On the Combined Effect of Directional Antennas and Imperfect Spectrum Sensing upon Ergodic Capacity of Cognitive Radio Systems

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***Abstract*—We consider a cognitive radio system, consisting of a primary transmitter (PU_{tx}), a primary receiver (PU_{rx}), a secondary transmitter (SU_{tx}), and a secondary receiver (SU_{rx}). The secondary users (SUs) are equipped with steerable directional antennas. We assume the SUs and primary users (PUs) coexist and the SU_{tx} knows the geometry of network. We find the ergodic capacity of the channel between SU_{tx} and SU_{rx}, and study how spectrum sensing errors affect the capacity. In our system, the SU_{tx} first senses the spectrum and then transmits data at two power levels, according to the result of sensing. The optimal SU_{tx} transmit power levels and the optimal directions of SU_{tx} transmit antenna and SU_{rx} receive antenna are obtained by maximizing the ergodic capacity, subject to average transmit power and average interference power constraints. To study the effect of fading channel, we considered three scenarios: 1) when SU_{tx} knows fading channels between SU_{tx} and PU_{rx}, PU_{tx} and SU_{rx}, SU_{tx} and SU_{rx}, 2) when SU_{tx} knows only the channel between SU_{tx} and SU_{rx}, and statistics of the other two channels, and, 3) when SU_{tx} only knows the statistics of these three fading channels. For each scenario, we explore the optimal SU_{tx} transmit power levels and the optimal directions of SU_{tx} and SU_{rx} antennas, such that the ergodic capacity is maximized, while the power constraints are satisfied.**

I. Introduction

Cognitive radio (CR) systems can alleviate spectrum scarcity problem by allowing an unlicensed user to access licensed bands under the condition that its imposed interference on the licensed users are limited [1]. Optimizing the transmission strategies of secondary users (SUs) in the presence of a primary user (PU) has attracted much research interests in industry and academia [2]–[10], where most of these works assume the SUs are equipped with *omni-directional* antennas and the result of spectrum sensing is *perfect*. However, spectrum sensing methods are prone to errors and their false alarm and detection probabilities should be incorporated in the design and performance analysis. Different from the bulk of the literature, in this paper we assume the SUs and PUs can coexist, the SU_{tx} knows the geometry of network. Also, SUs are equipped with *steerable directional* antennas and can use *spatial spectrum holes* [11]–[13] to increase spectrum utilization.

In this work, the SU transmitter (SU_{tx}) first senses the spectrum and then adapts its transmit power, according to the result of spectrum sensing, i.e., SU_{tx} transmits signal to secondary receiver (SU_{rx}) with power levels P_0 and P_1 when spectrum is sensed idle and busy, respectively. To study the effect of fading channels, we consider three scenarios: 1) when SU_{tx} has channel state information (CSI) of links between SU_{tx} and PU_{rx}, PU_{tx} and SU_{rx}, SU_{tx} and SU_{rx}, 2) when SU_{tx} knows only the CSI of link between SU_{tx} and SU_{rx}, and the statistics of the other two links, and, 3) when SU_{tx} only knows the statistics of these three fading channels. For each scenario, we establish the ergodic capacity of the channel between SU_{tx} and SU_{rx}, when spectrum sensing is *imperfect* and find the optimal directions of SU_{tx} and SU_{rx} antennas and optimal SU_{tx} power levels such that the ergodic capacity is maximized, subject to average transmit power and average interference power constraints.

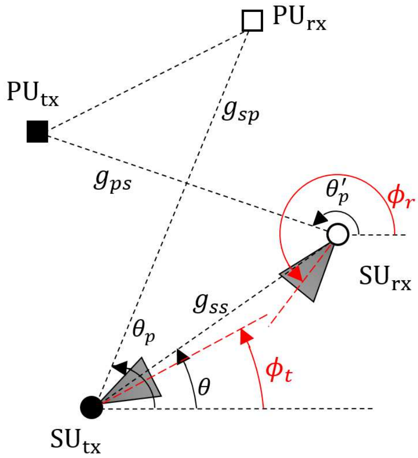


Fig. 1: Our cognitive radio system with directional antennas.

II. System Model

Our CR system model is shown in Fig. 1. The SUs are equipped with steerable directional antennas. The orientation of PU_{rx} and SU_{rx} with respect to SU_{tx} are denoted by θ_p and θ, receptively, and the orientation of PU_{tx} with respect to SU_{rx} is denoted by θ'_p. The boresight of SU_{tx} and SU_{rx} antennas in their local coordination are denoted by ϕ_t and ϕ_r, respectively. We assume θ_p, θ and θ'_p are known or can be estimated [14]. The antenna gain is given by $A(\phi) = A_1 + A_0 \ \exp\left(-B\left(\frac{\phi}{\phi_{3\text{dB}}}\right)^2\right)$ where $B = \ln(2)$, $\phi_{3\text{dB}}$ is the half-power beam-width, A_1 and A_0 are two constant parameters [12], [13]. Let d_{ps}, d_{sp} and d_{ss} be the distances between PU_{tx} and SU_{rx}, PU_{rx} and SU_{tx}, and SU_{tx} and SU_{rx}, respectively.

