

Quality and Diversity in Evolutionary Modular Robotics

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Abstract—In Evolutionary Robotics a population of solutions is evolved to optimize robots that solve a given task. However, in traditional Evolutionary Algorithms, the population of solutions tends to converge to local optima when the problem is complex or the search space is large, a problem known as premature convergence. Quality Diversity algorithms try to overcome premature convergence by introducing additional measures that reward solutions for being different while not necessarily performing better. In this paper we compare a single objective Evolutionary Algorithm with two diversity promoting search algorithms; a Multi-Objective Evolutionary Algorithm and MAP-Elites a Quality Diversity algorithm, for the difficult problem of evolving control and morphology in modular robotics. We compare their ability to produce high performing solutions, in addition to analyze the evolved morphological diversity. The results show that all three search algorithms are capable of evolving high performing individuals. However, the Quality Diversity algorithm is better adept at filling all niches with high-performing solutions. This confirms that Quality Diversity algorithms are well suited for evolving modular robots and can be an important means of generating repertoires of high performing solutions that can be exploited both at design- and runtime.

Index Terms—modular robotics, quality diversity, NSGA-II, comparison

I. INTRODUCTION

For many real-world robotics problems knowing the correct design of the robot’s body and control system in advance can be a difficult challenge. Ideally, we would like the robot to adapt itself to the task, which in many environments could also necessitate a change in the robot’s morphology. Modular robots are a class of robots that are comprised of several modules, which in total make up the morphology of the robot. Such robots, in addition to advances within 3D-printing technology, such as better materials, higher speeds, and increased portability, could be capable of repairing and/or producing new parts in situ to adapt to different problems [1].

Evolutionary Robotics (ER) tries to solve the problem of automatic design and optimization through the use of Evolutionary Algorithms (EAs) which are population-based search algorithms inspired by natural evolution [2]. While ER is not

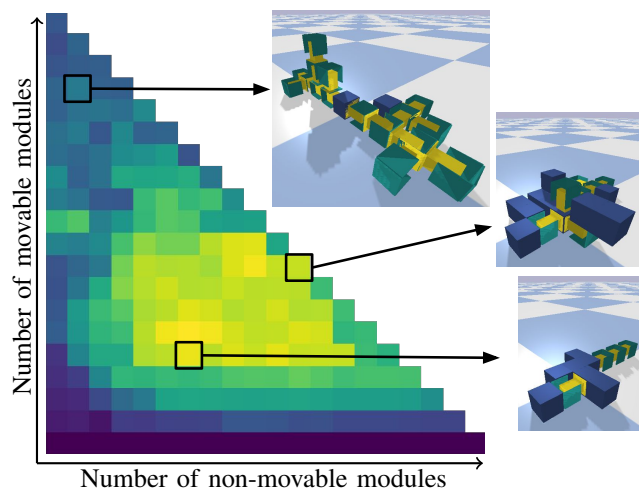


Fig. 1: Evolving a varied set of high performing modular robots utilizing the Quality Diversity paradigm.

solely focused on optimizing morphology and control, several prominent examples have shown that it is not only possible but also that the algorithms employed can exhibit a surprising amount of variation [3]–[5].

Several challenges exist when evolving morphology and control for modular robots. One challenge is how to encode the robot morphology when the topology is not fixed [6]. Another is the choice of control architecture which can range from simple open-loop wave generators [7], to interconnected pattern producing generators [8] and even more complex neural networks [3]. A third challenge is the problem of premature convergence, which is a challenge for all classes of EAs but is especially prominent in modular ER due to the large and complex search space resulting from the difficult and interconnected relationship of optimizing morphology and control simultaneously [3].

Overcoming premature convergence in modular ER require search algorithms that are not only capable of optimization, but

also exploration. In other words, the search algorithm needs to exhibit both quality and diversity. Different approaches to this challenge exist [9], such as utilizing the notion of heterogeneity to retain solutions based on attributes other than fitness [10]. Quality Diversity (QD) algorithms are a subset of these diversity promoting algorithms where explicit phenotypic feature descriptors are used to distinguish and group solutions [11]. QD algorithms have been shown to increase performance and maintain a diverse population when effective feature descriptors can be defined [12].

