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Brief Paper

Robust discrete variable structure control with finite-time approach to switching surface☆

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Abstract

By using a dead-beat control technique of discrete-time systems, a robust discrete variable structure control (DVSC) is developed for the linear discrete-time systems subject to input disturbance, measurement noise and uncertainty. The proposed control includes two parts: equivalent control and switching control. Based on the internal model principle, the input disturbance and the measurement noise modeled as pulse transfer functions, are rejected by the equivalent control. The unmatched uncertainty caused by the time-invariant parameter variations is also tackled by the equivalent control. If the inverse of stable characteristic polynomial of the real closed-loop system is a finite-degree polynomial, the trajectory reaches the switching surface in a finite-time step. Due to the subjection of input disturbance or measurement noise or uncertainty, a poor response occurs. Under these circumstances, a switching control based on Lyapunov redesign is employed to improve the system performance. The stability of the closed-loop system is then verified by Lyapunov stability theory. Simulations are also given to confirm the usefulness of the proposed controller.

Keywords: Discrete-time variable structure control; Dead-beat control; Internal model principle; Lyapunov redesign

1. Introduction

One of the most intriguing aspects of variable structure control is the discontinuous feature of control input whose primary function is to switch between distinctively different system structures such that a new type of system motion (i.e., sliding mode) exists in a manifold (Slotine & Sastry, 1983; Hwang, 1992; Hung, Gao, & Hung, 1993; Hwang & Hsu, 1995; Haskara, Özgüner, & Utkin, 1997; Young, Utkin, & Özgüner, 1999; Chern, Chang, Chen, & Su, 1999; Chan, 1999; Cheng, Lin, & Hsiao, 2000; Furuta & Pan, 2000; Hwang & Lin, 2000). This peculiar system feature is claimed to result in an excellent system performance including insensitivity to parameter variations and rejection of disturbances. Within control research community, the superior behavior invites criticism and skepticism, e.g., chattering control input, incomplete analysis for robustness. However, many methods have been developed to reduce the possibility of chattering control input, e.g., boundary layer (Haskara et al., 1997), forward control to attenuate the uncertainties (Hwang & Hsu, 1995; Hwang and Lin, 2000), sliding sector (Furuta & Pan, 2000). Moreover, the Lyapunov-like theorem can be applied to analyze the robust stability of variable structure control (VSC) (e.g., Khalil, 1996; Furuta & Pan, 2000; Hwang & Lin, 2000). Therefore, incomplete analysis for the robustness of VSC does not necessarily exist.

Due to the rapid development of personal computer and DSP chip, a discrete-time controller becomes more and more important nowadays. In the continuous-time VSC, the desired closed-loop bandwidth does not provide any useful guidelines for the selection of sampling rate. The implementation of the continuous VSC using microprocessor probably has some differences or difficulties as compared with the continuous-time controller design (Haskara et al., 1997; Young et al., 1999). Hence, the discrete-time variable structure control (DVSC) has its necessity for a discrete-time system.

In this paper, a finite-time step to reach the switching surface for the discrete-time systems subject to input

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disturbance, measurement noise and uncertainty, is designed by dead-beat control technique, internal model principle and Lyapunov redesign. First, the input disturbance and the measurement noise modeled by pulse transfer functions, are rejected by the equivalent control (Åström & Wittenmark, 1997). The concept is the same as "internal model principal" (e.g., Vidyasagar, 1986). In this situation, the trajectory reaches the switching surface in a finite-time step that is independent of the magnitude of input disturbance or measurement noise or reference input. However, the larger amplitude of input disturbance or measurement noise or reference input results in a larger control input. The uncertainty in many DVSCs needs to satisfy the matching condition (e.g., Haskara et al., 1997; Furuta & Pan, 2000; Cheng et al., 2000). In the current paper, the unmatched uncertainty caused by the time-invariant parameter variations is also tackled by the equivalent control. If the inverse of stable characteristic polynomial of the real closed-loop system is a finite-degree polynomial, the trajectory reaches the switching surface in a finite-time step (see Remark 1).

Due to the presence of input disturbance or measurement noise or uncertainty, a huge transient response or a poor steady-state performance occurs. In this situation, the switching control based on Lyapunov redesign (e.g., Khalil, 1996) is constructed to improve the system performance. The upper bound of the uncertainty for the design of switching control will vanish as the trajectory is on the switching surface; i.e., after transient response, the control input becomes an equivalent control only (Haskara et al., 1997). However, the upper bound of uncertainty has a limit (Furuta & Pan, 2000). The stability of the closed-loop system is also verified by Lyapunov stability theory. Since some concepts of robust control (e.g., internal model principle, variable structure system, Lyapunov redesign) are employed to develop an easy and effective controller for a class of linear discrete-time systems subject to input disturbance or measurement noise or uncertainty, the proposed control is different from the conventional dead-beat control which is not robust and whose transient response is often large.

2. Problem formulation

Consider the following discrete-time single-input–single-output systems:

$$A_r(q^{-1})y(k) = q^{-d}B_r(q^{-1})[u(k) + d_i(k)], \tag{1}$$

where $y(k)$, $u(k)$ and $d_i(k) \in \Re$ denote the system output, the system input and the input disturbance, respectively, q^{-1} is a backward-time shift operator, i.e., $q^{-1}y(k) = y(k-1)$, and the polynomials $A_r(q^{-1}) = A(q^{-1}) + \Delta A(q^{-1})$, $B_r(q^{-1}) = B(q^{-1}) + \Delta B(q^{-1})$ are described as follows:

$$\begin{aligned} &A(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_{n_a} q^{-n_a}, \\ &\Delta A(q^{-1}) = \Delta a_1 q^{-1} + \cdots + \Delta a_{\delta_a} q^{-\delta_a},\ \delta_a \geqslant n_a, \end{aligned} \tag{2}$$

$$\begin{aligned} &B(q^{-1}) = b_0 + b_1 q^{-1} + \cdots + b_{n_b} q^{-n_b}, \\ &\Delta B(q^{-1}) = \Delta b_1 q^{-1} + \cdots + \Delta b_{\delta_b} q^{-\delta_b},\ \delta_b \geqslant n_b, \end{aligned} \tag{3}$$

where the parameters a_i, b_j for $1 \leqslant i \leqslant n_a$, $0 \leqslant j \leqslant n_b$ are known, the parameters Δa_i, Δb_j for $1 \leqslant i \leqslant \delta_a$, $0 \leqslant j \leqslant \delta_b$ are unknown but bounded. In this paper, the similar symbol of $n_a = \deg\{A(q^{-1})\}$ is used. Assume that:

A1: n_a, n_b and $d \geqslant 1$ (i.e., $b_0 \neq 0$) are known.
A2: $A(q^{-1})$ and $B(q^{-1})$, $A_r(q^{-1})$ and $B_r(q^{-1})$ are coprime pairs.

