sigma_3^2 = 0.3$

Fig. 4: The rate performance versus SNR comparison of different strategies, averaged over 100 random channel realizations. K=3.

random relay". The users served as relaying users in \mathcal{K}_1 cannot achieve any common rate improvement while their private rates deteriorate due to the time slot allocation. To enhance the rate among users, reducing the size of \mathcal{K}_1 is preferred as more users can benefit from the cooperative transmission. Under the given "1 best relay" scheduling protocol, we compare the algorithms of optimizing precoders, message split and time resource allocation. In all subfigures, the proposed "CRS: Proposed SCA" algorithm achieves the same or even higher rate performance when compared with "CRS: WMMSE", but at a much lower computational complexity. In Fig. 4(a), the achievable max-min rate of "CRS: Proposed SCA" averaged across all SNRs attains relative rate gain of 143.4%, 41.4% and 166.1% when compared with that of "NRS: WMMSE", "ERS: WMMSE" and "SDMA: WMMSE", respectively. Therefore, the proposed CRS with SCA-based algorithm achieves explicit rate improvement over all the existing transmission schemes.

We further compare the rates achieved by different strategies in Fig. 4(a) and Fig. 4(b) with four or two transmit antennas. The relative rate gain of "CRS: Proposed SCA" (with "1 best relay" scheduling protocol) over "NRS: WMMSE" averaged across all SNRs increases from 143.4% in Fig. 4(a) to 192.9% in Fig. 4(b) as the number of transmit antennas decreases. This is due to the fact that multi-user interference received at each user in the overloaded case is more severe than that in the underloaded case. A larger portion of users' private streams is encoded in the common stream for each user to decode. As the relaying users in CRS re-transmit the common stream, the amount of interference to be decoded at each user is further enhanced. Hence, CRS has a greater capability to manage the interference and it is more suited to the scenarios when users suffer from stronger multi-user interference. We also observe that the rate gap between the proposed "CRS: Proposed SCA" (with "1 best relay") and "SDMA: WMMSE" increases as the number of transmit antennas decreases. The relative rate gain of "CRS: Proposed SCA" over "SDMA: WMMSE" averaged over SNRs increases from 166.1% in Fig. 4(a) to 266.1% in Fig. 4(b). SDMA is only suited to the underloaded scenario. As N_t decreases, SDMA cannot manage the multi-user interference coming from all other users and the rate saturates at high SNR. In comparison, RS assisted transmission strategies are more robust to the network load thanks to its ability to partially decode the interference and partially treat interference as noise. This observation coincides with the observations in [1], [10]. We should also notice that, with the assistance of cooperative transmission, the gain achieved by CRS over SDMA is larger than that achieved by NRS over SDMA at low SNR. When SNR is 10 dB, the relative rate gains between CRS and SDMA in three subfigures are 253.8%, 342.9%, 83.8%, respectively while the relative rate gains between NRS and SDMA in three subfigures are 8.04%, 9.2%, 35.6%, respectively.

Comparing the rate achieved by the strategies under the same network load but different channel strength disparities among users (subfigure (b) and (c) in Fig. 4), we observe that the rate region gap between "CRS: Proposed SCA" (with "1 best relay") and "NRS: WMMSE" increases from an average of 23.9% in Fig. 4(c) to an average of 192.9% in Fig. 4(b)

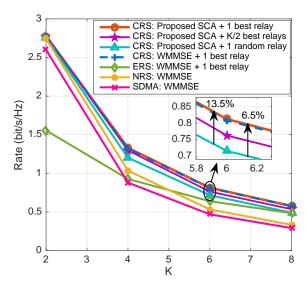
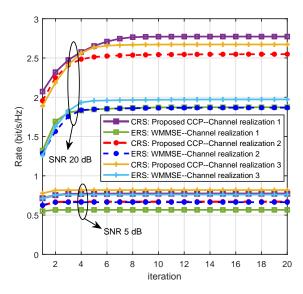


Fig. 5: The rate performance versus the number of users K of different strategies, averaged over 100 random channel realizations. SNR=10 dB.