In this paper we compare the quality of evolved solutions of two diversity promoting search algorithms, the Multi-Objective Evolutionary Algorithm (MOEA) Non-dominated Sorting Genetic Algorithm-II (NSGA-II) [13] with diversity as additional objectives and Multi-dimensional Archive of Phenotypic Elites (MAP-Elites) [14], with a traditional single objective EA for the difficult task of evolving control and morphology in modular ER. Our goal is to understand which attributes of the different search algorithms contribute to the performance of the systems and to which capacity QD algorithms can be applied to modular ER challenges. Because of the explicit definition of feature descriptors utilized in MAP-Elites we believe this algorithm is well suited for modular ER as it should be capable of evolving a repertoire of diverse high performing morphologies. The repertoire should aid in overcoming premature convergence in addition to serving as a store that can be utilized at design or runtime to select the best morphology for the given task [15], [16]. To group solutions, we utilize the number of movable and the number of non-movable modules within the morphology as illustrated in Figure 1. The selection of morphological feature descriptors to distinguish solutions, both in MAP-Elites and NSGA-II, is a difficult challenge with many possibilities [17]. For the experiments carried out in this paper, we kept the descriptors simple to avoid confounding factors. By keeping the feature descriptors simple we focus on the search algorithm’s capacity for evolving diverse solutions and not on the feature descriptors themselves [6].

Our results show that the different search algorithms are capable of achieving the same performance for this modular ER locomotion task. However, the results also show that the addition of diversity measures in both NSGA-II and MAP-Elites greatly increases morphological diversity. The QD algorithm is additionally able to evolve a full repertoire of high performing solutions showing that QD algorithms are appropriate for modular robotics tasks.

The contribution of our paper is a comparison between a single objective EA and two diversity promoting EAs for the difficult problem of optimizing morphology and control in modular ER. In the comparison, we elucidate both the differences between the search algorithms in addition to comparing the differences in evolved populations.

II. BACKGROUND

In this section relevant background information about modular ER and QD algorithms will be described.

A. Modular Robotics

Evolving body and control for artificial creatures have a long history in the field of Artificial Life [18]. Modular robotics is distinguished from these virtual creatures by comprising the morphology of re-usable homogeneous or heterogeneous building blocks, called modules [7], [19]. Modular robots provide a way to effectively transition from simulation to reality since modules can be fabricated individually and then combined based on designs optimized in simulation [20].

One of the challenges with modular robotics is the interconnected relationship between control and morphology [3], [21]. To overcome this challenge many different approaches such as generative encoding [22] and different control architectures [8], [23] have been applied.

B. Quality Diversity

QD algorithms emerged from the realization that optimization through diversity can yield high performing solutions and, more importantly, can be better suited to exploring the whole problem space [24]. By focusing on phenotypic diversity, QD algorithms search the space of possible solutions without constraining the search to only finding better-fit solutions [11]. This separates QD algorithms from traditional MOEAs since Pareto dominated solutions can be kept as long as their phenotypic expression is sufficiently different from other solutions in the population [14].

An interesting property of QD algorithms is the capability to produce a repertoire of different solutions for the same problem [25]. The repertoire can be exploited, either at design time [16] or during operation [15], to select different solutions depending on the circumstances of the situation.

Although QD algorithms have been applied to the evolution of artificial creatures [26] and morphological descriptors have been used to evolve modular robots [6], [27] few examples exist applying the QD paradigm to modular robotics. This makes our contribution valuable, opening up a new application area for QD and introducing a way to generate a repertoire of possible morphologies within modular robotics that can later be experimented on in the real world.

III. METHODS

To compare the different search algorithms, experiments were carried out to evolve both the control and morphology of a modular ER system. The main objective of the search algorithms is to evolve modular robots that locomote across a plane as quickly as possible. To measure quality in the algorithms, solutions are compared based on fitness. To ensure that the comparison is as fair as possible the parameters for the individual algorithms were optimized in advance and the number of fitness evaluations is kept constant between the different search algorithms. The parameter search consisted of testing all permutations of combinations from table I for all three search algorithms, giving a total of 241 combinations to test.

In the next section, the modular robots with their morphological encoding and control system will be described.

