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# A Mixed-Integer Linear Programming approach to wind farm layout and inter-array cable routing

Martina Fischetti[1], John Leth[2] and Anders Bech Borchersen[2]

*Abstract*— A Mixed-Integer Linear Programming (MILP) approach is proposed to optimize the turbine allocation and inter-array offshore cable routing. The two problems are considered with a two steps strategy, solving the layout problem first and then the cable problem. We give an introduction to both problems and present the MILP models we developed to solve them. To deal with interference in the onshore cases, we propose an adaptation of the standard Jensen's model, suitable for 3D cases. A simple Stochastic Programming variant of our model allows us to consider different wind scenarios in the optimization. For the inter-array cable routing, we propose a new MILP model able to deal with different constraints arsing in practical application, such as capacity limitations, substation limitations and non-crossing constraints. Computational results on real-world instances prove the practical viability of the approach.

## I. INTRODUCTION

Two topics of great interest in the wind energy sector are investigated in the present paper: wind turbine allocation and their optimal connection among cables. Our models for the wind farm allocation fit both onshore and offshore cases, while in the cable routing optimization we focused on the offshore case (the so called "inter-array" cable connection problem). In the offshore layout optimization the model we propose is also taking costs of foundation into account.

Wind farm layout optimization problems deal with the optimal placement of turbines in a wind farm field. Currently, metaheuristics and greedy approaches have been used. Some existing heuristic methods [14] [17] and Mixed-Integer Linear Programming models [4] [16] [18] [20] [13] have explored the problem with discretization, while a continuous approach has been used in [12] [11]. The continuous models are highly nonconvex and turn out to be intractable from a computational viewpoint when considering real-world cases, therefore we preferred a discrete programming approach. The MILP approach with some ad-hoc heursitics is able to solve large-instances problems as shown in [13]. An interesting feature of the wind farm optimization problem is that of dealing with the aerodynamic interaction among multiple turbines. In a simple scenario with only two turbines, the turbine downstream is said to be in the wake region of the upstream turbine, and it experiences a loss in energy production due to the reduction in wind speed and to the increase in turbulence intensity [10]. In practice, a turbine that is downstream of multiple turbines is affected by all upstream turbines simultaneously, and the overall effect is a nonlinear function of individual wakes. There are different analytical equations to describe the superposition of multiple wakes, some being closer to the physical reality than others [3]. It is however very difficult to incorporate the more accurate wake equations into a mathematical programming model due to their nonlinearity: currently, only heuristics [14] [17] [12] [2] are able to take accurate wake models into account. In our optimization we are therefore considering Jensen's model [10], adapting it to the onshore case, and we assume pairwise interference between sites (i.e., assuming the interferences as cumulative). We use Stochastic Programming to consider different wind scenarios in our optimization.

Another problem considered is the inter-array cable routing optimization. Different cables, with different capacities and costs, exist and a correct use of them can lead to large savings, since the cost of cables constitute a significant part of the establishment costs of a wind turbine park. This optimization problem has been studied in [1] and [16]. Here we developed a different model able to deal with different constraints appearing in practical application, such as non-crossing constraints.

Hence the contributions from the present work is the development of original MILP models to solve the two problems with specific constraints. In the present paper, for both models, some interesting tests are comparing how the optimal (from a wind-resources or cable-price point of view) wind farms should look like, compared with the ones actually build nowadays. A detailed use of MILP-based heuristics to speed up the current model in difficult test-cases is presented in [13]. An original adaptation of the classical Jensen's model is here presented.

The present paper is organized as follows: In Sect. II we introduce the MILP model we used for the layout optimization problem. The interference between turbines is taken into account in the model, and Sect. III explains how we adapted the Jensen's model to work in a 3d onshore scenario. In Sect. IV we report some tests for layout optimization. The cable routing optimization is treated in Sect. V, where we present our MILP model. Sect. VI compares the cable layout from our tool to existing ones, showing how the use of our tool can lead to large savings. Finally Sect. VII summarizes the

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contributions of the paper.

## II. LAYOUT OPTIMIZATION

This section is based on the original results presented in [6], [13], however here we also include, as a first result, the cost of foundations in the offshore case.