It is not necessarily to assume that the system (1) is minimum phase. The proposed control is assumed as the following form:

$$\begin{aligned} R(q^{-1})u(k) = &-S(q^{-1})[y(k) + m_n(k)] \\ &+ T(q^{-1})r(k) + \bar{u}(k), \end{aligned} \tag{4}$$

where $m_n(k)$ denotes the measurement noise, $R(q^{-1})$, $S(q^{-1})$ and $T(q^{-1})$ are the polynomials to be found to achieve an equivalent control of $u(k)$, and $\bar{u}(k)$ represents a switching control to improve the system performance. The reference input is assumed as follows:

$$r(k) = G_r(q^{-1})\delta(k)/F_r(q^{-1}), \tag{5}$$

where $G_r(q^{-1})$ and $F_r(q^{-1})$ are coprime, $\delta(k)$ is the Kronecker delta: $\delta(k) = 1$, if $k = 0$, $\delta(k) = 0$, otherwise. Then the output response $y(k)$ from the inputs $r(k)$, $d_i(k), m_n(k), \bar{u}(k)$ is achieved from (1) and (4); i.e.

$$\begin{aligned} y(k) = &\frac{q^{-d}B(q^{-1})T(q^{-1})}{A_c(q^{-1})}r(k) + \frac{q^{-d}B(q^{-1})R(q^{-1})}{A_c(q^{-1})}d_i(k) \\ &- \frac{q^{-d}B(q^{-1})S(q^{-1})}{A_c(q^{-1})}m_n(k) \\ &+ \frac{R(q^{-1})\{q^{-d}\Delta B(q^{-1})[u(k) + d_i(k)] - \Delta A(q^{-1})y(k)\}}{A_c(q^{-1})} \\ &+ \frac{q^{-d}B(q^{-1})}{A_c(q^{-1})}\bar{u}(k), \end{aligned} \tag{6}$$

where $A_c(q^{-1})$ denotes the characteristic polynomial of the nominal closed-loop system

$$A_c(q^{-1}) = A(q^{-1})R(q^{-1}) + q^{-d}B(q^{-1})S(q^{-1}). \tag{7}$$

Furthermore, the characteristic polynomial of the real closed-loop system is defined as $A_{cr}(q^{-1}) =$

$A_r(q^{-1})R(q^{-1}) + q^{-d}B_r(q^{-1})S(q^{-1})$. Define the following switching surface:

$$\sigma(k) = C(q^{-1})e(k), \tag{8}$$

where $e(k) = r(k) - y(k)$ and $C(q^{-1})$ is a stable monic polynomial with degree n_c. In addition, $d_i(k)$ and $m_n(k)$ are modeled as follows:

$$d_i(k) = G_i(q^{-1})\delta(k)/F_i(q^{-1}),$$
$$m_n(k) = G_m(q^{-1})\delta(k)/F_m(q^{-1}). \tag{9}$$

Many important engineering problems (e.g., Hwang, Wei, & Jieng, 1997; Chern et al., 1999; Lindquist & Yakubovich, 1999), can be modeled as the system (1) with the controller (4). The objective of the paper is to construct a robust DVSC (4) with $\bar{u}(k) = 0$ for the system (1) such that under suitable conditions the trajectory approaches the switching surface in a manner of finite-time step. Due to the subjection of input disturbance or measurement noise or uncertainty, a poor response often occurs. Under these circumstances, the switching control (i.e., $\bar{u}(k)$) according to Lyapunov redesign is applied to improve the system performance.

3. Controller design

The controller design includes the following three subsections.

3.1. Control for the systems without disturbance, noise and uncertainty

The case $\Delta a_i, \Delta b_j$ $(1 \leqslant i \leqslant \delta_a, 0 \leqslant j \leqslant \delta_b) = d_i(k) = m_n(k) = \bar{u}(k) = 0$ in (1) and (4), is considered in this section. First, one factories the polynomial $B(q^{-1})$ as $B(q^{-1}) = B^+(q^{-1})B^-(q^{-1})$, where $B^-(q^{-1})$ has all its zeros in $|q| \geqslant 1$ and $B^+(q^{-1})$ is the remaining part, i.e., $B^-(q^{-1})$ is unstable, $B^+(q^{-1})$ is stable and $B^+(0) = 1$. Similarly, $G_r(q^{-1}) = G_r^+(q^{-1})G_r^-(q^{-1})$. For the purpose of achieving the dead-beat (or finite-time to reach) the switching surface, the switching surface $\sigma(k)$ must have the following form:

$$\sigma(k) = H(q^{-1})\delta(k), \tag{10}$$

where $H(q^{-1})$ is a polynomial with the degree which is the same as the number of dead-beat steps.

Substituting (5) and (6) with $\Delta A(q^{-1}) = \Delta B(q^{-1}) = d_i(k) = m_n(k) = \bar{u}(k) = 0$ into (8) gives

$$\sigma(k) = C(q^{-1})\{A_c(q^{-1}) - q^{-d}B(q^{-1})T(q^{-1})\}$$
$$\times G_r(q^{-1})\delta(k)/\{A_c(q^{-1})F_r(q^{-1})\}. \tag{11}$$

Comparing (10) and (11) yields

$$\frac{T(q^{-1})}{A_c(q^{-1})} = \frac{G_r(q^{-1})C(q^{-1}) - F_r(q^{-1})H(q^{-1})}{q^{-d}B(q^{-1})G_r(q^{-1})C(q^{-1})}. \tag{12}$$

Since $A_c(q^{-1})$ is stable, the denominator of the right-hand side of (12) must be stable. Hence,

$$H(q^{-1}) = L(q^{-1})G_r^-(q^{-1}), \tag{13}$$

$$G_r^+(q^{-1})C(q^{-1}) - F_r(q^{-1})L(q^{-1}) = q^{-d}B^-(q^{-1})M(q^{-1}), \tag{14}$$

where $F_r(q^{-1})L(q^{-1})$ and $q^{-d}B^-(q^{-1})$ are coprime, the monic polynomial $L(q^{-1})$ has the degree $n_l = n_{b^-} + d - 1$, and the polynomial $M(q^{-1})$ has the minimum degree. Assume that

$$R(q^{-1}) = \bar{R}(q^{-1})B^+(q^{-1}). \tag{15}$$

From (7) and (12)–(15), the following equations are achieved:

$$A(q^{-1})\bar{R}(q^{-1}) + q^{-d}B^-(q^{-1})S(q^{-1})$$
$$= G_r^+(q^{-1})C(q^{-1})N(q^{-1}), \tag{16}$$

$$T(q^{-1}) = M(q^{-1})N(q^{-1}), \tag{17}$$

where $N(q^{-1})$ is a stable polynomial with the degree $n_n = n_a + n_{b^-} + d - n_{g_r^+} - n_c - 1$.