Spectrum sensing at the SU_{tx} can be formulated as a binary hypothesis testing problem in which $\mathcal{H}_0$ and $\mathcal{H}_1$ with prior probabilities π_0 and $\pi_1 = 1 - \pi_0$ denote the spectrum is truly idle and truly busy, respectively. When the spectrum is truly busy, the average transmit power of PU_{tx} is P_p and we assume SU_{tx} knows P_p. Let $\hat{\mathcal{H}}_1$ and $\hat{\mathcal{H}}_0$ with probabilities $\hat{\pi}_0 = \Pr\{\hat{\mathcal{H}}_0\}$ and $\hat{\pi}_1 = \Pr\{\hat{\mathcal{H}}_1\}$ denote that the result of spectrum sensing is busy and idle, respectively. When the spectrum is sensed idle and busy, SU_{tx} uses two power levels P_0 and P_1, respectively to transmit signal to SU_{rx}. The accuracy of spectrum sensing method is characterized by false alarm probability $P_f = \Pr\{\hat{\mathcal{H}}_1|\mathcal{H}_0\}$ and detection probability $P_d = \Pr\{\hat{\mathcal{H}}_1|\mathcal{H}_1\}$. We assume π_0, P_d, P_f are known.

The fading from SU_{tx} to SU_{rx}, and PU_{tx} to SU_{rx} are denoted by g_{ss} and g_{ps}, respectively, and g_{sp} is the fading from SU_{tx} to PU_{rx}. We assume g_{ss}, g_{ps} and g_{sp} are three independent exponentially distributed random variables with mean γ_{ss}, γ_{ps} and γ_{sp}, respectively. The path-loss is $L = (d_0/d)^\nu$, where d_0 is the reference distance, d is the distance between users, and ν is the path loss exponent. Our goal is to find the ergodic capacity of the channel between SU_{tx} and SU_{rx} and explore the optimal SU transmit power levels and the optimal directions of SU_{tx} and SU_{rx} antennas, such that this capacity maximized, subject to average transmit power and average interference power constraints.

III. Constrained Ergodic Capacity Maximization

When spectrum sensing is imperfect, depending on the true status of the PU and the spectrum sensing result, the ergodic capacity can be written as $C = \mathbb{E}_{\boldsymbol{g}}\Big\{ \sum_{i=0}^{1} \big(\alpha_i\, c_{0,i} + \beta_i\, c_{1,i}\big) \Big\}$, where $\mathbb{E}_{\boldsymbol{g}}\{.\}$ is the expectation operator over random fading coefficients $\boldsymbol{g} = \big(g_{ss}, g_{sp}, g_{ps}\big)$ and $c_{i,j}$ is instantaneous capacity corresponding to $\mathcal{H}_i$ and $\hat{\mathcal{H}}_j$ with probability $\alpha_i = \Pr\{\mathcal{H}_0, \hat{\mathcal{H}}_i\}$ and $\beta_i = \Pr\{\mathcal{H}_1, \hat{\mathcal{H}}_i\}$ for $i \in \{0,1\}$, given as

$$c_{0,i} = \log_2\left(1 + \frac{g_{ss}L_{ss}\; G(\theta,\phi_t,\phi_r) P_i(\boldsymbol{g})}{\sigma_n^2}\right) \quad (1)$$

$$c_{1,i} = \log_2\left(1 + \frac{g_{ss}L_{ss}\; G(\theta,\phi_t,\phi_r) P_i(\boldsymbol{g})}{\sigma_n^2 + P_p\; g_{ps}L_{ps}\; A(\phi_r - \theta_p')}\right). \quad (2)$$

In (1) and (2), $G(\theta,\phi_t,\phi_r) = A(\phi_t - \theta)A(\phi_r - \pi - \theta)$ is the product of SU_{tx} and SU_{rx} antennas' gain and σ_n^2 is the variance of additive zero-mean Gaussian noise at SU_{rx}. It is easy to verify

$$\alpha_0 = \pi_0(1 - P_f), \quad \alpha_1 = \pi_0 P_f,$$
$$\beta_0 = \pi_1(1 - P_d), \quad \beta_1 = \pi_1 P_d.$$