Our model determines a feasible allocation of turbines that maximizes power production. The building area (site) and its resource maps are given on input. The optimizer considers the following constraints:

a) a minimum and maximum number of turbines that can be built;

b) a minimum separation distance between any pair of turbines to ensure that the blades do not physically clash (turbine proximity constraints);

c) interference between installed turbines;

d) cost of foundations in the offshore case.

Let $V \subset \mathbb{R}^2$ denote the set of possible positions for a turbine, called "sites" in what follows, and let

- $I_{ij}$ be the interference (loss of power) experienced by site $j$ when a turbine is installed at site $i$, with $I_{jj} = 0$ for all $j \in V$;

- $P_i$ be the power that a turbine would produce if built (alone) at site $i$;

- $N_{MIN}$ and $N_{MAX}$ be the minimum and maximum number of turbines that can be built, respectively;

- $D_{MIN}$ be the minimum distance between two turbines;

- $dist(i, j)$ be the distance between sites $i$ and $j$.

In the sequel we assume that a numbering have been chosen among the elements of V and by abuse of notation we also let i denote the numbering of element $i \in V$. In addition, let $G_I = (V, E_I)$ denote the incompatibility graph with

$$E_I = \{\{i,j\} : i,j \in V, dist(i,j) < D_{MIN}, i \neq j\}$$

note that $\{i,j\} = \{j,i\}$ by convention. Let $n := |V|$ denote the total number of sites.

In our model, binary variables are defined for each $i \in V$:

$$x_i = \begin{cases} 1 & \text{if a turbine is built at site } i \in V; \\ 0 & \text{otherwise} \end{cases}$$

The original quadratic objective function (to be maximized)

$$\sum_{i \in V} P_i x_i - \sum_{i \in V} \left( \sum_{j \in V} I_{ij} x_j \right) x_i \qquad (1)$$

can be restated as

$$\sum_{i \in V} (P_i x_i - w_i) \qquad (2)$$

where

$$w_i := \left( \sum_{j \in V} I_{ij} x_j \right) x_i = \begin{cases} \sum_{j \in V} I_{ij} x_j & \text{if } x_i = 1; \\ 0 & \text{if } x_i = 0. \end{cases}$$

denotes the total interference caused by site $i$. Our compact model then reads

$$
\begin{align}
\max \quad & z = \sum_{i \in V}(P_i x_i - w_i) && (3) \\
\text{s.t.} \quad & N_{MIN} \leq \sum_{i \in V} x_i \leq N_{MAX} && (4) \\
& x_i + x_j \leq 1 && \forall \{i,j\} \in E_I & (5) \\
& \sum_{j \in V} I_{ij} x_j \leq w_i + M_i(1 - x_i) && \forall i \in V & (6) \\
& x_i \in \{0, 1\} && \forall i \in V & (7) \\
& w_i \geq 0 && \forall i \in V & (8)
\end{align}
$$

where the big-M term

$$M_i = \sum_{\substack{j \in V \\ [i,j] \notin E_I}} I_{ij}$$

is used to deactivate constraint (6) in case $x_i = 0$. The model above follows a recipe of Glover [9] which is widely used, e.g., in the Quadratic Assignment Problem [19], [8]. It allows our model to work on larger instances compared with equivalent models in the literature (we refer to [6] for details).

The definition of the turbine power $P_i$ and of the interference $I_{ij}$ depends on the wind scenario considered, that greatly varies in time. Let us introduce a new variable $z_{i,j}$ equal to 1 if turbines are built both in position $i$ and $j$, 0 otherwise. With respect to (1), $z_{i,j}$ as the same meaning of $x_i x_j$. Using statistical data, one can in fact collect a large number, say $K$, of wind scenarios $k$, each associated with $P_i^k, I_{i,j}^k$ and with a probability $\pi_k$. Using that data, one can write a Stochastic Programming variant of the previous model where only the objective function needs to be modified as

$$z = \sum_{k=1}^{K} \pi_k \left( \sum_{i \in V} P_i^k x_i - \sum_{i \in V} \sum_{j \in V} I_{ij}^k z_{ij} \right) \qquad (9)$$

