Decentralized formation control with connectivity maintenance and collision avoidance under limited and intermittent sensing

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***Abstract*—A decentralized switched controller is developed for dynamic agents to perform global formation configuration convergence while maintaining network connectivity and avoiding collision within agents and between stationary obstacles, using only local feedback under limited and intermittent sensing. Due to the intermittent sensing, constant position feedback may not be available for agents all the time. Intermittent sensing can also lead to a disconnected network or collisions between agents. Using a navigation function framework, a decentralized switched controller is developed to navigate the agents to the desired positions while ensuring network maintenance and collision avoidance.**

I. Introduction

Multi-agent systems have found applications in a wide range of situations. These include problems of consensus [1]–[3], rendezvous [4]–[6], and formation and flocking of multiple agents [7]–[10]. In these applications, a decentralized control structure has advantages over a centralized structure including: computational efficiency, robustness, and flexibility. Various decentralized approaches (cf. [11]–[15]) have been developed to perform cooperative objectives for a multi-agent system; however, network connectivity problems are often neglected. The loss of connectivity can arise through limited communications and limited sensing ranges and angles, and it can result in collisions as well as the loss of the formation or individual agents.

The broad applicability of dynamic network topologies helps explain a recent increase in its popularity. In particular the issues surrounding network connectivity has been gaining more focus. In [16] and [17], decentralized schemes addressing connectivity issues for dynamic topologies of formation and rendezvous problems were approached with a graph theoretic method. In these papers, the authors used a convergence analysis based on LaSalle's invariant theorem with common Lyapunov functions. In [18], the network connectivity issue was handled with a navigation function based controller using bounded control inputs for a formation problem using both static and dynamic graphs. However, these applications didn't consider the problem of collision avoidance. Both network connectivity and collision avoidance were addressed in [19], but only a fixed network topology is considered.

In some formation control problems, communication is not necessary, but in these cases local feedback information from sensors is required. Moreover, due to environment factors or limitations in the field-of-view of sensors, the interaction graphs can be intermittent and time-varying. Intermittent sensing problems were considered for formation control problems using graph-theoretic methods in [20] and [21]. These problems were solved based on the existence of a globally reachable node, but they didn't account for connectivity or collision avoidance problems. A switched control scheme is developed in [9] for formation problems, but the controller neglects network connectivity. In [22], a coordination algorithm was designed to stabilize the shape of the formation in a way that it was robust to the sensing link failure, but connectivity and collision avoidance were not considered. In [23] swarm aggregation problems were investigated within fixed and dynamic network topologies for both network connectivity and collision avoidance, but the dynamic topologies only resulted from link additions to the network. In [4] and [24], decentralized controllers were designed to address network connectivity. Unfortunately, the control strategies were specific to rendezvous problems, and collision avoidance was not considered.

This paper considers formation control problems under limited and intermittent sensing. Based on a navigation function framework, a decentralized hybrid controller is developed to ensure network connectivity and collision avoidance while controlling the formation. Nonsmooth navigation functions are used which result in the use of a common Lyapunov function, so the formation error of the entire configuration converges globally with sufficiently small error (i.e. converges to the neighborhood of the critical points) under arbitrary switching sequence. This paper is organized as follows. In Section II, the dynamics of the agents and the problem are formulated. Then the navigation function based controller is proposed in Section III. We perform a connectivity analysis in Section IV and a convergence analysis in Section V. Finally, the simulation results are presented in Section VI.

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II. Problem Formulation

Consider N dynamic point-mass agents in the finite workspace $\mathcal{F} \subset \mathbb{R}^2$ with motion governed by the following kinematics

$$\dot{q}_i = u_i, \; i = 1, \ldots, N \tag{1}$$

where $q_i \in \mathbb{R}^2$ represents the position of agent i in a two-dimensional space, and $u_i \in \mathbb{R}^2$ denotes the control input of agent i. The subsequent development is based on the assumption that each agent has a limited sensing range, which is encoded by a disk centered at the agent. Position feedback is only available for agents within the interior of the disk. Moreover, the sensing is assumed to be intermittent (i.e., existing links within the disk region may fail), which implies that two agents do not have continuous state feedback even if they remain within the sensing zone of each other.

Since sensing is intermittent, the set of neighbor nodes that can be successfully sensed by agent i at $t \in \mathbb{R}_{\geq 0}$ is denoted as the time-varying set $\mathcal{N}_i^s(t)$, where $\mathcal{N}_i^s : [0, \infty) \to \mathcal{V}$, where $\mathcal{V} \triangleq \{1, 2, \cdots, N\}$ is an index set of all agents in the system. As a result, the sensor graph of the network system is an undirected, time-varying graph that can be modeled as $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$, where $\mathcal{E}(t) \triangleq \{(i, j) \in \mathcal{V} \times \mathcal{V} | \, j \in \mathcal{N}_i^s(t), \, i \in \mathcal{V}, \, i \neq j\}$, where i and j represent nodes located at q_i and q_j, $d_{ij} \in \mathbb{R}_{\geq 0}$ denotes the distance between two nodes defined as $d_{ij} \triangleq \|q_i - q_j\|$, and R_s is the maximal sensing radius for every agent. To include all the time-varying graphs, a switched graph is defind as $G_{\sigma(t)}$, where $\sigma : [0, \infty) \to \mathcal{P}$ is a switching signal, and $\mathcal{P} \in \{1, 2, \ldots, P\}$ is a finite index set such that $\{G_p : p \in \mathcal{P}\}$ includes all possible graphs $\underset{t \geq 0}{\cup} \mathcal{G}(t)$.