Parameter	Values
Probability of morphological mutation	[0.05, 0.1, 0.2]
Crossover probability	[0.05, 0.1, 0.2]
Probability of controller mutation	[0.05, 0.1, 0.2]
Controller standard deviation	[0.01, 0.05, 0.1]

TABLE I: Description of parameters optimized through a pre-experiment parameter search.

Following that, the EAs will be described along with the parameters and experiment configurations.

A. Modular ER System

The morphological encoding employed is a tree-based direct encoding similar to [28]. The encoding allows for any combination of modules representable as a graph where each node in the graph is a module and each edge is a connection between two modules. For the experiments carried out in this paper two different modules were utilized, one non-movable rectangular module supporting 5 child modules and one servo module capable of moving one side back-and-forth and supporting 3 child modules [29], see Figure 1 for a small selection of robots. Each morphology starts with a single rectangular module as its root. To randomly initialize the morphology, a random size is selected between 1 and η (see Table II). Modules are then added to the tree at random locations until the size of the morphology equals the selected size.

The morphological encoding supports mutation- and crossover-operators. When mutating the morphology three possibilities exist 1) *Add a random module*. The tree is traversed and each available connection point is added as a possibility. A connection point is randomly selected along with a randomly selected module type before being inserted into the tree. 2) *Remove a module*. The tree is traversed adding all modules except the root into a list of candidates to remove. A module is randomly selected from the candidates before being removed along with any existing children. 3) *Mutate a module*. The two modules in use both support rotation around its connection axis and mutation will randomly select a new orientation in 90° increments. Note that only one of the three possibilities can happen per morphological mutation.

For crossover, a branch exchange is implemented. For both parent morphologies the tree is traversed adding all modules, except the root, to a list of candidates. A random candidate is selected from both morphologies before being exchanged. The candidate module, including its children, from the first morphology, is inserted into the place of the candidate from the second morphology and vice versa.

Lastly, the morphology is limited to a maximum size, η (see Table II), and a maximum depth δ , so that additional modules are not realized in the simulator. This limit ensures that morphologies do not grow unbounded and are feasible to simulate.

The control system of the modular robots is based on a decentralized wave pattern controller [7]. Each movable module in the morphology is given a controller which outputs

Parameter	Description	Value
η	Maximum module count	20
δ	Maximum module depth	4

TABLE II: Morphology parameters.

Parameter	Description	Range
θ	Set-point angle	$[-1.57, 1.57]$
α	Amplitude	$[-1.57, 1.57]$
ω	Frequency	$[0.2, 2]$
ϕ	Phase offset	$[-2\pi, 2\pi]$
o	Offset	$[-1.57, 1.57]$

TABLE III: Decentralized wave pattern controller parameters. The ranges are based on the real world servo used in the modules.

the desired angle of the joint, θ_i , according to the following equation

$$\theta_i = \alpha_i * \sin(\omega_i t + \phi) + o_i \quad (1)$$

where α_i is the amplitude, t is the time since the controller was initialized, ω_i is the frequency, ϕ_i is the phase offset and o_i is the amplitude offset for joint module i . The output, θ_i , is further limited to the minimum and maximum angle of the joint so that the control values do not exceed allowable set-points for the real-world equivalent. The parameters, and allowable range for each, are summarized in Table III.

Controllers are mutated separately from morphology and each parameter is perturbed using Gaussian noise, $\mathcal{N}(p, \sigma)$ where p is the mean and σ is the magnitude of the noise. The magnitude, σ , is specified for each search algorithm as a number in $[0, 1]$ and then scaled to the range of each parameter shown in Table III. To avoid mutating parameters outside the allowable range, a bounce-back function is applied according to the following equation

$$L(v, min, max) = \begin{cases} min + (min - v) & \text{if } v < min \\ max - (v - max) & \text{if } v > max \\ v & \text{otherwise} \end{cases} \quad (2)$$

where v is the parameter to mutate, min and max are the parameter's range taken from Table III. The effect of this function is to limit the parameters to their allowable range with a uniform distribution [30].

The fitness function used is based on the straight-line distance between the starting- and final position of the root module. To discourage optimization towards local optima, where the robot simply falls over, the selection of starting point is delayed in time so that early movement is discounted towards the total fitness. The distance calculation is only performed in the X and Y axis since we are mostly interested in distance on the surface plane. The evaluation parameters can be found in Table IV.

