

SUFFICIENT CONDITION FOR INVERTIBILITY OF SQUARE FIR MIMO SYSTEMS

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ABSTRACT

We derive a sufficient condition for a square FIR MIMO system to have a causal and stable IIR inverse. The condition requires that the spectral norm of the normalized channel impulse response (i.e., the first tap is the identity matrix) is below a certain bound. Intuitively, this means that the system has a strong first tap. This condition often is easier to check than the usual minimum-phasesness, where the roots of the systems determinant have to be computed. Simple approximations of the bound are found. Furthermore, we also give a negative result: the Wiener Filter, which approximates the inverse under low noise conditions, nevertheless always is non-causal. We apply our results to two inversion problems with causality and stability constraint. These problems arise in oversampled noise-shaping subband coding and residual interference cancellation in precoded systems, respectively.

Index Terms— MIMO Systems, Equalizers, IIR digital filters, Causality, Stability

I. INTRODUCTION

The problem of inverting a square finite impulse response (FIR) multiple-input multiple-output (MIMO) system

$$H(z) = \sum_{k=0}^K H_k z^{-k} \quad (H_k \in \mathbb{C}^{q \times q})$$

arises in various situations. A popular example is filter-bank design, where inverses are called perfect reconstruction filter banks [1]. Often, it is desirable to have a FIR inverse

$$G_{FIR}(z) = \sum_{m=0}^M G_m^{FIR} z^{-m} \text{ s.t. } G_{FIR}(z)H(z) = I,$$

because FIR inverses are inherently stable and simple to compute [2]. It is well-known that the inverse is FIR iff $\det[H(z)] = c \in \mathbb{C} \setminus \{0\}$ [1], [3]. However, recent results show that a square FIR MIMO system is generically (“almost surely”) not FIR invertible [4]. Thus, we cannot expect FIR invertibility of MIMO systems, in general. Infinite impulse response (IIR) invertibility, i.e., existence of a stable and causal inverse, is less restrictive. A stable IIR inverse (i.e., all poles are contained in the open unit disc $|z| < 1$ [5])

$$G(z) = \sum_{m=0}^{\infty} G_m z^{-m} \text{ stable s.t. } G(z)H(z) = I,$$

exists iff $\det[H(z)] \neq 0$ for all $1 \leq |z| \leq \infty$ [6], [3]. However, this condition is not trivial to check because it requires finding the roots of the systems determinant. Moreover, given typical characteristics of a FIR MIMO system, e.g., a power profile, it

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is usually impossible to say a priori whether the system will be IIR invertible or not. A simpler IIR invertibility condition, which is more compatible with standard system characteristics, would be useful. Thus, we make the following contributions.

- We derive a novel condition for IIR invertibility of square FIR MIMO systems. The condition is intuitive (strong first tap), and can be checked from standard system characteristics.
- Besides, we show that although the Wiener Filter approximates the inverse in environments with high signal-to-noise ratios, there can be no similar result for the Wiener Filter. The Wiener Filter is non-causal as soon as the channel is frequency-selective and IIR invertible.
- Finally, we give two applications of our results. First, we derive a sufficient condition that ensures that the feedback filter in oversampled noise-shaping subband coding results in a stable feedback loop. Second, we give a safety margin on the errors in the feedback loop of a wireless system with precoding that guarantees that the precoded channel (with perturbed precoder) still is invertible.

The main tools in our derivation will be state-space techniques which are especially popular in control [5], but are also commonly used in filter bank design [1]. In particular, we use results from robust stabilization [7] to analyze a state-space realization of a certain inner-outer factorization [8].

The paper is structured as follows. In Section II, we derive and discuss our novel sufficient invertibility condition. The non-causality of the Wiener Filter is discussed in Section III. Finally, in Section IV, we give some applications.