Network connectivity maintenance is ensured by preserving every existing link in the network. Particularly, the agents are considered connected if they stay within the sensing zone of the desired neighboring agents (even if there are intermittent sensing link failures), if they are neighbors initially, i.e.,

$$d_{ij}(t) < R_s, \, \forall t \geq 0. \tag{2}$$

The objective in this paper is to maintain network connectivity while also achieving a desired formation, which is specified by

$$\|q_i - q_j - c_{ij}\| \to 0 \text{ as } t \to \infty, \; j \in \mathcal{N}_i^f, \, i \in \mathcal{V}, \tag{3}$$

where $\mathcal{N}_i^f$ is the set of preassigned agents, and $c_{ij} \in \mathbb{R}^2$, satisfying $c_{ij} = -c_{ji}$, describes the desired relative position between node i and the adjacent node $j \in \mathcal{N}_i^f$. Different from $\mathcal{N}_i^s(t)$ which is time-varying due to the intermittent sensing, $\mathcal{N}_i^f$ is time-invariant.

Consider stationary obstacles $o_1, o_2, \ldots, o_m$ in the workspace $\mathcal{F}$, which are represented by a set of m points indexed by $\mathcal{M} = \{1, 2, \cdots, m\}$. To prevent collisions among agents and obstacles, a disk region centered at agent i with radius $\delta_1 < R_s$ is defined. Any agent or obstacle in this region is considered as a potential collision with agent i, and the potential collision set $\mathcal{N}_i : [0, \infty) \to \mathcal{V}$ is defined as

$$\mathcal{N}_i(t) \triangleq \{j \in \mathcal{V} \mid \|q_i - q_j\| \leq \delta_1\}. \tag{4}$$

Since only $j \in \mathcal{N}_i^f$ in (2) are required to maintain the neighborhood with agent i in the desired formation, (2) can be modified as

$$d_{ij}(t) < R_s, \, \forall t \geq 0, \; j \in \mathcal{N}_i^f, \, i \in \mathcal{V}. \tag{5}$$

In summary, the objective is to asymptotically achieve a formation configuration as in (3), while ensuring network connectivity as in (5) and collision avoidance between agents and stationary obstacles $o_1, o_2, \ldots, o_m$.

Assumption 1. The sensing link failures between agents happen a finite number of times in a finite time interval, (i.e., the switching signal σ has finite switches in any finite time interval.) Specifically, given any non-overlapping time interval $[t_k, t_{k+1})$, $k = 0, 1, \cdots$, then $0 < \tau < t_{k+1} - t_k < T$, where $\tau \in \mathbb{R}$ is the non-vanishing dwell-time, and $T \in \mathbb{R}$ is a positive constant. The graph G_σ is invariant for $t \in [t_k, t_{k+1})$, $\forall \, k = 0, 1, 2, \cdots$, and the switching sequence of σ is arbitrary.

Assumption 2. The desired formation neighbor set of agent i is initially inside its sensing zone, $\mathcal{N}_i^f \subset \mathcal{N}_i^s(t_0)$, $\forall i \in \mathcal{V}$, and the neighboring agents are not initially located at any unstable equilibria.

Assumption 3. The desired relative position described by c_{ij} is achievable (i.e., $\delta_1 < \|c_{ij}\| < R_s - \delta_2$, where $\delta_2 \in \mathbb{R}^+$ denotes a buffer distance for connectivity maintenance. So the relative position would not result in a partition of the graph or cause collision of any two agents.) and the agents do not take certain pathological configurations. One example would be all of the agents and goals being co-linear. However this and other such configurations are assumed to constitute a Lebesgue measure zero set in the space of all configurations, and are practically resolved by small perturbations.

III. Control development

Based on [19], a navigation function $\varphi_i : \mathcal{F} \to [0, 1]$ for each agent i is designed as,

$$\varphi_i = \frac{\gamma_i}{\left(\gamma_i^k + \beta_i\right)^{\frac{1}{k}}}, \tag{6}$$

where $k \in \mathbb{R}$ is an adjustable positive constant, $\gamma_i : \mathbb{R}^2 \to \mathbb{R}_{\geq 0}$ is a goal function, and $\beta_i : \mathbb{R}_{\geq 0} \to [0, 1]$ is a constraint function for agent i. Based on the objective in (3), the goal function γ_i in (6) is designed as

$$\gamma_i(q_i, q_j) \triangleq \sum_{j \in \mathcal{N}_i^f} \|q_i - q_j - c_{ij}\|^2. \tag{7}$$

The constraint function β_i is defined as

$$\beta_i \triangleq \prod_{j \in \mathcal{N}_i^f} b_{ij} \prod_{k \in \mathcal{N}_i \cup \mathcal{M}_i} B_{ik}, \tag{8}$$

which enables collision avoidance and connectivity maintenace. To maintain network connectivity, the nonsmooth function $b_{ij} : \mathbb{R}_{\geq 0} \to [0, 1]$ in (8) is designed as

$$b_{ij}(d_{ij}) \triangleq \begin{cases} 1, & d_{ij} < R_s - \delta_2, \\ -\frac{1}{\delta_2^2}(d_{ij} + 2\delta_2 - R_s)^2 \\ +\frac{2}{\delta_2}(d_{ij} + 2\delta_2 - R_s), & R_s - \delta_2 \leq d_{ij} \leq R_s, \\ 0, & d_{ij} > R_s, \end{cases} \tag{9}$$

where b_{ij} is not differentiable at R_s. Specifically, b_{ij} is designed to prevent node i from leaving the communication region of its formation neighbor $j \in \mathcal{N}_i^f$. Let $\mathcal{M}_i$ denote the set of stationary obstacles within the collision region of node i. In (8), for each node $k \in \mathcal{N}_i \cup \mathcal{M}_i$, $B_{ik} : \mathbb{R} \to [0, 1]$ is defined as

$$B_{ik}(d_{ik}) \triangleq \begin{cases} -\frac{1}{\delta_1^2} d_{ik}^2 + \frac{2}{\delta_1} d_{ik}, & 0 \leq d_{ik} \leq \delta_1, \\ 1, & d_{ik} > \delta_1. \end{cases}$$

Therefore, $\beta_i \to 0$ when node i enters the constraint region, (i.e. when node i approaches other nodes, stationary obstacles, or tries to leave the sensing range of their adjacent nodes $j \in \mathcal{N}_i^f$, $\forall t \geq 0$).

