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# An Improved Lattice-Based Ring Signature with Unclaimable Anonymity in the Standard Model

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Ring signatures enable a user to sign messages on behalf of an arbitrary set of users, called the ring, without revealing exactly which member of that ring actually generated the signature. The signer-anonymity property makes ring signatures have been an active research topic. Recently, Park and Sealfon (CRYPTO'19) presented an important anonymity notion named *signer-unclaimability* and constructed a lattice-based ring signature scheme with unclaimable anonymity in the standard model, however, it did not consider the unforgeable w.r.t. adversarially-chosen-key attack (the public key ring of a signature may contain keys created by an adversary) and the signature size grows quadratically in the size of ring and message.

In this work, we propose a new lattice-based ring signature scheme with unclaimable anonymity in the standard model. In particular, our work improves the security and efficiency of Park and Sealfon's work, which is unforgeable w.r.t. adversarially-chosen-key attack, and the ring signature size grows linearly in the ring size.

*Keywords: Lattice-Based; Ring Signature; Standard Model; Unclaimable Anonymity*

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## 1. INTRODUCTION

Ring signatures, introduced by Rivest et al. [1], allow a signer to hide in a *ring* of potential signers of which the signer is a member, without revealing which member actually produced the signature. Thereafter, ring signatures have been researched extensively [2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. Among these works, it is worth mentioning that Bender et al. [11] presented a hierarchy of security and privacy models which were widely used in many works [12, 13, 14, 15, 16, 17, 18, 19, 21]. Among these privacy models, the strongest one is anonymity w.r.t. full key exposure which allows that even if an adversary compromises the randomness used to produce these signing keys of all the ring members in a ring, the adversary cannot identify the real signers of past signatures. Recently, Park and Sealfon [18] presented a stronger privacy notion named *signer-unclaimability* and showed that signer-unclaimability implies the signer-anonymity w.r.t. full key exposure. The signer-unclaimability notion not only allows the adversary to compromise all the randomness used to produce the signing keys but also all the randomness used to produce the signatures.

Another important line of research is the ring

signature constructions from lattices [12, 18, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32], since lattice-based cryptography has attracted more attention due to its distinctive features especially quantum resistant. However, the security of the majority of these works relies on random oracle (ROM) heuristic. As shown by Katz [33] (Sect. 6.2.1), it will arouse concerns on the basic security of cryptosystems that rely on ROM. For instance, Leurent and Nguyen [34] presented the attacks extracting the secret keys on several hash-then-sign type signature schemes (including the lattice-based signature [35]) when the underlying hash functions are modeled as random oracle. The first lattice-based ring signature in the standard model proposed by Brakerski and Tauman-Kalai [12], however, it did not consider the security notion i.e., unforgeable w.r.t. *adversarially-chosen-key attack*, and the signature size grows quadratically in the ring and message size. Recently, Park and Sealfon [18] adapted the work [12] to a ring signature with unclaimable anonymity, which retained the merits of the standard model and standard lattice assumption, but was still not unforgeable w.r.t. adversarially-chosen keys attack and the signature size is still large.

Overcoming these two weaknesses is very necessary

for practical scenarios. Particularly, in cryptocurrencies, an attacker can create some public keys maliciously and put them into the blockchain as the normal ones, in this case, honest users may include these maliciously created keys in their rings to sign their transactions. Therefore, the security model must consider the attack in such a scenario, which is referred to as *adversarially-chosen-key attack*. Moreover, the signature size can not grow too fast with the ring size since the number of members in cryptocurrencies is usually huge.

### 1.1. Our Results

To address the above concerns, we propose a new lattice-based ring signature scheme with unclaimable anonymity in the standard model based on standard lattice assumptions (SIS and LWE). In particular, our work *simultaneously improves the security and efficiency* of Park and Sealfon's work [18].

On the security, our ring signature is unforgeable w.r.t. adversarially-chosen-key attack, which is a stronger security notion than the one used in Park and Sealfon's work [18]; On the efficiency, we eliminate the dependency between the message length and ring signature size, i.e., make the ring signature size grows linearly in the ring size.

Table 1 shows a comparison between our results in this work and the existing works on lattice-based ring signatures. We note that although the works [27, 28, 29] achieved sublinear-size ring signatures, they rely on the random oracle heuristic and the anonymity is not unclaimable. And even though the work [21] achieved logarithmic-size, their anonymity is not unclaimable and the construction employs many cost cryptographic building blocks and proof system which would likely render concrete instantiations inefficient for reasonable parameters.

### 1.2. Overview of Our Approach

To describe our approach, it is instructive to recall Park and Sealfon's work [18]. In the PS scheme, there are  $N$  users in a ring  $R$ , each user generates  $\mathbf{A}^{(i)}$  and trapdoor  $\mathbf{S}^{(i)}$  by trapdoor generation algorithm, and samples  $2t$  "message matching" matrices

$\{\mathbf{A}_{j,b}^{(i)}\}_{(j,b) \in [t] \times \{0,1\}} \stackrel{\$}{\leftarrow} \mathbb{Z}_q^{n \times m}$  two of them corresponding to each bit of the message. It additionally sample a vector  $\mathbf{y}^{(i)} \in \mathbb{Z}_q^n$ . Each user's verification key is  $\mathbf{vk}^{(i)} = (\mathbf{A}^{(i)}, \{\mathbf{A}_{j,b}^{(i)}\}_{(j,b) \in [t] \times \{0,1\}}, \mathbf{y}^{(i)})$ , signing key is  $\mathbf{sk}^{(i)} = \mathbf{S}^{(i)}$ . The ring  $R = \{\mathbf{vk}^{(1)}, \dots, \mathbf{vk}^{(N)}\}$ . Let  $\boldsymbol{\mu} = (\mu_1, \dots, \mu_t) \in \{0,1\}^t$  be the message. The signing procedure is one of the  $N$  users who samples  $\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(N)} \in \mathbb{Z}^{(t+1) \times m}$  such that the ring equation  $\bar{\mathbf{A}}^{(1)} \mathbf{x}^{(1)} + \dots + \bar{\mathbf{A}}^{(N)} \mathbf{x}^{(N)} = \mathbf{y} \pmod{q}$  holds, where  $\bar{\mathbf{A}}^{(i)} = [\mathbf{A}^{(i)} | \mathbf{A}_{\boldsymbol{\mu}}^{(i)}]$ ,  $\mathbf{A}_{\boldsymbol{\mu}}^{(i)} = [\mathbf{A}_{1,\mu_1}^{(i)} | \dots | \mathbf{A}_{t,\mu_t}^{(i)}]$ , and  $\mathbf{y} \leftarrow \{\mathbf{y}^{(i)}\}_{i \in [N]}$  selected in lexicographically first way. Finally, output  $(\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(N)})$  as the signature. The

verification procedure check if the signature is well-formed and if the ring equation holds, accept, otherwise reject.

In this setting, an adversary can easily forge a signature by querying the signing oracle with an adversarially-formed ring such as  $R^* = \{\mathbf{vk}^{(1)}, \mathbf{vk}^{(2)}, c_1 \mathbf{vk}^{(1)}, c_2 \mathbf{vk}^{(2)}\}$  where  $c_1, c_2$  are constants and

$$\begin{aligned} c_1 \mathbf{vk}^{(1)} &= \left( c_1 \mathbf{A}^{(1)}, \{c_1 \mathbf{A}_{j,b}^{(1)}\}_{(j,b) \in [t] \times \{0,1\}}, \mathbf{y}^{(1)} \right) \\ c_2 \mathbf{vk}^{(2)} &= \left( c_2 \mathbf{A}^{(2)}, \{c_2 \mathbf{A}_{j,b}^{(2)}\}_{(j,b) \in [t] \times \{0,1\}}, \mathbf{y}^{(2)} \right) \end{aligned}$$

Assume  $(\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \mathbf{x}^{(3)}, \mathbf{x}^{(4)})$  is the replied signature for that query,  $\boldsymbol{\mu}$  is the queried message, then the adversary can immediately obtain a forgery signature  $(\mathbf{x}^{(1)} + c_1 \mathbf{x}^{(3)}, \mathbf{x}^{(2)} + c_2 \mathbf{x}^{(4)})$  for the ring  $(\mathbf{vk}^{(1)}, \mathbf{vk}^{(2)})$  since the ring equation  $\bar{\mathbf{A}}^{(1)}(\mathbf{x}^{(1)} + c_1 \mathbf{x}^{(3)}) + \bar{\mathbf{A}}^{(2)}(\mathbf{x}^{(2)} + c_2 \mathbf{x}^{(4)}) = \mathbf{y} \pmod{q}$  holds for  $\mathbf{y} \leftarrow \{\mathbf{y}^{(1)}, \mathbf{y}^{(2)}\}$ .

To resolve the above problem, we use the key-homomorphic evaluation algorithm that developed from [36, 37, 38] to evaluate circuits of a PRF. Even though the method is inspired by the standard signature work [39], it is essentially different in ring signature setting, since its privacy and security requirements is more complex than standard signature. For our construction, we borrow the idea from the unforgeability simulation of [39]. In our setting, the verification key of each user is  $\mathbf{vk}^{(i)} = (\mathbf{A}^{(i)}, (\mathbf{A}_0^{(i)}, \mathbf{A}_1^{(i)}), \{\mathbf{B}_j^{(i)}\}_{j \in [k]}, (\mathbf{C}_0^{(i)}, \mathbf{C}_1^{(i)}))$  and the signing key is  $\mathbf{sk}^{(i)} = (\mathbf{S}^{(i)}, \mathbf{k}^{(i)})$ , where  $(\mathbf{A}^{(i)}, \mathbf{S}^{(i)})$  are generated by trapdoor generation algorithm,  $\mathbf{k}^{(i)} \in \{0,1\}^k$  is a PRF key, the remained matrices are used to construct the homomorphic evaluated matrix  $\mathbf{A}_{C_{\text{PRF}}, \boldsymbol{\mu}}^{(i)}$  in signing phase.

In the signing phase, the signer first construct a homomorphic evaluated matrix  $\mathbf{A}_{C_{\text{PRF}}, \boldsymbol{\mu}}^{(i)}$  which determined by the signer's PRF key, where  $C_{\text{PRF}}$  denote NAND Boolean circuit expression of the PRF function. Then sample a  $2Nm$ -dimensional vector  $\mathbf{x}' = ((\mathbf{x}^{(1)})^\top | \dots | (\mathbf{x}^{(N)})^\top)$  such that the following ring equation holds

$$\sum_{i \in [N]} [\mathbf{A}^{(i)} | \mathbf{A}_{C_{\text{PRF}}, \boldsymbol{\mu}}^{(i)}] \mathbf{x}^{(i)} = \mathbf{0} \pmod{q} \quad (1)$$

Finally, output  $\mathbf{x}'$  as the signature. In the verification phase, check if the input signature is well-formed and the ring equation (1) holds.

In this setting, it is effective against the adversary to forge signatures by adversarially forming a ring. Because  $\mathbf{A}_{C_{\text{PRF}}, \boldsymbol{\mu}}^{(i)}$  can not be predetermined i.e., is unpredictable for the adversary since  $\mathbf{A}_{C_{\text{PRF}}, \boldsymbol{\mu}}^{(i)}$  generated based on the PRF key that selected during signing phase. For the ring signature size, we process the message by puncturing it into specific matrices and homomorphically evaluate them using  $C_{\text{PRF}}$  which finally outputs only one matrix as result, rather than

**TABLE 1.** Comparison with existing lattice-based ring signature schemes

Lattice-based ring signature works	Standard model	Unclaimable anonymity	Unforgeability w.r.t. adversarially-chosen-key attack	Signature size
[22, 24]	✓	×	✓, flaws <sup>a</sup>	Linear
[23, 25, 26, 30, 31, 32]	×	×	✓	Linear
[27, 28, 29]	×	×	✓	Sub-linear
[21]	✓	×	✓	Logrithmic
[12, 18]	✓	✓	×	Quadratic
Our	✓	✓	✓	Linear

<sup>a</sup> The works [22, 24] are not unforgeable w.r.t. adversarially-chosen-key attack since the forger can trivially forge a signature by querying the signing oracle with his adversarially-chosen public keys ability (Refer to Sect. 1.2 for the details of the attack).

directly assigning a matrix to each message bit and finally outputs a concatenation of these matrices as [18], which eliminate the dependency between the message length and ring signature size and therefore our ring signature size is only linear with ring size.

## 2. DEFINITIONS

In this section, we define the ring signature system and then formalize the security and privacy models.

### 2.1. Algorithm Definition

**DEFINITION 2.1** (Ring Signature). *A ring signature RS scheme consists of the following algorithms:*

- $\text{Setup}(1^n) \rightarrow \text{PP}$ . *This is a probabilistic algorithm. On input a security parameter  $n$ , the algorithm outputs the public parameters PP.*  
The public parameters PP are common parameters used by all participants in the system, for example, the message space  $\mathcal{M}$ , the signature space, etc. In the following, PP are implicit input parameters to every algorithm.
- $\text{KeyGen}() \rightarrow (\text{vk}, \text{sk})$ . *This is a probabilistic algorithm. The algorithm outputs a verification key vk and a signing key sk.*  
Any ring member can run this algorithm to generate a pair of verification keys and signing keys.
- $\text{Sign}(\mu, \text{R}, \text{sk}) \rightarrow \Sigma$ . *This is a probabilistic algorithm. On input a message  $\mu \in \mathcal{M}$ , a ring of verification keys  $\text{R} = (\text{vk}^{(1)}, \dots, \text{vk}^{(N)})^3$  and a signing key sk. Assume that (1) the input signing key sk and the corresponding verification key vk is a valid key pair output by KeyGen and  $\text{vk} \in \text{R}$ , (2) the ring size  $|\text{R}| \geq 2$ , (3) each verification key in ring R is distinct. The algorithm outputs a signature  $\Sigma$ .*
- $\text{Ver}(\mu, \text{R}, \Sigma) \rightarrow 1/0$ . *This is a deterministic algorithm. On input a message  $\mu$ , a ring of verification keys  $\text{R} = (\text{vk}^{(1)}, \dots, \text{vk}^{(N)})$  and a signature  $\Sigma$ , the algorithm outputs 1 if the signature is valid, or 0 if the signature is invalid.*

<sup>3</sup>Below we regard the verification key ring as an ordered set, namely, it consists of a set of verification keys, and when it is used in Sign and Ver algorithms, the verification keys are ordered and each one has an index.