The feature descriptors used for NSGA-II and MAP-Elites are given as the tuple

$$b' = (m_i, j_i) \quad (3)$$

using the notation presented in [14], where m_i is the number of non-movable modules and j_i is the number of movable joint modules.

B. Evolutionary Algorithms

As a baseline a single objective EA is selected. The EA is (λ, μ) generational replacement strategy from [31] with tournament selection between two individuals. The EA has a single objective function that is set to the fitness function described in the previous section. Further configuration parameters can be found in Table IV. Note that the population size is selected to be similar to the maximum number of solutions in the MAP-Elites repertoire.

The first diversity promoting search algorithm is the MOEA — NSGA-II [13]. This search algorithm is used so that diversity metrics can be introduced into a purely optimizing MOEA [10]. The diversity metrics are based on the same morphological descriptors as MAP-Elites uses, but for NSGA-II to optimize these objectives they are recast as the sum of difference in diversity between individuals in the population, [26], according to the following equations

$$D(x) = \frac{1}{|P_n|} \sum_{y \in P_n} d(x, y) \quad (4)$$

$$d(x, y) = 1.0 - e^{-|(m_x, j_x) - (m_y, j_y)|} \quad (5)$$

where P_n is the population, x and y are solutions in P_n , m_i is the number of non-movable modules and j_i is the number of movable joint modules in the respective solutions. The output of equation 5 is in \mathbb{R}^2 giving a total of three dimensions to optimize with NSGA-II. Note that the diversity score is recalculated every time a change in the population occurs. It is also important to point out that equation 5 is altered compared to previous work [6], [26] as we experienced that the original equation leads to convergence in morphologies¹. The changes to the distance function, equation 5, weigh all changes to morphology equally which mitigates this convergence.

The last search algorithm used is MAP-Elites. This algorithm represents the QD paradigm and differs from traditional MOEAs in that the additional feature descriptors are not optimized, but rather differentiate solutions for storage in a repertoire. Central to the MAP-Elites algorithm is the notion of feature descriptors which are used as additional objectives for the search, however, these are not maximized nor minimized. As described before, we utilize morphological metrics as feature descriptors. We define the repertoire to contain individuals using the feature descriptors defined in equation 3. The range is set to the maximum number of modules, described in Table II, so that the repertoire potentially can contain any morphology representable within those limits, where the minimal morphology is simply the root module.

¹The convergence is most likely a result of the maximization of diversity, which leads to convergence at the morphological extremities.

Parameter	Applied to	Value
Evaluation time		20 seconds
Warm-up before start		2 seconds
Repetitions	Shared	30
Number of generations		500
Number of evaluations		100 000
Initial population size	EA NSGA-II MAP-Elites	200 1000
Population / Batch size	EA NSGA-II MAP-Elites	200
Selection	EA NSGA-II MAP-Elites	Tournament Uniform
Morphological mutation	EA NSGA-II MAP-Elites	0.2 0.05 0.2
Crossover rate	EA NSGA-II MAP-Elites	0.2 0.1 0.2
Controller mutation	EA NSGA-II MAP-Elites	0.2 0.2 0.1
Controller σ	EA NSGA-II MAP-Elites	0.05 0.1 0.1

TABLE IV: Experiment parameters.

IV. RESULTS

To understand the quality of the three search algorithms the maximum fitness in each repetition was recorded. In Figure 2a the trajectory of each algorithm is shown over evolutionary time and Figure 2b shows the full distribution of the last generation. From the fitness gradients it can be seen that the single objective EA more quickly finds fit solutions, while the two diversity promoting search algorithms take more time. We can also see that MAP-Elites has less variation across the different runs of the experiments.

When comparing the full distribution, shown in Figure 2b, through a pairwise Wilcoxon rank sum test with Holm [32] correction, statistical significant differences can be found between the single objective EA and NSGA-II and MAP-Elites and NSGA-II. Furthermore a Fligner-Killeen test of homogeneity of variances [33] shows statistical significant differences between the three algorithms. In other words, there is an observable difference in the mean and distribution of the three search algorithms.