Note that the optimal antenna directions ϕ_t and ϕ_r are expected to be functions of fading $\boldsymbol{g}$ and for simplicity, we dropped parameter $\boldsymbol{g}$. Also, for simplicity of presentation, we drop the parameters θ, ϕ_t and ϕ_r from $G(\theta,\phi_t,\phi_r)$ and define $a = g_{ss}L_{ss}G$ and $\sigma_p^2 = P_p\; g_{ps}L_{ps}A(\phi_r - \theta_p')$. The term σ_p^2 captures the interference on SU_{rx} due to PU activities. Then, we can rewrite (1) and (2) as $c_{0,i} = \log_2\left(1 + \frac{aP_i(\boldsymbol{g})}{\sigma_n^2}\right)$ and $c_{1,i} = \log_2\left(1 + \frac{aP_i(\boldsymbol{g})}{\sigma_n^2 + \sigma_p^2}\right)$, respectively.

Let $\bar{I}_{\text{av}}$ indicate the maximum allowed interference power of PU_{rx} and $\bar{P}_{\text{av}}$ denote the maximum allowed average transmit power of SU_{tx}. To satisfy the average interference power constraint, we have

$$\mathbb{E}_{\boldsymbol{g}}\Big\{ \Big(\beta_0 P_0(\boldsymbol{g}) + \beta_1 P_1(\boldsymbol{g})\Big)\; g_{sp}L_{sp}\; A(\phi_t - \theta_p) \Big\} \leq \bar{I}_{\text{av}}. \quad (3)$$

By defining $b_i = \beta_i g_{sp}L_{sp}A(\phi_t - \theta_p)$, (3) can be written as

$$\mathbb{E}_{\boldsymbol{g}}\Big\{ b_0 P_0(\boldsymbol{g}) + b_1 P_1(\boldsymbol{g}) \Big\} \leq \bar{I}_{\text{av}}. \quad (4)$$

In (4), $b_0 P_0(\boldsymbol{g})$ and $b_1 P_1(\boldsymbol{g})$ denote the imposed interference to PU_{rx} from SU_{tx} when channel is sensed idle and busy, respectively. To satisfy the average transmit power constraint, we have

$$\mathbb{E}_{\boldsymbol{g}}\{\hat{\pi}_0 P_0(\boldsymbol{g}) + \hat{\pi}_1 P_1(\boldsymbol{g})\} \leq \bar{P}_{\text{av}}. \quad (5)$$

The problem we consider is maximizing the ergodic capacity C over $P_0(\boldsymbol{g}), P_1(\boldsymbol{g}), \phi_t$ and ϕ_r subject to constraints (4) and (5). The expression C is concave with respect to $P_0(\boldsymbol{g})$, $P_1(\boldsymbol{g})$ and ϕ_r. However, it is not concave with respect to ϕ_t. The optimal ϕ_t can be obtained using one-dimensional search, i.e., we consider an initial value for ϕ_t and find $P_0(\boldsymbol{g})$, $P_1(\boldsymbol{g})$ and ϕ_r. Then, we find the value of ϕ_t which maximizes C. Given ϕ_t, we can solve this problem using the Lagrange multipliers method to find $P_0(\boldsymbol{g})$, $P_1(\boldsymbol{g})$ and ϕ_r. The Lagrangian is given as

$$L = -\mathbb{E}_{\boldsymbol{g}}\Big\{ \sum_{i=0}^{1} \big(\alpha_i\, c_{0,i} + \beta_i\, c_{1,i}\big) \Big\} + \lambda\Big(\mathbb{E}_{\boldsymbol{g}}\Big\{\hat{\pi}_0 P_0(\boldsymbol{g}) + \hat{\pi}_1 P_1(\boldsymbol{g})\Big\} - \bar{P}_{\text{av}}\Big) + \mu\Big(\mathbb{E}_{\boldsymbol{g}}\Big\{b_0 P_0(\boldsymbol{g}) + b_1 P_1(\boldsymbol{g})\Big\} - \bar{I}_{\text{av}}\Big) \quad (6)$$

where λ and μ are nonnegative Lagrange multipliers. In the following subsections, we address this constrained maximization problem when 1) SU_{tx} knows perfect CSI of $\boldsymbol{g}$, 2) when SU_{tx} knows only g_{ss}, and statistics of g_{ps} and g_{sp}, 3) when SU_{tx} only knows the statistics of $\boldsymbol{g}$.