while all constraints stay unchanged as they only involve "first-stage" variables $x$ and $z$. It is therefore sufficient to define

$$P_i := \sum_{k=1}^{K} \pi_k P_i^k \qquad \forall i \in V \qquad (10)$$

$$I_{ij} := \sum_{k=1}^{K} \pi_k I_{ij}^k \qquad \forall i, j \in V \qquad (11)$$

to obtain the same model (1)–(8) as before. The above model can be solved for large-scale instances (around $20\,000$ possible positions to locate turbines) in a few minutes on a standard PC, using some ad-hoc heuristics and a MILP-based heuristic called "Proximity Search" [7]. We refer to [13] for details.

In the offshore case, and contrary to [6], we here also add the cost of foundation, as the building cost highly depends on the sea depth and type of turbine, and this affects the final optimal turbine allocation (for further details see [15]). To consider that, the objective function (9) becomes

$$\text{maximize} \sum_{i \in V} \left[ \left( P_i - \frac{p(d_i)}{FACT} \right) x_i - w_i \right] \qquad (12)$$
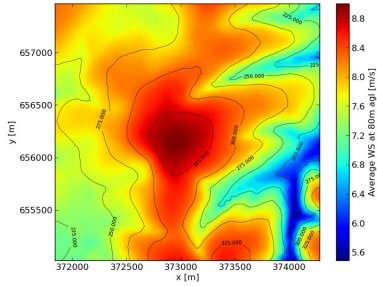
Fig. 1: Average wind speed [m/s] at 80m agl (above ground level) for our onshore test site

Where $p(d_i)$ is the price of foundation as a function of the depth $d_i$ of the $i$th position, while $FACT$ is a factor to scale the price from €/KW to MW in a typical 20-years production horizon. All the others constraints of the model stay unchanged.

## III. INTERFERENCE IN ONSHORE CASES

To compute the interference matrix we used the Jensen's model [10] , adapting it to a 3D scenario. That means that, under proper assumptions, Jensen's model can be as well used in an onshore setting, where the terrain topology plays a role. The offshore case is a 2D case since the sea level can be assumed as constant and the wind as equally distributed in the site. In this framework, the standard Jensen's formula computes the loss in wind speed $\delta V$. So if we want to compute the loss $\delta V$ on a turbine located in the position (say) $j$ due to the presence of an upwind turbine in position (say) $i$, we apply the following formula

$$\delta V = U(1 - \sqrt{1 - C_t})\left(\frac{D}{D + 2kX}\right)^2 \qquad (13)$$

where

$U$ = upwind speed at turbine $i$
$C_t$ = thrust coefficient corresponding to
       wind speed $U$
$D$ = rotor dimension (diameter)
$X$ = distance between the first coordinates
      of $i$ and $j$ (i.e. on the $x$-axis)
$k$ = wake decay constant, typically
$$k = \begin{cases} 0.075 & \text{onshore} \\ 0.050 & \text{offshore} \end{cases}$$

The wind speed at $j$ is computed as $V = U - \delta V$. Determining interference for onshore problems is actually much more difficult than in the offshore (2D) case, due to the presence of nonuniform wind.
Due to possible different heights even without any interference, wind is no longer uniform in the site. Figure 1, for example, shows how the wind is distributed in average in a real-world onshore site in United Kingdom.

Some assumptions therefore need be formulated to be able to extend Jensen's model to a 3D scenario. First, we are
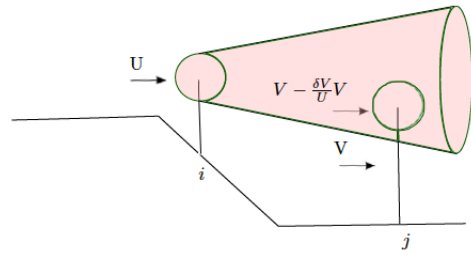


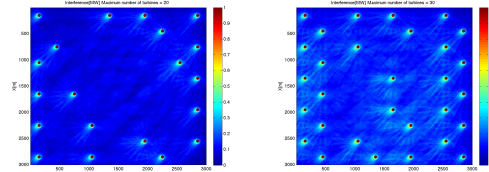Fig. 2: A simple scheme for our 3D Jensen's model



Fig. 3: Optimal layout solutions imposing maximum number of turbine equal to 20 (right) and 30 (left)

not considering wind flow inclination. That means that the interference cone is always horizontal and does not depend on the shape of mountains and hills. Due to this assumption, our 3D wake cone would look like the one in Figure 2.