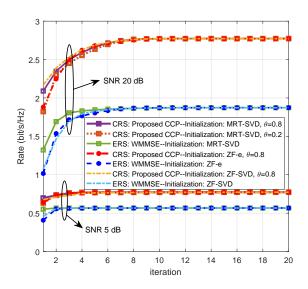
as the channel strength disparities among users increases. Without cooperative transmission, the achievable common rate in NRS is limited by the common rate of the worst-case user. The loss of common rate becomes more severe when there is a larger channel strength disparity. In comparison, CRS further enhances the common rate of the worst-case user by retransmitting the common stream to that user. When the user channel strength disparities among users are small, θ is much closer to 1. Therefore, the benefits of using CRS over NRS fall off in such circumstance.

Fig. 5 shows the average rate of different strategies versus number of users K over 100 random channel realizations when SNR is 10 dB. The variances of the user channels uniformly decrease from 1 with stepsize $\frac{1}{K}$. For example, $\sigma_1^2=1,\sigma_2^2=0.5$ when K=2 and $\sigma_1^2=1,\sigma_2^2=0.75,\sigma_3^2=0.5,\sigma_4^2=0.25$ when K=4. As the number of users increases, the optimal relaying protocol becomes computational prohibitive since $\sum_{k=1}^{K-1} {K \choose k}$ possible scheduling groups need to be considered. Under the given joint optimization algorithm "CRS: Proposed SCA", "1 best relay" is able to achieve an average of 57.83% rate improvement over "K/2 best relays" as well as an average of 13.5% rate improvement over "1 random relays". From the observations of the scheduling methods in Fig. 4 and Fig. 5, we conclude that a single relaying user selection based on channel strength is effective enough in the K-user CRS-assisted transmission network.

As the number of users increases, the rate loss of NRS becomes more obvious while the performance of ERS improves when compared with CRS. It implies that the optimal θ decreases as the number of users increases. This is due to the fact that multi-user interference increases with the number of users, and a larger portion of the user messages is transmitted via the common stream. CRS benefits from the ability to enhance the common rate and therefore boosts the max-min fairness of the system. We can also observe that CRS with one random relaying user selection achieves non-negligible rate



(a) Three different channel realizations with initialization method based on MRT-SVD.



(b) Three different initializations of **P** (namely, MRT-SVD, ZF-e, ZF-SVD) and two different initializations of θ ($\theta = 0.2, 0.8$) for one random channel realization

Fig. 6: Convergence comparison of different algorithms for different random channel realizations and different initializations of ${\bf P}$ and θ . K=3.

improvement over NRS and ERS. Comparing the RS-based schemes with the baseline SDMA, we confirm the superiority of RS over traditional SDMA in both underloaded ($K \le 4$ in Fig. 4) and overloaded (K > 4 in Fig. 4) MISO BC as demonstrated in the literature [1], [10], [11], [41]⁸.

As "CRS: Proposed SCA", "NRS: WMMSE" and "ERS: WMMSE" maintain the same low computation complexity,

⁸We should also notice that CRS has been shown to outperform C-NOMA where the message to be decoded first by all users is re-transmitted from the relaying users to other users. Similar observations are obtained in the non-cooperative transmission scenarios. Readers are referred to [5] for more information about the rate gain of CRS over C-NOMA as well as [1], [2], [6], [24] for the rate improvement of NRS over conventional NOMA without cooperative transmission.

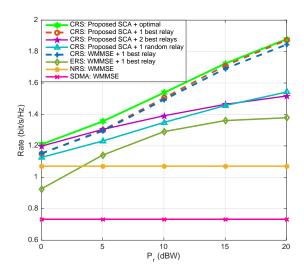


Fig. 7: The rate performance versus the transmit power constraint at the relaying users of different strategies, averaged over 100 random channel realizations. $P_t = 20$ dBW, $N_t = 2$, K = 3, $\sigma_1^2 = 1$, $\sigma_2^2 = 0.3$, $\sigma_3^2 = 0.1$.