Based on Assumption 3, γ_i and β_i will not be zero at the same time, and the navigation function φ_i reaches its maximum at 1 when $\beta_i = 0$ and its minimum at 0 when $\gamma_i = 0$.

Due to the intermittent sensing, consider the two sets $\mathcal{V}_f(t)$ and $\mathcal{V}_u(t)$, where $\mathcal{V}_f, \mathcal{V}_u : [0, \infty) \to \mathcal{V}$ are defined as $\mathcal{V}_f(t) \triangleq \left\{ i \in \mathcal{V} | \mathcal{N}_i^f = \mathcal{N}_i^s(t) \cap \mathcal{N}_i^f \right\}$ and $\mathcal{V}_u(t) \triangleq \mathcal{V} \setminus \mathcal{V}_f(t)$. The set $\mathcal{V}_f(t)$ includes agents that can sense all of the formation neighbors $\mathcal{N}_i^f$ at $t \in \mathbb{R}_{\geq 0}$. Otherwise, agent i will be in $\mathcal{V}_u(t)$ at some $t \in \mathbb{R}_{\geq 0}$. Using the navigation function in (6), the decentralized switched controller for agent i is designed as

$$u_i(t) = \begin{cases} -\Gamma \nabla_{q_i} \varphi_i, & i \in \mathcal{V}_f(t), \\ 0, & i \in \mathcal{V}_u(t), \end{cases} \tag{10}$$

where $\Gamma \in \mathbb{R}^+$ is a positive constant gain, and $\nabla_{q_i}(\cdot) \triangleq \frac{\partial}{\partial q_i}(\cdot)$. In (10), the control switching scheme of agent i is based on the sensing condition at time t. If all neighbor agents in $\mathcal{N}_i^f$ can be sensed by agent i, $u_i(t) = -\Gamma \nabla_{q_i} \varphi_i$, and $u_i(t) = 0$ otherwise.

IV. Connectivity Analysis

Lemma 1. *If the initial graph of the multi-agent system is connected, then the controller in (10) ensures agent i and j remain connected for all time.*

Proof: Consider an agent $i \in \mathcal{V}$ located at $q_0 \in \mathcal{F}$, where the sensing link is about to break, which implies $\prod_{j \in \mathcal{N}_i^f} b_{ij} \to 0$, then three cases must be considered.

Case 1. As agent $j \in \mathcal{N}_i^f$ approaches the sensing region (i.e., $\|q_i - q_j\|$ approaches R_s from the left), then β_i tends to zero. The gradient of φ_i is

$$\nabla_{q_i} \varphi_i = \frac{k \beta_i \nabla_{q_i} \gamma_i - \gamma_i \nabla_{q_i} \beta_i}{k(\gamma_i^k + \beta_i)^{\frac{1}{k}+1}}. \tag{11}$$

Consider

$$\nabla_{q_i} \beta_i = \sum_{h \in \mathcal{N}_i^f} \prod_{\substack{l \in \mathcal{N}_i^f, \\ l \neq h}} b_{il} (\nabla_{q_i} b_{ih}) \prod_{k \in \mathcal{N}_i \cup \mathcal{M}_i} B_{ik} + \sum_{h \in \mathcal{N}_i \cup \mathcal{M}_i} \prod_{j \in \mathcal{N}_i^f} b_{ij} \prod_{\substack{l \in \mathcal{N}_i \cup \mathcal{M}_i, \\ l \neq h}} B_{il} (\nabla_{q_i} B_{ih}).$$

Provided only agent j is near the boundary (i.e., $\|q_i - q_j\| \to R_s^-$), $\nabla_{q_i} \beta_i$ has only one dominant term:

$$\nabla_{q_i} \beta_i = \prod_{\substack{l \in \mathcal{N}_i^f, \\ l \neq j}} b_{il} (\nabla_{q_i} b_{ij}) \prod_{k \in \mathcal{N}_i \cup \mathcal{M}_i} B_{ik} + O(b_{ij}),$$

where $O(\cdot)$ is the Big O notation, which vanishes as b_{ij} approaches R_s. The other term in the numerator of $\nabla_{q_i} \varphi_i$ in (11) is $k \beta_i \nabla_{q_i} \gamma_i = O(b_{ij})$, hence $\nabla_{q_i} \varphi_i$ in (11) can be expressed as

$$\nabla_{q_i} \varphi_i = \frac{-\gamma_i \prod_{\substack{l \in \mathcal{N}_i^f, \\ l \neq j}} b_{il} \prod_{k \in \mathcal{N}_i \cup \mathcal{M}_i} B_{ik} (\nabla_{q_i} b_{ij}) + O(b_{ij})}{k(\gamma_i^k + \beta_i)^{\frac{1}{k}+1}}.$$

Note that the gradient of b_{ij} w.r.t. q_i can be determined as

$$\nabla_{q_i} b_{ij} = \begin{cases} 0, & d_{ij} < R_s - \delta_2 \text{ or } d_{ij} > R_s, \\ -\frac{2(d_{ij} + \delta_2 - R_s)(q_i - q_j)}{\delta_2^2 d_{ij}}, & R_s - \delta_2 \leq d_{ij} < R_s, \end{cases} \tag{12}$$

where γ_i, b_{il}, B_{ik}, k, δ_2, and R_s are positive constants. Thus, $\dot{q}_i = -\Gamma \nabla_{q_i} \varphi_i$ points in the direction of $q_j - q_i$, which forces nodes i to move toward node j.

