Multi-component system maintenance optimisation and masked data

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***Abstract*— The paper discusses he maintenance optimization of a multi-component system. The exact cause of failure is unknown and only some information about it is available. Opportunistic perfect preventive maintenance is considered and optimized using cost criteria. Numerical implementations illustrate the applicability of results.**

I. INTRODUCTION

Maintenance planning for multi-component systems is an important issue and not easy to address because of economic, structural and/or failure dependency between components [5], [6]. Maintenance policy optimization depends on the available information at our disposal, this paper discusses maintenance policy optimization in presence of incomplete information. In the case of multi-component systems, the maintenance can be perfect and maintain all the components which is very costly or imperfect in sens that only failed components are replaced. Imperfect maintenance can be applied when the system or the component is not as good as new after maintenance. Sometimes, the system is only repaired in order to continue to operate where the failure rate remains unchanged in comparison to the failure rate before maintenance, that is called minimal maintenance [1]. Moreover, preventive maintenance actions can be planned in order to avoid failure. The preventive action occurs before the failure an is more cost-efficient since it permits to avoid the failure and its induced cost and consequences. Preventive maintenance planning is an important issue since a frequent preventive maintenance could be costly and by reducing the frequency of these actions, the occurence of failures will be more likely which will induce higher costs and a period of unavailability. That is why preventive maintenance scheduling has attracted lot of attentions lately as well as in high-tech industry as in basic production and manufacturing. The preventive maintenance can be planned periodically called period maintenance, based on the age of the system called age-based maintenance, randomly according to the maintenance teams availability or at failure of some components called opportunistic maintenance or based on the health indicator of the component or the system called condition-based or predictive maintenance, for more details refer to [1], [2], [3], [4], [7], [8], [9].

The choice of maintenance planning is strongly related to the available data and information. In absence of regular monitoring of a health indicator or sensor measurements on this latter, it is not possible to plan condition-based or predictive maintenance actions. If only historical data is available, periodic or age-based maintenance is carried out. In the case of multi-component systems where it is not sensible to replace the whole system at each maintenance, at components failures some diagnosis can be made on other components and carry out opportunistic maintenance based on observations at maintenance date. However, the cause of the failure of a multi-component system (the failed component(s)) could be not always identified. In this framework, the available data is called masked data. Dealing with this kind of data in the framework of reliability calculation and maintenance planning is a big challenge, [11], [10]. Since it is important to take advantage of available information, in the framework of masked data, opportunistic maintenance polices could reduce maintenance costs.

In this paper, an opportunistic perfect preventive maintenance is considered in presence of masked data. The preventive maintenance action replaces the failed component as good as new. When the system fails besides the failed component, some endangered components are replaced by a new one thus the perfect preventive replacements are opportunistic. Inspections are periodic and inter-inspection interval is optimized. The rest of the paper is organized as follows. In section 2, the model is explained. The maintenance model is presented in section 3. A numerical example is conducted in order to illustrate the applicability of the proposed method in section 4. Finally, the conclusion is given in section 5.

II. MODEL DESCRIPTION

A series system with J components is considered. When the system fails we observe failure time, t, but the exact cause of failure might be unknown but it is known that it belongs to a subset of $\{1, 2, ..., J\}$ called MRS. Let M be the MRS corresponding to the failure time t for the system. The set M contains components that are possible to be the cause of system failure and if $M = \{1, ..., J\}$ then the system cause of failure is called to be completely masked.

Let $T_l;\ \ l = 1, 2, ..., J$ be the lifetimes of the lth component (independent components) and assume that the system fails only due to one of the J components, therefore the system failure time T is defined to be $T = min(T_1, ..., T_J)$. Let be $f_l(t)$ and $R_l(t)$ probability density and reliability functions denoted by , respectively. The reliability function of T is given by

$$R(t) = R(t;\theta) = \mathbb{P}_\theta(T > t) = \prod_{l=1}^{J} [1 - F_l(t)]$$

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$\boldsymbol{\theta} = (\theta_1, ..., \theta_J)$ and θ_l is the set of parameters related to the lth component and F_l is corresponding distribution function of lth component. Let's consider the cause of failure K as random variable. Then the joint probability density function of (T, K) is given by

$$f_{T,K}(t,l) = f_l(t) \prod_{j \neq l} [1 - F_j(t)] \quad (1)$$

and also $F(j,t) = \mathbb{P}(K = j, T \leq t)$, or equivalently $R(j,t) = \mathbb{P}(K = j, T > t)$ [12].

