# Controller Design for Trajectory Tracking of Autonomous Passenger Vehicles

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***Abstract*— This paper presents controllers design procedure for dynamic trajectory tracking of a highly automated vehicle. The main objective is to follow the planned trajectories generated by a co-pilot module in the safe way despite the presence of vehicle model uncertainties and also to guarantee a passenger comfort by generating soft actions on the steering wheel and accelerations. A decoupled design approach of longitudinal and lateral controller is adopted. For the longitudinal controller, a proportional including a feedforward terms is adopted. On the other hand, an adaptive backstepping approach is used in lateral case to deal with model nonlinearities and parameter uncertainties. The developed controller is integrated and tested in simulation environment. Performance of this controller are presented to demonstrate the effectiveness of the proposed controllers.**

## NOMENCLATURE

| | |
|---|---|
| $u/v$ | Longitudinal/lateral vehicle speeds |
| $m$ | Vehicle mass |
| $I_z$ | Inertia moment about the yaw axis through the vehicle center of gravity |
| $l_f/l_r$ | Distances of the front and rear tires from vehicle's center of gravity |
| $c_f/c_r$ | Cornering stiffness of the front and rear tires |
| $\delta_f$ | Steering angle |
| $l_s$ | Distance from the vehicle center of gravity to the vehicle mounted sensor |

## I. INTRODUCTION

Automotive has motivated a large scientific community to bring out solutions to arising problems. Indeed, ground vehicles are the most used means of transport for people traveling for short or long distances and safety is a critical issue to be addressed. To enhance passenger safety, several passive and active systems have been developped and integrated in serial passenger cars such as airbags, antilock braking systems, electronic stability systems, adaptive cruise control systems ... etc. With the development on the electronic devices for automobile, more sophisticated and intrusive safety systems like lane keeping assistance systems are developed and proposed (e.g. toyota Prius) to handle the vehicle when the driver's vigilance decreases and the probability to lane departure becomes important, [1]. Recently, some projects like the European project HAVEit (highly automated vehicle for intelligent transport) proposes a new vision of future cars with a virtual co-pilot that can share the driving task with the driver or completely substitute him, [2], [3]. As it will be presented in the sequel of this paper, the co-pilot is able to analyze the environment, to decide which maneuver is appropriate to bring the vehicle to a safe state, and interpret this decision by generating safe and feasible trajectories [3], [4]. All these tasks are executed in a short time in order to handle environment changes. The present paper is devoted to the design of controllers for ground vehicle trajectories tracking. The adopted approach is based on the separation between the longitudinal and lateral dynamics. This methodology involves several hypotheses, to reduce the complexity of vehicle's dynamics. For longitudinal case, a proportional on the velocity error with a feedforward desired acceleration term controller is implemented to control the vehicle velocity. The lateral controller is more complex and parameter dependent, which needs a more robust technique. An adaptive backstpping technique is adopted for the lateral controller synthesis.

The problem considered, in this paper is the design of a decoupled lateral and longitudinal controllers that enables the vehicle to track the reference trajectory by generating suitable steering angle, while maintaining a desired speed by through of throttle or braking .The present paper is organized as follows: section 2 consists in the vehicle modeling which makes appear the longitudinal and lateral dynamics and the yaw motion of the vehicle. Section 3 is focused on the problem statements. Section 4 deals with the synthesis of longitudinal and lateral controllers where the first one is based on proportional with feedforward action controller; the second one is based on adaptive backstepping technique. While the validation of this technique by some simulation scenarios of a driving is exposed in the section 5. Conclusion and perspective of this work are presented in the section 6

## II. HIGHLY AUTONOMOUS VEHICLE ARCHITECTURE

Highly automated vehicle consists of three principal layers, perception, decision and control, see Fig 1. The objective of the first is to assess the environment surrounding the vehicle and the driver state. This information is needed to make an appropriate decision in order to adapt the vehicle to its environment. Several sensors such as cameras, laser scanners and odometers are embedded in the vehicle where the provided signals are treated in the data processing module to extract information about the lanes, the obstacles in the area surrounding the vehicle and also the driver state.