Since we are interested in finding a diverse set of high-performing solutions and two of the search algorithms are able to utilize morphological descriptors to encourage diversity it is instructive to project the results along these dimensions. To project the population into the repertoires, as used in MAP-Elites, we create an empty repertoire at the start of evolution and insert solutions as they appear. For the single objective EA this means that solutions can be retained in the repertoire longer than it was kept in the population as long as the fitness is better than newer solutions with the same feature descriptors. Figure 3 shows a selection of these projections through time. The first row shows a single run, selected as the run closest to the median best in Figure 2b, exemplifying the

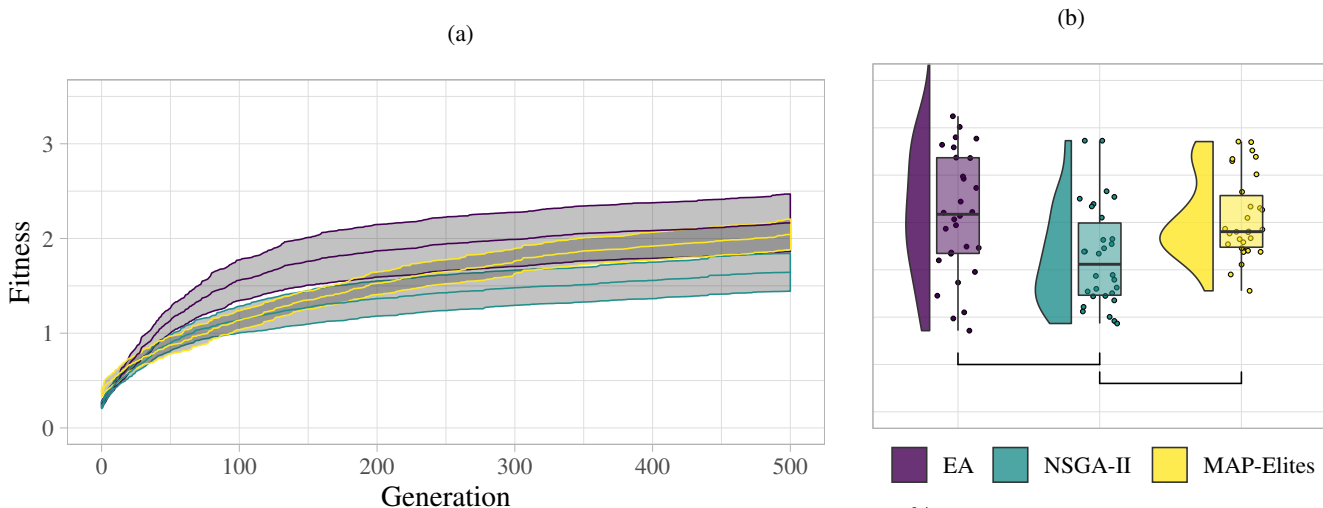


Fig. 2: Maximum fitness discovered in each run. In (a) the mean is shown with a 95% confidence interval while (b) shows the full distribution after the last evaluations. Statistical significant differences are noted in (b) using a Pairwise Wilcoxon Rank Sum test with Holm [32] correction.

output one could expect when running each search algorithm once. The next row shows a heatmap of the number of runs that found a solution for each morphological description. Finally the last row shows the average fitness of each morphological description where the solution is the cumulative best found in each run of the experiment. Note that the color gradient used changes scale. As can be seen from the heatmaps, MAP-Elites is able to fill out every niche in almost all runs. From the last row it can also be seen that MAP-Elites is effective, on average, at finding high performing solutions compared to the other search algorithms. When comparing NSGA-II and the single objective EA, we can see the effect of adding diversity measures to an optimizing EA, where NSGA-II is able to find a more diverse set of solutions.