Notation: The spectral norm $\|\Phi\|_2$ of a scalar matrix $\Phi \in \mathbb{C}^{n \times n}$ is given by its largest singular value, the Frobenius norm by $\|\Phi\|_F^2 = \text{tr}\{\Phi^* \Phi\}$. We denote the set of rational matrices by $\mathcal{R}^{n \times n}$, and the subset of causal and stable (i.e., all poles are contained in $|z| < 1$) rational matrices by $\mathcal{RH}_{\infty}^{n \times n}$. The \mathcal{L}_2 norm of a rational matrix $\Psi \in \mathcal{R}^{n \times n}$ is given by $\|\Psi\|_{\mathcal{L}_2}^2 = \int_{\theta=0}^{2\pi} \|\Psi(e^{i\theta})\|_F^2 \frac{d\theta}{2\pi}$. We use the usual shorthand

$$\left[\begin{array}{c|c} E & F \\ \hline G & H \end{array} \right] = H + G(zI - E)F \in \mathcal{R}^{n \times n}$$

to denote the transfer function of a state-space system [5]. The tilde operator $\Psi^{\sim}(z) := \Psi(\bar{z}^{-1})^*$ denotes the para-hermitian. Finally, an inner-outer factorization of a stable rational matrix $\Psi \in \mathcal{RH}_{\infty}^{n \times n}$ is a factorization $\Psi = \Psi_i \Psi_o$, where $\Psi_i, \Psi_o, \Psi_o^{-1} \in \mathcal{RH}_{\infty}^{n \times n}$ and $\Psi_i^{-1} = \Psi_i^{\sim}$.

II. SUFFICIENT INVERTIBILITY CONDITION

We consider a $q \times q$ FIR MIMO channel $H(z) = \sum_{k=0}^K H_k z^{-k}$ with H_0 invertible.¹ This goal of this section is to prove the following sufficient invertibility condition.

¹This means no loss of generality. If H^{-1} is causal then H_0 is invertible.

Proposition 1. Suppose that H_0 is invertible, and

$$\left\| \begin{bmatrix} H_1 H_0^{-1} \\ \vdots \\ H_K H_0^{-1} \end{bmatrix} \right\|_2 < \frac{1}{C_K}, \quad (1)$$

where

$$C_K := \left\| \begin{bmatrix} (-1)^0 & (-1)^1 & \dots & (-1)^{K-1} \\ & \ddots & \ddots & \vdots \\ & & \ddots & (-1)^1 \\ & & & (-1)^0 \end{bmatrix} \right\|_2. \quad (2)$$

Then, the inverse H^{-1} is causal and stable.

Remark 2. We point out that the upper bound does only depend on the number of taps K , but not the taps size q . Numerical evaluation of C_K has shown that C_K grows approximately linearly with K . A good approximation in the range $K = 2, \dots, 1000$ is $\tilde{C}_K := 0.254 + 0.683K$. Moreover, we have verified numerically that

$$\frac{1}{\tilde{C}_K} < \frac{1}{C_K} \text{ for all } K = 2, \dots, 1000.$$

Thus, if the channel length is below 1000, we can use $1/\tilde{C}_K$ instead of $1/C_K$ as upper bound Proposition 1. We have plotted both the exact and the approximate upper bound for different K in Fig. 1.

Before we can prove Proposition 1, some results on inner-outer factorization of $H(z)$ have to be established.

II-A. Inner-Outer Factorization of H

In this section, we establish state-space realizations for a special inner-outer factorization of $H(z)$. Let us first consider $z^{-1}H(z)$. We introduce a state-space realization

$$z^{-1}H(z) = \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array} \right] = \left[\begin{array}{ccc|ccc} 0_q & I_q & & & & H_0 \\ & \ddots & \ddots & & & \vdots \\ & & \ddots & & & \vdots \\ & & & \ddots & & \vdots \\ & & & & I_q & H_{K-1} \\ & & & & 0_q & H_K \\ \hline I_q & 0_q & \dots & 0_q & & 0_q \end{array} \right].$$

Suppose $X = X^*$ is the unique stabilizing solution to the discrete-time algebraic Riccati equation [9]

$$A^*XA - X - A^*XB(B^*XB)^{-1}B^*XA + C^*C = 0, \quad (3)$$

i.e., the eigenvalues of the matrix $A + BR$, where $R := -(B^*XB)^{-1}B^*XA$, are contained inside the complex unit circle $|z| < 1$. Introduce a second matrix $S := (B^*XB)^{1/2}$. Then, an inner-outer factorization of $z^{-1}H(z) = \mathcal{I}_1(z)\mathcal{O}(z)$ is given by [8]

$$\mathcal{I}_1(z) := \left[\begin{array}{c|c} A + BR & BS^{-1} \\ \hline C & 0 \end{array} \right], \mathcal{O}(z) := \left[\begin{array}{c|c} A & B \\ \hline -SR & S \end{array} \right],$$