*Remark:* Note that it is open on whether the Sign algorithm is probabilistic or deterministic, which may depend on the concrete construction.

**Correctness.** *A RS scheme is correct, if for all  $n \in \mathbb{N}$ , all  $N = \text{poly}(n)$ , all  $i \in [N]$ , all messages  $\mu \in \mathcal{M}$ , any  $\text{PP} \leftarrow \text{Setup}(1^n)$  as implicit input parameter to every algorithm, any  $N$  pairs  $\{\text{vk}^{(i)}, \text{sk}^{(i)}\}_{i \in [N]} \leftarrow \text{KeyGen}()$  and any  $\Sigma \leftarrow \text{Sign}(\mu, \text{R}, \text{sk}^{(i)})$  where  $\text{R} = \{\text{vk}^{(1)}, \dots, \text{vk}^{(N)}\}$ , it holds that*

$$\Pr[\text{Ver}(\mu, \text{R}, \Sigma) = 1] = 1 - \text{negl}(n)$$

where the probability is taken over the random coins used by Setup, KeyGen, and Sign.

### 2.2. Security and Privacy Models

Below we define the security and privacy models for RS. In both models, we give the randomness used in Setup to the adversary, which implies the Setup algorithm is public, does not rely on a trusted setup that may incur concerns on the existence of trapdoors hidden in the output parameters.

The security model i.e., unforgeability w.r.t. adversarially-chosen-key attack captures that only the ring member knowing the secret key for some verification key in a ring can generate a valid signature with respect to the ring, even though existing adversary is allowed to arbitrarily add some verification keys in the ring when querying the signing oracle. In other words, assuming there is a ring signature system that satisfies unforgeability w.r.t. adversarially-chosen-key attack, even some verification keys in the system were maliciously generated by an adversary, and these keys were used to issue signatures by some honest ring members, the unforgeability still holds.

The privacy model i.e., signer-unclaimability captures that given a valid signature with respect to a ring of verification keys and the randomness used to produce the signature, even though existing adversary obtained all the randomness that used to produce all the signing keys in the system, the adversary still can not identify the signer's verification key out of the ring. In other words, assuming an RS system with unclaimable anonymity revealed all ring members' signing keys, and allowed the adversary to obtain the signing random-

ness of any valid signature in the system, the signer-anonymity still holds.

**Unforgeability.** A RS scheme is unforgeability against adversarially-chosen-key attack (UnfAdvKey), if for any PPT adversary  $\mathcal{A}$ , it holds that  $\mathcal{A}$  has at most negligible advantage in the following experiment with a challenger  $\mathcal{C}$ .

- **Setup.**  $\mathcal{C}$  generates  $\text{PP} \leftarrow \text{Setup}(1^n; \gamma_{\text{st}})$  and  $(\text{vk}^{(i)}, \text{sk}^{(i)}) \leftarrow \text{KeyGen}()$  for all  $i \in [N]$ , where  $N = \text{poly}(n)$  and  $\gamma_{\text{st}}$  is the randomness used in Setup.  $\mathcal{C}$  sets  $S = \{\text{vk}^{(i)}\}_{i \in [N]}$  and initializes an empty set  $L$ . Finally,  $\mathcal{C}$  sends  $(\text{PP}, S, \gamma_{\text{st}})$  to  $\mathcal{A}$ .
- **Probing Phase.**  $\mathcal{A}$  is given access to a signing oracle  $\text{OSign}(\cdot, \cdot, \cdot)$ : On input a message  $\mu \in \mathcal{M}$ , a ring of verification keys  $R$  and an index  $s \in [N]$  such that  $\text{vk}^{(s)} \in R \cap S$ , this oracle returns  $\Sigma \leftarrow \text{Sign}(\mu, R, \text{sk}^{(s)})$  and adds the tuple  $(\mu, R, \Sigma)$  to  $L$ . Note that it only requires that the  $\text{vk}^{(s)}$  is in  $S$  without requiring that  $R \subseteq S$ . This captures that  $\mathcal{A}$  can obtain the signing oracle of its choice, in which the queried ring  $R$  may contain verification keys that are created by  $\mathcal{A}$  (referred to as adversarially-chosen-key attack).
- **Forge.**  $\mathcal{A}$  outputs a signature  $(\mu^*, R^*, \Sigma^*)$  and succeeds if (1)  $\text{Ver}(\mu^*, R^*, \Sigma^*) = 1$ , (2)  $R^* \subseteq S$ , and (3)  $(\mu^*, R^*, \Sigma^*) \notin L$ .

**Anonymity.** A RS scheme is signer-unclaimability, if for any PPT adversary  $\mathcal{A}$ , it holds that  $\mathcal{A}$  has at most negligible advantage in the following experiment with a challenger  $\mathcal{C}$ .

- **Setup.**  $\mathcal{C}$  generates  $\text{PP} \leftarrow \text{Setup}(1^n; \gamma_{\text{st}})$  and  $(\text{vk}^{(i)}, \text{sk}^{(i)}) \leftarrow \text{KeyGen}(\gamma_{\text{kg}}^{(i)})$  for all  $i \in [N]$ , where  $N = \text{poly}(n)$  and  $(\gamma_{\text{st}}, \{\gamma_{\text{kg}}^{(i)}\}_{i \in [N]})$  are randomness used in Setup and KeyGen, respectively.  $\mathcal{C}$  sets  $S = \{\text{vk}^{(i)}\}_{i \in [N]}$ . Finally,  $\mathcal{C}$  sends  $(\text{PP}, S, \gamma_{\text{st}}, \{\gamma_{\text{kg}}^{(i)}\}_{i \in [N]})$  to  $\mathcal{A}$ .
- **Challenge.**  $\mathcal{A}$  provides a challenge  $(\mu^*, R^*, s_0^*, s_1^*)$  to the challenger such that  $s_0^*, s_1^* \in [N]$ ,  $s_0^* \neq s_1^*$  and  $\text{vk}^{(s_0^*)}, \text{vk}^{(s_1^*)} \in S \cap R^*$ .  $\mathcal{C}$  chooses a random bit  $b \in \{0, 1\}$  and computes the signature  $\Sigma^*$  by invoking  $\Sigma^* \leftarrow \text{Sign}(\mu^*, R^*, \text{sk}^{(s_b^*)}; \gamma_{\text{sign}})$ . Finally, returns  $(\Sigma^*, \gamma_{\text{sign}})$  to  $\mathcal{A}$ . Note that we not only give  $\mathcal{A}$  the randomness  $\gamma_{\text{st}}$  and  $\{\gamma_{\text{kg}}^{(i)}\}_{i \in [N]}$  in Setup phase, but also give  $\mathcal{A}$  the signature  $\Sigma^*$  and the corresponding signing randomness  $\gamma_{\text{sign}}$  that used to produce  $\Sigma^*$  in Challenge phase (referred to as signer-unclaimability).
- **Guess.**  $\mathcal{A}$  outputs a guess  $b'$ . If  $b' = b$ ,  $\mathcal{C}$  outputs 1, otherwise 0.

### 3. PRELIMINARY

In this section, we first review some lattice-based backgrounds, then we review the key-homomorphic

evaluation algorithm which we will use as a building block for our construction.

**Notation.** We denote vectors as lower-case bold letters (e.g.  $\mathbf{x}$ ), and matrices by upper-case bold letters (e.g.  $\mathbf{A}$ ). We say that a function in  $n$  is negligible, written  $\text{negl}(n)$ , if it vanishes faster than the inverse of any polynomial in  $n$ . We say that a probability  $p(n)$  is overwhelming if  $1 - p(n)$  is negligible. We denote the horizontal concatenation of two matrices  $\mathbf{A}$  and  $\mathbf{B}$  as  $\mathbf{A} \parallel \mathbf{B}$ .

**Matrix Norms.** For a vector  $\mathbf{x}$ , we let  $\|\mathbf{x}\|$  denote its  $l_2$ -norm. For a matrix  $\mathbf{A}$  we denote two matrix norms:  $\|\mathbf{A}\|$  denotes the  $l_2$  length of the longest column of  $\mathbf{A}$ .  $\|\tilde{\mathbf{A}}\|$  denotes the result of applying Gram-Schmidt orthogonalization to the columns of  $\mathbf{A}$ .

**Lattices and Gaussian Distributions.** Let  $m \in \mathbb{Z}$  be a positive integer and  $\Lambda \subset \mathbb{R}^m$  be an  $m$ -dimensional full-rank lattice formed by the set of all integral combinations of  $m$  linearly independent basis vectors  $\mathbf{B} = (\mathbf{b}_1, \dots, \mathbf{b}_m) \subset \mathbb{Z}^m$ , i.e.,  $\Lambda = \mathcal{L}(\mathbf{B}) = \{\mathbf{B}\mathbf{c} = \sum_{i=1}^m c_i \mathbf{b}_i : \mathbf{c} \in \mathbb{Z}^m\}$ . For positive integers  $n, m, q$ , a matrix  $\mathbf{A} \in \mathbb{Z}_q^{n \times m}$ , and a vector  $\mathbf{y} \in \mathbb{Z}_q^m$ , the  $m$ -dimensional integer lattice  $\Lambda_q^\perp(\mathbf{A})$  is defined as  $\Lambda_q^\perp(\mathbf{A}) = \{\mathbf{x} \in \mathbb{Z}^m : \mathbf{A}\mathbf{x} = \mathbf{0} \pmod{q}\}$ .  $\Lambda_q^{\mathbf{y}}(\mathbf{A})$  is defined as  $\Lambda_q^{\mathbf{y}}(\mathbf{A}) = \{\mathbf{x} \in \mathbb{Z}^m : \mathbf{A}\mathbf{x} = \mathbf{y} \pmod{q}\}$ . For a vector  $\mathbf{c} \in \mathbb{R}^m$  and a positive parameter  $\sigma \in \mathbb{R}$ , define  $\rho_{\sigma, \mathbf{c}}(\mathbf{x}) = \exp(-\pi\|\mathbf{x} - \mathbf{c}\|^2/\sigma^2)$  and  $\rho_{\sigma, \mathbf{c}}(\Lambda) = \sum_{\mathbf{x} \in \Lambda} \rho_{\sigma, \mathbf{c}}(\mathbf{x})$ . For any  $\mathbf{y} \in \Lambda$ , define the discrete Gaussian distribution over  $\Lambda$  with center  $\mathbf{c}$  and parameter  $\sigma$  as  $\mathcal{D}_{\Lambda, \sigma, \mathbf{c}}(\mathbf{y}) = \rho_{\sigma, \mathbf{c}}(\mathbf{y})/\rho_{\sigma, \mathbf{c}}(\Lambda)$ . For simplicity,  $\rho_{\sigma, \mathbf{0}}$  and  $\mathcal{D}_{\Lambda, \sigma, \mathbf{0}}$  are abbreviated as  $\rho_\sigma$  and  $\mathcal{D}_{\Lambda, \sigma}$ , respectively.

The following Lemma 3.1 bounds the length of a discrete Gaussian vector with a sufficiently large Gaussian parameter.

LEMMA 3.1 ([40]). For any lattice  $\Lambda$  of integer dimension  $m$  with basis  $\mathbf{B}$ ,  $\mathbf{c} \in \mathbb{R}^m$  and Gaussian parameter  $\sigma > \|\tilde{\mathbf{B}}\| \cdot \omega(\sqrt{\log m})$ , we have  $\Pr[\|\mathbf{x} - \mathbf{c}\| > \sigma\sqrt{m} : \mathbf{x} \leftarrow \mathcal{D}_{\Lambda, \sigma, \mathbf{c}}] \leq \text{negl}(n)$ .

The following generalization of the leftover hash lemma is needed for our security proof.

LEMMA 3.2 ([41]). Suppose that  $m > (n+1)\log q + \omega(\log n)$  and that  $q > 2$  is prime. Let  $\mathbf{R}$  be an  $m \times k$  matrix chosen uniformly in  $\{1, -1\}^{m \times k} \pmod{q}$  where  $k = k(n)$  is polynomial in  $n$ . Let  $\mathbf{A}$  and  $\mathbf{B}$  be matrices chosen uniformly in  $\mathbb{Z}_q^{n \times m}$  and  $\mathbb{Z}_q^{n \times k}$ , respectively. Then, for all vectors  $\mathbf{w}$  in  $\mathbb{Z}_q^m$ , the distribution  $(\mathbf{A}, \mathbf{A}\mathbf{R}, \mathbf{R}^\top \mathbf{w})$  is statistically close to the distribution  $(\mathbf{A}, \mathbf{B}, \mathbf{R}^\top \mathbf{w})$ .