Theorem 1. *The system* (1) *and the controller* (4) *with* Δa_i, Δb_j $(1 \leqslant i \leqslant \delta_a, 0 \leqslant j \leqslant \delta_b) = d_i(k) = m_n(k) = \bar{u}(k) = 0$ *are considered. The polynomials* $R(q^{-1})$, $S(q^{-1})$ *and* $T(q^{-1})$ *are achieved from* (15)–(17). *If the assumptions* A1–A2 *are satisfied, then the trajectory arrives to the switching surface at most* $n_h + 1$ *steps* (*i.e.*, $\sigma(k) = 0$, *as* $k > n_h + 1$) *that are independent of the magnitude of reference input. Furthermore,* $\{u(k)\}$ *is bounded and* $e(k) \to 0$ *as* $k \to \infty$.

Proof. Since $r(k) = y(k) = 0$ as $k \leqslant 0$, $e(k) = 0$ as $k \leqslant 0$. Then from (10), $\sigma(0) = 0$, $\sigma(1) = h_0, \ldots, \sigma(n_h + 1) = h_{n_h}$, $\sigma(k) = 0$, $k > n_h + 1$. Since $C(q^{-1})$ is stable, $e(k) \to 0$ as $k \to \infty$. □

3.2. Control for the systems in the presence of disturbance, noise and uncertainty

In this section, the system (1) and the controller (4) with $\bar{u}(k) = 0$ are examined. Let

$$R(q^{-1}) = G_r^+(q^{-1})N(q^{-1})F_{ir}(q^{-1})F_i'(q^{-1})$$
$$\times F_r'(q^{-1})R'(q^{-1})B^+(q^{-1}), \tag{18}$$

$$S(q^{-1}) = G_r^+(q^{-1})N(q^{-1})F_m(q^{-1})S'(q^{-1}), \tag{19}$$

where $F_{ir}(q^{-1})$ denotes the great common divisor of the polynomials $F_i(q^{-1})$ and $F_r(q^{-1})$; $F_i(q^{-1}) = F_{ir}(q^{-1})F_i'(q^{-1})$, $F_r(q^{-1}) = F_{ir}(q^{-1})F_r'(q^{-1})$. After some mathematical manipulations, the following equation is achieved from (6), (8), (18) and (19):

$$\sigma(k) = H'(q^{-1})\delta(k) + d_\Delta(k), \tag{20}$$

where

$$H'(q^{-1}) = H(q^{-1}) - q^{-d}B^-(q^{-1})G_i(q^{-1})F_r'(q^{-1})R'(q^{-1}) + q^{-d}B^-(q^{-1})G_m(q^{-1})S'(q^{-1}), \tag{21}$$

$$d_\Delta(k) = F_{ir}(q^{-1})F_i'(q^{-1})F_r'(q^{-1})R'(q^{-1}) \times \{\Delta A(q^{-1})y(k) - q^{-d}\Delta B(q^{-1})[u(k) + d_i(k)]\}. \tag{22}$$

The simplified characteristic polynomial of the nominal closed-loop system (7) becomes

$$A(q^{-1})F_{ir}(q^{-1})F_i'(q^{-1})F_r'(q^{-1})R'(q^{-1}) + q^{-d}B^-(q^{-1})F_m(q^{-1})S'(q^{-1}) = C(q^{-1}), \tag{23}$$

where $n_{r'} = d + n_{b^-} + n_{f_m} - 1$, $n_{s'} = n_a + n_{f_{ir}} + n_{f_i'} + n_{f_r'} - 1$. Define the sensitivity functions of the nominal and the real closed-loop system as follows:

$$\hat{S}(q^{-1}) = 1/[1 + \hat{L}(q^{-1})], \quad \hat{S}_r(q^{-1}) = 1/[1 + \hat{L}_r(q^{-1})], \tag{24}$$

where $\hat{L}(q^{-1})$ and $\hat{L}_r(q^{-1})$ denote the nominal and the real loop pulse transfer functions

$$\hat{L}(q^{-1}) = q^{-d}B(q^{-1})S(q^{-1})/[A(q^{-1})R(q^{-1})],$$
$$\hat{L}_r(q^{-1}) = q^{-d}B_r(q^{-1})S(q^{-1})/[A_r(q^{-1})R(q^{-1})]. \tag{25}$$

The following theorem discusses the robustness of closed-loop system (Vidyasagar, 1986; Åström and Wittenmark, 1997).

Theorem 2. *Consider the closed-loop system* $\hat{S}(q^{-1})$, $\hat{S}_r(q^{-1})$, *where* $\hat{S}(q^{-1})$ *is stable. The number of zero of* $A(q^{-1})$ *and* $A_r(q^{-1})$ (*or* $B(q^{-1})$ *and* $B_r(q^{-1})$) *outside of unit circle is same. If the following condition*:

$$\left|\frac{e^{-id\theta}}{R(e^{-i\theta})}\left[\frac{B_r(e^{-i\theta})}{A_r(e^{-i\theta})} - \frac{B(e^{-i\theta})}{A(e^{-i\theta})}\right]\right| < |1 + \hat{L}(e^{-i\theta})|, \ \theta \in [0,2\pi],$$

$$(or\ |1/\hat{L}_r(e^{-i\theta}) - 1/\hat{L}(e^{-i\theta})| < |1 + 1/\hat{L}(e^{-i\theta})|, \ \theta \in [0,2\pi])$$

is satisfied, *then* $\hat{S}_r(q^{-1})$ *is stable*.

Theorem 3. *Assume that system* (1) *and controller* (4) *with* $\bar{u}(k) = 0$. *The polynomials* $R(q^{-1})$, $S(q^{-1})$ *and* $T(q^{-1})$ *are accomplished from* (17)–(19), *and* (23). *Assumptions* A1–A2, *the condition in* Theorem 2 *about the uncertainty*, *and* $1/A_{cr}(q^{-1}) = Q(q^{-1})$, *where* $Q(q^{-1})$ *is a polynomial of finite-degree*, *are satisfied*. *Then* $\sigma(k) = 0$ *as* $k > n_{h'} + n_{h''} + 1$, *where*

$$\begin{aligned} H''(q^{-1}) &= F_{ir}(q^{-1})F_i'(q^{-1})F_r'(q^{-1})\{\Delta A(q^{-1})[q^{-d}B_r(q^{-1}) \\ &\quad \times Q(q^{-1})G_r^+(q^{-1})N(q^{-1})S'(q^{-1})G_m(q^{-1}) \\ &\quad - q^{-d}B_r(q^{-1})Q(q^{-1})G_r^+(q^{-1})N(q^{-1}) \\ &\quad \times F_r'(q^{-1})R'(q^{-1})B^+(q^{-1})G_i(q^{-1})] + q^{-q}\Delta B(q^{-1}) \\ &\quad \times [A_r(q^{-1})Q(q^{-1})G_r^+(q^{-1})N(q^{-1})S'(q^{-1})G_m(q^{-1})]\}. \end{aligned} \tag{26}$$

Moreover, $\{u(k)\}$ *is bounded and* $e(k) \to 0$ *as* $k \to \infty$.