A. Perfect CSI for Three Fading Channels

In the first scenario, we assume SU_{tx} has perfect knowledge of g_{ss}, g_{ps} and g_{sp} and it maximizes the capacity for each realization of fading coefficients. Taking the derivative of Lagrangian in (6) with respect to $P_i(\boldsymbol{g})$ and equaling it to zero gives

$$\frac{\partial L}{\partial P_i(\boldsymbol{g})} = \frac{-a}{\sigma_n^2 \ln(2)} w_i(x,y) + \lambda\hat{\pi}_i + \mu b_i = 0 \quad (7)$$

where $y \triangleq \sigma_n^2/\sigma_p^2$, $x_i \triangleq \sigma_n^2/aP_i(\boldsymbol{g})$ and

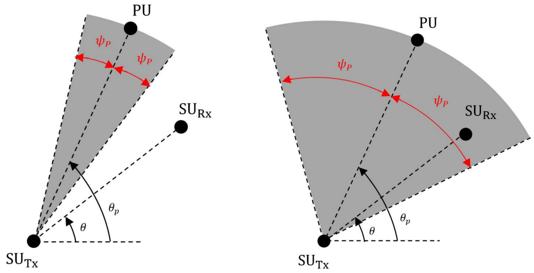
$$w_i(x,y) = x\left(\frac{\alpha_i}{x+1} + \frac{\beta_i y}{xy + x + y}\right).$$

Also, x^{-1} and y^{-1} are the received signal-to-noise-ratio (SNR) and interference-to-noise-ratio (INR) at SU_{rx}. By solving (7), the optimal transmit power levels can be written as

$$P_i(\boldsymbol{g}) = \left[\frac{F_i + \sqrt{\Delta_i}}{2}\right]^+ \qquad \text{for } i = 0,1 \quad (8)$$

where $[x]^+$ denotes $\max(x,0)$ and

$$F_i = \frac{\hat{\pi}_i}{\ln(2)\,(\lambda\hat{\pi}_i + \mu b_i)} - \frac{2\sigma_n^2 + \sigma_p^2}{a}$$



(a) when $|\theta_p - \theta| > \psi_p$ (b) when $|\theta_p - \theta| < \psi_p$

Fig. 2: Illustration of ϕ_t^{opt} for $0 < Z \leq 1$

$$\Delta_i = F_i^2 - \frac{4}{a}\left(\frac{\sigma_n^2(\sigma_n^2+\sigma_p^2)}{a} - \frac{\hat{\pi}_i\sigma_n^2+\beta_i\sigma_p^2}{\ln(2)\,(\lambda\hat{\pi}_i+\mu b_i)}\right).$$

The Lagrange multipliers λ and μ can be updated using subgradient method as follows [5]

$$\lambda^{(n+1)} = \left[\lambda^{(n)} + t_0\left(\mathbb{E}_{\boldsymbol{g}}\{\hat{\pi}_0 P_0(\boldsymbol{g}) + \hat{\pi}_1 P_1(\boldsymbol{g})\} - \bar{P}_{\text{av}}\right)\right]^+ \quad \text{(9a)}$$

$$\mu^{(n+1)} = \left[\mu^{(n)} + t_0\left(\mathbb{E}_{\boldsymbol{g}}\{b_0 P_0(\boldsymbol{g}) + b_1 P_1(\boldsymbol{g})\} - \bar{I}_{\text{av}}\right)\right]^+ \quad \text{(9b)}$$

where t_0 is the step size and λ and μ converge when for a small number δ we get

$$\lambda^{(n)}\left(\mathbb{E}_{\boldsymbol{g}}\{\hat{\pi}_0 P_0(\boldsymbol{g}) + \hat{\pi}_1 P_1(\boldsymbol{g})\} - \bar{P}_{\text{av}}\right) \leq \delta \quad \text{(10a)}$$

$$\mu^{(n)}\left(\mathbb{E}_{\boldsymbol{g}}\{b_0 P_0(\boldsymbol{g}) + b_1 P_1(\boldsymbol{g})\} - \bar{I}_{\text{av}}\right) \leq \delta. \quad \text{(10b)}$$

The optimal ϕ_r can be obtained by solving $\partial L/\partial\phi_r = 0$. There is no closed form solution for ϕ_r^{opt}, but, one can verify that when transmit power of PU$_{\text{tx}}$ is zero ($P_p = 0$), $\phi_r^{\text{opt}} = \pi + \theta$. We can reduce the computational complexity of one-dimensional search for finding ϕ_t^{opt} by finding a narrower interval to which ϕ_t^{opt} belongs to [13]. We define

$$Z = \frac{\bar{I}_{\text{av}}}{\pi_1 g_{sp} A_0 \bar{P}_{\text{av}}} - \frac{A_1}{A_0}. \quad (11)$$