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Based on this information, the decision module evaluates the situation and provides to the control module a trajectory that brings vehicle in a safe state. Besides, in robotics, traditional trajectory planning is time consuming and can not be integrated into applications that have fast environment changes and strong real time constraints. To solve this, authors in [3] have split the problem into two layers. The first one is a high level description decision in terms of maneuvers. The output of this layer is a grid of possible maneuvers combining longitudinal and lateral actions, i.e {change lane right, stay in current lane, change lane left} $\times$ {accelerate, keep speed, decelerate}. Other maneuvers are integrated to prevent failure safty such as {emmergency, minimum risk }. The best maneuver is selected by evaluating some risk indicators for each maneuver, e.g. collision risk. From the resulting maneuver and environment information, the lower level description of trajectory generates a set of trajectories ordered with respect to a combination of performance indicators such as: slipping risk, fuel consumption, comfort, traffic rule ...etc. The resultant trajectory is represented by a table of 40 points with the longitudinal, lateral positions and the desired velocity at each point. Finally, the last module contains the control algorithms that bring the vehicle to follow the planned trajectory. This issue constitutes the main subject of this paper.

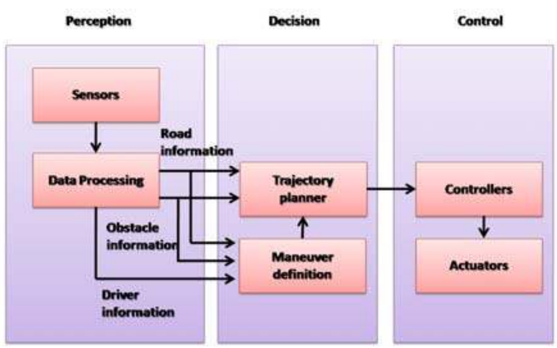


Fig. 1. Highly autonomous vehicle architecture

## III. Problem Statement

As was previously stated, the role of co-pilot module is to assess the environment and the driver states and computes a safe trajectories to ensure an adequate interaction of the vehicle with its environment. The computed trajectories are described by a list of longitudinal position, lateral position and velocity where each element is indexed by the time. In the current implementation, the trajectories are generated locally in the vehicle fixed frame and the co-pilot module is executed with the same frequency as the controller module. This constrains the trajectories to start always from the origin of the vehicle fixed frame (vehicle's center of gravity). In this case, it is convenient to control for the lateral behavior, not the center of gravity of vehicle but a point located at some distance ahead of the vehicle to track the planed trajectories, see Fig 2. Another problem arises from the trajectories planer is the trajectories discontinuity due to maneuver module state chaining. This fact can affect the controller and generate an abrupt steering wheel actions that effect passenger comfort. Furthermore for the longitudinal part, at this stage of the development a focus is made to control vehicle's longitudinal velocity to track a planned profile.

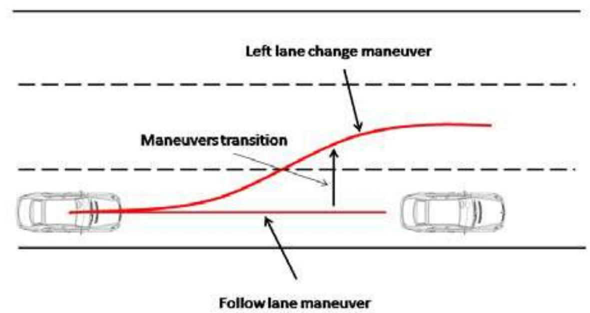


Fig. 2. Maneuvers transition discrete state

## IV. Control Design

In order to simplify controllers design, decoupling approach is adopted to control separately vehicle's longitudinal and lateral behaviors. The longitudinal controller is devoted to control the vehicle speed and the lateral controller is devoted to control the lateral offset and the relative heading between the planed trajectories and vehicle. In the sequel of this section we develop the controller design approach for both longitudinal and lateral dynamics.