Case 2. Now suppose several agents $j_1, j_2, \ldots, j_s \in \mathcal{N}_i^f$ are near the boundary of the sensing region. That is, d_{ij_m} is near R_s for each $m = 1, 2, \ldots, s$. For this case, $\nabla_{q_i} \varphi_i = \frac{-\gamma_i \sum_m \prod_{\substack{l \in \mathcal{N}_i^f, \\ l \neq j_m}} b_{il} \prod_{k \in \mathcal{N}_i \cup \mathcal{M}_i} B_{ik} (\nabla_{q_i} b_{ij_m})}{k(\gamma_i^k + \beta_i)^{\frac{1}{k}+1}} + O\left(\prod_m b_{ij_m} \right)$. The first term above in $\nabla_{q_i} \varphi_i$ tends to zero, however since the b_{ij_m} terms are quadratic near R_s, the order of the zero contributed by the first term is one degree less than $O\left(\prod_m b_{ij_m} \right)$, so the first term dominantes as each $d_{ij_m} \to R_s$. Hence $\dot{q}_i = -\Gamma \nabla_{q_i} \varphi_i$ is approximately a linear combination of the vectors $q_{j_1} - q_i, q_{j_2} - q_i, \ldots, q_{j_s} - q_i$, where the largest contribution comes from those j_m closest to the sensing boundary. Thus, node i moves almost toward j_m resulting in a largest decrease in d_{ij_m}, so the connectivity can be maintained.

Case 3. Consider a node $i \in \mathcal{V}_u$ (or more than one node in the set of $\mathcal{V}_u$). The controller will be $u_i = 0$ based on (10). Since both node i and its neighbor $j \in \mathcal{N}_i^f$ are in the undirected graph, node j can't sense node i, so $j \in \mathcal{V}_u$, thus $u_j = 0$. Since both i, j nodes have no control input, the distance between them remains the same.

By Assumption 2, $\mathcal{N}_i^f \subset \mathcal{N}_i^s(t_0)$, $i \in \mathcal{V}$. Furthermore, from *Case 1-Case 3*, the decentralized switched control policy in (10) ensures the distances between agent $i \in \mathcal{V}$ and its formation neighbors $j \in \mathcal{N}_i^f$ never increase under intermittent sensing conditions. As a result, the formation neighbors $j \in \mathcal{N}_i^f$ remain inside the sensing region of agent i for all time. Specifically,

$$d_{ij}(t) < R_s,\, j \in \mathcal{N}_i^f,\, i \in \mathcal{V},\, \forall t \geq 0. \tag{13}$$

■

V. Convergence Analysis

Definition 1. [25] Consider the following differential equation with a discontinuous right-hand side:

$$\dot{x} = f(x), \tag{14}$$

where $f : \mathbb{R}^n \to \mathbb{R}^n$ is measurable and essentially locally bounded, and $n \in \mathbb{N}$ is a finite constant. The vector function x is called a solution of (14) on $[t_0,\, t_1]$ if x is absolutely continuous on $[t_0,\, t_1]$ and for almost all $t \in [t_0,\, t_1]$

$$\dot{x} \in K[f](x)$$

$$K[f](x) \triangleq \bigcap_{\delta>0} \bigcap_{\mu N=0} \overline{co}\, f(B(x,\delta) \setminus N), \tag{15}$$

where $\bigcap_{\mu N=0}$ denotes the intersection over all sets N of Lebesgue measure zero.

To prove the convergence of the agents to the desired formation, an invariance principle for switched systems is applied to a common Lyapunov function candidate $V : \mathbb{R}^{2N} \to \mathbb{R}$ given by

$$V(q) \triangleq \sum_{i=1}^{N} \varphi_i, \tag{16}$$

where q is the stack state vector, and V reaches its minimum value of 0 if the desired formation is achieved.

Theorem 1. *[26] Let $x(\cdot)$ be a Filippov solution to $\dot{x} = f(x)$ on an interval containing t and $V : \mathbb{R}^n \to \mathbb{R}$ be a Lipschitz and regular function. Then $V(x(t))$ is absolutely continuous, $\frac{d}{dt}V(x(t))$ exists almost everywhere (a.e.) and*

$$\frac{d}{dt}V(x(t)) \overset{a.e.}{\in} \dot{\tilde{V}}(x) \triangleq \bigcap_{\xi \in \partial V(x(t))} \xi^T K[f](x(t)).$$

Based on Definition 1 and Theorem 1 , the main result of this paper is provided as follows.

Theorem 2. *Given (10), the maximum relative position errors of any two formation neighbors of the network system in (1) converges to $\max\limits_{j\in\mathcal{N}_i^f} \|q_i - q_j - c_{ij}\| = \sqrt{\frac{c\underline{max}}{\underline{N}}}$, $i \in \mathcal{V}$ provided that the adjustable gain k in (6) is selected sufficiently large and every agent can sense all its formation neighbors in the finite time interval $\bigcup\limits_{t\in[t_k,\, t_{k+n})} \left(\mathcal{N}_i^f \cup \{i\}\right) = \mathcal{V}$, where $n \in \mathbb{N}$ is finite.*

Proof: Consider the common Lyapunov function candidate V defined in (16), where V can be minimized at the critical points as shown in [19], and V reaches its minimum value of 0 when the desired formation is achieved. Based on Theorem 1,

$$\frac{d}{dt}V(q(t)) \overset{a.e.}{\in} \dot{\tilde{V}}(q) \triangleq \bigcap_{\xi \in \partial V(x(t))} \xi^T K[\dot{q}]. \tag{17}$$

The finite sums property of the generalized gradient defined in [27] gives

$$\partial V \subset \left[\partial_{q_1} V^T,\, \partial_{q_2} V^T,\, \ldots,\, \partial_{q_N} V^T\right]^T. \tag{18}$$