If $Z > 1$, it means that PU$_{\text{rx}}$ can tolerate an interference power that is larger than the interference power imposed by SU$_{\text{tx}}$, constraint (4) is loose and $\phi_t^{\text{opt}} = \theta$. When $0 < Z \leq 1$, we define $\psi_p = \phi_{3\text{dB}}\sqrt{\frac{-1}{B}\ln(Z)}$ and consider two cases. When $|\theta_p - \theta| > \psi_p$, ϕ_t^{opt} has to lie outside the shaded area shown in Fig. 2a. Since the unshaded area in Fig. 2a includes the line of sight (LOS) between SU$_{\text{tx}}$ and SU$_{\text{rx}}$, $\phi_t^{\text{opt}} = \theta$. When $|\theta_p - \theta| < \psi_p$, which is shown in Fig. 2b, ϕ_t^{opt} lies in the

$$\begin{cases} \phi_t^{\text{opt}} \in [\theta_p - \psi_p, \theta], & \text{if } \theta_p > \theta \\ \phi_t^{\text{opt}} \in [\theta, \theta_p + \psi_p], & \text{if } \theta_p < \theta. \end{cases}$$

If $Z \leq 0$, we cannot find a narrower interval. Algorithm 1 summarizes our proposed approach to find the optimal solutions ϕ_t^{opt}, ϕ_r^{opt}, P_0^{opt} and P_1^{opt}.

B. Perfect CSI for g_{ss} and Statistical CSI for Other Channels

For the second scenario, we assume that SUs cannot cooperate with PUs and as a result, SU$_{\text{tx}}$ and SU$_{\text{rx}}$ cannot estimate the fading coefficients g_{sp} and g_{ps}, respectively and they only

Algorithm 1: Optimization Algorithm

$k \leftarrow 0$
$\phi_r^{(0)} = \pi + \theta$
repeat
  $\lambda^{(0)} = \lambda_{\text{init}}$, $\mu^{(0)} = \mu_{\text{init}}$
  $n \leftarrow 0$
  repeat
    calculate $P_0^{(k)}$ and $P_1^{(k)}$ using (8).
    update λ and μ using (9).
    $n \leftarrow n + 1$
  until (10) *is satisfied*;
  solve $\partial L/\partial\phi_r = 0$ and update $\phi_r^{(k+1)}$.
  $k \leftarrow k + 1$
until *the differences of* $\phi_r^{(k)}$, $P_0^{(k)}$ *and* $P_1^{(k)}$ *in two consecutive iterations is less than some pre-determined values*;
$\phi_t^{\text{opt}} = \text{argmax}\,\{C\}$ using bisection search
$P_i^{\text{opt}} = [P_i]_{\phi_t = \phi_t^{\text{opt}}}$
$\phi_r^{\text{opt}} = [\phi_r]_{\phi_t = \phi_t^{\text{opt}}}$

know the statistics of fading coefficients g_{sp} and g_{ps}. On the other hand, we assume that SU$_{\text{tx}}$ has perfect knowledge of fading coefficient g_{ss}. Therefore, at first we take expectation with respect to g_{sp} and g_{ps} in ergodic capacity expression and then maximize the capacity. In this case the optimal transmit power levels and the optimal antenna directions are functions of g_{ss}. The instantaneous capacity $c_{0,i}$ is independent of g_{sp} and g_{ps} and $\mathbb{E}_{g_{ps},g_{sp}}\{c_{0,i}\} = c_{0,i}$. The expectation of $c_{1,i}$ can be written as

$$\mathbb{E}_{g_{ps},g_{sp}}\{c_{1,i}\} = \mathbb{E}_{g_{ps}}\left\{\log_2\left(1 + \frac{g_{ss}L_{ss}GP_i(g_{ss})}{\sigma_n^2 + P_p\, g_{ps}L_{ps}A(\phi_r - \theta_p')}\right)\right\}$$
$$= \frac{1}{\ln(2)}\left[\ln\left(1 + \frac{1}{x_i}\right) + T(\bar{y}) - T\left(\bar{y} + \frac{\bar{y}}{x_i}\right)\right] \quad (12)$$

where $T(z) = e^z\text{Ei}(-z)$ and $\text{Ei(z)} = -\int_{-z}^{\infty} e^{-t}\, t^{-1}dt$ is the exponential integration [15]. In (12), $x_i = \sigma_n^2/aP_i(g_{ss})$, $\bar{y} = \sigma_n^2/\bar{\sigma}_p^2$ and $\bar{\sigma}_p^2 = \mathbb{E}_{g_{ps}}\{\sigma_p^2\} = P_p\gamma_{ps}L_{ps}A(\phi_r - \theta_p')$. Finally, the ergodic capacity in this scenario is

$$C = \mathbb{E}_{g_{ss}}\left\{\sum_{i=0}^{1}\left[\hat{\pi}_i\log_2\left(1 + \frac{1}{x_i}\right) + \frac{\beta_i}{\ln(2)}\left(T(\bar{y}) - T\left(\bar{y} + \frac{\bar{y}}{x_i}\right)\right)\right]\right\}$$