### A. Vehicle speed controller

The longitudinal controller is designed in such way to control a vehicle speed profile. In this case we assume that vehicle acceleration/deceleration are controlled by lower level controllers acting directly on throttle and brake pedals. In this case, the longitudinal model can be written as:

$$a_x = F \tag{1}$$

where $a_x$ is vehicle's longitudinal acceleration and $F$ is velocity controller output. In this case, velocity controller provides a requested acceleration/deceleration with respect to the error between the vehicle and desired speed. For this purpose a proportional controller with respect to the velocity error and a feedforward term is implemented:

$$F = a_d - K_v(v - v_d) \tag{2}$$

where $a_d$ and $v_d$ are respectively the desired acceleration and velocity, $v$ is vehicle velocity. $K_v$ is the controller parameter that can be chosen appropriately to get a satisfactory velocity tracking behavior.

### B. Lateral controller

*1) Vehicle lateral model:* For vehicle's lateral control design, a linear "bicycle model" with two degrees of freedom is considered. The two degrees of freedom are represented by the vehicle lateral position $y$ and the vehicle yaw angle $\psi$ of a lookahead point situated at a front distance of $l_s$ from the vehicle center of gravity, see Fig 3.

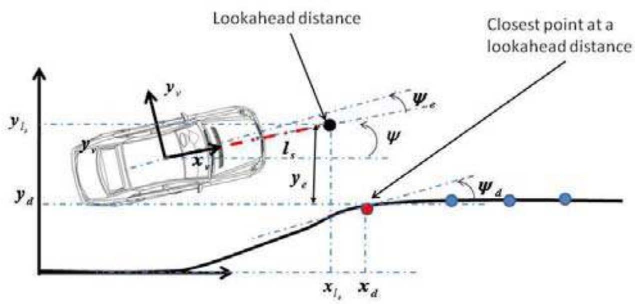


Fig. 3. Vehicle configuration

The only control input to the vehicle lateral model is the steering angle of the front wheels $\delta_f$.
Lateral vehicle dynamics is non-linear and complex to be used for lateral controller design, [7]. Several assumptions are needed to be taken into account in order to simplify the model which they can be summarized as follows:

- The road is supposed plane with neither bank angle nor road slope.
- The vehicle is moving, e.g. $u \neq 0$.
- The pitch, the roll and the vertical dynamics are neglected.
- The vehicle side slip angle is small.

Defining the state space vector for lateral steering dynamics as $x = [y_{l_s}, \psi, v, r]^T$, where $y_{l_s}$ is the vehicle lookahead lateral position and $r$ is the vehicle yaw rate, see Fig 3. Based on the previously announced assumptions, the vehicle lateral model related to the trajectory frame can be expressed as [8]:

$$\begin{cases} \dot{y}_{l_s} = \sin(\psi)u + \cos(\psi)v + l_s\cos(\psi)r \\ \dot{\psi} = r \\ \dot{v} = \dfrac{a_{11}}{u}v + \dfrac{a_{12}}{u}r - ru + b_1\delta_f \\ \dot{r} = \dfrac{a_{21}}{u}v + \dfrac{a_{22}}{u}r + b_2\delta_f \end{cases} \quad (3)$$

where:

$$a_{11} = -\frac{2\left(c_r - c_f\right)}{m}, \qquad a_{12} = \frac{2(l_r c_r - l_f c_f)}{m},$$
$$a_{21} = \frac{2\left(l_r c_r - l_f c_f\right)}{I_z}, \qquad a_{22} = -\frac{2(l_r^2 c_r - l_f^2 c_f)}{I_z},$$
$$b_1 = \frac{2c_f}{m}, \qquad b_2 = \frac{2l_f c_f}{I_z}$$

*2) Lateral controller design:* The controller's objective is to minimize the lateral offset between the lookahead point and the trajectory. Let we define the lateral offset as:

$$y_e = (y_{l_s} - y_d) \quad (4)$$

define the scalar positive definite function as:

$$V_1 = \frac{1}{2}y_e^2 \quad (5)$$

The derivation of this function with time leads to:

$$\dot{V}_1 = y_e\left(\dot{y}_{l_s} - \dot{y}_d\right) \quad (6)$$

replacing $\dot{y}_{l_s}$ by its expression from (3):

$$\dot{V}_1 = y_e\left(\sin(\psi)u + \cos(\psi)v + l_s\cos(\psi)r - \dot{y}_d\right) \quad (7)$$