we further compare the number of iterations required for the proposed SCA-based algorithm and the WMMSE algorithm to converge. Fig. 6 illustrates the convergence results of "CRS: Proposed SCA" and "ERS: WMMSE" algorithms for different random channel realizations as well as different initializations of **P** and θ when SNR is equal to 5 dB or 20 dB. "1 best relay" relaying protocol is adopted for both CRS and ERS. In Fig. 6(a), the rate achieved by the two algorithms are compared for three different random channel realizations. The initialization method is inline with the method used for Fig. 4 and Fig. 5, where the precoders \mathbf{p}_c and $\mathbf{p}_k, k \in \mathcal{K}$ are respectively initialized by MRT and SVD while the time slot allocation θ is initialized as 0.8. It is evident that both algorithms converge within a few iterations. Though the proposed SCAbased algorithm jointly optimizes θ , P, c, it is able to converge quickly within a few iterations at both high and low SNR. The convergence speed is similar to WMMSE which only jointly optimizes P, c. In Fig. 6(b), the rate achieved by the two algorithms with different initialization of P and θ are compared for one certain random channel realization. Three different initialization methods of P are used, namely, MRT-SVD, ZF-e and ZF-SVD. ZF-e is a DoF-motivated precoder design as adopted in [10] where $\mathbf{p}_k^{[0]} = \sqrt{p_k} \bar{\mathbf{p}}_k^{[0]}, \forall k \in \mathcal{K}$ and $\mathbf{p}_c^{[0]} = \sqrt{p_c} \bar{\mathbf{p}}_c^{[0]}$. $[\bar{\mathbf{p}}_1^{[0]}, \dots, \bar{\mathbf{p}}_K^{[0]}]$ are normalized ZF beamforming vectors constructed by using $\mathbf{H} = [\mathbf{h}_1, \dots, \mathbf{h}_K]$ and $ar{\mathbf{p}}_c^{[0]}$ is a randomly generated normalized vector. ZF-SVD is a modification of ZF-e where $\mathbf{p}_c^{[0]}$ is initialized by SVD. $\theta^{[0]}$ is initialized as 0.2 or 0.8. Though the initializations of P and θ influence the convergence speed, both algorithms still converge within limited iterations and different initialization methods have very close convergence rate at both high and low SNR.

The results shown in Fig. 4–Fig. 6 assume equal transmit power constraints at BS and the relaying users, i.e., $P_t = P_j, \forall j \in \mathcal{K}_1$. We further study the impact of different transmit power constraints at the relaying users on the rate performance

in Fig. 7. The transmit power constraint at BS is fixed to $P_t = 20$ dBW while the transmit power constraints at the relaying users are assumed to be equal to $P_r = P_j, \forall j \in \mathcal{K}_1$ and P_r varies from 0 dBW to 20 dBW. As is illustrated in Fig. 7, the non-cooperative SDMA and NRS strategies remain constant as P_r increases since both schemes only consider the use of the direct transmission phase from BS to the users. P_r is not utilized at all. The cooperative transmission achieves a higher rate performance at the sacrifice of investing additional transmit power at the relaying users. We observe that as P_r increases, the rate performance of the proposed "1 best relay" scheduling protocol is much closer to that of the optimal one. It achieves 95.6% of the average rate of the optimal scheduling protocol and 105.6% relative rate improvement over SDMA. Therefore, it is the preferred selection protocol when considering the trade-off between the total power consumption $(P_t + K_1 P_r)$ and max-min rate.

VI. CONCLUSIONS

To conclude, we investigate the max-min fairness of Kuser CRS for MISO BC with user relaying by designing the precoder, the RS message split and time slot allocation as well as the relaying user selection scheme in order to maximize the minimum QoS rate among users subject to a total transmit power constraint. We propose a two-stage low-complexity algorithm to solve the problem. In the first stage, channelgain based centralized and decentralized relaying protocols are proposed and compared. In the second stage, a joint time slot allocation, precoder and message split optimization algorithm based on SCA method is proposed to solve the max-min fairness problem. Numerical results show that the proposed relaying protocols with single relaying user selection achieves nearly the same rate performance as the optimal relaying scheduling algorithm. RS simplifies the scheduling procedure in the cooperative transmission networks and only the CSI between BS and the user is required at each user for decentralized relaying user scheduling. We further show through numerical results that the proposed SCA-based algorithm solves the problem much more efficiently than the algorithm in the literature, and without any rate loss. Finally, the influence of SNR, the number of transmit antennas, channel strength disparities among users as well as the number of users to the CRS performance is studied. We find that as the number of transmit antenna decreases, or the channel strength disparity among users increases or the number of serving users increases or at low SNR, the rate improvement of CRS over noncooperative RS and SDMA become more significant. Due to its ability to enhance the transmission rate of the common stream, CRS is more capable of managing strong multi-user interference and disparity of channel strengths. Hence, we draw the conclusion that K-user CRS is superior than existing transmission schemes and has a great potential to improve the system performance of future communication networks.