Moreover, the constraints in (4) and (5) can be written as

$$\mathbb{E}_{g_{ss}}\{\bar{b}_0 P_0(g_{ss}) + \bar{b}_1 P_1(g_{ss})\} \leq \bar{I}_{\text{av}} \quad \text{(13a)}$$

$$\mathbb{E}_{g_{ss}}\{\hat{\pi}_0 P_0(g_{ss}) + \hat{\pi}_1 P_1(g_{ss})\} \leq \bar{P}_{\text{av}} \quad \text{(13b)}$$

where $\bar{b}_i = \mathbb{E}_{g_{sp}}\{b_i\} = \beta_i\gamma_{sp}L_{sp}A(\phi_t - \theta_p)$. The optimal transmit power levels $P_i(g_{ss})$ can be obtained by solving the following equation

$$\frac{\partial L}{\partial P_i(g_{ss})} = \frac{-a}{\sigma_n^2\ln(2)}f_i(x_i,\bar{y}) + \lambda\hat{\pi}_i + \mu\bar{b}_i = 0$$

where

$$f_i(x,\bar{y}) = \frac{\alpha_i x}{x+1} - \beta_i\bar{y}\,T\left(\bar{y} + \frac{\bar{y}}{x}\right).$$

This equation has no closed form solution and has to be solved numerically. Furthermore, the parameter Z in (11) for this scenario is modified to

$$\bar{Z} = \frac{\bar{I}_{\text{av}}}{\pi_1 \gamma_{sp} A_0 \bar{P}_{\text{av}}} - \frac{A_1}{A_0}. \tag{14}$$

Algorithm 1 can be used for this scenario with some modifications.

C. Statistical CSI for All Fading Channels

In the third scenario we assume that SU_{tx} cannot estimate g_{ss} and it knows only the statistical CSI of all fading channels. Even if SU_{tx} can estimate g_{ss}, when we maximize the capacity for each realization of g_{ss}, the optimal ϕ_t and ϕ_r will be a function of g_{ss} and as a result they may change very fast in a fast fading environment. In some cases where antennas are steered mechanically, their rotation speeds are limited and cannot adapt themselves according to channel variations. Thus, in this scenario we wish the optimal directions to be independent of the realizations of fading coefficients. Hence, we take expectation with respect to all fading coefficients and then maximize capacity. The expectation of $c_{0,i}$ is equal to $\mathbb{E}_{g_{ss}}\{c_{0,i}\} = -T(\bar{x}_i)/\ln(2)$, where $\bar{x}_i = \sigma_n^2/\bar{a}P_i$ and $\bar{a} = \mathbb{E}\{a\} = \gamma_{ss} L_{ss} G$. Similar to previous section, we can write $\mathbb{E}_{\boldsymbol{g}}\{c_{1,i}\} = -U(\bar{x}_i, \bar{y})/\ln(2)$ where

$$U(\bar{x}_i, \bar{y}) = \begin{cases} \frac{-\bar{y}}{\bar{y}-\bar{x}_i}\left[T(\bar{y}) - T(\bar{x}_i)\right], & \text{if } \bar{x}_i \neq \bar{y} \\ -\bar{x}_i T(\bar{x}_i) - 1. & \text{if } \bar{x}_i = \bar{y} \end{cases}$$

The ergodic capacity is

$$C = \frac{-1}{\ln(2)} \sum_{i=0}^{1} \left[\alpha_i T(\bar{x}_i) + \beta_i U(\bar{x}_i, \bar{y})\right]$$

and the constraints in (4) and (5) can be written as

$$\bar{b}_0 P_0 + \bar{b}_1 P_1 \leq \bar{I}_{\text{av}} \tag{15a}$$

$$\hat{\pi}_0 P_0 + \hat{\pi}_1 P_1 \leq \bar{P}_{\text{av}}. \tag{15b}$$

The optimal transmit power levels can be obtained by solving the following equation numerically

$$\frac{\partial L}{\partial P_i} = \frac{-\bar{a}}{\sigma_n^2 \ln(2)} h_i(\bar{x}_i, \bar{y}) + \lambda \hat{\pi}_i + \mu \bar{b}_i = 0$$

where

$$h_i(\bar{x}, \bar{y}) = \bar{x}^2 \left(\alpha_i \frac{\partial T(\bar{x})}{\partial x} + \beta_i \frac{\partial U(\bar{x}, \bar{y})}{\partial x}\right).$$

Algorithm 1 can be used for this scenario.