The projections shown in Figure 3 can further be summarized through methods developed within the QD paradigm [11]. The QD-score calculation, shown in Figure 4, summarizes the total fitness of all solutions in the map projection according to the following equation

$$\text{QD - score}(m) = \sum_{x \in m} Q_x \quad (6)$$

where m is a projection, x is a solution in the projection and Q_x is the quality of solution x . The metric gives a good balance between exploring the search space and exploiting already found solutions and can be better at comparing QD algorithms than earlier *precision* and *coverage* plots. Figure 4 demonstrates that MAP-Elites is able to evolve a more diverse set of high-performing solutions compared to the other two search algorithms. For the single objective EA and NSGA-II the difference is not statistically significant which is interesting when compared to the projections in Figure 3. This shows that while NSGA-II is able to find more diverse solutions their performance are not enough to offset the better-performing solutions found through the single objective EA on the QD-

score metric. To elucidate this difference further, we have plotted cumulative coverage which shows the number of filled morphological niches, normalized to the maximum number of possible niches, in Figure 5. This figure, together with maximum fitness, shows the trade-off between finding diverse solutions, NSGA-II, and finding high performing solutions, EA.

Although Figure 3 gives an overview of how the different search algorithms unfold, it is not well suited to show the distribution of the morphologies in the population over time. In Figure 6 the full distribution of morphologies are shown. The figure is comprised of individual vertical bars that show the number of each type of module, where each bar is normalized to sum to one. The figure shows that MAP-Elites and NSGA-II are able to evolve diverse solutions in most runs, while the single objective EA focuses on fewer morphologies. Furthermore it can be seen that NSGA-II has more fluctuation in the population over time compared to MAP-Elites. Lastly, it is interesting to note that the single objective EA is able to rediscover morphologies, which can be seen as bands of colors that appear, vanish and then re-appear throughout the search.

V. DISCUSSION

The results for the locomotion task in this paper show that both the single objective EA and MAP-Elites are capable of promoting the same quality. While NSGA-II did not achieve the same fitness, the difference was not immense and can presumptively be attributed to the number of dimensions to optimize and limitations of the initial parameter sweep. This shows that for simple locomotion, all three search algorithms can be useful tools in modular ER.

When looking at the evolution of fitness over time, illustrated in Figure 2a, we can see that fitness of the single objective EA quickly grows in contrast with the two other

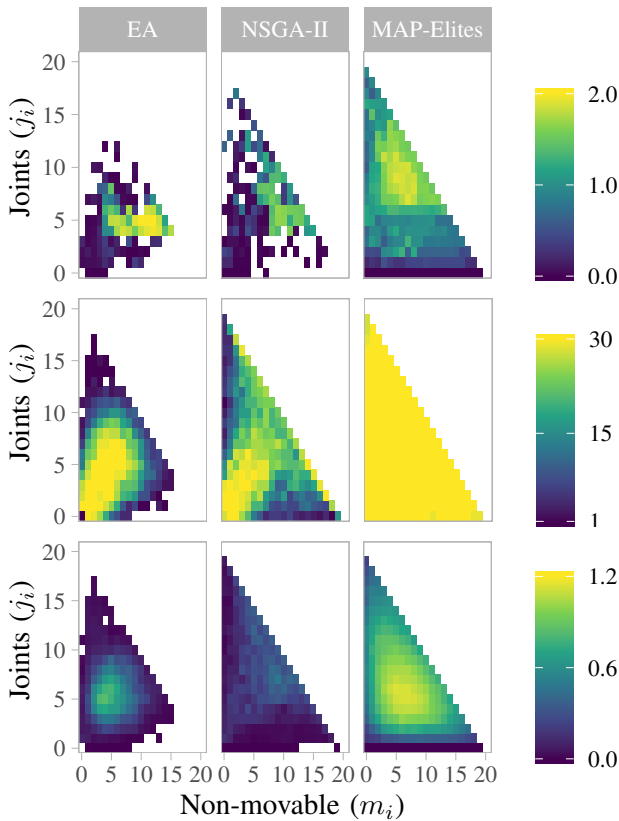


Fig. 3: Projection of the solutions found for each morphological description throughout the experiment. The top row shows a single representative run to give an impression of the expected outcome. The next row shows the number of runs that found a solution for the given morphological description. The last row shows the mean fitness of the best individuals over all runs.

algorithms. This is in line with previous results comparing EAs and QD algorithms where QD algorithms tend to have slower growth [16]. In the results shown here the growth is likely due to the relatively easier task of finding new morphological niches to occupy compared to increasing fitness of already discovered solutions. If we take the number of filled niches, shown in Figure 5, into account we can see the rapid increase in filling out new niches taking place early in the search for MAP-Elites further reinforcing this explanation. This could indicate that the MAP-Elites search could benefit from adding *curiosity* [25] or *dynamic mutation* [34]. From Figure 2a it can also be seen that MAP-Elites has a lower between-run variance than the two other algorithms which indicates that MAP-Elites more consistently find high performing solutions.