The security of our RS construction is based on the following Small Integer Solution (SIS) assumption and the security of PRF.

DEFINITION 3.1 (SIS Assumption [35, 40]). Let  $q$  and  $\beta$  be functions of  $n$ . An instance of the  $\text{SIS}_{q, \beta}$  problem is

a uniformly random matrix  $\mathbf{A} \xleftarrow{\$} \mathbb{Z}_q^{n \times m}$  for any desired  $m = \text{poly}(n)$ . The goal is to find a nonzero integer vector  $\mathbf{x} \in \mathbb{Z}^m$  such that  $\mathbf{A}\mathbf{x} = \mathbf{0} \pmod{q}$  and  $\|\mathbf{x}\| \leq \beta$ .

For  $\beta = \text{poly}(n)$ ,  $q \geq \beta \cdot \omega(\sqrt{n \log n})$ , no (quantum) algorithm can solve  $\text{SIS}_{q,\beta}$  problem in polynomial time.

**DEFINITION 3.2** (Pseudorandom Functions). For a security parameter  $n > 0$ , let  $k = k(n)$ ,  $t = t(n)$  and  $c = c(n)$ . A pseudorandom function  $\text{PRF} : \{0, 1\}^k \times \{0, 1\}^t \rightarrow \{0, 1\}^c$  is an efficiently computable, deterministic two-input function where the first input, denoted by  $K$ , is the key. Let  $\Omega$  be the set of all functions that map  $\ell$  bits strings to  $c$  bits strings. There is a negligible function  $\text{negl}(n)$  such that:

$$\left| \Pr[\mathcal{A}^{\text{PRF}(K, \cdot)}(1^n) = 1] - \Pr[\mathcal{A}^{F(\cdot)}(1^n) = 1] \right| \leq \text{negl}(n)$$

where the probability is taken over a uniform choice of key  $K \xleftarrow{\$} \{0, 1\}^k$ ,  $F \xleftarrow{\$} \Omega$ , and the randomness of  $\mathcal{A}$ .

*Remark:* The PRF that we employed is Lai et al.'s work [42], which is based on standard lattice assumption i.e., learning with errors (LWE) assumption.

**Algorithms on Lattices.** Our work will use the following lattice algorithms.

**LEMMA 3.3** (TrapGen Algorithm [43]). Let  $n \geq 1$ ,  $q \geq 2$  and  $m = O(n \log q)$  be integers. There is a probabilistic algorithm  $\text{TrapGen}(1^n, 1^m, q)$  that outputs a matrix  $\mathbf{A} \in \mathbb{Z}_q^{n \times m}$  and a trapdoor  $\mathbf{S}_{\mathbf{A}} \subset \Lambda_q^\perp(\mathbf{A})$ , the distribution of  $\mathbf{A}$  is statistically close to the uniform distribution over  $\mathbb{Z}_q^{n \times m}$  has  $\|\widetilde{\mathbf{S}}_{\mathbf{A}}\| \leq O(\sqrt{n \log q})$  and  $\|\mathbf{S}_{\mathbf{A}}\| \leq O(n \log q)$  with all but negligible probability in  $n$ .

**LEMMA 3.4** (BasisExt Algorithm [44]). For  $i = 1, 2, 3$ , let  $\mathbf{A}_i$  be a matrix in  $\mathbb{Z}_q^{n \times m_i}$  whose columns generate  $\mathbb{Z}_q^n$  and let  $\mathbf{A}' = [\mathbf{A}_1 | \mathbf{A}_2 | \mathbf{A}_3]$ . Let  $\mathbf{S}_{\mathbf{A}_2}$  be a basis of  $\Lambda^\perp(\mathbf{S}_{\mathbf{A}_2})$ . There is a deterministic algorithm  $\text{BasisExt}(\mathbf{A}', \mathbf{S}_{\mathbf{A}_2})$  that outputs a basis  $\mathbf{S}_{\mathbf{A}'}$  for  $\Lambda^\perp(\mathbf{A}')$  such that  $\|\widetilde{\mathbf{S}}_{\mathbf{A}'}\| = \|\widetilde{\mathbf{S}}_{\mathbf{A}_2}\|$ .

**LEMMA 3.5** (BasisRand Algorithm [44]). Let  $\mathbf{S}_{\mathbf{A}'} \in \mathbb{Z}^{m' \times m'}$  be an extended basis of  $\Lambda^\perp(\mathbf{A}')$  output by  $\text{BasisExt}$ . There is a probabilistic algorithm  $\text{BasisRand}(\mathbf{S}_{\mathbf{A}'}, \sigma)$  which takes as input a basis  $\mathbf{S}_{\mathbf{A}'}$  and a parameter  $\sigma \geq \|\widetilde{\mathbf{S}}_{\mathbf{A}'}\| \cdot \omega(\sqrt{\log m})$ , outputs a basis  $\mathbf{S}_{\mathbf{A}''} \in \mathbb{Z}^{m' \times m'}$  of  $\Lambda^\perp(\mathbf{A}')$  which is statistically independent with the original basis  $\mathbf{S}_{\mathbf{A}'}$ , and has  $\|\widetilde{\mathbf{S}}_{\mathbf{A}''}\| \leq \sigma \cdot \sqrt{m'}$  holds.

The following lemma is a property of the  $\text{BasisRand}$  algorithm, which will be used in our signer-anonymity proof.

**LEMMA 3.6** (Trapdoor Indistinguishability of  $\text{BasisRand}$  [44]). For any two bases  $\mathbf{S}_0, \mathbf{S}_1$  of the same lattice and any  $\sigma \geq \max\{\|\widetilde{\mathbf{S}}_0\|, \|\widetilde{\mathbf{S}}_1\|\} \cdot \omega(\sqrt{\log m})$ , the outputs of  $\text{BasisRand}(\mathbf{S}_0, \sigma)$  and  $\text{BasisRand}(\mathbf{S}_1, \sigma)$  are within  $\text{negl}(n)$  statistical distance.

The following lattice basis extension algorithm is also needed for our security proof, which was presented by Agrawal, Boneh, and Boyen [41], so we abbreviate that as  $\text{BasisExtABB}$  algorithm.

**LEMMA 3.7** ( $\text{BasisExtABB}$  Algorithm [41]). Let  $q$  be a prime,  $n, m$  be integers with  $m > n$ . There is a probabilistic algorithm  $\text{BasisExtABB}(\mathbf{A}, \mathbf{B}, \mathbf{R}, \mathbf{S}_{\mathbf{B}})$  which takes as input two matrices  $\mathbf{A}, \mathbf{B} \in \mathbb{Z}_q^{n \times m}$  whose columns generate  $\mathbb{Z}_q^n$ , a matrix  $\mathbf{R} \in \mathbb{Z}^{m \times m}$ , and a basis  $\mathbf{S}_{\mathbf{B}} \in \Lambda_q^\perp(\mathbf{B})$ , outputs a basis  $\mathbf{S}_{\mathbf{F}}$  of  $\Lambda_q^\perp(\mathbf{F})$  such that  $\|\widetilde{\mathbf{S}}_{\mathbf{F}}\| < (\|\mathbf{R}\| + 1) \cdot \|\widetilde{\mathbf{S}}_{\mathbf{B}}\|$  where  $\mathbf{F} = [\mathbf{A} | \mathbf{A}\mathbf{R} + \mathbf{B}] \in \mathbb{Z}_q^{n \times 2m}$ .

**LEMMA 3.8** ( $\text{SampleGaussian}$  Algorithm [35]). Let  $q > 2$ ,  $m > n$  be integers. There is a probabilistic algorithm  $\text{SampleGaussian}(\mathbf{A}, \mathbf{S}_{\mathbf{A}}, \mathbf{y}, \sigma)$  which takes as input a matrix  $\mathbf{A} \in \mathbb{Z}_q^{n \times m}$  whose columns generate  $\mathbb{Z}_q^n$ , and a basis  $\mathbf{S}_{\mathbf{A}}$  of  $\Lambda_q^\perp(\mathbf{A})$ , a vector  $\mathbf{y} \in \mathbb{Z}_q^n$ , and a Gaussian parameter  $\sigma \geq \|\widetilde{\mathbf{S}}_{\mathbf{A}}\| \cdot \omega(\sqrt{\log m})$ , outputs a vector  $\mathbf{x} \in \Lambda_q^\perp(\mathbf{A})$  sampled from a distribution which is statistically close to  $\mathcal{D}_{\Lambda_q^\perp(\mathbf{A}), \sigma}$ .

Given output values of the algorithms  $\text{SampleGaussian}$ , the following algorithm  $\text{ExplainGaussian}$  is used to sample the randomness under which the  $\text{SampleGaussian}$  algorithm produces the desired output.

**LEMMA 3.9** ( $\text{ExplainGaussian}$  Algorithm [18]). There is a probabilistic algorithm  $\text{ExplainGaussian}(\mathbf{A}, \mathbf{S}_{\mathbf{A}}, \mathbf{x}, \sigma, \mathbf{y})$  that on input a pair of matrices  $(\mathbf{A}, \mathbf{S}_{\mathbf{A}})$  from  $\text{TrapGen}$ , preimage vector  $\mathbf{x}$ , a parameter  $\sigma$  and a image vector  $\mathbf{y} \in \mathbb{Z}^m$ , samples randomness  $\gamma$  that yields output  $\mathbf{x}$  under algorithm  $\text{SampleGaussian}$ , i.e., samples from the distribution  $\{\gamma | \text{SampleGaussian}(\mathbf{A}, \mathbf{S}_{\mathbf{A}}, \mathbf{y}, \sigma; \gamma) = \mathbf{x}\}$ .

The following lemma is a property of the  $\text{ExplainGaussian}$  algorithm, which will be used in our signer-anonymity proof.

**LEMMA 3.10** (Randomness Indistinguishability of  $\text{ExplainGaussian}$  [18]). Let  $\mathcal{R}$  be the randomness space, let  $(\mathbf{A}_0, \mathbf{S}_{\mathbf{A}_0})$  and  $(\mathbf{A}_1, \mathbf{S}_{\mathbf{A}_1})$  be two pairs of matrices from  $\text{TrapGen}$ , let  $\mathbf{F} = [\mathbf{A}_0 | \mathbf{A}_1]$ , let  $\mathbf{S}_{\mathbf{F}_0}$  and  $\mathbf{S}_{\mathbf{F}_1}$  be the extended basis from  $\mathbf{S}_{\mathbf{A}_0}$  and  $\mathbf{S}_{\mathbf{A}_1}$  respectively. The distribution of randomness  $\gamma^{(0)} \xleftarrow{\$} \mathcal{R}$  for  $\mathbf{x} \leftarrow \text{SampleGaussian}(\mathbf{F}, \mathbf{S}_{\mathbf{F}_0}, \mathbf{y}, \sigma; \gamma^{(0)})$  and  $\gamma^{(1)} \leftarrow \text{ExplainGaussian}(\mathbf{F}, \mathbf{S}_{\mathbf{F}_1}, \mathbf{x}, \sigma, \mathbf{y})$  are statistically indistinguishable.

**Gadget Matrix.** The ‘‘gadget matrix’’  $\mathbf{G}$  defined in the work [45]. We recall the following one fact.

**LEMMA 3.11** ([45]). Let  $q$  be a prime, and  $n, m$  be integers with  $m = n \log q$ . There is a fixed full-rank matrix such that the lattice  $\Lambda_q^\perp(\mathbf{G})$  has a publicly known basis  $\mathbf{S}_{\mathbf{G}} \in \mathbb{Z}^{m \times m}$  with  $\|\widetilde{\mathbf{S}}_{\mathbf{G}}\| \leq \sqrt{5}$ .

### 3.1. Key-Homomorphic Evaluation Algorithm

In our construction, we borrow the idea from the standard signature work [39], that is employing the key-homomorphic evaluation algorithm  $\text{Eval}(\cdot, \cdot)$  from [36, 37, 38] to evaluate circuits of a PRF. In particular, they used the Brakerski and Vaikuntanathan's evaluation algorithm [37]. The inputs of  $\text{Eval}(\cdot, \cdot)$  are  $C$  and a set of  $\ell$  different matrices  $\{\mathbf{A}^{(i)}\}_{i \in [\ell]}$ , where  $C : \{0, 1\}^\ell \rightarrow \{0, 1\}$  is a fan-in-2 Boolean NAND circuit expression of some functions such as a PRF, and each  $\mathbf{A}^{(i)} = \mathbf{A}\mathbf{R}^{(i)} + b^{(i)}\mathbf{G} \in \mathbb{Z}_q^{n \times m}$  corresponds to each input wire of  $C$ , and where  $\mathbf{A} \xleftarrow{\$} \mathbb{Z}_q^{n \times m}$ ,  $\mathbf{R}^{(i)} \xleftarrow{\$} \{1, -1\}^{m \times m}$ ,  $b^{(i)} \in \{0, 1\}$  and  $\mathbf{G} \in \mathbb{Z}_q^{n \times m}$  is the gadget matrix. The algorithm deterministically output a matrix  $\mathbf{A}_C = \mathbf{A}\mathbf{R}_C + C(b^{(1)}, \dots, b^{(\ell)})\mathbf{G} \in \mathbb{Z}_q^{n \times m}$ . In the analysis of our unforgeability proof, we will use the following lemma to show  $\mathbf{R}_C$  is short enough.