Proof. The following signals of closed-loop system are obtained from Fig. 1.

$$\begin{aligned} y(k) &= \{q^{-d}B_r(q^{-1})T(q^{-1})r(k) + q^{-d}B_r(q^{-1})S(q^{-1})m_n(k) \\ &\quad - q^{-d}B_r(q^{-1})R(q^{-1})d_i(k)\}/A_{cr}(q^{-1}), \\ u(k) &= \{A_r(q^{-1})T(q^{-1})r(k) - A_r(q^{-1})S(q^{-1})m_n(k) \\ &\quad - q^{-d}B_r(q^{-1})S(q^{-1})d_i(k)\}/A_{cr}(q^{-1}). \end{aligned} \tag{27}$$

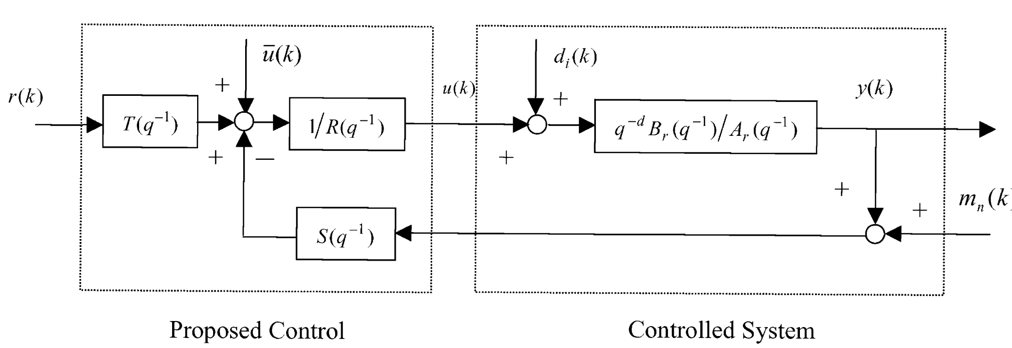

Fig. 1. Control block diagram.

According to the result of Theorem 2, $A_{cr}(q^{-1})$ is stable. Substituting (9), (18), (19), $1/A_{cr}(q^{-1}) = Q(q^{-1})$ and $F_i(q^{-1}) = F_{ir}(q^{-1})F_i'(q^{-1})$ into (27) gives

$$y(k) = q^{-d}B_r(q^{-1})T(q^{-1})Q(q^{-1})r(k) + q^{-d}B_r(q^{-1}) \times Q(q^{-1})G_r^+(q^{-1})N(q^{-1})S'(q^{-1})G_m(q^{-1})\delta(k) - q^{-d}B_r(q^{-1})Q(q^{-1})G_r^+(q^{-1})N(q^{-1})F_r'(q^{-1}) \times R'(q^{-1})B^+(q^{-1})G_i(q^{-1})\delta(k),$$

$$u(k) = A_r(q^{-1})T(q^{-1})Q(q^{-1})r(k) - A_r(q^{-1})Q(q^{-1}) \times G_r^+(q^{-1})N(q^{-1})S'(q^{-1})G_m(q^{-1})\delta(k) - q^{-d}B_r(q^{-1})Q(q^{-1})S(q^{-1})d_i(k). \quad (28)$$

Substituting (28) into (22) with the facts $F_{ir}(q^{-1})F_i'(q^{-1})F_r'(q^{-1})r(k) = 0$ and $F_{ir}(q^{-1})F_i'(q^{-1})F_r'(q^{-1})d_i(k) = 0$ gives

$$d_\Delta(k) = H''(q^{-1})\delta(k), \quad (29)$$

where the polynomial $H''(q^{-1})$ is described in (26). Combining (20) and (29) yields

$$\sigma(k) = [H'(q^{-1}) + H''(q^{-1})]\delta(k). \quad (30)$$

Then the other results are achieved. □

Remark 1. Since $A_{cr}(q^{-1})$ is a stable and monic polynomial, $A_{cr}(q^{-1}) = \prod_{n=1}^{\alpha}(1 - a_n q^{-1})(1 - a_n^* q^{-1})\prod_{m=1}^{\beta}(1 - b_m q^{-1})$, where a_n, a_n^* $(1 \leqslant n \leqslant \alpha)$ are mutually complex conjugate, $|a_n| < 1$, b_m $(1 \leqslant m \leqslant \beta)$ is real, $|b_m| < 1$, and $2\alpha + \beta = n_{cr}$. Let a_k,where $1 \leqslant k \leqslant \alpha$, be the maximum absolute coefficient of the polynomial $A_{cr}(q^{-1})$. In this situation, $1/(1 - a_k q^{-1}) = 1 + \sum_{n=1}^{\infty}(a_k)^n q^{-n} \approx 1 + \sum_{n=1}^{N}(a_k)^n q^{-n}$, where N is an enough large and finite number. The smaller $|a_k|$ is assigned, the smaller N is achieved. Furthermore, the multiplications of many polynomials $(1 - a_n q^{-1})$, $(1 - a_n^* q^{-1})$ and $(1 - b_m)$ with not all $\Re e\{a_n, b_m\} > 0$ or $\Re e\{a_n, b_m\} < 0$, for $1 \leqslant n \leqslant \alpha$, $1 \leqslant m \leqslant \beta$, make the fact $1/1A_{cr}(q^{-1}) \approx Q(q^{-1})$, where $n_q \leqslant N$. Then an almost finite-time step to arrive the switching surface is accomplished; the corresponding simulations confirm the conclusion.