We can transform the inner-outer factorization $z^{-1}H(z) = \mathcal{I}_1(z)\mathcal{O}(z)$ into an inner-outer factorization of $H(z)$. Since \mathcal{I}_1 is strictly causal, we can absorb the inner factor z^{-1} into \mathcal{I}_1 ,

$$\mathcal{I}(z) := z\mathcal{I}_1(z) = \left[\begin{array}{c|c} A + BR & BS^{-1} \\ \hline C(A + BR) & CBS^{-1} \end{array} \right]. \quad (4)$$

Then, $H(z) = \mathcal{I}(z)\mathcal{O}(z)$ is an inner-outer factorization of $H(z)$.

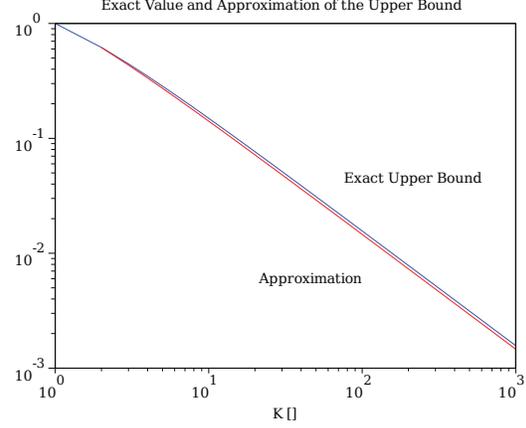


Fig. 1. Plot of the upper bound $1/C_K$ and its approximation $1/\tilde{C}_K$.

II-B. Sufficient Condition for X_1 being the Stabilizing Solution

In this section, we derive a sufficient condition for

$$X_1 = X_1^* = \begin{bmatrix} I_q & \\ & 0 \end{bmatrix} \quad (5)$$

being the stabilizing solution to the Riccati equation (3). It is simple to check that X_1 does always solve the Riccati equation,

$$\begin{aligned} & A^*X_1A - X_1 - A^*X_1B(B^*X_1B)^{-1}B^*X_1A + C^*C \\ &= A^*X_1A - A^*X_1B(B^*X_1B)^{-1}B^*X_1A \\ &= A^*X_1A - A^* \begin{bmatrix} H_0(H_0^*H_0)^{-1}H_0^* & 0 \\ 0 & 0 \end{bmatrix} A = 0. \end{aligned}$$

However, this solution is not always stabilizing. The matrix X_1 is a stabilizing solution to the Riccati equation if and only if $A + BR_1$ is stable (i.e., all eigenvalues are located inside the open unit disc $|z| < 1$), where

$$R_1 := -(B^*X_1B)^{-1}B^*X_1A = \begin{bmatrix} 0 & -H_0^{-1} & 0 & \dots & 0 \end{bmatrix}. \quad (6)$$

A sufficient condition for

$$\begin{aligned} & A + BR_1 = \\ & \underbrace{\begin{bmatrix} 0_q & 0_q & & & \\ & 0_q & I_q & & \\ & \vdots & \ddots & \ddots & \\ & 0_q & & I_q & \\ & 0_q & & & 0_q \end{bmatrix}}_{=: \Gamma_1(q, K) \in \mathbb{C}^{(K+1)q \times (K+1)q}} + \underbrace{\begin{bmatrix} 0_q & 0_q & & & \\ & -H_1H_0^{-1} & 0_q & & \\ & \vdots & & \ddots & \\ & -H_{K-1}H_0^{-1} & & & 0_q \\ & -H_KH_0^{-1} & & & 0_q \end{bmatrix}}_{=: \Gamma_2(q, H) \in \mathbb{C}^{(K+1)q \times (K+1)q}} \end{aligned} \quad (7)$$

being stable is [7]

$$\begin{aligned} \|\Gamma_2(q, H)\|_2 &= \left\| \begin{bmatrix} H_1H_0^{-1} \\ \vdots \\ H_KH_0^{-1} \end{bmatrix} \right\|_2 \\ &< \frac{1}{\|(e^{i\theta}I_{(K+1)q} - \Gamma_1(q, K))^{-1}\|_2}, \quad \forall \theta \in [0, \pi]. \end{aligned} \quad (8)$$

The next lemma shows that the right-hand side of this inequality is

independent of θ and q . The proof is a simple extension of Lemma 1 in [10]. Thus, we give no proof.