Using (17) and (18), the generalized time derivative of V in (17) can be expressed as

$$\dot{\tilde{V}} \subset \sum_{i\in\mathcal{V}} \left(\bigcap_{\xi_i} \xi_i^T K[\dot{q}_i]\right). \tag{19}$$

where $\xi_i \in \partial_{q_i} V$. To turn the generalized gradient into the gradient, the points at which V is not differentiable and Lebesgue measure zero need to be considered. From the inequality in (13), d_{ij} never takes on the value $d_{ij} = R_s,\, j \in \mathcal{N}_i^f,\, i \in \mathcal{V}$, at the nonsmooth point of b_{ij}, so b_{ij} is differentiable w.r.t. q_i along the solution of the closed-loop system. Since B_{ik} and γ_i are differentiable functions, V is differentiable w.r.t. q_i along the solution of the closed-loop system for $i \in \mathcal{V}$. Therefore, the generalized gradient can be expressed as

$$\partial_{q_i} V(q) = \{\nabla_{q_i} V(q)\},\, i \in \mathcal{V}. \tag{20}$$

Based on (20), (19) can be rewritten as

$$\dot{\tilde{V}} \subset \sum_{i\in\mathcal{V}} \left(\nabla_{q_i} V^T K[\dot{q}_i]\right). \tag{21}$$

By segregating $\mathcal{V}$ into the sets, $\mathcal{V}_f$ and $\mathcal{V}_u$, (21) can be rewritten as

$$\dot{\tilde{V}} \subset \sum_{i\in\mathcal{V}_f} \left(\nabla_{q_i} V^T K[\dot{q}_i]\right) + \sum_{i\in\mathcal{V}_u} \left(\nabla_{q_i} V^T K[\dot{q}_i]\right). \tag{22}$$

From Assumption 1, the switching graph $G_{\sigma(t)}$ is invariant for $t \in [t_k,\, t_{k+1})$, so the set $\mathcal{V}_f$ is also invariant during that time period. Based on the switched control scheme in (10), the second term on the RHS of (22) will be zero, therefore,

$$\dot{\tilde{V}} \subset \sum_{i\in\mathcal{V}_f} \left(\nabla_{q_i} V^T K[\dot{q}_i]\right),\, t \in [t_k,\, t_{k+1}). \tag{23}$$

In addition, by the definition of $K[\dot{q}_i]$ in (15), the switched controller in (10) can be expressed as

$$K[\dot{q}_i] \subset \overline{co}\left\{-\Gamma \nabla_{q_i}\varphi_i,\, \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right\}. \tag{24}$$

Also based on Assumption 1, the switching time instance is Lebesgue measure zero, so (24) can be further expressed as $K[\dot{q}_i] \subset \{-\Gamma \nabla_{q_i}\varphi_i\}$. Thus, by using the gradient of V, (17) and (23) can be used to conclude that

$$\dot{V} \overset{a.e.}{\leq} -\sum_{i\in\mathcal{V}_f} \left(\Gamma \left(\sum_{j=1}^{N} \nabla_{q_i}\varphi_j\right)^T \nabla_{q_i}\varphi_i\right), \tag{25}$$

where $t \in [t_n, t_{n+1})$, $n \in \mathbb{N}$. An equivalent way to prove $\dot{V} \overset{a.e.}{<} 0$ is to show $\sum_{i \in \mathcal{V}_f} \left(\Gamma \left(\sum_{j=1}^{N} \nabla_{q_i} \varphi_j \right)^T \nabla_{q_i} \varphi_i \right) > 0$, and based on the development in the appendix, its sufficient condition is

$$\sum_{i \in \mathcal{V}_f} \left(4\underline{\beta} \left\| \sum_{j \in \mathcal{N}_i^f} (q_i - q_j - c_{ij}) \right\|^2 - \frac{\rho_{1,i}}{2k} - \frac{\rho_{2,i}}{2k^2} \right) > 0, \tag{26}$$

for $t \in [t_n, t_{n+1})$. In (26), $\rho_{1,i}, \rho_{2,i} \in \mathbb{R}$ are functions defined as $\rho_{1,i} \triangleq c_{1,i}\gamma_i + c_{2,i}\gamma_i^2 + c_{3,i}\left(\sum_{k=1}^{N} \gamma_k\right)^2$, $\rho_{2,i} \triangleq c_{4,i}\gamma_i^2 + c_{5,i}\left(\sum_{k=1}^{N} \gamma_k\right)^2$, where $c_{p,i} \in \mathbb{R}$, $p = 1-5$, are positive constants. To develop a further sufficient condition for (26), we exploit the facts from [28] that $\nabla_{q_i}\gamma_i \triangleq 2 \sum_{j \in \mathcal{N}_i^f} (q_i - q_j - c_{ij})$ and $\|\nabla_{q_i}\gamma_i\| \geq \frac{\gamma_i}{R}$, where $R \triangleq \max \|q_i - q_j\|$, $q_i, q_j \in \mathcal{F}$, $\forall j \in \mathcal{N}_i^f$. Hence, from (13)

$$\|\nabla_{q_i}\gamma_i\| \geq \frac{\gamma_i}{R_s}, \tag{27}$$

and a sufficient condition for (26) can be developed as

$$\sum_{i \in \mathcal{V}_f} \left(\underline{\beta}\frac{\gamma_i^2}{R_s^2} - \frac{\rho_{1,i}}{2k} - \frac{\rho_{2,i}}{2k^2} \right) > 0. \tag{28}$$

By solving (28) for γ_i and using (7), a further sufficient condition for (26) is

$$\sum_{j \in \mathcal{N}_i^f} \|q_i - q_j - c_{ij}\|^2 > c_{\max},\ i \in \mathcal{V}_f, \tag{29}$$

where $c_{\max} \triangleq \sqrt{\frac{R_s^2}{\underline{\beta}}\left(\frac{\overline{\rho}_1}{2k} + \frac{\overline{\rho}_2}{2k^2}\right)}$, and $\overline{\rho}_1, \overline{\rho}_2, \underline{\beta} \in \mathbb{R}_{>0}$ are positive constants defined as $\overline{\rho}_1 \triangleq \max_{i \in \mathcal{V}} \rho_{1,i}$, $\overline{\rho}_2 \triangleq \max_{i \in \mathcal{V}} \rho_{2,i}$, and $\underline{\beta} \triangleq \min_{j \in \mathcal{N}_i^f, i \in \mathcal{V}} \beta_i\beta_j$. Additionally, $\beta_i, \beta_j \neq 0$ due to the fact that no open set of initial solutions can be attracted to the maxima of φ_i (i.e., $\beta_i = 0$) along the negative gradient motion $-\frac{\partial \varphi_i}{\partial q_i}$ [29]. Recall that V in (16) is a common Lyapunov function, so the switching signal σ of the time-varying graphs G_σ can have arbitrary sequence provided that (29) holds. Additionally, (29) can be extended to global (i.e., $i \in \mathcal{V}$) formation configuration convergence if the switching signal σ switches in the way that satisfies the following condition