IV. Numerical Results

We numerically show the effect of using directional antennas on the ergodic capacity of the considered CR system when spectrum sensing is imperfect. Assume $\sigma_n^2 = 1$, $\phi_{3\text{dB}} = 45°$, $A_0 = 9.8$, $A_1 = 0.2$, $\gamma_{ss} = \gamma_{sp} = \gamma_{ps} = 1$, $\pi_1 = 0.3$, $\theta_p = 90°$ and $\theta'_p = 130°$. For fair comparisons, we consider a fixed spectrum sensing method with $P_d = 0.9$ and $P_f = 0.1$.

Suppose $C_{\text{opt}}^{\text{Dir}}$ denote the optimal capacity when we use directional antennas. Fig. 3 shows $C_{\text{opt}}^{\text{Dir}}$ versus θ for $P_p = 0.4, 3$ watts for all three scenarios when $\bar{P}_{\text{av}} = 12$ dB. When θ increases from 0° to 40°, SU_{rx} receives less interference from PU_{tx} and SU_{tx} can increase transmit power and as a result the capacity increases. However, when θ increases from 40° to 80°, SU_{tx} imposes more interference on PU_{rx} and the optimal capacity decreases. Furthermore, we observe that the capacity for scenario 3 is always smaller than that of scenarios 1 and 2. Increasing P_p doesn't have any impact on constraints, however, the capacity expression depends on P_p and as it can be seen in Fig. 3, increasing P_p decreases the capacity. Fig. 4 shows the optimal capacity for all three scenarios when $\bar{P}_{\text{av}} = 15$ dB. Comparing Figs. 3 and 4, we can see that when the maximum allowed average transmit power of SU_{tx} ($\bar{P}_{\text{av}}$) increases, the capacity increases as well, provided that the constraint (4) is not violated. Fig. 5 which plots $C_{\text{opt}}^{\text{Dir}}$ versus $\bar{P}_{\text{av}}$ when $\theta = 50°$ and $\bar{I}_{\text{av}} = 0$ dB also shows the similar fact.

Let $C_{\text{opt}}^{\text{Omn}}$ denote the capacity when SU_{tx} and SU_{rx} have omni-directional antennas and only transmit power levels P_0 and P_1 are optimized subject to constraints (4) and (5). Note that P_0^{opt} and P_1^{opt} are constant for all θ when SUs use omni-directional antennas and $C_{\text{opt}}^{\text{Omn}}$ is independent of θ. Furthermore, let $C_{\text{opt}}^{\text{LOS}}$ be the capacity when directional antennas of SU_{tx} and SU_{rx} are exactly pointed at each other ($\phi_t = \theta$, $\phi_r = \pi + \theta$) and only P_0 and P_1 are optimized subject to constraints (4) and (5). We compare $C_{\text{opt}}^{\text{Dir}}$, $C_{\text{opt}}^{\text{Omn}}$ and $C_{\text{opt}}^{\text{LOS}}$.

We define three capacity ratios $\Gamma_{\text{D2O}} = C_{\text{opt}}^{\text{Dir}}/C_{\text{opt}}^{\text{Omn}}$, $\Gamma_{\text{L2O}} = C_{\text{opt}}^{\text{LOS}}/C_{\text{opt}}^{\text{Omn}}$ and $\Gamma_{\text{D2L}} = C_{\text{opt}}^{\text{Dir}}/C_{\text{opt}}^{\text{LOS}}$. Fig. 6 plots Γ_{D2O} and Γ_{L2O} versus θ when $\bar{P}_{\text{av}} = 12, 15$ dB. We observe that when $\theta \approx \theta_p$, $C_{\text{opt}}^{\text{Dir}} \approx C_{\text{opt}}^{\text{LOS}}$ and as $|\theta - \theta_p|$ increases, the capacity gain increases. When PU_{rx} and SU_{rx} are close, using directional

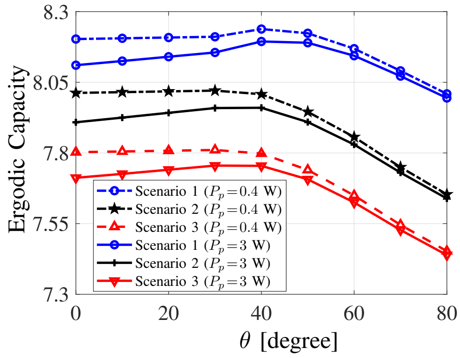


Fig. 3: $C_{\text{opt}}^{\text{Dir}}$ versus θ for three scenarios when $\bar{P}_{\text{av}} = 12$ dB.