**LEMMA 3.12.** *Let  $C : \{0, 1\}^\ell \rightarrow \{0, 1\}$  be a NAND boolean circuit which has depth  $d = c \log \ell$  for some constant  $c$ . Let  $\{\mathbf{A}^{(i)} = \mathbf{A}\mathbf{R}^{(i)} + b^{(i)}\mathbf{G} \in \mathbb{Z}_q^{n \times m}\}_{i \in [\ell]}$  be  $\ell$  different matrices correspond to each input wire of  $C$  where  $\mathbf{A} \xleftarrow{\$} \mathbb{Z}_q^{n \times m}$ ,  $\mathbf{R}^{(i)} \xleftarrow{\$} \{1, -1\}^{m \times m}$ ,  $b^{(i)} \in \{0, 1\}$  and  $\mathbf{G} \in \mathbb{Z}_q^{n \times m}$  is the gadget matrix. There is an efficient deterministic evaluation algorithm  $\text{Eval}(C, (\mathbf{A}^{(1)}, \dots, \mathbf{A}^{(\ell)}))$  runs in time  $\text{poly}(4^d, \ell, n, \log q)$ , the inputs are  $C$  and  $\{\mathbf{A}^{(i)}\}_{i \in [\ell]}$ , the output is a matrix*

$$\begin{aligned} \mathbf{A}_C &= \mathbf{A}\mathbf{R}_C + C(b^{(1)}, \dots, b^{(\ell)})\mathbf{G} \\ &= \text{Eval}(C, (\mathbf{A}^{(1)}, \dots, \mathbf{A}^{(\ell)})) \end{aligned}$$

where  $C(b^{(1)}, \dots, b^{(\ell)})$  is the output bit of  $C$  on the arguments  $(b^{(1)}, \dots, b^{(\ell)})$  and  $\mathbf{R}_C \in \mathbb{Z}^{m \times m}$  is a low norm matrix has  $\|\mathbf{R}_C\| \leq O(\ell^{2c} \cdot m^{3/2})$ .

## 4. OUR SCHEME

In this section, we present the construction of our RS scheme in Sect. 4.1, and give the concrete parameters in Sect. 4.2. Then we prove the unforgeability and anonymity in Sect. 4.3 and Sect. 4.4, respectively.

### 4.1. Construction

$\text{Setup}(1^n; \gamma_{\text{st}})$

1. On input a security parameter  $n$ , sets the modulo  $q$ , lattice dimension  $m$ , PRF key length  $k$ , message length  $t$ , let  $\gamma_{\text{st}}$  be the randomness that use to choose Gaussian parameters and then chooses Gaussian parameters  $\sigma$  and  $\sigma'$  as specified in Sect. 4.2 below.
2. Select a secure PRF  $: \{0, 1\}^k \times \{0, 1\}^t \rightarrow \{0, 1\}$ , express it as a NAND Boolean circuit  $C_{\text{PRF}}$ .
3. Output  $\text{PP} = (q, m, k, t, \sigma, \sigma', \text{PRF}, \gamma_{\text{st}})$ .

Note that including the randomness  $\gamma_{\text{st}}$  in  $\text{PP}$  is to guarantee the public has no concerns about the existence of trapdoors.

In the following,  $\text{PP}$  are implicit input parameters to every algorithm.

$\text{KeyGen}()$

1. Sample  $(\mathbf{A}, \mathbf{S}_{\mathbf{A}}) \leftarrow \text{TrapGen}(1^n, 1^m, q)$  where  $\mathbf{A} \in \mathbb{Z}_q^{n \times m}$ ,  $\mathbf{S}_{\mathbf{A}} \in \mathbb{Z}^{m \times m}$ .
2. Select a PRF key  $\mathbf{k} = (k_1, k_2, \dots, k_k) \xleftarrow{\$} \{0, 1\}^k$ .
3. Select  $\mathbf{A}_0, \mathbf{A}_1, \mathbf{C}_0, \mathbf{C}_1 \xleftarrow{\$} \mathbb{Z}_q^{n \times m}$ .
4. For  $j = 1$  to  $k$ , select  $\mathbf{B}_j \xleftarrow{\$} \mathbb{Z}_q^{n \times m}$ .
5. Output  $\text{vk} = (\mathbf{A}, (\mathbf{A}_0, \mathbf{A}_1), \{\mathbf{B}_j\}_{j \in [k]}, (\mathbf{C}_0, \mathbf{C}_1))$  and  $\text{sk} = (\mathbf{S}_{\mathbf{A}}, \mathbf{k})$ .

$\text{Sign}(\boldsymbol{\mu}, \mathbf{R}, \text{sk})$

1. On input a message  $\boldsymbol{\mu} = (\mu_1, \dots, \mu_t) \in \{0, 1\}^t$ , a ring of verification keys  $\mathbf{R} = (\text{vk}^{(1)}, \dots, \text{vk}^{(N)})$  where

$$\text{vk}^{(i)} = (\mathbf{A}^{(i)}, (\mathbf{A}_0^{(i)}, \mathbf{A}_1^{(i)}), \{\mathbf{B}_j^{(i)}\}_{j \in [k]}, (\mathbf{C}_0^{(i)}, \mathbf{C}_1^{(i)}))$$

and a signer's signing key  $\text{sk} := \text{sk}^{(\bar{i})}$  where  $\bar{i} \in [N]$  be the index of the signer in the ring  $\mathbf{R}$ .

2. Compute  $b = \text{PRF}(\mathbf{k}^{\bar{i}}, \boldsymbol{\mu})$ .
3. For  $i = 1$  to  $N$ , compute  $\mathbf{A}_{C_{\text{PRF}}, \boldsymbol{\mu}}^{(i)} \in \mathbb{Z}_q^{n \times m}$  by

$$\mathbf{A}_{C_{\text{PRF}}, \boldsymbol{\mu}}^{(i)} = \text{Eval}(C_{\text{PRF}}, (\{\mathbf{B}_j^{(i)}\}_{j \in [k]}, \mathbf{C}_{\mu_1}^{(i)}, \dots, \mathbf{C}_{\mu_t}^{(i)}))$$

and set  $\mathbf{F}_{C_{\text{PRF}}, \boldsymbol{\mu}, 1-b}^{(i)} = [\mathbf{A}^{(i)} | \mathbf{A}_{1-b}^{(i)} - \mathbf{A}_{C_{\text{PRF}}, \boldsymbol{\mu}}^{(i)}]$ .

4. Let  $\mathbf{F}'_{1-b} = [\mathbf{F}_{C_{\text{PRF}}, \boldsymbol{\mu}, 1-b}^{(1)} | \dots | \mathbf{F}_{C_{\text{PRF}}, \boldsymbol{\mu}, 1-b}^{(N)}]$ . Compute

$$\mathbf{S}_{\mathbf{F}'_{1-b}} \leftarrow \text{BasisRand}(\text{BasisExt}(\mathbf{F}'_{1-b}, \mathbf{S}_{\mathbf{A}^{(\bar{i})}}), \sigma).$$

5. Compute  $\mathbf{x}' \leftarrow \text{SampleGaussian}(\mathbf{F}'_{1-b}, \mathbf{S}_{\mathbf{F}'_{1-b}}, \mathbf{0}, \sigma')$  such that  $\mathbf{F}'_{1-b} \cdot \mathbf{x}' = \mathbf{0} \pmod{q}$ .
6. Output the signature  $\Sigma = \mathbf{x}'$ .

$\text{Ver}(\boldsymbol{\mu}, \mathbf{R}, \Sigma)$

1. On input a message  $\boldsymbol{\mu} = (\mu_1, \dots, \mu_t) \in \{0, 1\}^t$ , a ring of verification keys  $\mathbf{R} = (\text{vk}^{(1)}, \dots, \text{vk}^{(N)})$ , and a signature  $\Sigma = \mathbf{x}'$ .
2. For  $i = 1$  to  $N$ , check if  $\|\mathbf{x}'\| \leq \sigma\sqrt{2Nm}$  holds, otherwise return 0.
3. For  $i = 1$  to  $N$ , compute  $\mathbf{A}_{C_{\text{PRF}}, \boldsymbol{\mu}}^{(i)}$  as in  $\text{Sign}$  algorithm.
4. For  $b \in \{0, 1\}$ , set  $\mathbf{F}_{C_{\text{PRF}}, \boldsymbol{\mu}, b}^{(i)} = [\mathbf{A}^{(i)} | \mathbf{A}_b^{(i)} - \mathbf{A}_{C_{\text{PRF}}, \boldsymbol{\mu}}^{(i)}]$  and  $\mathbf{F}'_b = [\mathbf{F}_{C_{\text{PRF}}, \boldsymbol{\mu}, b}^{(1)} | \dots | \mathbf{F}_{C_{\text{PRF}}, \boldsymbol{\mu}, b}^{(N)}]$ .
5. Check if  $\mathbf{F}'_b \cdot \mathbf{x}' = \mathbf{0} \pmod{q}$  holds for  $b = 0$  or  $1$ , return 1, otherwise return 0.

### 4.2. Correctness and Parameters

We now show the correctness of RS. By Lemma 3.8 the signature  $\Sigma = \mathbf{x}'$  follows the distribution  $\mathcal{D}_{\Lambda_q^\perp(\mathbf{F}'_b), \sigma'}$ .

By Lemma 3.1, the length of  $\mathbf{x}'$  at most  $\sigma\sqrt{2Nm}$  with overwhelming probability. Therefore, the signature is accepted by the Ver algorithm.

We then explain the parameters choosing. We employ the work [42] to instantiate our PRF, which based on standard LWE assumption with polynomial modulus  $q = n^{\omega(1)}$ . Let  $n$  be the security parameter, let the message length be  $t = t(n)$  and the secret key length of PRF be  $k = k(n)$ . Let  $\ell = t + k$  be the input length of PRF. To ensure that hard lattices with good short bases can be generated by TrapGen in Lemma 3.3, we need to set  $m = 6n^{1+\delta}$  where  $\delta > 0$  is a constant such that  $n^\delta > O(\log n)$ . To ensure the randomized basis is statistically independent with the original basis as required in Lemma 3.5, we need to set  $\sigma = O(\ell^{2c} \cdot m^{3/2}) \cdot \omega(\sqrt{\log Nm})$  (see the unforgeability proof below). To ensure that the distribution on the output of SampleGaussian statistically close to the distribution  $\mathcal{D}_{\Lambda_q^\pm(\mathbf{F}'), \sigma'}$ , we need to set  $\sigma'$  sufficiently large that is  $\sigma' = \sqrt{N} \cdot O(\ell^{2c} \cdot m^2) \cdot \omega(\log Nm)$  (see the unforgeability proof below). To ensure that vectors sampled using a trapdoor are difficult SIS solutions, we need to set  $\beta \geq O(\ell^{2c} \cdot m^{3/2}) \cdot \sigma\sqrt{2m}$  for some constant  $c$  (see the unforgeability proof below). To ensure our construction based on SIS has a worst-case lattice reduction as defined in Definition 3.1, we need to set  $q \geq \beta \cdot \omega(\sqrt{n \log n})$ .

To satisfy the above requirements, let  $n$  be the security parameter, the other parameters can be instantiated in various ways. For a typical choice, we choose a function  $\omega(\sqrt{\log m})$ ,  $N = N(n)$ , set the parameters  $(m, \sigma, \sigma', \beta, q)$  as follows

$$\begin{aligned} m &= 6n^{1+\delta} \\ \sigma &= O(\ell^{2c} \cdot m^{3/2}) \cdot \omega(\sqrt{\log Nm}) \\ \sigma' &= \sqrt{N} \cdot O(\ell^{2c} \cdot m^2) \cdot (\omega(\sqrt{\log Nm}))^2 \\ \beta &= N \cdot O(\ell^{4c} \cdot m^{7/2}) \cdot \omega(\sqrt{\log Nm}) \\ q &= N \cdot O(\ell^{4c} \cdot m^4) \cdot (\omega(\sqrt{\log Nm}))^2 \end{aligned}$$

### 4.3. Unforgeability

We now prove the unforgeability of RS.

**THEOREM 4.1 (Unforgeability).** *Let  $m, q, \beta, \sigma$  be some polynomials in the security parameter  $n$ . For large enough  $\sigma = O(\ell^{2c} \cdot m^{3/2}) \cdot \omega(\sqrt{\log Nm})$ ,  $\sigma' = \sqrt{N} \cdot O(\ell^{2c} \cdot m^2) \cdot (\omega(\sqrt{\log Nm}))^2$  and  $\beta \geq N \cdot O(\ell^{2c} \cdot m^{3/2}) \cdot \sigma\sqrt{2m}$ , if the hardness assumption  $\text{SIS}_{q, \beta}$  holds and the based PRF is secure, the RS scheme is  $\text{UnfAdvKey}$  secure.*

*Proof.* Consider the following security game between a adversary  $\mathcal{A}$  and a simulator  $\mathcal{S}$ . Upon receiving a challenge  $\mathbf{A} \in \mathbb{Z}_q^{n \times m'}$  that is formed by  $m' = m \cdot N$  uniformly random and independent samples from  $\mathbb{Z}_q^n$ , parsing  $\mathbf{A}$  as  $\mathbf{A} = [\mathbf{A}^{(1)} | \dots | \mathbf{A}^{(N)}]$ ,  $\mathcal{S}$  simulates as follows.