$$\bigcup_{t \in [t_k, t_{k+n})} \mathcal{V}_f = \mathcal{V},\ n \in \mathbb{N}, \tag{30}$$

where n is a finite positive constant. Based on (29), and the ultimate maximum formation error for the entire switched system can be expressed as

$$\max_{j \in \mathcal{N}_i^f} \|q_i - q_j - c_{ij}\| = \sqrt{\frac{c_{\max}}{\underline{N}}},\ i \in \mathcal{V}, \tag{31}$$

where $\underline{N} \triangleq \min_{i \in \mathcal{V}} \left|\mathcal{N}_i^f\right|$. ■

VI. Simulation

To validate the proposed switched controller, we performed a simulation with 5 dynamic agents and 3 obstacles. The parameters used in the simulation are given by $R_S = 20$, $\delta_1 = 8$, $\delta_2 = 2$, $k = 1$, $\Gamma = 10$, $c_{12} = [0,\ 5]^T$, $c_{23} = [-5,\ 5]^T$, $c_{34} = [-5,\ -5]^T$, $c_{45} = [0,\ -5]^T$. Initially the agents are located within the sensing region of their formation neighbors. Fig. 1 illustrates that the agents avoid collisions with other agents and stationary obstacles. Moreover, they eventually achieve an approximation of their goal formation under arbitrary switching sequence that satisfies (30).

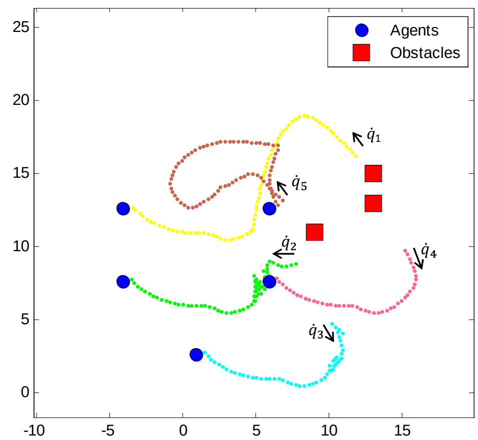

Figure 1. Trajectories of dynamic agents achieving formation configuration.

As indicated in Fig. 2, d_{ij} can increase during operation. However, these distances always remain smaller than the sensing range R_s (i.e., remain connected). Recall that the relative distance in our goal formations are given by $\|c_{12}\| = \|c_{45}\| = 5$, and $\|c_{23}\| = \|c_{34}\| = 5\sqrt{2}$. Fig. 2 indicates that the final distances approximate these values, and the position errors remain sufficiently small.

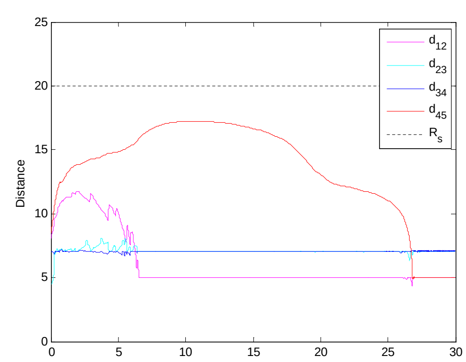

Figure 2. d_{ij} and R_s

VII. Conclusion

A switched controller is developed to achieve convergence of a network formation using only local feedback under both limited and intermittent sensing. At the same time, network connectivity is maintained and collisions between agents and obstacles are avoided. A common Lyapunov function approach is used to ensure convergence under an arbitrary switching sequence. Moreover the entire formation configuration converges globally, if the switching signal satisfies (30). The neighborhood of convergence can be made arbitrarilly small with sufficiently large gains. Finally, the effectiveness of the proposed controller is verified by simulation results.

Appendix

This section develops a sufficient condition for $\Gamma\left(\nabla_{q_i}\varphi_i\right)^T\left(\sum_{j=1}^{N}\nabla_{q_i}\varphi_j\right) > 0$, so that $\dot{V}$ in (25) is negative definite almost everywhere. We consider the equation $\left(\nabla_{q_i}\varphi_i\right)^T\left(\sum_{j=1}^{N}\nabla_{q_i}\varphi_j\right) = \left(\frac{\beta_i\left(\nabla_{q_i}\gamma_i\right)-\frac{\gamma_i}{k}\left(\nabla_{q_i}\beta_i\right)}{\left(\gamma_i^k+\beta_i\right)^{\frac{1}{k}+1}}\right)^T\left(\sum_{j=1}^{N}\frac{\beta_j\left(\nabla_{q_i}\gamma_j\right)-\frac{\gamma_j}{k}\left(\nabla_{q_i}\beta_j\right)}{\left(\gamma_j^k+\beta_j\right)^{\frac{1}{k}+1}}\right)$, and decompose this into smaller pieces. Using [19] as inspiration, it is sufficient to ensure the term

$$A^TC-\frac{\left(\|B\|\,\|C\|+\|A\|\,\|D\|\right)}{k}-\frac{\|B\|\,\|D\|}{k^2}>0, \tag{32}$$

where $A, B, C, D \in \mathbb{R}^2$ are from the numerator terms of $\left(\nabla_{q_i}\varphi_i\right)^T\left(\sum_{j=1}^{N}\nabla_{q_i}\varphi_j\right)$ and are defined as $A \triangleq \beta_i\left(\nabla_{q_i}\gamma_i\right)$, $B \triangleq \gamma_i\left(\nabla_{q_i}\beta_i\right)$, $C \triangleq \sum_{j=1}^{N}\beta_j\left(\nabla_{q_i}\gamma_j\right)$, and $D \triangleq \sum_{j=1}^{N}\gamma_j\left(\nabla_{q_i}\beta_j\right)$. We now proceed to find upper bounds for $\|A\|^2$, $\|B\|^2$, $\|C\|^2$, and $\|D\|^2$ so that we can satisfy $A^TC-\frac{\|B\|^2+\|C\|^2+\|A\|^2+\|D\|^2}{2k}-\frac{\|B\|^2+\|D\|^2}{2k^2}>0$, which is the upper bound of (32).