By tacking the yaw rate $r$ as a virtual control input, the stabilizing control law can be proposed as follows:

$$r_d = \alpha_r = \frac{1}{l_s cos(\psi)}\left(-\sin(\psi)u - \cos(\psi)v + \dot{y}_d - K_{y_e}y_e\right) \quad (8)$$

Let we define $\xi$ as a deviation of $r$ from its desired value $r_d$ as:

$$\xi = r - \alpha_r \quad (9)$$

Differentiating the previous equation, the dynamic of $\xi$ can be expressed as:

$$\begin{aligned} \dot{\xi} &= \dot{r} - \dot{\alpha}_r \\ &= \left(\frac{a_{21}}{u}v + \frac{a_{22}}{u}r + b_2\delta_f - \dot{\alpha}_r\right) \end{aligned} \quad (10)$$

Where

$$\begin{aligned} \dot{\alpha}_r = &-\tan(\psi)\alpha_r + \frac{1}{l_s\cos(\psi)}\{-\dot{u}\sin(\psi) - u\dot{\psi}\cos(\psi) \\ &- \dot{v}\cos(\psi) + v\dot{\psi}\sin(\psi) + \ddot{y}_d - K_{y_e}\dot{y}_e\ \} \end{aligned} \quad (11)$$

Or in compact form:

$$\dot{\alpha}_r = \lambda - \frac{\dot{v}}{l_s}. \quad (12)$$

where

$$\begin{aligned} \lambda = &-\tan(\psi)\alpha_r + \frac{1}{l_s\cos(\psi)}\{-\dot{u}\sin(\psi) - u\dot{\psi}\cos(\psi) \\ &+ v\dot{\psi}\sin(\psi) + \ddot{y}_d - K_{y_e}\dot{y}_e\ \} \end{aligned} \quad (13)$$

The objective now is to stabilize the system in the $(y_{l_s}, \xi)$ coordinates. We need to select a Lyapunov function candidate by augmenting $V_1$ with a quadratic term in the error variable $\xi$:

$$V_2 = V_1 + \frac{1}{2}\xi^2 \quad (14)$$

The derivative of $V_2$ is computed as:

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + \xi\dot{\xi} \\ &= y_e\left(\dot{y}_{l_s} - \dot{y}_d\right) + \xi\left(\frac{a_{21}}{u}v + \frac{a_{22}}{u}r + b_2\delta_f - \dot{\alpha}_r\right) \end{aligned} \quad (15)$$

From equation (9) we can write $r = \xi + \alpha_r$. Replacing this last equation in (15), we can write:

$$\dot{V}_2 = -k_e y_e^2 + \xi \left( \frac{a_{21}}{u} v + \frac{a_{22}}{u} r + b_2 \delta_f + l_s y_e cos(\psi) - \dot{\alpha}_r \right) \tag{16}$$

the expression of $\dot{V}_2$ can be rewritten as:

$$\begin{aligned} \dot{V}_2 =& -k_e y_e^2 + \xi \left( \frac{a_{21}}{u} v + \frac{a_{22}}{u} r + b_2 \delta_f + a_{11} \frac{v}{l_s u} \right. \\ & \left. + a_{12} \frac{r}{l_s u} - \frac{u}{l_s} r + \frac{b_1}{l_s} \delta_f + l_s y_e cos(\psi) - \lambda \right) \\ =& -k_e y_e^2 + \xi \left( \left( \left( \frac{a_{11}}{l_s} + a_{21} \right) \frac{v}{u} + \left( \frac{a_{12}}{l_s} + a_{22} \right) \frac{r}{u} \right. \right. \\ & \left. \left. - \frac{u}{l_s} r + \left( \frac{b_1}{l_s} + b_2 \right) \delta_f + y_e cos(\psi) - \lambda \right) \right) \end{aligned} \tag{17}$$

in this case the control law that make $\dot{V}_2$ negative can be formulated as follows:

$$\begin{aligned} \delta_f =& \frac{1}{\frac{b_1}{l_s} + b_2} \left( -k_\xi \xi - \left( \left( \frac{a_{11}}{l_s} + a_{21} \right) \frac{v}{u} \right. \right. \\ & + \left( \frac{a_{12}}{l_s} + a_{22} \right) \frac{r}{u} - \frac{u}{l_s} r + y_e cos(\psi) - \lambda)) \end{aligned} \tag{18}$$