3.3. Switching control for improving the system performance

The proposed switching control in (31) is designed to improve the system performance including transient response and tracking accuracy.

$$\bar{u}(k) = -A_c(q^{-1})\{\sigma(k) + W(q^{-1})v_{sw}(k)\}/ \times \{C(q^{-1})B^+(q^{-1})\bar{B}^-(q^{-1})\} \quad (31)$$

where $A_c(q^{-1}) = G_r^+(q^{-1})N(q^{-1})B^+(q^{-1})C(q^{-1})$, $\bar{B}^-(q^{-1}) = q^{-n_{b^-}}B^-(q)$ is a causal and stable polynomial, $W(q^{-1})$ is a causal and (inverse) stable rational weighting function and $v_{sw}(k)$ will be discussed later. Substituting (31), (7) and (6) into (8) gives

$$\sigma(k) = [H'(q^{-1}) + H''(q^{-1})]\delta(k^{-1}) + B^-(q^{-1})\sigma(k-d)/\bar{B}^-(q^{-1}) + \Psi(q^{-1})v_{sw}(k-d), \quad (32)$$

where $\Psi(q^{-1}) = W(q^{-1})B^-(q^{-1})/\bar{B}^-(q^{-1})$ denotes the gain of switching control. The weighting function $W(q^{-1})$ is selected such that $|\Psi(e^{-i\theta})|$, where $\theta \in [0,2\pi]$, is a low-pass filter to attenuate the high-frequency component of switching control. The difference of $\sigma(k)$ is defined as follows:

$$\Delta\sigma(k) = \sigma(k) - \sigma(k-d) \quad (33)$$

Then from (32) and (33)

$$\Delta\sigma(k) = \Omega(k) + [1 - \Sigma(q^{-1})]v_{sw}(k-d), \quad (34)$$

where $\Sigma(q^{-1}) = 1 - \Psi(q^{-1})$ and $\Omega(k) = [H'(q^{-1}) + H''(q^{-1})]\delta(k) - [\bar{B}^-(q^{-1}) - B^-(q^{-1})]\sigma(k-d)/\bar{B}^-(q^{-1})$.

The rational weighting function $W(q^{-1})$ is also selected such that

$$\|\Sigma(q^{-1})\|_\infty \leqslant \gamma < 1 \quad \text{on } D, \quad (35)$$

where $D = \{q \in C \| q| < \bar{r} < 1\}$ is the domain containing the poles of $W(q^{-1})$ and the zeros of $\bar{B}^-(q^{-1})$, $\|\Sigma(q^{-1})\|_\infty = ess.\sup_{0 \leqslant \theta \leqslant 2\pi} \bar{\lambda}\{\Sigma(e^{-i\theta})\}$, where $\bar{\lambda}(\bullet)$ denotes the maximum singular eigenvalue. The stable all-pass signal $[\bar{B}^-(q^{-1}) - B^-(q^{-1})]\sigma(k-d)/\bar{B}^-(q^{-1})$ vanishes in a steady state. Since the signal $\Omega(k)$ will disappear after transient response, the upper bound of $\Omega(k)$ is assumed as follows:

$$\|\Omega(k)\| \leqslant h_\Omega(k) = \zeta|\sigma(k)|, \quad (36)$$

where ζ satisfies the inequality: $[(1-\gamma)^2 - \mu(1+\gamma)^2]/[4(1+\gamma)] > \zeta$, where $1 > (1-\gamma)^2/(1+\gamma)^2 > \mu > 0$. That is, the upper bound of uncertainty is limited (Furuta & Pan, 2000). Then the control $v_{sw}(k)$ in (37) is employed to deal with the uncertainty $\Omega(k)$.

$$v_{sw}(k) = \begin{cases} -\xi(k)h_\Omega(k)\sigma(k)/[(1-\gamma)(1+\gamma)|\sigma(k)|] \\ \quad \text{if } |\sigma(k)| > 4(1+\gamma)h_\Omega(k)/[(1-\gamma)^2 - \mu(1+\gamma)^2] \\ 0 \quad \text{otherwise.} \end{cases} \quad (37)$$

The switching gain of (37) satisfies the following inequality:

$$\xi_2(k) > \xi(k) > \xi_1(k) \geqslant 0, \tag{38a}$$

where

$$\xi_{1,2}(k) = \eta_1(k) \pm \sqrt{\eta_1^2(k) - \eta_2(k)}, \tag{38b}$$

$$\eta_1(k) = (1-\gamma)^2|\sigma(k)|/[(1+\gamma)h_\Omega(k)] - (1-\gamma), \tag{38c}$$

$$\eta_2(k) = (1-\gamma)^2[h_\Omega^2(k) + 2h_\Omega(k)|\sigma(k)| + \mu|\sigma(k)|^2]/h_\Omega^2(k). \tag{38d}$$

Theorem 4. *Suppose the system* (1) *and the controller* (4) *with* $\bar{u}(k)$ *in* (31) *and* $v_{sw}(k)$ *in* (37). *The same conditions of* Theorem 3 *are satisfied. Then* $\sigma(k) = 0$ and $v_{sw}(k) = 0$ *as* $k > n_{h'} + n_{h''} + 1, \{u(k)\}$ *is bounded and* $e(k) \to 0$ *as* $k \to \infty$.

Proof. Define the following Lyapunov function:

$$V(k) = \sigma^2(k)/2 > 0, \quad \text{as } \sigma(k) \neq 0. \tag{39}$$

Then the change rate of (39) is given as follows:

$$\Delta V(k) = V(k) - V(k-d) = \sigma(k)\Delta\sigma(k) + \Delta\sigma^2(k)/2. \tag{40}$$

First, the case: $|\sigma(k)| > 4(1+\gamma)h_\Omega(k)/[(1-\gamma)^2 - \mu(1+\gamma)^2]$, is considered. Suppose $\Delta V(k) \leqslant -\mu V(k)$, where $0 < \mu < (1-\gamma)^2/(1+\gamma)^2 < 1$. Then the following equation is achieved by using (35)–(38) and (40).

$$\begin{aligned}\Delta\bar{V}(k) &= \Delta V(k) + \mu V(k) \\ &= \sigma(k)\{\Omega(k) + [1 - \Sigma(q^{-1})]v_{sw}(k-d)\} + \{\Omega(k) \\ &\quad + [1 - \Sigma(q^{-1})]v_{sw}(k-d)\}^2/2 + \mu\sigma^2(k)/2 \\ &\leqslant |\sigma(k)|h_\Omega(k) - \frac{\xi(k)h_\Omega(k)|\sigma(k)|}{1+\gamma} + \frac{h_\Omega^2(k)}{2} \\ &\quad + \frac{\xi(k)h_\Omega^2(k)}{1-\gamma} + \frac{\xi^2(k)h_\Omega^2(k)}{2(1-\gamma)^2} + \mu\sigma^2(k)/2 \\ &= h_\Omega^2(k)H(\xi)/\{2(1-\gamma)^2\},\end{aligned} \tag{41}$$

where

$$H(\xi) = \xi^2(k) - 2\eta_1(k)\xi(k) + \eta_2(k). \tag{42}$$

If $H(\xi) \leqslant 0$, then $\Delta\bar{V}(k) \leqslant 0$ (or $\Delta V(k) \leqslant -\mu V(k)$). Since $|\sigma(k)| > 4(1+\gamma)h_\Omega(k)/[(1-\gamma)^2 - \mu(1+\gamma)^2]$, the results $\eta_1(k) > 0$ and $\eta_1^2(k) - \eta_2(k) > 0$ are obtained. In summary, the switching gain chosen from (38) makes $\Delta V(k) \leqslant -\mu V(k)$. Hence, the operating point is driven into the invariant set: $|\sigma(k)| \leqslant 4(1+\gamma)h_\Omega(k)/[(1-\gamma)^2 - \mu(1+\gamma)^2]$. Based on the result of Theorem 2 and (37), $v_{sw}(k) = 0$ as $k > n_{h'} + n_{h''} + 1$.