**Setup Phase.**  $\mathcal{S}$  takes as input a security parameter  $n$  and a randomness  $\gamma_{\text{st}}$  to invoke  $\text{PP} \leftarrow \text{Setup}(1^n; \gamma_{\text{st}})$  algorithm. Then  $\mathcal{S}$  simulates as follows:

Select a PRF key  $\mathbf{k} = (k_1, k_2, \dots, k_k) \xleftarrow{\$} \{0, 1\}^k$ .  
For  $i = 1$  to  $N$ ,  $b \in \{0, 1\}$ :

- Choose  $\mathbf{R}_{\mathbf{A}_b}^{(i)}, \mathbf{R}_{\mathbf{C}_b}^{(i)} \xleftarrow{\$} \{1, -1\}^{m \times m}$ .
- Construct  $\mathbf{A}_b^{(i)} = \mathbf{A}^{(i)} \mathbf{R}_{\mathbf{A}_b}^{(i)} + b\mathbf{G}$  and  $\mathbf{C}_b^{(i)} = \mathbf{A}^{(i)} \mathbf{R}_{\mathbf{C}_b}^{(i)} + b\mathbf{G}$  where  $\mathbf{G}$  is the gadget matrix.

For  $j = 1$  to  $k$ :

- Choose  $\mathbf{R}_{\mathbf{B}_j}^{(i)} \xleftarrow{\$} \{1, -1\}^{m \times m}$  and construct  $\mathbf{B}_j^{(i)} = \mathbf{A}^{(i)} \mathbf{R}_{\mathbf{B}_j}^{(i)} + k_j \mathbf{G}$ .

$\mathcal{S}$  sets  $\text{vk}^{(i)} = (\mathbf{A}^{(i)}, (\mathbf{A}_0^{(i)}, \mathbf{A}_1^{(i)}), \{\mathbf{B}_j^{(i)}\}_{j \in [k]}, (\mathbf{C}_0^{(i)}, \mathbf{C}_1^{(i)}))$  and  $\mathcal{S} = \{\text{vk}^{(i)}\}_{i \in [N]}$ , then sends  $(\text{PP}, \mathcal{S}, \gamma_{\text{st}})$  to  $\mathcal{A}$ .

**Probing Phase.**  $\mathcal{A}$  adaptively issues tuples for querying the signing oracle  $\text{OSign}(\cdot, \cdot, \cdot)$ . For simplicity, here consider only one tuple  $(\boldsymbol{\mu}, \mathbf{R}, s)$  where  $s \in [N]$ , and requires that  $\text{vk}^{(s)} \in \mathcal{S} \cap \mathbf{R}$ . Assume the ring  $\mathbf{R} = (\text{vk}^{(1)}, \dots, \text{vk}^{(N')})$ , parse the  $\text{vk}^{(s)} = (\mathbf{A}^{(s)}, (\mathbf{A}_0^{(s)}, \mathbf{A}_1^{(s)}), \{\mathbf{B}_j^{(s)}\}_{j \in [k]}, (\mathbf{C}_0^{(s)}, \mathbf{C}_1^{(s)}))$  and let  $N' = |\mathbf{R}|$ .  $\mathcal{S}$  does the following to response the signature.

Compute  $b = \text{PRF}(\mathbf{k}, \boldsymbol{\mu})$ .

For  $i' = 1$  to  $N'$ , compute the evaluated matrix  $\mathbf{A}_{C_{\text{PRF}, \boldsymbol{\mu}}}^{(i')}$  by  $\text{Eval}(C_{\text{PRF}}, (\{\mathbf{B}_j^{(i')}\}_{j \in [k]}, \mathbf{C}_{\mu_1}^{(i')}, \dots, \mathbf{C}_{\mu_t}^{(i')}))$ . Then set

$$\begin{aligned} \mathbf{F}_{C_{\text{PRF}, \boldsymbol{\mu}, 1-b}}^{(i')} &= [\mathbf{A}^{(i')} | \mathbf{A}_{1-b}^{(i')} - \mathbf{A}_{C_{\text{PRF}, \boldsymbol{\mu}}}^{(i')}] \\ &= [\mathbf{A}^{(i')} | \mathbf{A}^{(i')} (\mathbf{R}_{1-b}^{(i')} - \mathbf{R}_{C_{\text{PRF}, \boldsymbol{\mu}}}^{(i')}) + (1 - 2b)\mathbf{G}] \end{aligned}$$

Let  $\mathbf{F}'_{1-b} = [\mathbf{F}_{C_{\text{PRF}, \boldsymbol{\mu}, 1-b}}^{(1)} | \dots | \mathbf{F}_{C_{\text{PRF}, \boldsymbol{\mu}, 1-b}}^{(N')}]$  and  $\bar{\mathbf{R}}^{(s)} = \mathbf{R}_{1-b}^{(s)} - \mathbf{R}_{C_{\text{PRF}, \boldsymbol{\mu}}}^{(s)}$ . Invoking

$$\begin{aligned} \mathbf{S}_{C_{\text{PRF}, \boldsymbol{\mu}, 1-b}}^{(i')} &\leftarrow \text{BasisExtABB}(\mathbf{A}^{(s)}, \mathbf{G}, \bar{\mathbf{R}}^{(s)}, \mathbf{S}_{\mathbf{G}}) \\ \mathbf{S}_{\mathbf{F}'_{1-b}} &\leftarrow \text{BasisExt}(\mathbf{S}_{C_{\text{PRF}, \boldsymbol{\mu}, 1-b}}^{(i')}, \mathbf{F}'_{1-b}) \\ \mathbf{S}_{\mathbf{F}'_{1-b}} &\leftarrow \text{BasisRand}(\bar{\mathbf{S}}_{\mathbf{F}'_{1-b}}, \sigma) \end{aligned}$$

then compute  $\mathbf{x}' \leftarrow \text{SampleGaussian}(\mathbf{F}'_{1-b}, \mathbf{S}_{\mathbf{F}'_{1-b}}, \mathbf{0}, \sigma')$  such that  $\mathbf{F}'_{1-b} \cdot \mathbf{x}' = \mathbf{0} \pmod{q}$ .

$\mathcal{S}$  responses the signature  $\Sigma = \mathbf{x}'$  for the query tuple  $(\boldsymbol{\mu}, \mathbf{R}, s)$  to  $\mathcal{A}$  and adds  $(\boldsymbol{\mu}, \mathbf{R}, \Sigma)$  to a list  $\mathbf{L}$  which  $\mathcal{S}$  initialized in prior.

**Exploiting the forgery.**  $\mathcal{A}$  outputs a forgery signature tuple  $(\boldsymbol{\mu}^*, \mathbf{R}^*, \Sigma^*)$ . Let  $N^* = |\mathbf{R}^*|$ . Parse  $\boldsymbol{\mu}^* = (\mu_1^*, \dots, \mu_t^*)$ ,  $\mathbf{R}^* = (\text{vk}^{(1)}, \dots, \text{vk}^{(N^*)})$ ,  $\text{vk}^{(i^*)} = (\mathbf{A}^{(i^*)}, (\mathbf{A}_0^{(i^*)}, \mathbf{A}_1^{(i^*)}), \{\mathbf{B}_j^{(i^*)}\}_{j \in [k]}, (\mathbf{C}_0^{(i^*)}, \mathbf{C}_1^{(i^*)}))$  and  $\Sigma^* = \mathbf{x}'^*$  where  $\mathbf{x}'^* = ((\mathbf{x}_1^{(1)})^\top | \dots | (\mathbf{x}_1^{(N^*)})^\top)^\top$ . Separate  $\mathbf{x}'^*(i^*)$  into  $((\mathbf{x}_1^{*(i^*)})^\top | (\mathbf{x}_2^{*(i^*)})^\top)^\top$ .  $\mathcal{S}$  does the following to exploit the forgery.

- Check if  $(\boldsymbol{\mu}^*, \mathbf{R}^*, \Sigma^*) \in \mathcal{L}$  or  $\|\mathbf{x}^{j*}\| > \sigma\sqrt{2N^*m}$ ,  $\mathcal{S}$  aborts.
- For  $i^* = 1$  to  $N^*$ , compute the matrix  $\mathbf{A}_{\text{CPRF}, \boldsymbol{\mu}^*}^{(i^*)}$  as in the probing phase above. Then, compute the matrix  $\mathbf{F}_{\text{CPRF}, \boldsymbol{\mu}^*, 1-b^*}^{(i^*)} = \left[ \mathbf{A}_{1-b^*}^{(i^*)} \mid \mathbf{A}_{1-b^*}^{(i^*)} - \mathbf{A}_{\text{CPRF}, \boldsymbol{\mu}^*}^{(i^*)} \right]$  (resp.,  $\mathbf{F}_{\text{CPRF}, \boldsymbol{\mu}^*, b^*}^{(i^*)} = \left[ \mathbf{A}_{b^*}^{(i^*)} \mid \mathbf{A}_{b^*}^{(i^*)} - \mathbf{A}_{\text{CPRF}, \boldsymbol{\mu}^*}^{(i^*)} \right]$ ) and  $\mathbf{F}'_{1-b^*} = \left[ \mathbf{F}_{\text{CPRF}, \boldsymbol{\mu}^*, 1-b^*}^{(1)} \mid \dots \mid \mathbf{F}_{\text{CPRF}, \boldsymbol{\mu}^*, 1-b^*}^{(N^*)} \right]$  (resp.,  $\mathbf{F}'_{b^*} = \left[ \mathbf{F}_{\text{CPRF}, \boldsymbol{\mu}^*, b^*}^{(1)} \mid \dots \mid \mathbf{F}_{\text{CPRF}, \boldsymbol{\mu}^*, b^*}^{(N^*)} \right]$ ).
- Check if  $\mathbf{F}'_{1-b^*} \cdot \mathbf{x}^{j*} = \mathbf{0} \pmod{q}$  holds,  $\mathcal{S}$  aborts. Therefore, it holds that  $\mathbf{F}'_{b^*} \cdot \mathbf{x}^{j*} = \sum_{i^* \in [N^*]} \left[ \mathbf{A}_{1-b^*}^{(i^*)} \mid \mathbf{A}_{1-b^*}^{(i^*)} (\mathbf{R}_{\mathbf{A}_{b^*}^{(i^*)}} - \mathbf{R}_{\text{CPRF}, \boldsymbol{\mu}^*}^{(i^*)}) \right] \cdot \mathbf{x}^{j*} = \mathbf{0} \pmod{q}$ .

Therefore, we have  $\sum_{i^* \in [N^*]} \mathbf{A}^{(i^*)} \cdot \bar{\mathbf{x}}^{*(i^*)} = \mathbf{0} \pmod{q}$  where  $\bar{\mathbf{x}}^{*(i^*)} = \left( \mathbf{x}_1^{*(i^*)} + (\mathbf{R}_{\mathbf{A}_{b^*}^{(i^*)}} - \mathbf{R}_{\text{CPRF}, \boldsymbol{\mu}^*}^{(i^*)}) \cdot \mathbf{x}_2^{*(i^*)} \right)$  then we have  $\sum_{i^* \in [N^*]} \mathbf{A}^{(i^*)} \cdot \bar{\mathbf{x}}^{*(i^*)} = \mathbf{0} \pmod{q}$ . Let  $\bar{\mathbf{x}}^*$  be the concatenation of  $\{\bar{\mathbf{x}}^{*(i^*)}\}$  i.e.,  $\bar{\mathbf{x}}^* = \left( (\bar{\mathbf{x}}^{*(1^*)})^\top \mid \dots \mid (\bar{\mathbf{x}}^{*(N^*)})^\top \right)^\top$ . Note that  $\{\mathbf{A}^{(i^*)}\}_{i^* \in [N^*]}$  is a subset of  $\{\mathbf{A}^{(i)}\}_{i \in [N]}$ , and we know  $\mathbf{A} = [\mathbf{A}^{(1)} \mid \dots \mid \mathbf{A}^{(N)}]$ . Therefore, by inserting zeros into  $\bar{\mathbf{x}}^*$ ,  $\mathcal{S}$  can obtain a nonzero  $\hat{\mathbf{x}}^*$  such that  $\mathbf{A}\hat{\mathbf{x}}^* = \mathbf{0} \pmod{q}$ . Therefore,  $\mathcal{S}$  can output  $\hat{\mathbf{x}}^*$  as a  $\text{SIS}_{q, \beta}$  solution.

CLAIM 1. The set of verifications keys  $\mathcal{S}$  that simulated by  $\mathcal{S}$  is statistically close to those in the real attack.

*Proof.* In the real scheme, the matrices  $\{\mathbf{A}^{(i)}\}_{i \in [N]}$  generated by  $\text{TrapGen}$ . In the simulation,  $\{\mathbf{A}^{(i)}\}_{i \in [N]}$  have uniform distribution as it comes from the  $\text{SIS}$  challenger that are formed by  $m'$  uniformly random and independent samples from  $\mathbb{Z}_q^n$ . By Lemma 3.3,  $\{\mathbf{A}^{(i)}\}_{i \in [N]}$  generated in the simulation has right distribution except a negligibly statistical error. For the matrices  $(\mathbf{A}_0^{(i)}, \mathbf{A}_1^{(i)})$ ,  $\{\mathbf{B}_j^{(i)}\}_{j \in [k]}$  and  $(\mathbf{C}_0^{(i)}, \mathbf{C}_1^{(i)})$  generated in the simulation have distribution that is statistically close to uniform distribution in  $\mathbb{Z}_q^{n \times m}$  by Lemma 3.2. Therefore, the set of verifications keys  $\mathcal{S}$  given to  $\mathcal{A}$  is statistically close to those in the real attack.