### C. Lateral control law robustification

In the previous section, we have proposed a static control for lateral vehicle steering assuming that vehicle parameters are constant and known. In real situation, these parameters are varying for several causes. In order to overcome this situation and to robustify the lateral control law, an estimation module for these parameter is added. In this paper, we have considered only the variation of the $a_{ij}$ parameters. Further work will be take into account the variation of $b_i$ parameters. Let $\hat{a}_{ij}$ be an estimation of $a_{ij}$. The estimation error can be defined as:

$$\tilde{a}_{ij} = a_{ij} - \hat{a}_{ij} \tag{19}$$

Let $\theta = [\tilde{a}_{11}, \tilde{a}_{12}, \tilde{a}_{21}, \tilde{a}_{22}]^T$ be the parameter estimation error vector. To design the adaptation law for $a_{ij}$, The Lyapunov function $V_2$ in (14) is augmented with a quadratic term in the parameter $\tilde{\theta}$:

$$V_2 = V_1 + \frac{1}{2}\xi^2 + \frac{1}{2\gamma}\tilde{\theta}^2 \tag{20}$$

Differentiating the last equation with respect to time:

$$\dot{V}_2 = \dot{V}_1 + \xi \dot{\xi} + \frac{1}{\gamma} \tilde{\theta} \dot{\tilde{\theta}} \tag{21}$$

by replacing the control law with the estimated parameter into the previous equation we can get:

$$\begin{aligned} \dot{V}_2 =& -k_e y_e^2 - k_\xi \xi^2 + \xi \left( \left( \frac{\tilde{a}_{11}}{l_s} + \tilde{a}_{21} \right) \frac{v}{u} \right. \\ & \left. + \left( \frac{\tilde{a}_{12}}{l_s} + \tilde{a}_{22} \right) \frac{r}{u} \right) + \frac{1}{\gamma} \tilde{a}_{11} \dot{\tilde{a}}_{11} + \frac{1}{\gamma} \tilde{a}_{12} \dot{\tilde{a}}_{12} \\ & + \frac{1}{\gamma} \tilde{a}_{21} \dot{\tilde{a}}_{21} + \frac{1}{\gamma} \tilde{a}_{22} \dot{\tilde{a}}_{22} \\ =& -k_e y_e^2 - k_\xi \xi^2 + \tilde{a}_{11} \left( \frac{1}{\gamma} \dot{\tilde{a}}_{11} + \xi \frac{v}{l_s u} \right) \\ & + \tilde{a}_{12} \left( \frac{1}{\gamma} \dot{\tilde{a}}_{12} + \xi \frac{r}{l_s u} \right) + \tilde{a}_{21} \left( \frac{1}{\gamma} \dot{\tilde{a}}_{21} + \xi \frac{v}{u} \right) \\ & + \tilde{a}_{22} \left( \frac{1}{\gamma} \dot{\tilde{a}}_{22} + \xi \frac{v}{u} \right) \end{aligned} \tag{22}$$

To make the Lyapunov function $V_2$ decreasing, it is adequate to cancel the estimation errors factors which leads to the following estimation errors dynamics:

$$\begin{cases} \dot{\tilde{a}}_{11} = -\gamma \xi \frac{v}{l_s u} \\ \dot{\tilde{a}}_{12} = -\gamma \xi \frac{r}{l_s u} \\ \dot{\tilde{a}}_{21} = -\gamma \xi \frac{v}{u} \\ \dot{\tilde{a}}_{22} = -\gamma \xi \frac{r}{u} \end{cases} \tag{23}$$

The adaptation laws for the parameters' estimator can be done by:

$$\begin{cases} \dot{\hat{a}}_{11} = \gamma \xi \frac{v}{u} \\ \dot{\hat{a}}_{12} = \gamma \xi \frac{r}{u} \\ \dot{\hat{a}}_{21} = \gamma l s \xi \frac{v}{u} \\ \dot{\hat{a}}_{22} = \gamma l s \xi \frac{r}{u} \end{cases} \tag{24}$$