In addition to the focus on fitness, it can be beneficial to encourage diversity to avoid premature convergence and increase robustness to noise. Figure 3, Figure 4, and Figure 5 explored how the three search algorithms compared when the populations are projected into a grid of the two morphological descriptors and how these projections can be summarized.

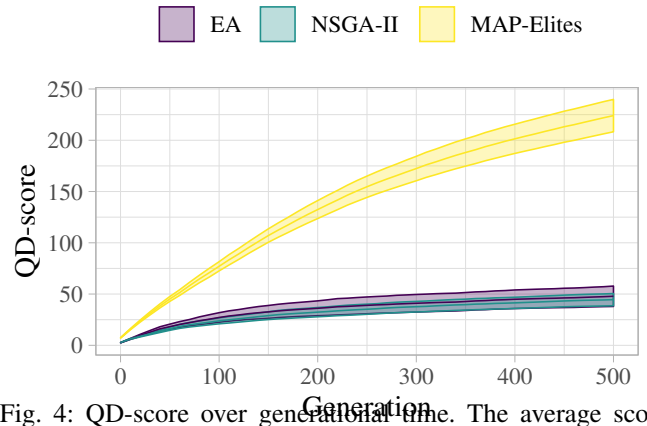


Fig. 4: QD-score over generational time. The average score is shown together with a 95% confidence interval. Pairwise Wilcoxon Rank sum testing shows significant differences between MAP-Elites and the two other search algorithms with $p < 2e^{-16}$ in the final generation.

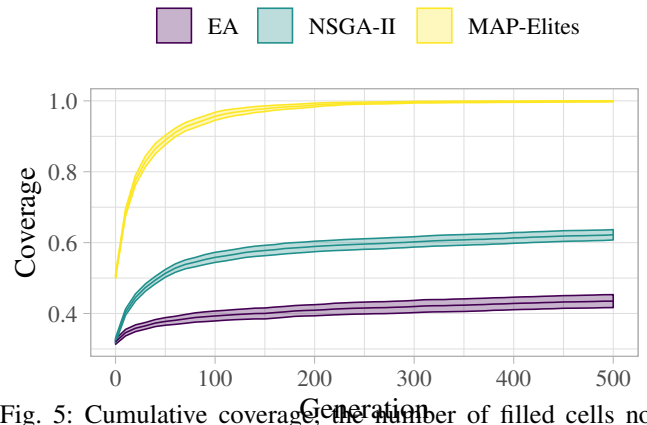


Fig. 5: Cumulative coverage, the number of filled cells normalized to the maximum possible filled, throughout evolution. The mean and a 95% confidence interval is shown. The difference between all search algorithms in the last generation is statistically significant according to a Wilcoxon Rank Sum test using Holm correction.

The projections show that MAP-Elites is better at exploring the search landscape of different morphologies, finding nearly every possible morphology in all repetitions of the search. While NSGA-II is able to find every expression across all runs, as can be seen in the middle row of Figure 3, it did not manage the same reliability per run as MAP-Elites. This is most likely caused by the complex Pareto front which NSGA-II maintains in its population, which works well for optimization problems, but is more difficult to apply to explicit morphological diversity measures. It is also possible that NSGA-II’s diversity maintenance conflicts with morphological diversity resulting in a slower search.

To understand how the population evolves in each search algorithm we plotted the distribution of morphologies in Figure 6. From this plot, it can be seen that both MAP-Elites