Lemma 3. *The singular values of $e^{i\theta}I_{(K+1)q} - \Gamma_1(q, K)$ coincide for all $\theta \in [0, \pi]$ and $q \in \mathbb{N}$.*

II-C. Proof and Discussion of Proposition 1

We are now ready to prove our main result.

Proof: (of Prop. 1) Suppose (1) holds. Since

$$C_K \stackrel{(C_K \geq 1)}{=} \left\| \begin{bmatrix} 1 & & & & \\ & (-1)^0 & (-1)^1 & \dots & (-1)^{K-1} \\ & & \ddots & \ddots & \vdots \\ & & & \ddots & (-1)^1 \\ & & & & (-1)^0 \end{bmatrix} \right\|_2$$

$$= \|(e^{i\theta}I_{(K+1)1} - \Gamma_1(1, K))^{-1}\|_2,$$

Lemma 3 implies that (8) holds. Thus, the results in Section II-B show that X_1 given in (5) is the stabilizing solution to the Riccati equation (3). Therefore, \mathcal{I} and \mathcal{O} given in Section II-A constitute an inner-outer factorization of $H(z)$. Now, note that in (4), we have

$$C(A + BR_1) = [I_q \ 0] \begin{bmatrix} 0 & 0 & & & \\ & -H_1H_0^{-1} & I & & \\ & \vdots & 0 & \ddots & \\ & -H_{K-1}H_0^{-1} & & \ddots & I \\ & -H_KH_0^{-1} & & & 0 \end{bmatrix} = 0$$

because of (6) and (7). Using that $S_1 := (B^*X_1B)^{1/2} = (H_0^*H_0)^{1/2}$, we see that the inner-outer factorization reduces to

$$\mathcal{I}(z) = CBS_1^{-1} = H_0(H_0^*H_0)^{-1/2},$$

$$\mathcal{O}(z) = \mathcal{I}^{\sim}(z)H(z) = (H_0^*H_0)^{-1/2}H_0^*H(z).$$

The proposition now follows from $H^{-1} = \mathcal{O}^{-1}\mathcal{I}^{\sim}$ because \mathcal{O}^{-1} is stable and causal by definition of the inner-outer factorization, and $\mathcal{I}^{\sim} = \mathcal{I}^{-1}$ is a scalar matrix. ■

The alert reader probably wonders at this point whether computation of the inner-outer factorization actually was necessary because state-space formulas for inverses are well known [5, Lem. 3.15],

$$H^{-1}(z) = \left[\begin{array}{ccc|ccc} -I & I & & & -I & \\ -H_1H_0^{-1} & 0 & \ddots & & -H_1H_0^{-1} & \\ \vdots & & \ddots & & \vdots & \\ -H_KH_0^{-1} & & & I & -H_KH_0^{-1} & \\ \hline & H_0^{-1} & 0 & \dots & 0 & H_0^{-1} \end{array} \right].$$

Therefore, note that our sufficient condition cannot be achieved from this realization of the inverse. To see this, decompose the state transition matrix of this realization in two ways, $(A - X_1) + P_1$ and $A + P_2$, where

$$\underbrace{\begin{bmatrix} -I & I & & & \\ & 0 & \ddots & & \\ & & \ddots & I & \\ & & & & 0 \end{bmatrix}}_{=:A-X_1}, \underbrace{\begin{bmatrix} 0 & 0 & \dots & 0 \\ -H_1H_0^{-1} & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ -H_KH_0^{-1} & 0 & \dots & 0 \end{bmatrix}}_{=:P_1},$$

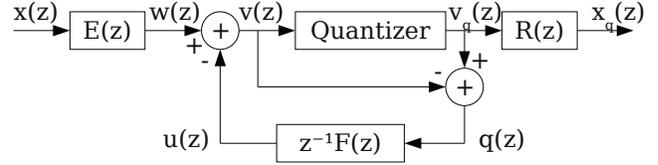


Fig. 2. Oversampled noise-shaping subband coder

$$\underbrace{\begin{bmatrix} 0 & I & & & \\ & \ddots & \ddots & & \\ & & & I & \\ & & & & 0 \end{bmatrix}}_{=:A}, \underbrace{\begin{bmatrix} -I & 0 & \dots & 0 \\ -H_1H_0^{-1} & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ -H_KH_0^{-1} & 0 & \dots & 0 \end{bmatrix}}_{=:P_2}.$$