The probability

$$p_l^t(M_i) = \mathbb{P}(M = M_i | T = t, K = l)$$

is called the masking probability, where M_i is an observation of M. Some authors such as Mukhopadhyay [13], Kuo and Yang [14] and Cai and et al. [15], assumed

$$\mathbb{P}(M = M_i | T = t, K = l) = \mathbb{P}(M = M_i | K = l) = p_l(M_i),$$

that is, the masking probability is independent of failure time, but is dependent to the causes of failure. Similar to a new approach that was presented to model the dependency of the masking probability on the failure time and its exact cause using the multinomial logistic regression model [16], we also assume that the masking probability depends on the failure time and its exact cause.

Suppose $\mathfrak{M}$ be the set of all nonempty subsets of $\{1, ..., J\}$ that have $2^J - 1$ members. For $l = 1, ..., J$, define by

$$\mathfrak{M}_l = \{M \in \mathfrak{M} : l \in M\}$$

the elements of $\mathfrak{M}$ that include l, thus

$$p_l^t(M_i) = \mathbb{P}(M = M_i | K = l, T = t) = 0$$

$\forall M_i \in \mathfrak{M}_l^c = \mathfrak{M} - \mathfrak{M}_l$ and

$$\sum_{M_i \in \mathfrak{M}} p_l^t(M_i) = \sum_{M_i \in \mathfrak{M}_l} {}_l^t(M_i) = 1, l = 1, ..., J$$

denote by $\mathbb{P}_l = \{p_l^t(M_i) : M_i \in \mathfrak{M}_l\}, l = 1, 2, \ldots, J$ then the set of all masking probabilities is $\mathbb{P} = (\mathbb{P}_1, ..., \mathbb{P}_J)$.

The reliability function of the multi-component system is given by

$$R(t) = \int_t^\infty \sum_{l=1}^J f_l(u) \prod_{j \neq l} R_j(u) du.$$

This function depends on the reliability of components and therefore on lifetime distribution of the components which are unknown.

III. Maintenance Modeling

The hypothesis of the maintenance framework are as follows.

- The system is inspected periodically at times $k\tau$; $k = 1, 2, ...$, with cost c_{ins} for the system and M_k the corresponding masked sets.
- The time interval $((k-1)\tau, k\tau]$ is called the kth period.
- Inspections are carried out at the end of each period
- The time required for inspection and maintenance actions is negligible.
- The system failure is not self-announced
- The components are maintained independently.
- At the kth inspection time, $k\tau$, a maintenance action is performed if the system has been failed during $((k-1)\tau, k\tau]$ interval. The probability of each cause in M_k given possibly masked set and interval censored failure time is given by

$$p_{jM_k} = \mathbb{P}(K = j | M_k, u \in ((k-1)\tau, k\tau])$$
$$= \frac{\int_{(k-1)\tau}^{k\tau} \mathbb{P}(M_k | j, u) f_{T,K}(u, j) du}{\int_{(k-1)\tau}^{k\tau} \sum_{l' \in M_k} \mathbb{P}(M_k | l', u) f_{T,K}(u, l') du}$$