CLAIM 2. The replies of the signing oracle  $\text{OSign}(\cdot, \cdot, \cdot)$  simulated by  $\mathcal{S}$  is statistically close to those in the real attack when set  $\sigma = O(\ell^{2c} \cdot m^{3/2}) \cdot \omega(\sqrt{\log Nm})$  and  $\sigma' = \sqrt{N} \cdot O(\ell^{2c} \cdot m^2) \cdot (\omega(\sqrt{\log Nm}))^2$ .

*Proof.* By Lemma 3.8, for sufficient large Gaussian parameter  $\sigma'$ , the distribution of the  $\mathbf{x}'$  generated in the simulation by  $\text{SampleGaussian}$  is statistically close to the distribution of signatures (i.e.,  $\mathcal{D}_{\mathbf{A}_+^{(i')}, \sigma}$ ) generated in the real scheme. So we next analyze how to set the parameter  $\sigma'$ . In the  $\text{Simulating Signing Oracle}$  phase, we constructed

$$\mathbf{F}_{\text{CPRF}, \boldsymbol{\mu}, 1-b}^{(i')} = \left[ \mathbf{A}^{(i')} \mid \mathbf{A}^{(i')} (\mathbf{R}_{1-b}^{(i')} - \mathbf{R}_{\text{CPRF}, \boldsymbol{\mu}}^{(i')}) + (1-2b)\mathbf{G} \right]$$

Let  $\bar{\mathbf{R}}^{(i')} = \mathbf{R}_{1-b}^{(i')} - \mathbf{R}_{\text{CPRF}, \boldsymbol{\mu}}^{(i')}$ . By Lemma 3.12, we know  $\|\bar{\mathbf{R}}^{(i')}\| \leq O(\ell^{2c} \cdot m^{3/2})$  for some constant  $c$ . By Lemma 3.7, we know  $\|\widetilde{\mathbf{S}}_{\text{CPRF}, \boldsymbol{\mu}, 1-b}^{(i')}\| < (\|\bar{\mathbf{R}}^{(i')} + \mathbf{1}\|) \cdot \|\widetilde{\mathbf{S}}_{\mathbf{G}}\|$ . Let  $\mathbf{F}'_{1-b} = \left[ \mathbf{F}_{\text{CPRF}, \boldsymbol{\mu}, 1-b}^{(1)} \mid \dots \mid \mathbf{F}_{\text{CPRF}, \boldsymbol{\mu}, 1-b}^{(N)} \right] \in \mathbb{Z}_q^{n \times 2Nm}$  be the extended basis output by  $\text{BasisExt}$ . By Lemma 3.5, it requires to set  $\sigma > \|\widetilde{\mathbf{S}}_{\mathbf{F}'_{1-b}}\| \cdot \omega(\sqrt{\log m})$ . Let  $\mathbf{F}''_{1-b}$  be the randomized basis output by  $\text{BasisRand}$ . By Lemma 3.5, we know  $\|\widetilde{\mathbf{S}}_{\mathbf{F}''_{1-b}}\| \leq \sigma\sqrt{Nm}$ . By Lemma 3.8, it requires to set  $\sigma' > \|\widetilde{\mathbf{S}}_{\mathbf{F}''_{1-b}}\| \cdot \omega(\sqrt{\log Nm})$ . Therefore, to satisfy these requirements, set  $\sigma = O(\ell^{2c} \cdot m^{3/2}) \cdot \omega(\sqrt{\log Nm})$  and  $\sigma' = \sqrt{N} \cdot O(\ell^{2c} \cdot m^2) \cdot (\omega(\sqrt{\log Nm}))^2$  is sufficient.

CLAIM 3. It's hard for  $\mathcal{A}$  to find a messages  $\boldsymbol{\mu}'$  such that each  $\mathbf{A}_{\text{CPRF}, \boldsymbol{\mu}}^{(i)} = \mathbf{A}_{\text{CPRF}, \boldsymbol{\mu}'}^{(i)}$  holds.

*Proof.* In our construction, note that  $\mathbf{F}_{\text{CPRF}, \boldsymbol{\mu}, 1-b}^{(i')} = \left[ \mathbf{A}^{(i')} \mid \mathbf{A}_{1-b}^{(i')} - \mathbf{A}_{\text{CPRF}, \boldsymbol{\mu}}^{(i')} \right]$ , one attacking method is to find a messages  $\boldsymbol{\mu}'$  and a  $\bar{k}'$  such that each  $\mathbf{A}_{\text{CPRF}, \boldsymbol{\mu}}^{(i)} = \mathbf{A}_{\text{CPRF}, \boldsymbol{\mu}'}^{(i)}$  holds. Assume an efficient adversary can do that, with the public parameters constructed above, it holds that  $\mathbf{A}^{(i')} \mathbf{R}_{\text{CPRF}, \boldsymbol{\mu}}^{(i')} + \text{PRF}(\mathbf{k}^{(i')}, \boldsymbol{\mu}) \mathbf{G} = \mathbf{A}^{(i')} \mathbf{R}_{\text{CPRF}, \boldsymbol{\mu}'}^{(i')} + \text{PRF}(\mathbf{k}^{(i')}, \boldsymbol{\mu}') \mathbf{G}$ . Assume the based PRF is secure, with 1/2 probability that  $\text{PRF}(\mathbf{k}^{(i')}, \boldsymbol{\mu}) \neq \text{PRF}(\mathbf{k}^{(i')}, \boldsymbol{\mu}')$  holds. In this case, we have  $\mathbf{R}_{\text{CPRF}, \boldsymbol{\mu}}^{(i')} \neq \mathbf{R}_{\text{CPRF}, \boldsymbol{\mu}'}^{(i')}$  and  $\mathbf{A}^{(i')} (\mathbf{R}_{\text{CPRF}, \boldsymbol{\mu}}^{(i')} - \mathbf{R}_{\text{CPRF}, \boldsymbol{\mu}'}^{(i')}) \pm \mathbf{G} = \mathbf{0} \pmod{q}$  holds. By Lemma 3.8 and 3.11, a low-norm vector  $\mathbf{e} \in \mathbb{Z}^m$  can be efficiently found such that  $\mathbf{G}\mathbf{e} = \mathbf{0} \pmod{q}$  where  $\mathbf{e} \neq \mathbf{0}$  and  $\|\mathbf{e}\| \leq \sigma_{\mathbf{G}}\sqrt{m}$  for some parameter  $\sigma_{\mathbf{G}} \geq \sqrt{5} \cdot \omega(\sqrt{\log m})$ . Then multiply  $\mathbf{e}$  to the both sides of the above equation, we have  $\mathbf{A}^{(i')} (\mathbf{R}_{\text{CPRF}, \boldsymbol{\mu}}^{(i')} - \mathbf{R}_{\text{CPRF}, \boldsymbol{\mu}'}^{(i')}) \mathbf{e} = \mathbf{0} \pmod{q}$  holds, which means the  $(\mathbf{R}_{\text{CPRF}, \boldsymbol{\mu}}^{(i')} - \mathbf{R}_{\text{CPRF}, \boldsymbol{\mu}'}^{(i')}) \mathbf{e}$  is a non-zero vector with all but negligible probability and, therefore, a valid the SIS solution for  $\mathbf{A}^{(i')}$ .

CLAIM 4. It's hard for  $\mathcal{A}$  to forge a signature by adversarially choosing keys.

*Proof.* Note that we allowed the adversary  $\mathcal{A}$  has the ability to adversarially choosing keys, one attacking method to exploit that is  $\mathcal{A}$  can adversarially provides a ring  $\mathbf{R} = (\mathbf{vk}^1, \mathbf{vk}^2, \mathbf{vk}^3, \mathbf{vk}^4)$ , suppose only the  $(\mathbf{vk}^1, \mathbf{vk}^2)$  are honest generated which in the verification keys set  $\mathcal{S}$ .  $\mathcal{A}$  will successfully forge a signature by querying the signing oracle if  $\mathbf{F}_{\text{CPRF}, \boldsymbol{\mu}, 1-b}^{(1)} = c_1 \mathbf{F}_{\text{CPRF}, \boldsymbol{\mu}, 1-b}^{(3)}$  and  $\mathbf{F}_{\text{CPRF}, \boldsymbol{\mu}, 1-b}^{(2)} = c_2 \mathbf{F}_{\text{CPRF}, \boldsymbol{\mu}, 1-b}^{(4)}$  holds, where  $c_1, c_2$  are some constants. It means that  $\mathcal{A}$  found the ring  $\mathbf{R}$  such that each  $\mathbf{A}_{\text{CPRF}, \boldsymbol{\mu}}^{(i)} = c \mathbf{A}_{\text{CPRF}, \boldsymbol{\mu}'}^{(i)}$  holds for some constant  $c$ . Assume the based PRF is secure, with 1/2 probability that  $1 = \text{PRF}(\mathbf{k}^{(i')}, \boldsymbol{\mu}) = \text{PRF}(\mathbf{k}^{(i')}, \boldsymbol{\mu}')$  holds. In this case,  $\mathbf{R}_{\text{CPRF}, \boldsymbol{\mu}}^{(i')} \neq \mathbf{R}_{\text{CPRF}, \boldsymbol{\mu}'}^{(i')}$  and  $\mathbf{A}^{(i')} (\mathbf{R}_{\text{CPRF}, \boldsymbol{\mu}}^{(i')} - \mathbf{R}_{\text{CPRF}, \boldsymbol{\mu}'}^{(i')}) \pm \mathbf{G}$



$(1-c)\mathbf{G} = \mathbf{0} \pmod{q}$  holds. Then we also can giving a reduction from the  $\text{SIS}_{q,\beta}$  assumption as same as in Claim 3.

CLAIM 5.  $\mathcal{A}$  can produce a valid  $\text{SIS}_{q,\beta}$  solution with overwhelming probability.

*Proof.* We argue that  $\hat{\mathbf{x}}^*$  that  $\mathcal{S}$  finally output in the simulation is a valid  $\text{SIS}_{q,\beta}$  solution in two steps. We first explain  $\hat{\mathbf{x}}^*$  is sufficiently short, note that  $\hat{\mathbf{x}}^*$  is consisted by at most  $N$  components  $\bar{\mathbf{x}}^{*(i^*)} = \left( \bar{\mathbf{x}}_1^{*(i^*)} + \left( \mathbf{R}_{\mathbf{A}_{b^*}}^{(i^*)} - \mathbf{R}_{C_{\text{PRF},\mu^*}}^{(i^*)} \right) \cdot \bar{\mathbf{x}}_2^{*(i^*)} \right)$  where  $\bar{\mathbf{x}}_1^{*(i^*)}$  and  $\bar{\mathbf{x}}_2^{*(i^*)}$  follow the distribution  $\mathcal{D}_{\mathbb{Z}^m, \sigma}$ . By Lemma 3.1,  $\|\bar{\mathbf{x}}_1^{*(i^*)}\|, \|\bar{\mathbf{x}}_2^{*(i^*)}\| \leq \sigma\sqrt{m}$ . By Lemma 3.12, we know the norm bound on  $\|\bar{\mathbf{R}}^{(i^*)}\| = \left\| \left( \mathbf{R}_{\mathbf{A}_{b^*}}^{(i^*)} - \mathbf{R}_{C_{\text{PRF},\mu^*}}^{(i^*)} \right) \right\| \leq O(\ell^{2c} \cdot m^{3/2})$ . Therefore, it requires to set  $\beta \geq N \cdot O(\ell^{2c} \cdot m^{3/2}) \cdot \sigma\sqrt{2m}$ .

Then we prove  $\hat{\mathbf{x}}^*$  is a non-zero with overwhelming probability. Suppose that the  $\{\bar{\mathbf{x}}_2^{*(i^*)}\}_{i^* \in [N^*]} = \mathbf{0}$ , then for a valid forgery we must have at least one  $\bar{\mathbf{x}}_1^{*(i^*)} \neq \mathbf{0}$  in  $\{\bar{\mathbf{x}}_1^{*(i^*)}\}_{i^* \in [N^*]}$  and thus  $\hat{\mathbf{x}}^*$  is non-zero. Suppose on the contrary, there exists one  $\bar{\mathbf{x}}_2^{*(i^*)} \neq \mathbf{0}$  in  $\{\bar{\mathbf{x}}_2^{*(i^*)}\}_{i^* \in [N^*]}$ , then we need to argue that the corresponding  $\bar{\mathbf{x}}^{*(i^*)} = \bar{\mathbf{x}}_1^{*(i^*)} + \bar{\mathbf{R}}^{(i^*)} \cdot \bar{\mathbf{x}}_2^{*(i^*)}$  is non-zero with overwhelming probability. Due to we assume  $\bar{\mathbf{x}}_2^{*(i^*)} = (x_1, \dots, x_m) \neq \mathbf{0}$  which means at least one coordinate of  $\bar{\mathbf{x}}_2^{*(i^*)}$ , denote as  $x_o$  where  $o \in [m]$ , such that  $x_o \neq 0$ . We write  $\bar{\mathbf{R}}^{(i^*)} = (\mathbf{r}_1 | \dots | \mathbf{r}_m)$  and so  $\bar{\mathbf{R}}^{(i^*)} \cdot \bar{\mathbf{x}}_2^{*(i^*)} = \mathbf{r}_o x_o + \sum_{\bar{o} \in [m] \setminus o} \mathbf{r}_{\bar{o}} x_{\bar{o}}$ . Note that for the fixed message  $\mu^*$  on which  $\mathcal{A}$  made the forgery,  $\bar{\mathbf{R}}^{(i^*)}$  (therefore  $\mathbf{r}_o$ ) depends on the low-norm matrices  $(\mathbf{R}_{\mathbf{A}_0^{(i^*)}}, \mathbf{R}_{\mathbf{A}_1^{(i^*)}}, \{\mathbf{R}_{\mathbf{B}_j^{(i^*)}}\}_{j \in [k]}, (\mathbf{R}_{\mathbf{C}_0^{(i^*)}}, \mathbf{R}_{\mathbf{C}_1^{(i^*)})$  and PRF key  $\mathbf{k}^*$ . The information about  $x_o$  for  $\mathcal{A}$  is from the public matrices in the verification set  $\mathbf{S}$  that given to the  $\mathcal{A}$ , note that the PRF keys  $\mathbf{k}^*$  which is not included in  $\mathbf{S}$ . So by the pigeonhole principle there is a (exponentially) large freedom to pick a value to  $\mathbf{r}_o$  which is compatible with  $\mathcal{A}$ 's view. This completes the proof.