The wind speeds $U$ and $V$ are the average wind speed given on input for points $i$ and $j$ respectively (i.e., we take the values shown in Figure 1 for the real-world test site). Because of this, value $\delta V$ computed by (13) cannot be used as it is, as this could even produce a negative wind value at point $j$. Therefore a proportion has been used to compute wind reduction at point $j$ due to the presence of a turbine at point $i$. To be more specific, $\delta V$ is still calculated according to (13) but the wind reduction applied in $j$ is $\frac{\delta V}{U}V$, i.e., the new wind, $V'$, in $j$ is now computed as

$$V' = V - \frac{\delta V}{U}V \qquad (14)$$

Wind direction is also different from point to point. However, considering a wind direction time series for each point would be computationally too heavy, so we consider only one of such series to compute the average interference.

## IV. TESTS

Our model was first tested on an offshore grid of $10 \times 10$ possible positions in a square of $3000 \times 3000\,m^2$, with $D_{MIN}$ 400 m. Wind scenarios used to compute the interference matrix are taken from the European Wind Energy Association (EWEA) data, aggregated in bins according to 24 wind-angles (15 degrees each) and wind speeds in 1m/s per bin.

The first set of plots (Figure 3 ) shows how the optimal layout changes when imposing a different maximum number of turbines. Red circles represent the built turbines, while the color in the background refers to the interference induced by those turbines.

It interesting to notice that the optimal solutions have as many turbines as possible on the boundaries: those are, indeed, the positions in which a turbine would cause less
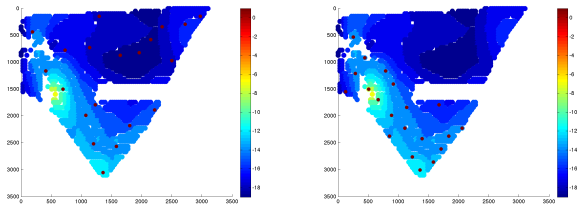
Fig. 4: Allocation of wind turbine in the Horns Rev 3 test case. The colors in the background represent the water depth. The first plot is the solution non considering cost of foundations, the second one is the solution considering cost of foundations. In the second case the turbines are concentrated in the less deep area

interference to the others. This means that, from a wind-resources point of view, turbines should not be allocated regularly, as we are used to see nowadays.

If we use the objective function (12) the solution layouts change due to the cost of foundations. If we for example consider the real-world case of Horns Rev 3, which information are available in [5], Figure 4 shows how the optimal layout would change using the model without and with cost of foundations. In this test we imposed the construction of 20 Vestas V164 wind turbines in the site, minimum distance of 10 rotor diameters.

In the onshore case, instead, the wind is not homogeneously distributed, i.e., a turbine can give different power productions depending on its position (even without considering any interference). As a consequence, turbines are concentrated in the areas with the best wind. We considered a real-world site in United Kingdom, where the wind is distributed as shown in Figure 1. Some positions (red in the first plot of 5) have not been considered in the optimization since unavailable for building turbines (due to extreme wind speed, extreme turbulance, high inclination and too high shear coefficient). The allowed positions turn out to identify 3103 potential points (blue area in the first plot in Figure 5). The potential power distribution in this area is shown in the second plot of Figure 5.

We compared our onshore solutions with the solutions given by a commercial software for the same study-case. This commercial software is using heuristics to determine the layout. In the tests we impose for both software a maximum number of turbine to built in the area (5,10,15,20,22,25,unlimited). The results of the comparison, in Table I, show the effectiveness of our method, since we are able to outperform the competitor in all tests.