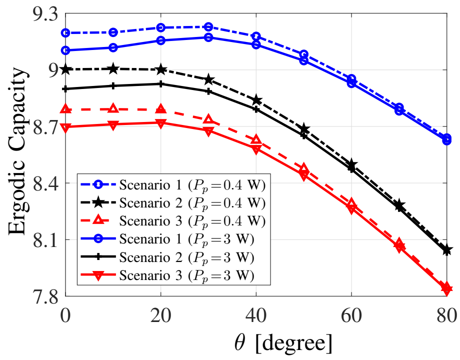


Fig. 4: $C_{\text{opt}}^{\text{Dir}}$ versus θ for three scenarios when $\bar{P}_{\text{av}} = 15$ dB.

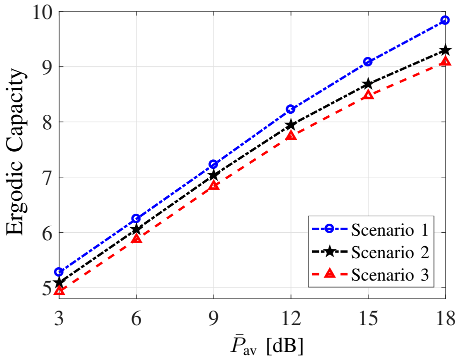


Fig. 5: $C_{\text{opt}}^{\text{Dir}}$ versus $\bar{P}_{\text{av}}$ for three scenarios when $\theta = 50°$, $\bar{I}_{\text{av}} = 0$.

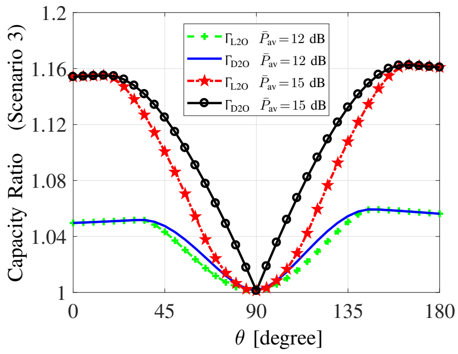


Fig. 6: Capacity ratio versus θ when $\bar{I}_{\text{av}} = 0$ dB.

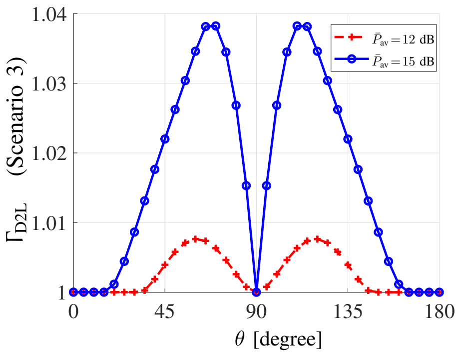


Fig. 7: Γ_{D2L} versus θ when $\bar{I}_{\text{av}} = 0$ dB.

antennas does not enhance the ergodic capacity (with respect to using omni-directional antennas). The capacity gain in Fig. 6 finally saturates, since the direction of SU_{tx} goes sufficiently away from PU_{rx} and directional antenna of SU_{tx} reduces the interference imposed on PU_{rx}. In addition, we can see that when $\bar{P}_{\text{av}}$ of SU_{tx} increases , the ergodic capacity increases, while constraints (4) and (5) still hold true.

The effect of optimizing the orientation of directional antennas on ergodic capacity is illustrated in Fig. 7, where the capacity gain Γ_{D2L} versus θ is plotted for $\bar{P}_{\text{av}} = 12, 15$ dB. We note that when we optimize the angles ϕ_t and ϕ_r, SU_{tx} can use more power for transmission (i.e., use higher power levels P_0 and P_1) without violating constraints (4) and (5) and, hence, the capacity increases.

V. Conclusion

In this paper, we considered a CR system, where the SUs are equipped with steerable directional antennas. The SU_{tx} first senses the spectrum (with error) and then transmits data at two power levels, according to the result of sensing. The optimal SU_{tx} transmit power levels and the optimal directions of SU_{tx} transmit antenna and SU_{rx} receive antenna are obtained by maximizing the ergodic capacity, subject to average transmit power and average interference power constraints. To study the effect of fading channels, we considered three scenarios: 1) when SU_{tx} knows fading channels between SU_{tx} and PU_{rx}, PU_{tx} and SU_{rx}, SU_{tx} and SU_{rx}, 2) when SU_{tx} knows only the channel between SU_{tx} and SU_{rx}, and statistics of the other two channels, and, 3) when SU_{tx} only knows the statistics of these three fading channels. Through simulations, we showed that directional antennas significantly enhance the ergodic capacity, without violating the power constraints.

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