#### 4.4. Anonymity

We now prove the anonymity of RS.

THEOREM 4.2 (Anonymity). *Let  $n$  be a security parameter. The parameters  $q, m, \sigma, \sigma', \beta$  are chosen as the Sect. 4.2. If the Trapdoor Indistinguishability property of BasisRand and Randomness Indistinguishability property of ExplainGaussian holds, the RS scheme is signer-unclaimability.*

*Proof.* The proof proceeds in two experiments  $\mathbf{E}_0, \mathbf{E}_1$  such that  $\mathbf{E}_0$  (resp.,  $\mathbf{E}_1$ ) corresponds to the experiment of Anonymity in Definition 2.1 with  $b = 0$  (resp.,  $b = 1$ ), and such that each experiment is statistically indistinguishable from the one before it. This implies that  $\mathcal{A}$  has negligible advantage in distinguishing  $\mathbf{E}_0$  from  $\mathbf{E}_1$ , as desired.

$\mathbf{E}_0$  : This experiment firstly generate  $\text{PP} \leftarrow \text{Setup}(1^n; \gamma_{\text{st}})$ , and  $\{\text{vk}^{(i)}, \text{sk}^{(i)}\}_{i \in [N]}$  by repeatedly invoking  $\text{KeyGen}(\gamma_{\text{kg}}^{(i)})$ , and  $\mathcal{A}$  is given  $(\text{PP}, \mathbf{S} = \{\text{vk}^{(i)}\}_{i \in [N]})$  and the randomness  $(\gamma_{\text{st}}, \{\gamma_{\text{kg}}^{(i)}\}_{i \in [N]})$ . Then  $\mathcal{A}$  outputs a tuple  $(\mu^*, \mathbf{R}^*, \text{vk}_0^*, \text{vk}_1^*)$  where  $\text{vk}_0^*, \text{vk}_1^* \in \mathbf{S} \cap \mathbf{R}^*$ . Finally,  $\mathcal{A}$  is given  $\Sigma^*$  and  $\gamma_{\text{sign}}$  that computed by algorithms  $\text{Sign}$  and  $\text{ExplainGaussian}$  with  $\text{sk}_0^*$ , respectively.

$\mathbf{E}_1$  : This experiment is the same as experiment  $\mathbf{E}_0$  except that the  $(\Sigma^*, \gamma_{\text{sign}})$  given to  $\mathcal{A}$  computed by  $\text{sk}_1^*$ .

It remains to show that  $\mathbf{E}_0$  and  $\mathbf{E}_1$  are statistically indistinguishable for  $\mathcal{A}$ , which we do by giving a reduction from the Trapdoor Indistinguishability property of BasisRand and Randomness Indistinguishability property of ExplainGaussian.

**Reduction.** Suppose  $\mathcal{A}$  has non-negligible advantage in distinguishing  $\mathbf{E}_0$  and  $\mathbf{E}_1$ . We use  $\mathcal{A}$  to construct an algorithm  $\mathcal{S}$  for the Trapdoor Indistinguishability property of BasisRand.

**Simulating Setup Phase.**  $\mathcal{S}$  generates  $(\text{PP}, \mathbf{S} = \{\text{vk}^{(i)}\}_{i \in [N]})$  exactly as in experiments  $\mathbf{E}_0$  and  $\mathbf{E}_1$ , and gives  $(\text{PP}, \mathbf{S} = \{\text{vk}^{(i)}\}_{i \in [N]})$  and the appropriate associated randomness  $(\gamma_{\text{st}}, \{\gamma_{\text{kg}}^{(i)}\}_{i \in [N]})$  to  $\mathcal{A}$ .

**Challenge.**  $\mathcal{A}$  provides a challenge  $(\mu^*, \mathbf{R}^*, s_0^*, s_1^*)$  to  $\mathcal{S}$ .  $\mathcal{S}$  does the following to response the challenge:

- Parse the message  $\mu^* = (\mu_1^*, \dots, \mu_t^*)$ , ring  $\mathbf{R}^* = (\text{vk}^{*(1)}, \dots, \text{vk}^{*(N)})$ , parse each  $\text{vk}^{(i^*)} = (\mathbf{A}^{(i^*)}, (\mathbf{A}_0^{(i^*)}, \mathbf{A}_1^{(i^*)}), \{\mathbf{B}_j^{(i^*)}\}_{j \in [k]}, (\mathbf{C}_0^{(i^*)}, \mathbf{C}_1^{(i^*)}))$ .
- Let  $N^* = |\mathbf{R}^*|$ .  $\mathcal{S}$  checks if  $s_0^*, s_1^* \in [N^*]$ ,  $s_0^* \neq s_1^*$  and  $\text{vk}^{(s_0^*)}, \text{vk}^{(s_1^*)} \in \mathbf{S} \cap \mathbf{R}^*$ , otherwise  $\mathcal{S}$  aborts the simulation.
- Let  $b^* = b_0^* = b_1^*$ . For  $i^* = 1$  to  $N^*$ , compute  $\mathbf{A}_{C_{\text{PRF},\mu^*}}^{(i^*)} = \text{Eval}(C_{\text{PRF}}, (\{\mathbf{B}_j^{(i^*)}\}_{j \in [k]}, \mathbf{C}_{\mu_1^*}^{(i^*)}, \dots, \mathbf{C}_{\mu_t^*}^{(i^*)}))$  and set the matrix

$$\mathbf{F}_{C_{\text{PRF},\mu^*}, 1-b^*}^{(i^*)} = [\mathbf{A}^{(i^*)} | \mathbf{A}_{1-b^*}^{(i^*)} - \mathbf{A}_{C_{\text{PRF},\mu^*}}^{(i^*)}]$$

- Let  $\mathbf{F}_{1-b^*}^* = [\mathbf{F}_{C_{\text{PRF},\mu^*}, 1-b^*}^{(1)} | \dots | \mathbf{F}_{C_{\text{PRF},\mu^*}, 1-b^*}^{(N^*)}]$ . Compute  $\mathbf{S}_{\mathbf{F}_{1-b^*}^*}^{(s_0^*)} \leftarrow \text{BasisExt}(\mathbf{S}_{\mathbf{A}^{(s_0^*)}}, \mathbf{R}^*)$  and  $\mathbf{S}_{\mathbf{F}_{1-b^*}^*}^{(s_1^*)} \leftarrow \text{BasisExt}(\mathbf{S}_{\mathbf{A}^{(s_1^*)}}, \mathbf{R}^*)$ .
- Send  $\mathbf{S}_{\mathbf{F}_{1-b^*}^*}^{(s_0^*)}$  and  $\mathbf{S}_{\mathbf{F}_{1-b^*}^*}^{(s_1^*)}$  to the challenger  $\mathcal{C}$ . Then  $\mathcal{C}$  chooses a random bit  $b \xleftarrow{\$} \{0, 1\}$ , responses  $\mathbf{S}'_{\mathbf{F}_{1-b^*}^*} \leftarrow \text{BasisRand}(\mathbf{S}_{\mathbf{F}_{1-b^*}^*}^{(s_b^*)}, \sigma)$ .
- Compute  $\gamma_{\text{sign}} \leftarrow \text{ExplainGaussian}(\mathbf{F}_{1-b^*}^*, \mathbf{S}'_{\mathbf{F}_{1-b^*}^*}, \mathbf{x}', \sigma', \mathbf{0})$ ,  $\mathbf{x}' \leftarrow \text{SampleGaussian}(\mathbf{F}_{1-b^*}^*, \mathbf{S}'_{\mathbf{F}_{1-b^*}^*}, \mathbf{0}, \sigma')$  such that  $\mathbf{F}_{1-b^*}^* \cdot \mathbf{x}' = \mathbf{0} \pmod{q}$ .
- Response  $(\mathbf{x}', \gamma_{\text{sign}})$  to  $\mathcal{A}$ .

**Guess.** When  $\mathcal{A}$  outputs the guess  $b'$ ,  $\mathcal{S}$  outputs the guess  $b'$ .

Note that if the random bit  $b$  that challenger selected s.t.  $b = 0$  then the view of  $\mathcal{A}$  is

distributed exactly according to experiment  $E_0$ , while if the random bit  $b$  that challenger selected s.t.  $b = 1$  then the view of  $\mathcal{A}$  is distributed exactly according to experiment  $E_1$ . By the Trapdoor Indistinguishability property of BasisRand (Lemma 3.6) and Randomness Indistinguishability property of ExplainGaussian (Lemma 3.10),  $E_0$  and  $E_1$  are statistical indistinguishability. This completes the proof.

## 5. CONCLUSION AND FUTURE WORKS

In this paper, we present a new lattice-based ring signature scheme with unclaimable anonymity. Particularly, our work simultaneously improves the security and efficiency of the work [18]. We proved that the scheme is unforgeable w.r.t. adversarially-chosen-key attack in the standard model based on standard lattice assumptions. The comparison shows that our work is the first lattice-based ring signature scheme with unclaimable anonymity in the standard model and with competitive efficiency. As for future works, it is interesting to focus on how to improve our work with the signature size that is logarithmic in the number of ring members, while at the same time relying on standard lattice assumptions and in the standard model.

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## REFERENCES

- [1] Rivest, R.L., Shamir, A. and Tauman, Y. (2001) How to leak a secret. In *Advances in Cryptology – ASIACRYPT 2001, 7th Int. Conf. on the Theory and Application of Cryptology and Information Security*, Gold Coast, Australia, December 9-13, 2001, Proc. LNCS, vol. 2248, pp. 552–565. Springer-Verlag, Berlin.
- [2] Abe, M., Ohkubo, M. and Suzuki, K. (2002) 1-out-of-n Signatures from a Variety of Keys. In *Advances in Cryptology – ASIACRYPT 2002, 8th Int. Conf. on the Theory and Application of Cryptology and Information Security*, Queenstown, New Zealand, December 1–5, 2002, Proc. LNCS, vol. 2501, pp. 415–432. Springer-Verlag, Berlin.
- [3] Bresson, E., Stern, J. and Szydło, M. (2002) Threshold Ring Signatures and Applications to Ad-Hoc Groups. In *Advances in Cryptology – CRYPTO 2002, 22nd Annual Int. Cryptology Conf.*, Santa Barbara, California, USA, August 18–22, 2002, Proc. LNCS, vol. 2442, pp. 465–480. Springer-Verlag, Berlin.
- [4] Naor, M. (2002) Deniable Ring Authentication. In *Advances in Cryptology – CRYPTO 2002, 22nd Annual Int. Cryptology Conf.*, Santa Barbara, California, USA, August 18–22, 2002, Proc. LNCS, vol. 2442, pp. 481–498. Springer-Verlag, Berlin.
- [5] Zhang, F.G. and Kim, K. (2002) ID-Based Blind Signature and Ring Signature from Pairings. In *Advances in Cryptology – ASIACRYPT 2002, 8th Int. Conf. on the Theory and Application of Cryptology and Information Security*, Queenstown, New Zealand, December 1–5, 2002, Proc. LNCS, vol. 2501, pp. 533–547. Springer-Verlag, Berlin.
- [6] Boneh, D., Gentry, C., Lynn, B. and Shacham, H. (2003) Aggregate and Verifiably Encrypted Signatures from Bilinear Maps. In *Advances in Cryptology – EUROCRYPT 2003, 22nd Int. Conf. on the Theory and Applications of Cryptographic Techniques*, Warsaw, Poland, May 4–8, 2003, Proc. LNCS, vol. 11478, pp. 416–432. Springer-Verlag, Berlin.
- [7] Herranz, J. and Sáez, G. (2003) Forking Lemmas for Ring Signature Schemes. In *Progress in Cryptology – INDOCRYPT 2003, 4th Int. Conf. on Cryptology in India*, New Delhi, India, December 8–10, 2003, Proc. LNCS, vol. 2904, pp. 266–279. Springer-Verlag, Berlin.
- [8] Dodis, Y., Kiayias, A., Nicolosi, A. and Shoup, V. (2004) Anonymous Identification in Ad-Hoc Groups. In *Advances in Cryptology – EUROCRYPT 2004, 23rd Int. Conf. on the Theory and Applications of Cryptographic Techniques*, Interlaken, Switzerland, May 2–6, 2004, Proc. LNCS, vol. 3027, pp. 281–311. Springer-Verlag, Berlin.
- [9] Liu, J.K., Wei, V.K. and Wong, D.S. (2004) Linkable Spontaneous Anonymous Group Signature for Ad-Hoc Groups. In *Information Security and Privacy – 9th Australasian Conf., ACISP 2004*, Sydney, Australia, July 13–15, 2004, Proc. LNCS, vol. 3108, pp. 325–335. Springer-Verlag, Berlin.
- [10] Chow, S.S., Wei, K., Liu, J.K. and Yuen, T.H. (2006) Ring Signatures Without Random Oracles. In *Proc. of the 2006 ACM Symposium on Information, Computer and Communications Security, ASIACCS 2006*, Taipei, Taiwan, March 21–24, 2006, Proc. LNCS, vol. 11478, pp. 281–311. Springer-Verlag, Berlin.
- [11] Bender, A., Jonathan Katz, J. and Morselli, R. (2006) Ring Signatures: Stronger Definitions, and Constructions Without Random Oracles. In *Theory of Cryptography, Third Theory of Cryptography Conference, TCC 2006*, New York, NY, USA, March 4–7, 2006, Proc. LNCS, vol. 11478, pp. 60–79. Springer-Verlag, Berlin.
- [12] Brakerski, Z. and Kalai, Y.T. (2010) A Framework for Efficient Signatures, ring signatures and identity based encryption in the standard model. *IACR Cryptol. ePrint Archive* 2010, 86.
- [13] Libert, B., Ling, S., Nguyen, K. and Wang, H.X. (2016) Zero-Knowledge Arguments for Lattice-Based Accumulators: Logarithmic-Size Ring Signatures and Group Signatures Without Trapdoors. In *Advances in Cryptology – EUROCRYPT 2016, 35th Annual Int. Conf. on the Theory and Applications of Cryptographic Techniques*, Vienna, Austria, May 8-12, 2016, Proc. LNCS, vol. 9666, pp. 1–31. Springer-Verlag, Berlin.
- [14] Malavolta, G. and Schröder, D. (2017) Efficient Ring Signatures in the Standard Model. In *Advances in Cryptology – ASIACRYPT 2017, 23rd Int. Conf. on the Theory and Applications of Cryptology and Information Security*, Hong Kong, China, December 3–7, 2017, Proc. LNCS, vol. 10625, pp. 128–157. Springer-Verlag, Berlin.