Then, we cannot consider P_1 as a perturbation of $A - X_1$ because the matrix $A - X_1$ is not stable. This prevents application of the robust stabilization results in [7]. On the other hand, when we consider P_2 as a perturbation of the stable matrix A , then there is a problem with the obtained condition

$$\left\| \begin{bmatrix} I \\ H_1H_0^{-1} \\ \vdots \\ H_0H_0^{-1} \end{bmatrix} \right\|_2 < \left\| I - \begin{bmatrix} 0 & I & & & \\ & \ddots & \ddots & & \\ & & & I & \\ & & & & 0 \end{bmatrix} \right\|_2^{-1}.$$

The left hand side is always larger than one, but in numerical experiments the right-hand side was always lower than one.

III. NON-CAUSALITY OF THE WIENER FILTER

In this section we consider the question whether the Wiener Filter, which is a regularization of the inverse and approximates it in low noise environments, can become causal and stable, too. The next proposition shows that this can only happen in the trivial case, i.e., flat fading channels.

Proposition 4. *Suppose that the inverse H^{-1} is causal and stable, and $\sigma > 0$. Then, the Wiener Filter $(H^{\sim}H + \sigma^2I)^{-1}H^{\sim}$ is causal and stable if and only if the system is flat-fading, i.e., $H(z) \equiv C$.*

Proof: The " \Leftarrow " part of the proof is trivial, we only treat the " \Rightarrow " part. Consider $H_{\Delta} := H + \sigma^2H^{\sim}$, and note that

$$H_{\Delta}^{-1} = [H^{\sim}(H^{\sim}H + \sigma^2I)]^{-1} = (H^{\sim}H + \sigma^2I)^{-1}H^{\sim}$$

is the Wiener Filter. The Wiener Filter H_{Δ}^{-1} is causal, thus also

$$H_{\Delta}^{-1}H = [H^{-1}(H + \sigma^2H^{\sim})]^{-1} = [I + \sigma^2(H^{\sim}H)^{-1}]^{-1},$$

is causal. However, $H_{\Delta}^{-1}H$ is also (non-strictly) anti-causal because $H_{\Delta}^{-1}H = (H_{\Delta}^{-1}H)^{\sim}$. Thus, $H_{\Delta}^{-1}H$ and in particular $H^{\sim}H =: C_1$ are constant scalar matrices. This shows that the inverse $H^{-1} = C_1^{-1}H^{\sim}$, which is causal, also is anti-causal. Again, this implies that H^{-1} and therefore H are constant scalar matrices. ■

IV. APPLICATIONS

IV-A. Oversampled Noise-Shaping Subband Coders

Oversampled noise-shaping subband coders have been introduced in [11] as a combination of oversampled noise-shaping A/D converters and perfect reconstruction filter banks (see Fig. 2). Given perfect reconstruction analysis and synthesis filter banks $E(z)$ and $R(z)$ the task is to design an optimal FIR feedback filter $F(z)$. In order to obtain a tractable optimization problem, the quantization

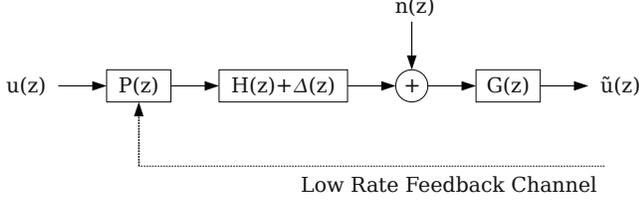


Fig. 3. Wireless communications system

noise $v - v_q$ is modeled as additive random noise with spectral density $W(z)W^*(z)$. Then, computation of an optimal feedback filter is to find a system $F(z) = \sum_{k=0}^K F_k z^{-k}$ such that

$$\|R(I - z^{-1}F)W\|_{\mathcal{L}_2} \quad (9)$$

is minimized. An explicit expression for the optimal FIR feedback filter is obtained in [11]. However, as noted in [11, p. 168], [12], the solution in [11] has a shortcoming. While most internal signals in Fig. 2 are always stable, the stability of some signals does depend on the stability of $[I - z^{-1}F(z)]^{-1}$. E.g., we have

$$\begin{aligned} v(z) &= w(z) - z^{-1}F(z)[v_q(z) - v(z)] \\ \Rightarrow v(z) &= [I - z^{-1}F(z)]^{-1}[w(z) - z^{-1}F(z)v_q(z)], \end{aligned}$$

which is only stable if $[I - z^{-1}F(z)]^{-1}$ is stable. However, the solution in [11] does not guarantee stability of $[I - z^{-1}F(z)]^{-1}$. Proposition 1 can be used to obtain a sufficient stability condition.