## V. Simulation Results

Controllers' performances are tested in simulation environment developed in the HaveIt European project used as a validation tool. The track contains three lanes and three vehicle are used as obstacles. The scenario starts with the vehicle in the middle lane. When there is no obstacle, the trajectory planner requests to vehicle to go to the right lane, see Fig 4. The second part of this scenario consists in the presence of two obstacles. The first one located in the same lane of the vehicle and the second one in the most left lane. In this situation, the vehicle overtakes the obstacle present in its lane by going to the middle lane. The presence of a second obstacle in the most left lane brings the vehicle to reduce its velocity to a prescribed value, $3.6(m/s)$ see Fig 7, and overtake it by the right, see Fig 5. In the third part of this scenario, after it performs the overtaking maneuver, the vehicle returns to the most right lane and accelerate before detecting an other obstacle in the middle lane which bring the vehicle to reduce its velocity to $3.6(m/s)$, see Fig 7, and overtaking this obstacle in the right, see Fig 6. Lastly we have requested to the vehicle to slowdown until standstill. Fig 8 and 9 illustrate the lateral and heading angle errors.

They show a convergent behavior with peaks corresponding to the moment where the maneuver changes from keep lane to change lane. Fig 10 shows the profile of the controller output. To evaluate if the controller fulfills the passenger comfort, vehicle's lateral acceleration does not exceed a limit value chosen in this paper by $0.2g$, [9], [10]. In Fig 11 we have sketched the lateral acceleration of the vehicle during the previous scenario. It is obvious that, this controller preserves the passenger's comfort for the previous scenario by producing a maximum value of lateral acceleration of $0.1g$.

**vehicle's parameters used in the simulation:**

$c_f = 63000, \quad c_r = 63000,$
$l_f = 1.014, \quad l_r = 1.676,$
$Iz = 2741, \quad M = 1750.$