Similarly, the result for the case: $|\sigma(k)| \leqslant 4(1+\gamma)h_\Omega(k)/[(1-\gamma)^2 - \mu(1+\gamma)^2]$, can be obtained. □

4. Illustrative examples

Consider the following second-order nominal system: $a_1 = -0.3$, $a_2 = 0.32$, $b_0 = 1$, $b_1 = 2$ and $d = 1$. The uncertainty caused by the parameter variations are $\Delta a_1 = 0.06$, $\Delta a_2 = -0.064$, $\Delta a_3 = 0.02$, $\Delta b_0 = 0.2$, $\Delta b_1 = -0.4$ and $\Delta b_2 = 0.02$. The reference input to be tracked is assigned as $r(k) = 1$. Let $d_i(k) = -1$ and $m_n(k) = 0.5(-1)^k$. First, the following PID controller (e.g., Åström & Wittenmark, 1997, Chapter 8) is employed to control the above system.

$$\begin{aligned}u(k) &= (1 + a_d)u(k-1) - a_d u(k-2) \\ &\quad + t_0 r(k) + t_1 r(k-1) + t_2 r(k-2) \\ &\quad - s_0 y(k) - s_1 y(k-1) - s_2 y(k-2),\end{aligned} \tag{43a}$$

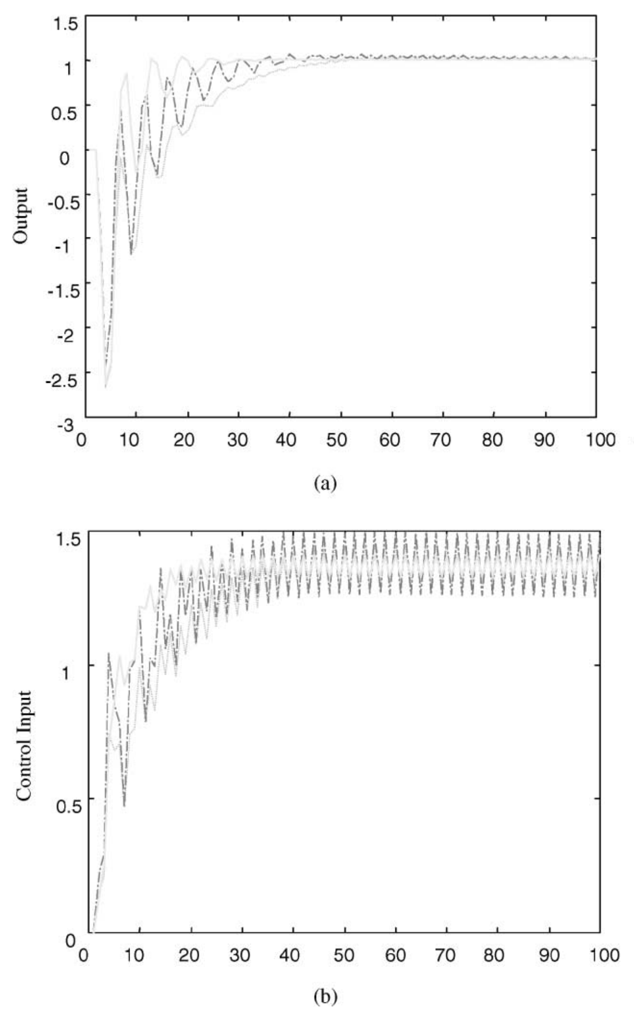

Fig. 2. The responses for PID control. (a) $y(k)$; (b) $u(k)$.

where

$$a_d = (2T_d - \bar{N}T_c)/(2T_d + \bar{N}T_c),$$

$$b_d = 2\bar{N}T_d/(2T_d + \bar{N}T_c), \quad b_i = T_c/(2T_i), \tag{43b}$$

$$t_0 = k_p(b + b_i), \quad t_1 = -k_p[b(1 + a_d) - b_i(1 - a_d)],$$

$$t_2 = k_p a_d(b - b_i), \tag{43c}$$

$$s_0 = k_p(a_d + b_d + b_i a_d),$$

$$s_1 = -k_p[1 + a_d + 2b_d - b_i(1 - a_d)],$$

$$s_2 = k_p(a_d + b_d - b_i a_d). \tag{43d}$$

After a little trial-and-error, the better responses for the following three sets of parameters:

(i) $k_p = 0.04,\ T_i = 0.01,\ T_d = 1,\ \bar{N} = 3,\ b = 1,\ T_c = 0.01$, using symbol… ,
(ii) $k_p = 0.06,\ T_i = 0.01,\ T_d = 1,\ \bar{N} = 3,\ b = 1,\ T_c = 0.01$, using symbol-.-. ,
(iii) $k_p = 0.04,\ T_i = 0.005,\ T_d = 1,\ \bar{N} = 1,\ b = 1,\ T_c = 0.01$, using symbol— ,

are shown in Fig. 2. The corresponding transient and steady-state responses are not good due to the existence of input disturbance, measurement noise, and uncertainty. This is one of motivations for the paper to provide a more effective controller.

According to the above requirements, the equivalent control design should include the mode of input disturbance and measurement noise: $F_r(q^{-1}) = F_i(q^{-1}) = F_{ir}(q^{-1}) = 1 - q^{-1}$, $F_m(q^{-1}) = 1 + q^{-1}$. In addition, the switching surface with the coefficients: $c_1 = -0.6$, $c_2 = 0.09$ (poles at 0.3,0.3) and $N(q^{-1}) = 1$ in Diophantine equation, are selected. The switching control $\bar{u}(k)$ (or $v_{sw}(k)$) uses the following control parameters: $\gamma = 0.01$, $\zeta = 0.01$, $\mu = 0.4$, $\xi(k) = \xi_1(k) + 1.5\eta_1(k)\{1 - 0.98e^{-1000|\sigma(k)|}\}$, and $W(q^{-1}) = (1.5 - 0.3q^{-1})/(1.4 - 0.4q^{-1})$. The following two cases are first examined: (i) case 1: $d_i(k) = -1$, $m_n(k) = 0.5(-1)^k$, $\Delta A(q^{-1}) = \Delta a_1 q^{-1} + \Delta a_2 q^{-2} + \Delta a_3 q^{-3}$, $\Delta B(q^{-1}) = \Delta b_0 q^{-1} + \Delta b_1 q^{-2} + \Delta b_2 q^{-3}$, $\bar{u}(k) = 0$, (ii) case 2: case 1 with $\bar{u}(k)$ in (31).