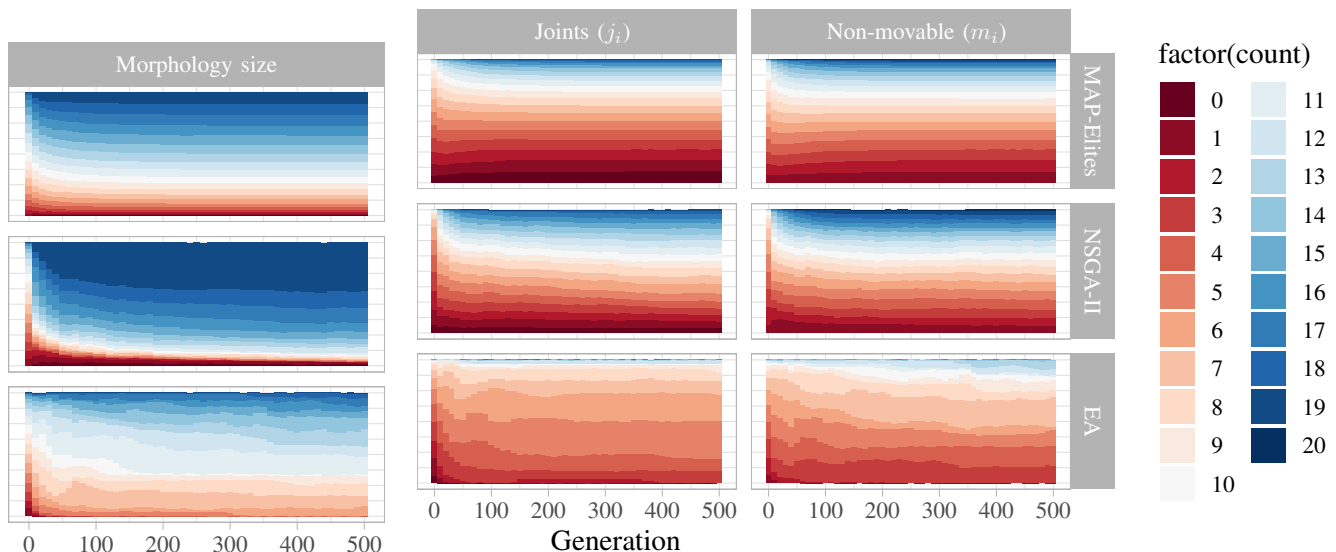


Fig. 6: Distribution of modules in the population over time separated by search algorithm. On the left the full size of the modular robots is displayed while the right shows the size separated into the two different module types. The figure is a sequential collection of vertical histogram bars that show the normalized distribution of modules at intervals during the search. The underlying data is collected from all 30 runs.

and NSGA-II quickly fills out all morphological niches. The same trend was seen in Figure 5 where both quickly converge to their respective maximum in contrast to fitness, Figure 2a, which slowly grows throughout the experiment. In comparison we can see that NSGA-II has more variation across morphologies and the focus of the search is on larger morphologies, both in terms of joints and non-movable modules. As one would expect, the single objective EA focuses on fewer morphologies over evolutionary time with some combination vanishing and re-appearing. The reduction in diversity for the single objective EA could be a sign of premature convergence. In contrast, both MAP-Elites and NSGA-II do not converge to a few solutions which could be a sign of a more sound search.

The next step for this research is to more closely look into why MAP-Elites is able to evolve both high quality and diverse solutions. Early indications point to the idea of *stepping-stones* where MAP-Elites, due to the elitist definition of the search, is capable of retaining and promoting better solutions that further the search [35]. By understanding the genealogy of the search we hope to generate objective statistics that can discern such details and give a better understanding of MAP-Elites.

To extend on the foundation of modular robotics, experiments including QD algorithms and indirect encodings would be a logical extension of this work. Additionally, testing different control schemes and morphological - behavioral metrics could improve the results even further.

VI. CONCLUSION

In this paper we compared a single objective EA with two diversity promoting search algorithms, a MOEA and a QD algorithm, on their capacity for evolving a diverse set of high performing solutions over time on the difficult task of

optimizing morphology and control in modular robotics. The result shows that the different algorithms have nearly the same capacity for quality, however, morphological diversity can be greatly improved, without affecting the maximum fitness obtained, by utilizing morphological descriptors to aid the search. The results also show that the method of applying morphological descriptors can impact performance and MAP-Elites, due to its simplicity, is well suited for application in modular robotics, achieving both high fitness and large diversity.

The work in this paper is a supplement both to the work on modular robotics and application areas for QD algorithms. By demonstrating that both high fitness as well as large diversity can be promoted simultaneously, future research on modular robotics can evolve repertoires of morphologies that can be exploited for different purposes. For QD this work opens up a new application domain in which rapid exploration of real-world robotics can be experimented with.

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