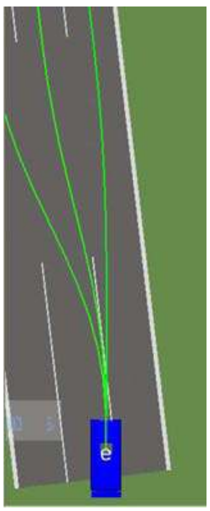


Fig. 4. Lane change maneuver the right lane

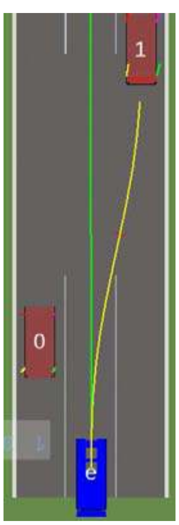


Fig. 5. Overtake and slowdown maneuver

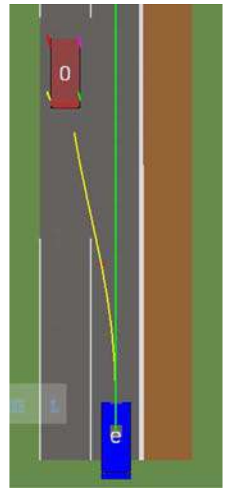


Fig. 6. Slowdown maneuver

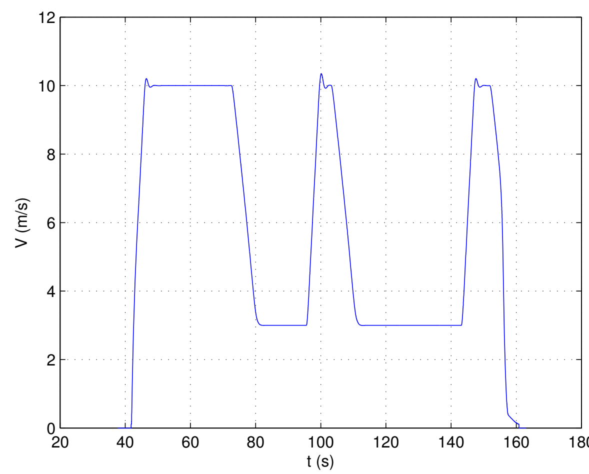


Fig. 7. Vehicle Velocity profile

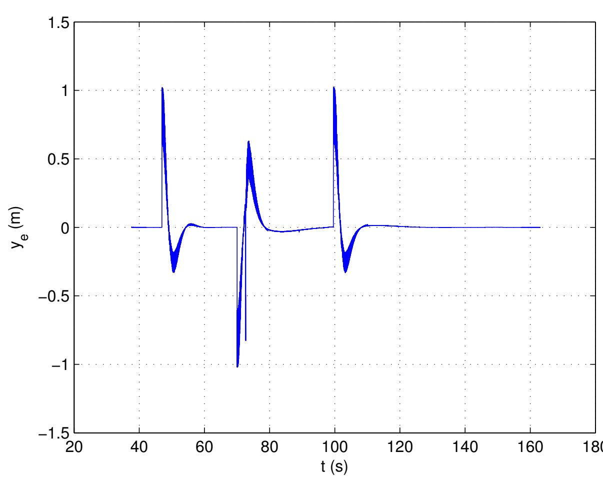


Fig. 8. Lateral offset profile

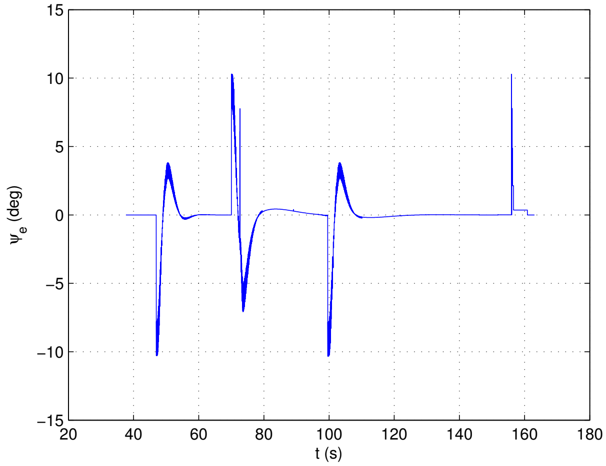


Fig. 9. Heading angle offset profile

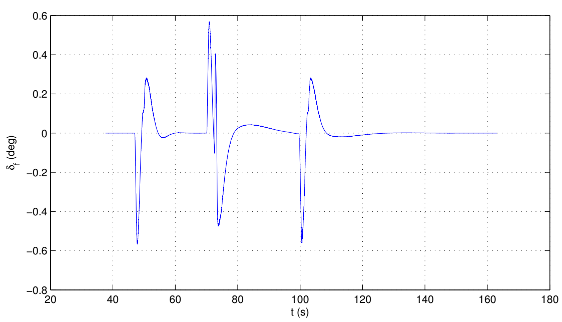


Fig. 10. Lateral controller output

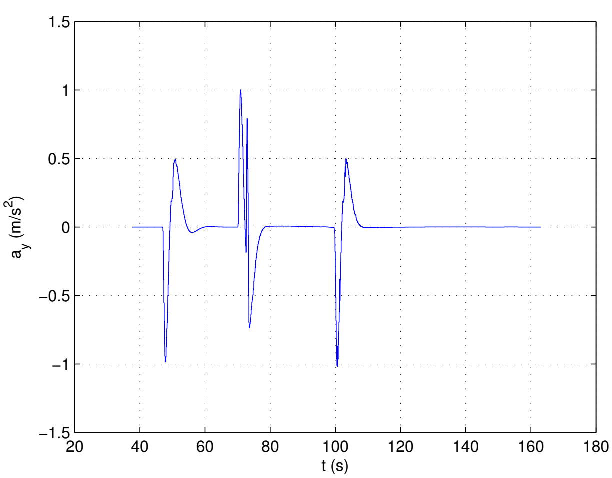


Fig. 11. Lateral acceleration profile

## VI. Conclusion

In this paper we have proposed a decoupled longitudinal and lateral controllers. For the longitudinal part a proportional on the velocity error including a desired acceleration feedforward term is proposed. Besides, the case of controlling the lateral dynamics seems to be more complicated that integrate a nonlinearities and parameters uncertainties. In this case, an Adaptive Backstepping technique is used to derive lateral control law. Both controller are implemented in the simulation environment using C code. A combination of lane keeping and overtaking in the left and right scenario is used to validate the controllers which show a good tracking performances and comfort preserving. In future work, an experimental validation on a test vehicle will be tested.

## VII. Acknowledgments

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