VII. ACKNOWLEDGEMENT

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REFERENCES

- [1] Y. Mao, B. Clerckx, and V. O. K. Li, "Rate-splitting multiple access for downlink communication systems: bridging, generalizing, and outperforming SDMA and NOMA," EURASIP J. Wireless Commun. Netw., vol. 2018, no. 1, p. 133, May 2018.
- [2] —, "Rate-splitting for multi-antenna non-orthogonal unicast and multicast transmission: Spectral and energy efficiency analysis," *IEEE Trans. Commun.*, 2019.
- [3] —, "Energy efficiency of rate-splitting multiple access, and performance benefits over SDMA and NOMA," in *Proc. IEEE Int. Symp. Wireless Commun. Syst. (ISWCS)*, Aug 2018, pp. 1–5.
- [4] —, "Rate-splitting multiple access for coordinated multi-point joint transmission," in *Proc. IEEE Int. Conf. Commun. (ICC) Workshop*, May 2019, pp. 1–6.
- [5] J. Zhang, B. Clerckx, J. Ge, and Y. Mao, "Cooperative rate-splitting for MISO broadcast channel with user relaying, and performance benefits over cooperative NOMA," *IEEE Signal Process. Lett.*, 2019.
- [6] B. Clerckx, Y. Mao, R. Schober, and H. V. Poor, "Rate-splitting unifying SDMA, OMA, NOMA, and multicasting in MISO broadcast channel: A simple two-user rate analysis," *IEEE Wireless Commun. Lett.*, vol. 9, no. 3, pp. 349–353, 2020.
- [7] T. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inf. Theory*, vol. 27, no. 1, pp. 49–60, Jan 1981.
- [8] C. Hao, Y. Wu, and B. Clerckx, "Rate analysis of two-receiver MISO broadcast channel with finite rate feedback: A rate-splitting approach," *IEEE Trans. Commun.*, vol. 63, no. 9, pp. 3232–3246, Sept 2015.
- [9] B. Clerckx, H. Joudeh, C. Hao, M. Dai, and B. Rassouli, "Rate splitting for MIMO wireless networks: A promising PHY-layer strategy for LTE evolution," *IEEE Commun. Mag.*, vol. 54, no. 5, pp. 98–105, May 2016.
- [10] H. Joudeh and B. Clerckx, "Sum-rate maximization for linearly precoded downlink multiuser MISO systems with partial CSIT: A rate-splitting approach," *IEEE Trans. Commun.*, vol. 64, no. 11, pp. 4847–4861, Nov 2016.
- [11] —, "Robust transmission in downlink multiuser MISO systems: A rate-splitting approach," *IEEE Trans. Signal Process.*, vol. 64, no. 23, pp. 6227–6242, Dec 2016.
- [12] E. Piovano, H. Joudeh, and B. Clerckx, "Overloaded multiuser MISO transmission with imperfect CSIT," in *Proc. 50th Asilomar Conf. Sig*nals, Syst. Comput., Nov 2016, pp. 34–38.
- [13] E. Piovano and B. Clerckx, "Optimal DoF region of the K-user MISO BC with partial CSIT," *IEEE Commun. Lett.*, vol. 21, no. 11, pp. 2368–2371, Nov 2017.
- [14] C. Hao and B. Clerckx, "MISO networks with imperfect CSIT: A topological rate-splitting approach," *IEEE Trans. Commun.*, vol. 65, no. 5, pp. 2164–2179, May 2017.
- [15] M. Medra and T. N. Davidson, "Robust downlink transmission: An offset-based single-rate-splitting approach," in *Proc. IEEE Int. Workshop* Signal Process. Adv. Wireless Commun. (SPAWC), June 2018, pp. 1–5.
- [16] G. Lu, L. Li, H. Tian, and F. Qian, "MMSE-based precoding for rate splitting systems with finite feedback," *IEEE Commun. Lett.*, vol. 22, no. 3, pp. 642–645, March 2018.
- [17] S. Yang and Z. Lit, "A constant-gap result on the multi-antenna broadcast channels with linearly precoded rate splitting," in *Proc. IEEE Int. Workshop Signal Process. Adv. Wireless Commun. (SPAWC)*, June 2018, pp. 1–5.
- [18] M. Dai, B. Clerckx, D. Gesbert, and G. Caire, "A rate splitting strategy for massive MIMO with imperfect CSIT," *IEEE Trans. Wireless Commun.*, vol. 15, no. 7, pp. 4611–4624, July 2016.
- [19] A. Papazafeiropoulos, B. Clerckx, and T. Ratnarajah, "Rate-splitting to mitigate residual transceiver hardware impairments in massive MIMO systems," *IEEE Trans. Veh. Technol.*, vol. 66, no. 9, pp. 8196–8211, Sept 2017.
- [20] M. Dai and B. Clerckx, "Multiuser millimeter wave beamforming strategies with quantized and statistical CSIT," *IEEE Trans. Wireless Commun.*, vol. 16, no. 11, pp. 7025–7038, Nov 2017.
- [21] O. Kolawole, A. Papazafeiropoulos, and T. Ratnarajah, "A rate-splitting strategy for multi-user millimeter-wave systems with imperfect CSI," in *Proc. IEEE Int. Workshop Signal Process. Adv. Wireless Commun.* (SPAWC), June 2018, pp. 1–5.