## V. MILP MODEL FOR CABLE ROUTING OPTIMIZATION

As a second result we now present inter-array cable optimization. Here we consider the turbine layout as given and we want to find an optimal cable connection between all turbines and the given substations, minimizing the total cable cost. The optimization considers that:

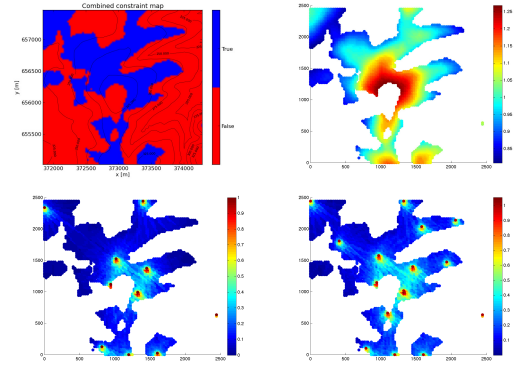- the energy flow leaving a node must be supported by a single cable



Fig. 5: The first plot shows feasible positions while the second shows the power distribution within the site. All other plots show our layout solutions when considering a maximum number of turbines equal to 10 and 15, respectively

| max n. turb. | WindPRO (WP) | | | MILP heuristic | |
|---|---|---|---|---|---|
| | layout production by WindPRO (MWh/y) | layout production (our obj value) (MWh/y) | production difference | layout production (our obj value) (MWh/y) | Improvement wrt WindPRO |
| 5 | 51 695 | 51 776 | 0.2% | 52 123 | 0.7% |
| 10 | 99 330 | 99 577 | 0.2% | 101 024 | 1.5% |
| 15 | 143 727 | 144 789 | 0.7% | 146 454 | 1.1% |
| 20 | 185 285 | 185 252 | 0.0% | 188 782 | 1.9% |
| 22 | 200 699 | 200 412 | -0.1% | 205 039 | 2.3% |
| 25 | 200 699 | 200 412 | -0.1% | 228 270 | 13.9% |
| (unlimited) 28 | 200 699 | 200 412 | -0.1% | 247 569 | 23.5% |

TABLE I: Summary of the comparison between *Optimize* and our tool. Note that *WindPRO's Optimize* is not able to locate more than 22 turbines.

- the energy flow in each connection cannot exceed the capacity of the installed cables
- different cables, with different capacities and costs, can be installed
- cable crossing should be avoided since it is expensive (requires an expensive bridge structure, it decreases the capacity of the crossing cables and drastically increase the risk of damages)
- a given maximum number of strings can be connected to each substation

Let us consider the turbine positions as the nodes of a directed graph $G = (V, A)$ and all possible connections between them as directed arcs. Some nodes correspond to the substations that are considered as the roots of the trees, and are the only nodes that collect energy. We can therefore indicate with $P_h$ the power production at node, so

$$P_h \begin{cases} \geq 0 & \text{if the } h\text{-th node is a turbine} \\ = -1 & \text{if the } h\text{-th node is a substation} \end{cases} \quad (h \in V)$$

Suppose that $T$ different types of cable can be used. Each type of cable $t$ has its given capacity $k_t$ and its cost $c_{i,j}^t$. This cost can be defined as $c_{i,j}^t = dist(i,j)u_t$ for each arc $(i,j)$ and for each type $t \in T$, where $dist(i,j)$ is the distance between turbine $i$ and turbine $j$ and $u_t$ is the unit cost of cable type $t$.

We used the continuous variables $f_{i,j} \geq 0$ to indicate the flow from $i$ to $j$, and the binary variables $x_{i,j}^t$ to define the cable layout, such that

$$x_{i,j}^t = \begin{cases} 1 & \text{if arc } (i,j) \text{ is built with the cable type } t \\ 0 & \text{otherwise} \end{cases}$$
$$(i, j \in V, \quad t \in T)$$

Finally, variables $y_{i,j}$ indicate whether an arc between turbine $i$ and $j$ is built (with any type of cable). Consequently, $y_{i,j}$ is a binary variable

$$y_{i,j} = \begin{cases} 1 & \text{if arc } (i,j) \text{ is built} \\ 0 & \text{otherwise} \end{cases} \quad (i, j \in V, \quad t \in T)$$