The responses for $d_i(k) = m_n(k) = \Delta A(q^{-1}) = \Delta B(q^{-1}) = 0$ and $d_i(k) = -1$, $m_n(k) = 0.5(-1)^k$,

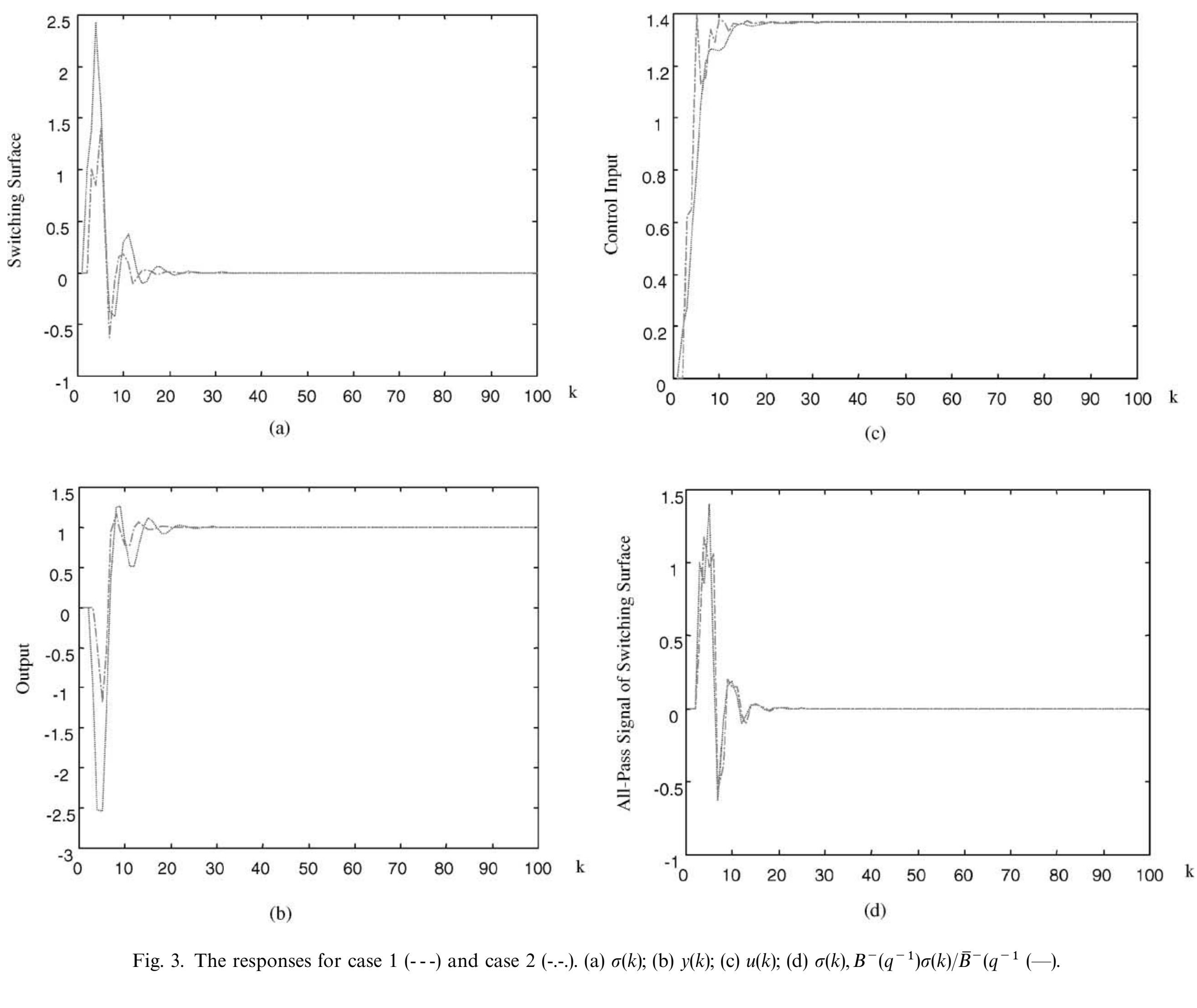

Fig. 3. The responses for case 1 (- - -) and case 2 (-.-.). (a) $\sigma(k)$; (b) $y(k)$; (c) $u(k)$; (d) $\sigma(k), B^-(q^{-1})\sigma(k)/\bar{B}^-(q^{-1}$ (—).

$\Delta A(q^{-1}) = \Delta B(q^{-1}) = 0$ have the results $\sigma(k) = 0$ as $k \geqslant 3$ and $\sigma(k) = 0$ as $k \geqslant 6$, respectively. These are omitted due to space limits. The responses for case 1 (---) and case 2 (-.-.) are presented in Fig. 3. The responses in Fig. 3(a) have $\sigma(k) = 0$ as $k \geqslant 34$ for case 1 and $\sigma(k) = 0$ as $k \geqslant 23$ for case 2. The transient responses of case 1 are improved by the case 2 (see Figs. 3(b) and (c)). Furthermore, the all-pass signal of switching surface (i.e., $B^-(q^{-1})\sigma(k)/\bar{B}^-(q^{-1})$) and the switching surface are shown in Fig. 3(d). They are almost the same for the transient response but the same for the steady-state response. Since the gain of switching control in Fig. 4 is a low-pass filter, the switching control of high-frequency component is attenuated. In addition, $\|\Sigma(a e^{-i\theta})\|_\infty < 1$, where $a \leqslant 0.5$ and $0 \leqslant \theta \leqslant 2\pi$.

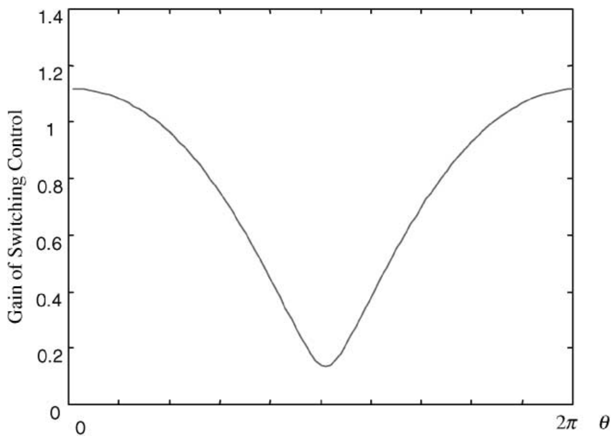

Fig. 4. The gain of switching control $|\Psi(e^{-i\theta})|$.