- [22] H. Joudeh and B. Clerckx, "Sum rate maximization for MU-MISO with partial CSIT using joint multicasting and broadcasting," in *Proc. IEEE Int. Conf. Commun. (ICC)*, June 2015, pp. 4733–4738.
- [23] O. Tervo, L. Trant, S. Chatzinotas, B. Ottersten, and M. Juntti, "Multi-group multicast beamforming and antenna selection with rate-splitting in multicell systems," in *Proc. IEEE Int. Workshop Signal Process. Adv. Wireless Commun. (SPAWC)*, June 2018, pp. 1–5.
- [24] Y. Mao, B. Clerckx, and V. O. K. Li, "Rate-splitting for multi-user multi-antenna wireless information and power transfer," in *Proc. IEEE Int. Workshop Signal Process. Adv. Wireless Commun. (SPAWC)*, July 2019, pp. 1–5.
- [25] A. A. Ahmad, H. Dahrouj, A. Chaaban, A. Sezgin, and M. Alouini, "Interference mitigation via rate-splitting in cloud radio access networks," in *Proc. IEEE Int. Workshop Signal Process. Adv. Wireless Commun. (SPAWC)*, June 2018, pp. 1–5.
- [26] A. Papazafeiropoulos and T. Ratnarajah, "Rate-splitting robustness in multi-pair massive MIMO relay systems," *IEEE Transactions on Wireless Communications*, vol. 17, no. 8, pp. 5623–5636, Aug 2018.
- [27] Z. Zhang, X. Chai, K. Long, A. V. Vasilakos, and L. Hanzo, "Full duplex techniques for 5G networks: self-interference cancellation, protocol design, and relay selection," *IEEE Commun. Mag.*, vol. 53, no. 5, pp. 128–137, May 2015.
- [28] R. Dabora and S. D. Servetto, "Broadcast channels with cooperating decoders," *IEEE Trans. Inf. Theory*, vol. 52, no. 12, pp. 5438–5454, Dec 2006.
- [29] Y. Liang and G. Kramer, "Rate regions for relay broadcast channels," IEEE Trans. Inf. Theory, vol. 53, no. 10, pp. 3517–3535, Oct 2007.
- [30] Y. Liang and V. V. Veeravalli, "Cooperative relay broadcast channels," IEEE Trans. Inf. Theory, vol. 53, no. 3, pp. 900–928, March 2007.
- [31] J. Y. Ryu, E. Y. Ahn, and W. Choi, "User relaying in a two-user MIMO broadcast channel," in *Proc. IEEE Glob. Commun. Conf. (GLOBECOM)*, Dec 2011, pp. 1–5.
- [32] Z. Zhou and B. Vucetic, "A cooperative beamforming scheme in MIMO relay broadcast channels," *IEEE Trans. Wireless Commun.*, vol. 10, no. 3, pp. 940–947, March 2011.
- [33] S. Serbetli and A. Yener, "Relay assisted F/TDMA ad hoc networks: node classification, power allocation and relaying strategies," *IEEE Trans. Commun.*, vol. 56, no. 6, pp. 937–947, June 2008.
- [34] W. Mesbah and T. N. Davidson, "Power and resource allocation for orthogonal multiple access relay systems," EURASIP J. Advances in Signal Process., vol. 2008, no. 1, p. 476125, 2008.
- [35] R. Hasanizadeh and T. N. Davidson, "Jointly optimal power and resource allocation for orthogonal NDF relay systems with QoS constraints," in Proc. IEEE Int. Conf. Commun. (ICC), May 2010, pp. 1–5.
- [36] D. Gunduz and E. Erkip, "Opportunistic cooperation by dynamic resource allocation," *IEEE Trans. Wireless Commun.*, vol. 6, no. 4, pp. 1446–1454, April 2007.
- [37] O. Amin and L. Lampe, "Opportunistic energy efficient cooperative communication," *IEEE Wireless Commun. Lett.*, vol. 1, no. 5, pp. 412– 415, October 2012.
- [38] J. W. Heron and H. Sun, "Dynamic time and power allocation for opportunistic energy efficient cooperative relay," in *Proc. IEEE 86th Veh. Technol. Conf. (VTC Fall)*, Sep. 2017, pp. 1–5.
- [39] T. Chang, Z. Luo, and C. Chi, "Approximation bounds for semidefinite relaxation of max-min-fair multicast transmit beamforming problem," *IEEE Trans. Signal Process.*, vol. 56, no. 8, pp. 3932–3943, Aug 2008.
- [40] S. Shi, M. Schubert, and H. Boche, "Rate optimization for multiuser MIMO systems with linear processing," *IEEE Trans. Signal Process.*, vol. 56, no. 8, pp. 4020–4030, Aug 2008.
- [41] H. Joudeh and B. Clerckx, "Rate-splitting for max-min fair multigroup multicast beamforming in overloaded systems," *IEEE Trans. Wireless Commun.*, vol. 16, no. 11, pp. 7276–7289, Nov 2017.
- [42] A. Bletsas, A. Khisti, D. P. Reed, and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 3, pp. 659–672, March 2006.
- [43] A. Lim and V. K. N. Lau, "On the fundamental tradeoff of spatial diversity and spatial multiplexing of MISO/SIMO links with imperfect CSIT," *IEEE Trans. Wireless Commun.*, vol. 7, no. 1, pp. 110–117, Jan 2008.
- [44] Y. Fu, L. Yang, and W. Zhu, "A nearly optimal amplify-and-forward relaying scheme for two-hop MIMO multi-relay networks," *IEEE Commun. Lett.*, vol. 14, no. 3, pp. 229–231, March 2010.
- [45] S. Zhang and V. K. N. Lau, "Multi-relay selection design and analysis for multi-stream cooperative communications," *IEEE Trans. Wireless Commun.*, vol. 10, no. 4, pp. 1082–1089, April 2011.