Corollary 5. Suppose $F(z) = \sum_{k=0}^K F_k z^{-k}$ satisfies

$$\left\| \begin{bmatrix} F_0 \\ \vdots \\ F_K \end{bmatrix} \right\|_2 < \frac{1}{C_{K+1}} \quad (10)$$

for C_{K+1} given by (2). Then, $[I - z^{-1}F(z)]^{-1}$ is causal and stable.

IV-B. Residual Interference Cancellation in Precoded Systems

In wireless systems, a precoder can be used to predistort the data signals such that after transmission through the wireless channel the undistorted data signals are received by the receiver. Therefore, the transmitter requires knowledge about the wireless channel, which may be obtained e.g. using reciprocity in TDMA systems or via a low-rate feedback channel. However, the channel data used by the transmitter to compute the precoder usually is erroneous e.g. due to channel estimation errors, out-dating, or quantization. Thus, there is residual interference in the channel, which may be canceled by applying a channel inverse to the precoded channel. Of course, therefore the precoded channel has to be causally and stably invertible. In the following, we will design a bound on the disturbance that guarantees invertibility of the precoded channel.

Consider the wireless communications system depicted in Fig. 3. The input-output relation of the system is

$$y(z) = G(z)[H(z) + \Delta(z)]P(z)u(z) + G(z)n(z),$$

where

- $H(z) = \sum_{k=0}^K H_k z^{-k}$ is the channel,
- $\Delta(z) = \sum_{k=0}^M \Delta_k z^{-k}$ is a perturbation,
- $P(z) = \sum_{k=0}^M P_k z^{-k}$ is a precoder, i.e., $\|P\|_{\mathcal{L}_2} = 1$ and $HP = cI$ for some $c > 0$, and
- $G = [(H + \Delta)P]^{-1}$ cancels the residual interference.

Now, let

$$R(z) := \sum_{k=0}^{K+M} R_k z^{-k} := [H(z) + \Delta(z)]P(z) = cI + \Delta(z)P(z)$$

denote the precoded channel. Proposition 1 shows that $G = R^{-1}$ is stable and causal if R holds (1). The left-hand side of (1) is upper bounded by

$$\begin{aligned} \left\| \begin{bmatrix} R_1 R_0^{-1} \\ \vdots \\ R_{K+M} R_0^{-1} \end{bmatrix} \right\|_2 &\leq \left\| \begin{bmatrix} \Delta_0 P_0 \\ R_1 \\ \vdots \\ R_{K+M} \end{bmatrix} \right\|_2 \underbrace{\|R_0^{-1}\|_2}_{\leq \|R_0^{-1}\|_F} \\ &\leq c^{-1} \|\Delta\|_{\mathcal{L}_2} \underbrace{\|P\|_{\mathcal{L}_2}}_{=1} \underbrace{\|(I + c^{-1}\Delta_0 P_0)^{-1}\|_F}_{=R_0^{-1}} \\ &\leq \frac{\|\Delta\|_{\mathcal{L}_2}}{c} \left(\sum_{k=0}^{\infty} c^{-k} \|\Delta_0\|_F^k \underbrace{\|P_0\|_F^k}_{\leq 1} \right) \leq \frac{\|\Delta\|_{\mathcal{L}_2}}{c - \|\Delta_0\|_F}. \end{aligned}$$

Thus, we obtain the following sufficient condition.

Corollary 6. Suppose that H^{-1} is stable and causal, and

$$\|\Delta_0\|_F < \frac{c}{2}, \quad \|\Delta\|_{\mathcal{L}_2} = \sum_{k=0}^K \|\Delta_k\|_F < \frac{c}{2C_K},$$

where C_K is given by (2). Then, $G = [(H + \Delta)P]^{-1}$ is causal and stable.

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