  where u is the exact failure time. Note that $p_{jM_k} = 0$ for $j \notin M_k$.
- If the system fails at $((k-1)\tau, k\tau]$ a maintenance action is carried out for each component in M_k according to a predetermined value of ρ; $0 < \rho < 1$, as follows:
  - If $T_l > (k-1)\tau$ and $T_l < k\tau$ then perfect corrective maintenance (PCM) action is done for component l with cost c_{lc} (that is, the failed component l is replaced by a new one).
  - If $T_l > k\tau$ and $p_{lM_k} > \rho$ then opportunistic perfect preventive maintenance (OPPM) action is done for component l with cost $c_{lp} < c_{lc}$ (that is, the degraded component l is replaced by a new one).
- A perfect corrective repair (PCR) is done for component l, $l \in M_k$, if the system fails at $((k-1)\tau, k\tau]$ and

$$T_l > (k-1)\tau \quad \& \quad T_l < k\tau.$$

  Define $\mathbb{P}_{cl}(k\tau)$ as probability of perfect corrective repair for component l; $l = 1, 2, ..., J$, at kth inspection time.
- An opportunistic perfect preventive repair (OPPR) is done for component l, $l \in M_k$, if the system fails at $((k-1)\tau, k\tau]$ and

$$T_l > k\tau \quad \& \quad p_{lM_k} > \rho.$$

  Define $P_{pl}(k\tau)$ as the probability of the opportunistic perfect preventive repair for component l; $l = 1, 2, ..., J$, at kth inspection time,

The time from the component installation to its first replacement or the time between two successive replacement of each component is referred to as a renewal cycle. Let L and L_j denote the average long-run maintenance cost per unit of time for the system and component j, respectively. Therefore, based on the renewal reward theorem, [20], the expected long-run maintenance cost rate for component j is

$$L_j(\tau, \rho) = \lim_{t \to \infty} \frac{C_j(t)}{t} = \frac{E(C_{rj})}{E(T_{rj})}$$

where $E(C_{rj})$ and $E(T_{rj})$ are total expected cost during a replacement cycle and expected length of the replacement cycle for component j, respectively such that

$$E(C_{rj}) = \sum_{k=1}^\infty \left[(\frac{kc_{ins}}{J} + c_{jp}) P_{pj}(k\tau) + (\frac{kc_{ins}}{J} + c_{jc}) P_{cj}(k\tau) \right]$$

and

$$E(T_{rj}) = \sum_{k=1}^{\infty} k\tau\big[P_{pj}(k\tau) + P_{cj}(k\tau)\big].$$

Finally, the total expected long-run maintenance cost rate for the series system until time t is given by [18], [19]

$$L(\tau,\rho) = \sum_{j=1}^{J} L_j(\tau,\rho).$$

The aim is to propose an inspection interval τ which minimises $L(\tau,\rho)$:

$$\tau^* = \text{argmin} L(\tau,\rho) = \sum_{j=1}^{J} L_j(\tau,\rho)$$

This joint optimisation of costs permits to take into account all the components reliability in the optimisation procedure.

IV. Numerical Example

A series system with $J = 3$ components is considered. The system is inspected periodically at times $k\tau; \quad k = 1,2,...$, with cost c_{ins} for the system and $M_k; \quad k = 1,2,...$ are corresponding masked sets, thus the collected data are $(k\tau, M_k); \quad k = 1,2,....$ When the system fails, an opportunistic perfect preventive replacement is carried out for components that are in the masked set with $p_l > \rho$ and a perfect corrective replacement is made for failed component with costs c_{lp} and c_{lc}, respectively. Two illustrative examples have been constructed to clarify previous sections. In both of them, it is assumed that the lifetime of components are independent and follow the Weibull distribution with parameter set $(\alpha_j, \beta_j); \quad j = 1,2,3$, as follows:

$$f_{T_j}(t_j) = \frac{\alpha_j}{\beta_j}\Big(\frac{t_j}{\beta_j}\Big)^{(\alpha_j - 1)} \exp\Big(-\Big(\frac{t_j}{\beta_j}\Big)^{\alpha_j}\Big).$$

We set parameters as in Table I

TABLE I: The optimal inter-inspection interval value (τ) as decision parameter with different cost rates and ρ

unit costs	
c_{ins}	1
c_{1p}	1.5
c_{2p}	2.5
c_{3p}	3.5
c_{1c}	2
c_{2c}	4
c_{3c}	5