- [46] B. Klaiqi, X. Chu, and J. Zhang, "Energy-efficient and low signaling overhead cooperative relaying with proactive relay subset selection," *IEEE Trans. Commun.*, vol. 64, no. 3, pp. 1001–1015, March 2016.
- [47] S. Boyd and L. Vandenberghe, Convex optimization. Cambridge University Press, 2004.
- [48] B. R. Marks and G. P. Wright, "A general inner approximation algorithm for nonconvex mathematical programs," *Oper. Res.*, vol. 26, no. 4, pp. 681–683, 1978.
- [49] W. Li, T. Chang, C. Lin, and C. Chi, "Coordinated beamforming for multiuser MISO interference channel under rate outage constraints," *IEEE Trans. Signal Process.*, vol. 61, no. 5, pp. 1087–1103, March 2013
- [50] A. Ben-Tal and A. Nemirovski, "On polyhedral approximations of the second-order cone," Math. Oper. Res., vol. 26, no. 2, pp. 193–205, 2001.
- [51] O. Tervo, L. N. Tran, and M. Juntti, "Optimal energy-efficient transmit beamforming for multi-user MISO downlink," *IEEE Trans. Signal Process.*, vol. 63, no. 20, pp. 5574–5588, Oct 2015.
- [52] S. S. Christensen, R. Agarwal, E. D. Carvalho, and J. M. Cioffi, "Weighted sum-rate maximization using weighted MMSE for MIMO-BC beamforming design," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 4792–4799, Dec 2008.
- [53] J. Lofberg, "YALMIP: a toolbox for modeling and optimization in MATLAB," in *Proc. CACSD Conf.*, 2004, pp. 284–289.