Note that variables $y_{i,j}$ are related to variables $x_{i,j}^t$, namely $\sum_{t \in T} x_{ij}^t = y_{ij}$ for all $i \neq j$ in $V$. Our model then reads:

$$\min \quad \sum_{i,j \in V} \sum_{t \in T} c_{i,j}^t x_{i,j}^t \quad (15)$$
$$\text{s.t.} \quad \sum_{t \in T} x_{i,j}^t = y_{i,j} \quad \forall i, j \in V : j \neq i \quad (16)$$
$$\sum_{i:i \neq h} (f_{h,i} - f_{i,h}) = P_h \quad \forall h \in V : P_h \geq 0 \quad (17)$$
$$\sum_{t \in T} k_t x_{i,j}^t \geq f_{i,j} \quad \forall i, j \in V : j \neq i, \quad (18)$$
$$\sum_{j:j \neq h} y_{h,j} = 1 \quad \forall h \in V : P_h > 0 \quad (19)$$
$$\sum_{j:j \neq h} y_{h,j} \leq 1 \quad \forall h \in V : P_h = 0 \quad (20)$$
$$\sum_{j:j \neq h} y_{h,j} = 0 \quad \forall h \in V : P_h < 0 \quad (21)$$
$$\sum_{i \neq h} y_{i,h} \leq MS \quad \forall h \in V : P_h < 0 \quad (22)$$
$$x_{i,j}^t \in \{0,1\} \quad \forall i, j \in V, t \in T \quad (23)$$
$$y_{i,j} \in \{0,1\} \quad \forall i, j \in V \quad (24)$$
$$f_{i,j} \geq 0 \quad \forall i, j \in V, j \neq i \quad (25)$$

The objective function (15) minimizes the total cable layout cost. Constraints (16) impose that only one type of cable can be selected for each built arc, and defines the $y_{i,j}$ variables. Constraints (17) are flow conservation constraints: the energy (flow) exiting each node $h$ is equal to the energy entering $h$ plus the power production of that node (if $P_h \geq 0$). Note that the constraint is not imposed if $h$ is a substation. Constraints (18) ensure that the flow does not exceed the capacity of the installed cable, while constraints (19), (20), and (21) impose that only one cable can exit a turbine and none can exit the substation (tree structure). Finally, constraint (22) imposes the maximum number of strings ($MS$) that can enter the substation(s).

Non-crossing constraints are difficult constraints from the optimization point of view. In the model we have to consider the complete set of all possible connections, so theoretically we should impose that each two arcs of the complete graph $G$ should not cross. Of course this implies to deal with a huge number of constraints. We have, therefore, used an approach on the fly, as suggested in [1], where the optimizer will consider model (15) - (25) and add on the fly the new constraints: whenever two build arcs $(i,j)$ and $(h,k)$ cross

$$y_{i,j} + y_{j,i} + y_{h,k} + y_{k,h} \leq 1 \quad (26)$$

Using this approach, the number of non-crossing constraints actually added to the model decreases dramatically, making the optimization feasible.

## VI. COMPARISON WITH EXISTING LAYOUT

We compared our cable routing results with some existing layouts, kindly provided us by Vattenfall AB [21]. The cable

| Cost (€) | Strings to substation | Crossings | Cost saving (€) | Cost saving (%) |
|---|---|---|---|---|
| 18 918 866 | 8 | 3 | 0 | 0 % |
| 17 522 596 | 8 | 0 | 1 396 270 | 7.38 % |

TABLE II: The first line records the cost of the layout proposal by Vattenfall (manual solution). Second line refers to the layout found by our tool when considering non-crossing constraint and maximum number of strings to the substation equal to 8.
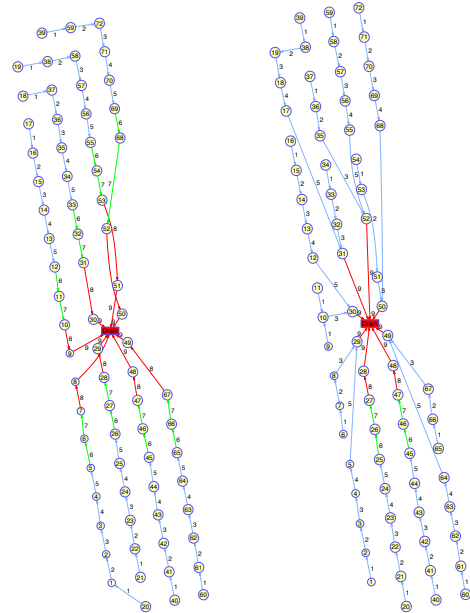