Property 1. $\|A\|^2 \le 4\beta_i^2\left|\mathcal{N}_i^f\right|\gamma_i$.

Proof: By definition $A = \beta_i\left(\nabla_{q_i}\gamma_i\right) = \beta_i\left(2\sum_{j\in\mathcal{N}_i^f}\left(q_i-q_j-c_{ij}\right)\right) = 2\beta_i\sum_{j\in\mathcal{N}_i^f}\left(q_i-q_j-c_{ij}\right)$, from which it follows that $\|A\|^2 =$

$$\|A\|^2 = 4\beta_i^2\left\|\sum_{j\in\mathcal{N}_i^f}\left(q_i-q_j-c_{ij}\right)\right\|^2. \tag{33}$$

Taking $X_j \triangleq \left[x_{j1},\, x_{j2}\right]^T$ we can bound the sum, first by using the triangle inequality as

$$\left\|\sum_{j\in\mathcal{N}_i^f}X_j\right\|^2 \le \left(\sum_{j\in\mathcal{N}_i^f}\|X_j\|\right)^2 = \left(\sum_{j\in\mathcal{N}_i^f}\|X_j\|\cdot 1\right)^2. \tag{34}$$

Next we apply the Cauchy–Schwarz inequality, and bound (34) as

$$\begin{aligned}\left\|\sum_{j\in\mathcal{N}_i^f}X_j\right\|^2 &\le \left(\sum_{j\in\mathcal{N}_i^f}\|X_j\|^2\right)\left(\sum_{j\in\mathcal{N}_i^f}1\right)\\ &\le \left|\mathcal{N}_i^f\right|\sum_{j\in\mathcal{N}_i^f}\|X_j\|^2.\end{aligned} \tag{35}$$

We can bound $\|A\|^2$ by using (35) to establish Property 1. ■

Property 2. $\|B\| \le \gamma_i\left(\left|\mathcal{N}_i^f\right|\frac{2}{\delta_2}+\left|\mathcal{N}_i\cup\mathcal{M}_i\right|\frac{2}{\delta_1}\right)$.

Proof: Given the definition: $B = \gamma_i\left(\nabla_{q_i}\beta_i\right) = \gamma_i\left(\sum_{j\in\mathcal{N}_i^f}\left(\nabla_{q_i}b_{ij}\right)\bar{b}_{ij}+\sum_{k\in\mathcal{N}_i\cup\mathcal{M}_i}\left(\nabla_{q_i}B_{ik}\right)\bar{B}_{ik}\right)$, where we take $\bar{B}_{ik} \triangleq \prod_{j\in\mathcal{N}_i^f}b_{ij}\prod_{h\in\mathcal{N}_i\cup\mathcal{M}_i,\, h\neq k}B_{ih}$, since b_{ij} and $B_{ik} \in [0,1]$, then $\bar{b}_{ij},\, \bar{B}_{ik} \in [0,1]$. Thus, we can develop the following inequality for $\|B\|$:

$$\|B\| \le \gamma_i\left(\sum_{j\in\mathcal{N}_i^f}\|\nabla_{q_i}b_{ij}\|+\sum_{k\in\mathcal{N}_i\cup\mathcal{M}_i}\|\nabla_{q_i}B_{ik}\|\right). \tag{36}$$

By using (12), $\|\nabla_{q_i}b_{ij}\| \le \frac{2}{\delta_2}$. In a similar manner, $\|\nabla_{q_i}B_{ik}\| \le \frac{2}{\delta_1}$. Property 2 is proven by applying these inequalities term by term to (36). ■

Property 3. $\|C\|^2 \le 4\left|\mathcal{N}_i^f\right|\gamma_i$.

Proof: Recall that C is defined as $C \triangleq \sum_{j=1}^{N}\beta_j\left(\nabla_{q_i}\gamma_j\right) = \sum_{j\in\mathcal{V}}\beta_j\left(\nabla_{q_i}\gamma_j\right) = \sum_{j\in\mathcal{N}_i^f}\beta_j\left(\nabla_{q_i}\gamma_j\right) + \sum_{j\in\mathcal{V}\setminus\mathcal{N}_i^f}\beta_j\left(\nabla_{q_i}\gamma_j\right)$. Since the graph is undirected, whenever j in $\mathcal{N}_i^f$, we have i in $\mathcal{N}_j^f$. Therefore, for any agent i in $\mathcal{N}_j^f$

$$\begin{aligned}\nabla_{q_i}\gamma_j &= \nabla_{q_i}\left(\|q_j-q_i-c_{ji}\|^2\right)\\ &\quad+\nabla_{q_i}\left(\sum_{\substack{h\in\mathcal{N}_j^f\\ h\neq i}}\|q_j-q_h-c_{jh}\|^2\right)\\ &= \nabla_{q_i}\left(\sum_{i\in\mathcal{N}_j^f}\|q_j-q_i-c_{ji}\|^2\right)\\ &= -2\left(q_j-q_i-c_{ji}\right) = 2\left(q_i-q_j-c_{ij}\right).\end{aligned} \tag{37}$$

By using (37)

$$\begin{aligned}\sum_{j\in\mathcal{N}_i^f}\beta_j\left(\nabla_{q_i}\gamma_j\right) &= \sum_{j\in\mathcal{N}_i^f}\beta_j\left(2\left(q_i-q_j-c_{ij}\right)\right)\\ &= 2\sum_{j\in\mathcal{N}_i^f}\beta_j\left(q_i-q_j-c_{ij}\right).\end{aligned} \tag{38}$$