- [15] Torres, W.A., Steinfeld, R., Sakzad, A., Liu, J.K., Kuchta, V., Bhattacharjee, N., Au, M.H. and Cheng, J. (2018) Post-Quantum One-Time Linkable Ring Signature and Application to Ring Confidential Transactions in Blockchain (Lattice RingCT v1. 0). In *Information Security and Privacy - 23rd Australasian Conf., ACISP 2018*, Wollongong, NSW, Australia, July 11–13, 2018, Proc. LNCS, vol. 10946, pp. 558–576. Springer-Verlag, Berlin.
- [16] Backes, M., Hanzlik, L., Kluczniak, K. and Schneider, J. (2018) Signatures With Flexible Public Key: Introducing Equivalence Classes for Public Keys. In *Advances in Cryptology – ASIACRYPT 2018, 24th Int. Conf. on the Theory and Application of Cryptology and Information Security*, Brisbane, QLD, Australia, December 2–6, 2018, Proc. LNCS, vol. 11273, pp. 405–434. Springer-Verlag, Berlin.
- [17] Torres, W.A., Kuchta, V., Steinfeld, R., Sakzad, A., Liu, J.K. and Cheng, J. (2019) Lattice RingCT v2. 0 With Multiple Input and Multiple Output Wallets. In *Information Security and Privacy – 24th Australasian Conf., ACISP 2019*, Christchurch, New Zealand, July 3–5, 2019, Proc. LNCS, vol. 11547, pp. 156–175. Springer-Verlag, Berlin.
- [18] Park, S. and Sealfon, A. (2019) It Wasn’t Me! Repudiability and Unclaimability of Ring Signatures. In *Advances in Cryptology – CRYPTO 2019, 39th Annual Int. Cryptology Conf.*, Santa Barbara, CA, USA, August 18–22, 2019, Proc. LNCS, vol. 11694, pp. 159–190. Springer-Verlag, Berlin.
- [19] Backes, M., Döttling, N., Hanzlik, L., Kluczniak, K. and Schneider, J. (2019) Ring Signatures: Logarithmic-Size, No Setup—from Standard Assumptions. In *Advances in Cryptology – EUROCRYPT 2019, 38th Annual Int. Conf. on the Theory and Applications of Cryptographic Techniques*, Darmstadt, Germany, May 19–23, 2019, Proc. LNCS, vol. 11478, pp. 281–311. Springer-Verlag, Berlin.
- [20] Libert, B., Nguyen, K., Peters, T. and Yung, M. (2020) Compact Simulation–Sound NIZK Arguments of Composite Residuosity and Applications to Logarithmic-Size Ring Signatures. *IACR Cryptol. ePrint Archive* 2020, 1334.
- [21] Chatterjee, R., Garg, S., Hajiabadi, M., Khurana, D., Liang, X., Malavolta, G., Pandey, O. and Shiehian, S. (2021) Compact Ring Signatures from Learning With Errors. In *Advances in Cryptology – CRYPTO 2021, 41st Annual Int. Cryptology Conf.*, Virtual Event, August 16–20, 2021, Proc. LNCS, vol. 12825, pp. 282–312. Springer-Verlag, Berlin.
- [22] Wang, J. and Sun, B. (2011) Ring Signature Schemes From Lattice Basis Delegation. In *Information and Communications Security - 13th Int. Conf., ICICS 2011*, Beijing, China, November 23–26, 2011, Proc. LNCS, vol. 7043, pp. 281–311. Springer-Verlag, Berlin.
- [23] Melchor, C.A., Bettaieb, S., Boyen, X., Fousse, L. and Gaborit, P. (2013) Adapting Lyubashevsky’s Signature Schemes to the Ring Signature Setting. In *Progress in Cryptology – AFRICACRYPT 2013, 6th Int. Conf. on Cryptology in Africa*, Cairo, Egypt, June 22–24, 2013, Proc. LNCS, vol. 7918, pp. 1–25. Springer-Verlag, Berlin.
- [24] Noh, G., Chun, J.Y. and Jeong, I.R. (2014) Strongly Unforgeable Ring Signature Scheme From Lattices in the Standard Model. *Journal of Applied Mathematics*, 1, 1–12.
- [25] Zhang, Y.H., Hu, Y.P., Xie, J. and Jiang, M.M. (2016) Efficient Ring Signature Schemes Over NTRU lattices. *Security Comm. Networks*, 9, 5252–5261.
- [26] Wang, S.P., Zhao, R. and Zhang, Y.L. (2018) Lattice-Based Ring Signature Scheme Under The Random Oracle Model. *IJHPCN 2018*, 11, 281–332.
- [27] Esgin, M.F., Steinfeld, R., Sakzad, A., Liu, J.K. and Liu, D.X. (2019) Short Lattice-Based One-out-of-Many Proofs and Applications to Ring Signatures. In *Applied Cryptography and Network Security - 17th Int. Conf., ACNS 2019*, Bogota, Colombia, June 5–7, 2019, Proc. LNCS, vol. 11464, pp. 281–311. Springer-Verlag, Berlin.
- [28] Esgin, M.F., Steinfeld, R., Liu, J.K. and Liu, D.X. (2019) Lattice-Based Zero-Knowledge Proofs: New Techniques for Shorter and Faster Constructions and Applications. In *Advances in Cryptology – CRYPTO 2019, 39th Annual Int. Cryptology Conf.*, Santa Barbara, CA, USA, August 18–22, 2019, Proc. LNCS, vol. 11692, pp. 115–146. Springer-Verlag, Berlin.
- [29] Esgin, M.F., Zhao, R.K., Steinfeld, R., Liu, J.K. and Liu, D.X. (2019) MatRiCT: Efficient, Scalable and Post-Quantum Blockchain Confidential Transactions Protocol. In *Proc. of the 2019 ACM SIGSAC Conf. on Computer and Communications Security, CCS 2019*, London, UK, November 11–15, 2019, LNCS, vol. 11478, pp. 281–311. ACM, New York.
- [30] Lu, X.Y., Au, M.H. and Zhang, Z.F. (2019) Raptor: A Practical Lattice-Based (Linkable) Ring Signature. In *Applied Cryptography and Network Security - 17th Int. Conf., ACNS 2019*, Bogota, Colombia, June 5–7, 2019, Proc. LNCS, vol. 11478, pp. 110–130. Springer-Verlag, Berlin.
- [31] Liu, J.H., Yu, Y., Jia, J.W., Wang, S.J., Fan, P.R., Wang, H.Z. and Zhang, H.G. (2019) Lattice-Based Double-Authentication-Preventing Ring Signature for Security and Privacy in Vehicular Ad-Hoc Networks. *Tsinghua Science and Technology*, 24, 575–584.
- [32] Mundhe, P., Yadav, V.K., Verma, S. and Venkatesan, S. (2020) Efficient Lattice-Based Ring Signature for Message Authentication in Vanets. *IEEE Systems Journal*, 14, 5463–5474.
- [33] Katz, J. (2010) Digital Signatures. Springer-Verlag, Berlin.
- [34] Leurent, G., Nguyen, P.Q.: How risky is the random-oracle model ?. In *Advances in Cryptology – CRYPTO 2009, 29th Annual Int. Cryptology Conf.*, Santa Barbara, CA, USA, August 16–20, 2009, Proc. LNCS, vol. 5677, pp. 445–464. Springer-Verlag, Berlin.
- [35] Gentry, C., Peikert, C. and Vaikuntanathan, V. (2008) Trapdoors for Hard Lattices and New Cryptographic Constructions. In *Proc. of the 40th Annual ACM Symposium on Theory of Computing*, Victoria, British Columbia, Canada, May 17–20, 2008, LNCS, vol. 11478, pp. 197–206. ACM, New York.
- [36] Gentry, C., Sahai, A. and Waters, B. (2013) Homomorphic Encryption from Learning With Errors: Conceptually-Simpler, Asymptotically-Faster, Attribute-Based. In *Advances in Cryptology –*

- CRYPTO 2013, 33rd Annual Cryptology Conf.*, Santa Barbara, CA, USA, August 18–22, 2013, Proc. LNCS, vol. 8042, pp. 75–92. Springer-Verlag, Berlin.
- [37] Brakerski, Z. and Vaikuntanathan, V. (2014) Lattice-Based FHE as Secure as PKE. In *Innovations in Theoretical Computer Science, ITCS'14*, Princeton, NJ, USA, January 12–14, 2014, LNCS, vol. 11478, pp. 1–12. Springer-Verlag, Berlin.
- [38] Boneh, D., Gentry, C., Gorbunov, S., Halevi, S., Nikolaenko, V., Segev, G., Vaikuntanathan, V. and Vinayagamurthy, D. (2014) Fully Key-Homomorphic Encryption, Arithmetic Circuit ABE and Compact Garbled Circuits. In *Advances in Cryptology – EUROCRYPT 2014, 33rd Annual Int. Conf. on the Theory and Applications of Cryptographic Techniques*, Copenhagen, Denmark, May 11–15, 2014, Proc. LNCS, vol. 11478, pp. 281–311. Springer-Verlag, Berlin.
- [39] Boyen, X. and Li, Q.Y. (2016) Towards Tightly Secure Lattice Short Signature and ID-Based Encryption. In *Advances in Cryptology – ASIACRYPT 2016, 22nd Int. Conf. on the Theory and Application of Cryptology and Information Security*, Hanoi, Vietnam, December 4–8, 2016, Proc. LNCS, vol. 11478, pp. 404–434. Springer-Verlag, Berlin.
- [40] Micciancio, D. and Regev., O. (2007) Worst-Case to Average-Case Reductions Based on Gaussian Measures. *SIAM Journal on Computing*, 37, 267–302.
- [41] Agrawal, S., Boneh, D. and Boyen, X. (2010) Efficient Lattice (H)IBE in The Standard Model. In *Advances in Cryptology – EUROCRYPT 2010, 29th Annual International Conference on the Theory and Applications of Cryptographic Techniques*, Monaco / French Riviera, May 30 – June 3, 2010, Proc. LNCS, vol. 6110, pp. 553–572. Springer-Verlag, Berlin.
- [42] Lai, Q.Q., Liu, F.H. and Wang, Z.D. (2020) Almost Tight Security in Lattices With Polynomial Moduli–PRF, IBE, All-But-Many LTF, and More. In *Public-Key Cryptography - PKC 2020 – 23rd IACR International Conference on Practice and Theory of Public-Key Cryptography*, Edinburgh, UK, May 4–7, 2020, Proc. LNCS, vol. 12110, pp. 652–681. Springer-Verlag, Berlin.
- [43] Alwen, J. and Peikert, C. (2011) Generating Shorter Bases for Hard Random Lattices. In *Theory of Computing Systems*, 48, 535–553.
- [44] Cash, D., Hofheinz, D., Kiltz, E. and Peikert, C. (2010) Bonsai Trees, or How to Delegate a Lattice Basis. In *Advances in Cryptology – EUROCRYPT 2010, 29th Annual Int. Conf. on the Theory and Applications of Cryptographic Techniques*, Monaco / French Riviera, May 30 – June 3, 2010, Proc. LNCS, vol. 6110, pp. 523–552. Springer-Verlag, Berlin.
- [45] Micciancio, D. and Peikert, C. (2012) Trapdoors for Lattices: Simpler, Tighter, Faster, Smaller. In *Advances in Cryptology – EUROCRYPT 2012, 31st Annual Int. Conf. on the Theory and Applications of Cryptographic Techniques*, Cambridge, UK, April 15–19, 2012, Proc. LNCS, vol. 7237, pp. 700–718. Springer-Verlag, Berlin.