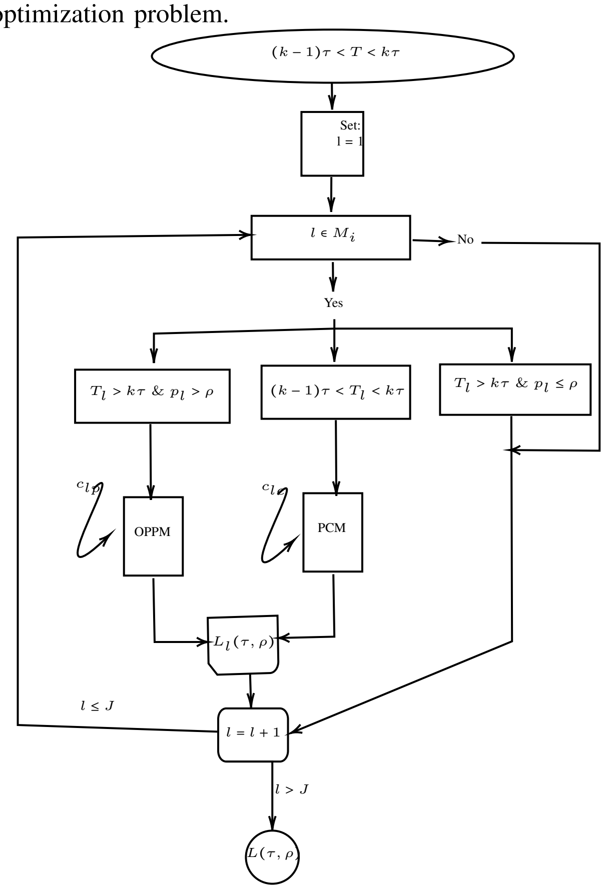
All masked sets are considered as completely masked sets which is similar to missing setup, that is, $M_k = \{1,2,...,J\}; \quad k = 1,2,...$, thus

$$\begin{aligned} p_{jM_k} =& P(K = j | M_k, u \in ((k-1)\tau, k\tau]) \\ =& \frac{\int_{(k-1)\tau}^{k\tau} f_{T,K}(u,j)du}{\int_{(k-1)\tau}^{k\tau} \sum_{l' \in M_k} f_{T,K}(u,l')du} \end{aligned}$$

since $\forall j \in \{1,2,...,J\}$ and $t \in [0,\infty)$

$$P(M_k = M | K = j, t) = \begin{cases} 1 & \text{if } M = \{1,2,...,J\} \\ 0 & \text{if } M \neq \{1,2,...,J\} \end{cases}$$

where the inter-inspection interval, τ, is considered as decision parameter and should be optimized through maintenance optimization problem.



Algorithm 1: Imperfect Corrective Maintenance Policy

TABLE II: The optimal inter-inspection interval value (τ) as decision parameter with different cost rates and ρ considering a constant failure rate ($\alpha_j = 1$ for $j = 1,2,3$)

τ^*	ρ	Cost Rate
0.03	0.00	154.38
0.04	0.00	125.23
0.05	0.10	102.16
0.06	0.10	92.90
0.07	0.10	86.11
0.09	0.10	69.64
0.11	0.10	62.42
0.14	0.10	55.72
0.17	0.10	48.73
0.26	0.10	36.90
0.27	0.10	34.14
0.31	0.10	32.96
0.42	0.10	24.21
0.44	0.10	23.16
0.50	0.10	21.13
0.53	0.10	20.10
0.54	0.10	19.90

It can be noticed in Tables II and III that the cost rate is very sensitive to the failure rate distribution. Indeed with the same maintenance unit costs, the total cost rate can be very high for an increasing failure rate. When the failure rate is increasing a with the same masking probability threshold the impact of a missing failure can be very high between two

TABLE III: The inter-inspection interval value (τ) as decision parameter with different cost rates and ρ considering an increasing failure rate

τ	ρ	Cost Rate
0.01	0.00	368.44
0.01	0.20	445.45
0.02	0.20	234.48
0.03	0.10	161.29
0.04	0.10	123.22
0.10	0.20	56.15
0.30	0.20	24.18
0.40	0.20	19.50
0.50	0.20	16.30
0.70	0.20	12.72
0.80	0.20	11.42
0.82	0.20	11.20
0.85	0.20	10.88
0.86	0.20	10.77
0.87	0.20	10.68

inspections.