For further demonstrating the usefulness of the proposed control, the case 3: the sinusoidal reference input $r(k) = 0.5 + \sin(0.06\pi k)$, the input disturbance $d_i(k) = -0.5 + \sin(0.06\pi k)$, the measurement noise $m_n(k) = 0.5(-1)^k$, the uncertainty $\Delta A(q^{-1}) = \Delta a_1 q^{-1} + \Delta a_2 q^{-2} + \Delta a_3 q^{-3}$, $\Delta B(q^{-1}) = \Delta b_0 q^{-1} + \Delta b_1 q^{-2} + \Delta b_2 q^{-3}$, are studied. The study is the same as the noncircular cutting on lathe (e.g., Hwang et al., 1997). The

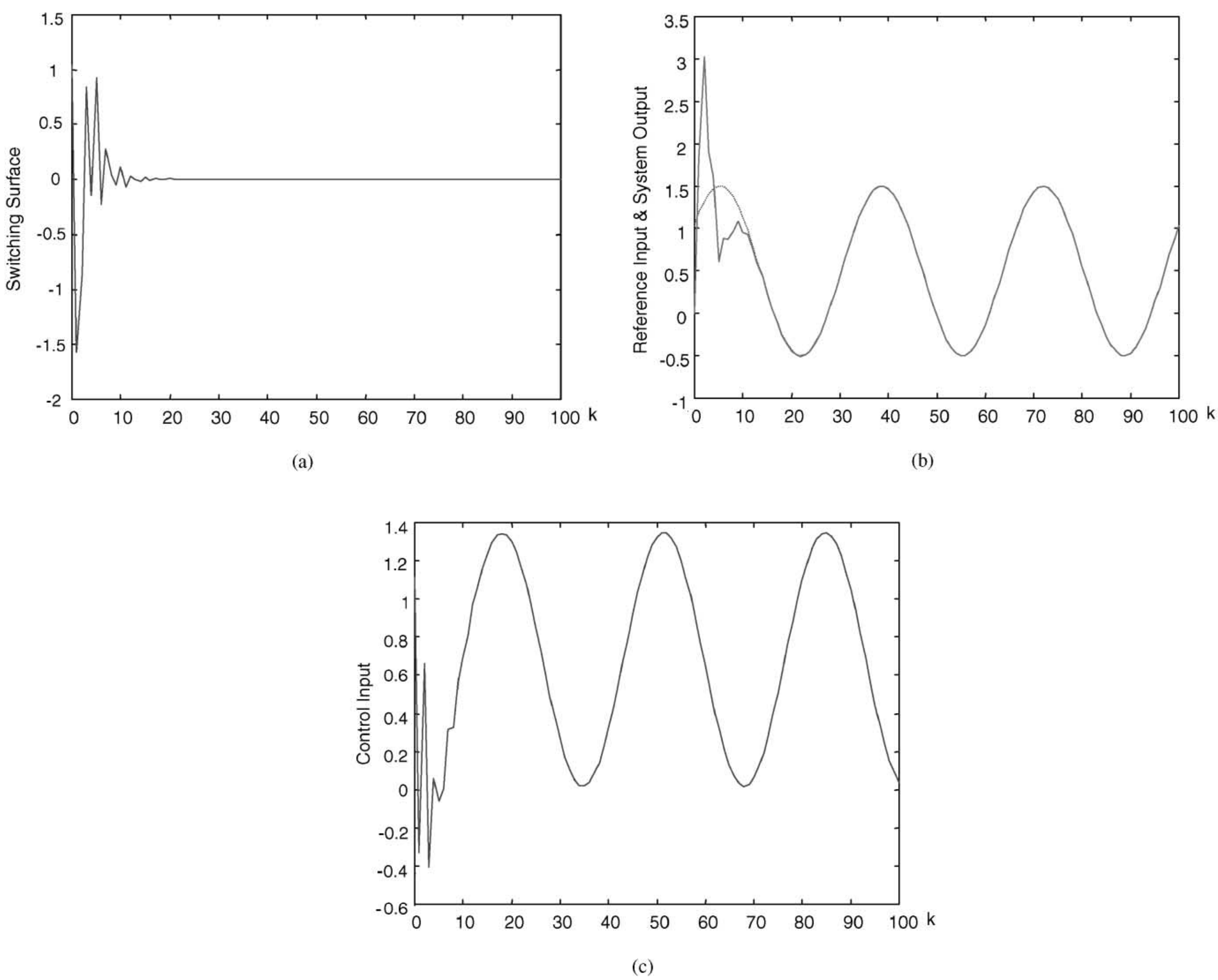

Fig. 5. The responses for case 3. (a) $\sigma(k)$; (b) $y(k)(-)$, $r(k)(\ldots)$; (c) $u(k)$.

equivalent control design must contain the following modes: $F_r(q^{-1}) = F_i(q^{-1}) = F_{ir}(q^{-1}) = (1 - q^{-1})(1 - 2\cos(0.06\pi)q^{-1} + q^{-2})$, $F_m(q^{-1}) = 1 + q^{-1}$. The responses for the modified switching surface: $c_1 = -0.9$, $c_2 = 0.27$, $c_3 = -0.027$ (poles at 0.3,0.3,0.3), are shown in Fig. 5. The switching surface in Fig. 5(a) reaches $\sigma(k) = 0$ as $k \geqslant 25$. After transient response, the tracking performance is excellent and the control input is smooth enough (see Figs. 5(b) and (c)).

5. Conclusions

A new DVSC with a finite-time step to reach the switching surface is constructed by using dead-beat control technique, internal model principle, and Lyapunov redesign. First, a dead-beat to switching surface for a nominal system is accomplished. Then the internal model principle is used for the redesign of the equivalent control to deal with input disturbance and measurement noise. Without the requirement of matching condition, the uncertainty is also tackled by the equivalent control. From the practical viewpoint in Remark 1, the characteristic polynomial of the real closed-loop system is stabilized and its inverse is approximated as a finite-degree polynomial. In this situation, the trajectory almost reaches the switching surface in a finite-time step. Based on the concept of Lyapunov redesign, a switching control is designed to reinforce the system performance. The upper bound of the uncertainty for the switching control vanishes after transient response because the trajectory reaches the switching surface in the sense of finite-time step. The simulations confirm the usefulness of the proposed controller.

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