(a) Vattenfall's proposal (18 918 866 €)    (b) The layout from our tool (17 522 596 €)

Fig. 6: Comparison of the layouts for Sandbank's cable routing. Our layout (on the right) is 1 396 270 € (7.38 %) less expensive.

routing design is nowadays carried out manually, so we compared our results with manual solutions. Note that some crossings are allowed in the manual layouts with the idea to avoid them at construction time. Those crossings are easy to avoid by just laying cables in curved trajectories. The tests prove that the use of our automatized tool provides big savings.

Table II and Figures 6a-6b compare the cable layouts for Sandbank offshore wind farm. This wind farm contains 72 wind turbines and one substation. The substation allows up to 8 strings to be connected. Three types of cables are considered in the optimization:

- the first type (blue) has a price of 135€/m and can support up to 5 turbines
- the second type (green) costs 250€/m and can support up to 7 turbines
- the third type (red) costs 370€/m and can support up to 9 turbines

Those significant savings are achieved by a smart use of the less expensive cables. This results clear from Figures 6a-6b Table III shows how much the use of the expensive cables impacts the final cost of the Sandbank solution.

| Cost (€) | tot.length blue cables(m) | tot.length green cables(m) | tot.length red cables(m) | Cost saving (€) |
|---|---|---|---|---|
| 18 918 866 | 40 493.37 | 18 264 | 24 016 | 0 |
| 17 522 596 | 83 611 | 4 021 | 14 134 | 1 396 270 |

TABLE III: The first line refers to the layout proposed by Vattenfall for Sandbank. The second line (yellow) refers to the solution found by our tool. Second, third and fourth columns record the total length of the three different type of cables.

| Cost (€) | Strings to substation | Crossings | Cost saving (€) | Cost saving (%) |
|---|---|---|---|---|
| 23 515 587 | 12 | 2 | 0 | 0 % |
| 21 490 400 | 12 | 0 | 2 025 187 | 8.6 % |

TABLE IV: The first line records the cost of the layout proposal by Vattenfall (manual solution). Second line refers to the layout found by our tool when considering non-crossing constraint and maximum number of strings to the substation equal to 12.

Table IV refers to a real-world wind farm containing around 50 8MW turbines[1] and one substation with at most 12 connections to the substation. In our tests we considered two types of cables:

- the less expensive type of cable (180.0€/m) can support up to 3 turbines
- the most expensive (370€/m) can support up to 5 turbines

Table V refers to another real-world wind farm consisting of around 70 6MW turbines [1] and 1 substation. The maximum number of strings connected to the substation is still 12. The two types of cables are considered:

- the less expensive cable (180.0€/m) can support up to 4 turbines
- the most expensive cable (370€/m) can support up to 6 turbines

## VII. Conclusion

In the present paper a Mixed-Integer Linear Programming (MILP) approach has been used to optimize turbine allocation and inter-array offshore cable routing. The use of Stochastic Programming allowed to take into consideration the variability of the wind in the layout optimization. Different constraints arsing in practical application have been considered in the optimization. A specific strategy has been developed, in particular, to deal with the no-crossing constraints for cable routing. Computational results on real-world instances prove the practical viability of the approach and show how the optimized wind-farm should look like.

[1] the exact number of turbines used in the optimization is not disclosed due to privacy issues

| Cost (€) | Strings to substation | Crossings | Cost saving (€) | Cost saving (%) |
|---|---|---|---|---|
| 25 365 822 | 12 | 3 | 0 | 0 % |
| 23 848 104 | 12 | 0 | 1 517 717 | 5.98 % |

TABLE V: Summary of the results. The first line reports the cost of the layout found manually. The second line (yellow) refers to the solution found by our tool when considering non-crossing constraint and maximum number of strings to the substation equal to 12.

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