On the contrary, if j is not in $\mathcal{N}_i^f$, then $\nabla_{q_i}\gamma_j = \nabla_{q_i}\left(\sum_{i\in\mathcal{N}_j^f}\|q_j - q_i - c_{ji}\|^2\right) = 0$, which indicates that $\sum_{j\in\mathcal{V}\setminus\mathcal{N}_i^f}\beta_j\left(\nabla_{q_i}\gamma_j\right) = 0$. Finally, using (38)

$$C = \sum_{j\in\mathcal{N}_i^f}\beta_j\left(\nabla_{q_i}\gamma_j\right) = \sum_{j\in\mathcal{N}_i^f}\beta_j\left(2\left(q_i - q_j - c_{ij}\right)\right).$$

According to $\beta_j \in [0,1]\,, \forall j \in \mathcal{V}$, $\|C\|$ can be bounded by $\|C\| \le 2\left\|\sum_{j\in\mathcal{N}_i^f}(q_i - q_j - c_{ij})\right\|$, and $\|C\|^2$ can be further bounded by

$$\|C\|^2 \le 4\left\|\sum_{j\in\mathcal{N}_i^f}(q_i - q_j - c_{ij})\right\|^2.$$

By using (35), $\|C\|^2$ can be bounded by

$$\|C\|^2 \le 4\left|\mathcal{N}_i^f\right|\sum_{j\in\mathcal{N}_i^f}\|(q_i - q_j - c_{ij})\|^2 = 4\left|\mathcal{N}_i^f\right|\gamma_i.$$

■

Property 4. $\|D\| \le \left(\frac{2}{\delta_2} + \frac{2}{\delta_1}\right)\sum_{j=1}^N \gamma_j$.

Proof: By using the definition of $D = \sum_{j=1}^N \gamma_j\left(\nabla_{q_i}\beta_j\right)$ and applying the same inequalities used in the proof of Property 2

$$\begin{aligned}\|D\| &= \left\|\sum_{j=1}^N \gamma_j\left(\nabla_{q_i}\beta_j\right)\right\| \le \sum_{j=1}^N \|\gamma_j\|\,\|(\nabla_{q_i}\beta_j)\| \\ &\le \sum_{j=1}^N \|\gamma_j\|\left(\frac{2}{\delta_2} + \frac{2}{\delta_1}\right).\end{aligned}$$

Since $\gamma_j \in \mathbb{R}_{\ge 0}$ (i.e., $\gamma_j = \|\gamma_j\|$), $\|D\|$ can be further bounded by $\|D\| \le \left(\frac{2}{\delta_2} + \frac{2}{\delta_1}\right)\sum_{j=1}^N \gamma_j$. ■

Property 5. $\gamma_i \le \left|\mathcal{N}_i^f\right|(R_s + \bar{c}_i)^2$, where $\bar{c}_i = \max_{j\in\mathcal{N}_i^f}\|c_{ij}\|$.

Proof: From (13), $\|q_i - q_j\| \le R_s,\, j \in \mathcal{N}_i^f$, then $\|q_i - q_j - c_{ij}\| \le \|q_i - q_j\| + \|c_{ij}\| \le R_s + \|c_{ij}\|$, which implies $\gamma_i = \sum_{j\in\mathcal{N}_i^f}\|q_i - q_j - c_{ij}\|^2 \le \sum_{j\in\mathcal{N}_i^f}\|R_s + \|c_{ij}\|\|^2$. By choosing the $\bar{c}_i = \max_{j\in\mathcal{N}_i^f}\|c_{ij}\|$, then

$$\gamma_i \le \left|\mathcal{N}_i^f\right|(R_s + \bar{c}_i)^2.$$

■

Recall that our goal is to establish (32). We will instead establish this for the smaller equation obtained by way of Young's inequality: $A^TC - \frac{\|B\|^2+\|C\|^2+\|A\|^2+\|D\|^2}{2k} - \frac{\|B\|^2+\|D\|^2}{2k^2} \le A^TC - \frac{(\|B\|\|C\|+\|A\|\|D\|)}{k} - \frac{\|B\|\|D\|}{k^2}$. By using the upper bounds established in Property 1-4, we find:

$$\begin{aligned}A^TC &- \frac{\|B\|^2+\|C\|^2+\|A\|^2+\|D\|^2}{2k} - \frac{\|B\|^2+\|D\|^2}{2k^2} \\ &\ge 4\underline{\beta}\left\|\sum_{j\in\mathcal{N}_i^f}(q_i - q_j - c_{ij})\right\|^2 - \frac{\rho_{1,i}}{2k} - \frac{\rho_{2,i}}{2k^2},\end{aligned} \qquad (39)$$

where $\rho_{1,i}$ and $\rho_{2,i}$ are defined below (26). In other words, if the right hand side of (39) is positive, then $A^TC - \frac{(\|B\|\|C\|+\|A\|\|D\|)}{k} - \frac{\|B\|\|D\|}{k^2} > 0$. In addition, we would have a sufficient condition for $(\nabla_{q_i}\varphi_i)^T\left(\sum_{j=1}^N \nabla_{q_i}\varphi_j\right) > 0$. Thus by (39) it suffices to show

$$\left(4\underline{\beta}\left\|\sum_{j\in\mathcal{N}_i^f}(q_i - q_j - c_{ij})\right\|^2 - \frac{\rho_{1,i}}{2k} - \frac{\rho_{2,i}}{2k^2}\right) > 0. \qquad (40)$$

Based on Property 5, γ_i can be bounded above by a constant, which means $\rho_{1,i}$ and $\rho_{2,i}$ both have upper bounds of $\overline{\rho}_1$ and $\overline{\rho}_2$ defined below (29). In addition, in (40) $\underline{\beta} \in \mathbb{R}$ is a positive constant defined below (29).

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