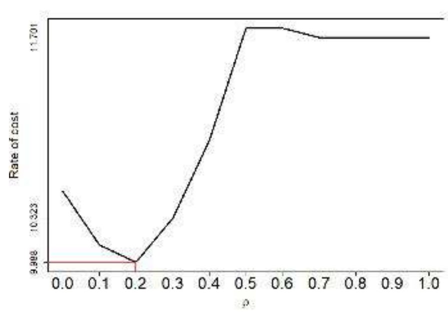


Fig. 1: The optimal value of ρ as decision parameter considering $(\alpha_1, \alpha_2, \alpha_3)$ = $(1.25, 2, 2.5)$ and $(\beta_1, \beta_2, \beta_3)$ = $(2.5, 2.25, 2.75)$ for the optimal inspection interval τ = 0.67.

In Figure 1, 2 and 3, the cost rate is depicted in function of component cause probability threshold ρ considering the optimal inspection. In Figure 1, since all the failure rates are non-decreasing the cost rate is convex for decision parameters and we can easily identify an optimal value. The optimal inspection interval is relatively small which requires frequent inspections and the cause probability threshold is close to 50%. Since the inspections are frequent the risk of missing the cause is not very high.

In Figure 2 and 3, since all the failure rates are decreasing the minimal cost rate exists but the cost rate variations around the optimal value are not very high. Indeed in this framework the system is getting better even though the failures occurs, by relatively frequent inspections and a low cause probability the maintenance operations are occurs frequently. However, since failures are less frequent the maintenance decision rule is more often postponed.

As the failure rate is decreasing (the failure rates in Figure 3 decrease faster than in Figure 2) the flatness around the optimal value is more obvious. The inspections are less frequent and the cause probability threshold still very low.

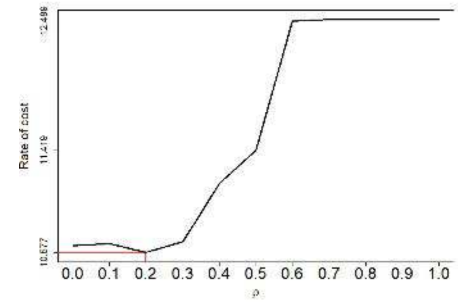


Fig. 2: The optimal value of ρ as decision parameter considering $(\alpha_1, \alpha_2, \alpha_3)$ = $(0.5, 0.25, 0.7)$ and $(\beta_1, \beta_2, \beta_3)$ = $(1.5, 1.25, 1.75)$ for the optimal inspection interval τ = 0.88.

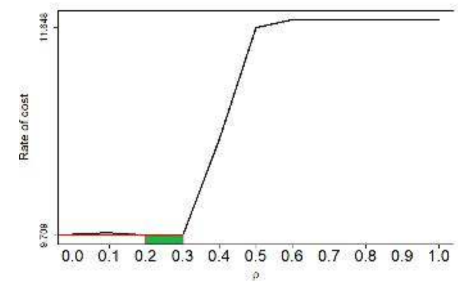


Fig. 3: The optimal value of ρ as decision parameter considering α = $(0.5, 0.25, 0.7)$ and $(\beta_1, \beta_2, \beta_3)$ = $(0.25, 0.5, 1.5)$ under different failure rates, for the optimal inspection interval τ = 1.11.

V. CONCLUSIONS

In this paper an opportunistic maintenance is proposed for a multi-component system with masked data. The maintenance policy is described and a cost criterion optimisation is proposed. A numerical example illustrate the methoddology.

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