# Efficiency ranking using Dominance Network and Multiobjective Optimization indexes

Sebastián Lozano[a]♣ and Laura Calzada-Infante[b]

[a] Dept. of Industrial Management, University of Seville, Spain, E-mail: slozano@us.es

[b] Engineering School, University of Oviedo, Spain, E-mail: calzadalaura@uniovi.es

♣Corresponding author:

Address: Escuela Superior de Ingenieros
Isla de la Cartuja, s/n
41092 Seville, Spain

E-mail: slozano@us.es

Phone: +34-954487208

ARTÍCULO PUBLICADO EN LA REVISTA

*Expert Systems with Applications* (2019), 126, 83-91

doi: https://doi.org/10.1016/j.eswa.2019.02.016

# Efficiency ranking using Dominance Network and Multiobjective Optimization indexes

*Abstract*

This paper presents a new approach for ranking organizational units within a benchmarking context. Instead of the conventional optimization-based techniques, the proposed approach uses Social network analysis and Multiobjective optimization concepts to extract second-order features from the input-output data, integrating the resulting multidimensional information using TOPSIS. It shows how several expert and intelligent systems techniques can be harmoniously integrated and applied to performance assessment. The proposed approach has been used for ranking the performance of 27 major US airlines, comparing the results with some existing Data Envelopment Analysis methods. It is shown that the use of a richer information set instead of the raw input-output data leads to an innovative and more effective way of discriminating between efficient units.

Keywords: Efficiency; Ranking; TOPSIS; PageRank; Crowding distance; Airlines

## 1. Introduction

Assessing the efficiency of organizational units is an important, well-researched issue. The most common approach is a non-parametric benchmarking technique known as Data Envelopment Analysis (DEA). DEA is a data-driven mathematical tool that only uses data on the inputs consumed and the outputs produced by the units under study, which are commonly termed Decision Making Units (DMUs). From the observed data and using some standard axioms (such as free disposability and convexity), DEA infers the Production Possibility Set (PPS, a.k.a. DEA technology) which represents all feasible operating points. The non-dominated set within this PPS corresponds to the Efficient Frontier (EF). The DMUs can be projected onto the EF using different DEA approaches (e.g. Hladík 2018, Lozano and Soltani 2018, Lozano and Calzada-Infante 2018a). For each inefficient DMU an efficiency score and an efficient target are provided by DEA. Efficient DMUs are non-dominated and thus projected onto themselves, getting an efficiency score of one. This lack of discriminatory power among the efficient DMUs motivates the existence of a number of DEA approaches aimed at discriminating and ranking efficient DMUs. There are a number of review papers on the subject (e.g. Adler et al. 2002, Jahanshahloo et al. 2008, Hosseinzadeh Lofti et al. 2013). In particular, we refer the reader to the recent paper by Aldamak and Zolfaghari (2017), which reviews up to ten different categories of DEA ranking methods, such as super-efficiency, cross-efficiency, common set of weights, cross-influence, etc. Most of the existing methods are based on optimization models that directly process the input and output data without analysing it to extract higher-level information. This paper follows a different path, analysing the available data from different perspectives and computing a multidimensional set of indexes that provide a richer view of the problem and lead to a more effective ranking of the efficient DMUs. It also shows how several expert and intelligent systems (EIS) techniques can be harmoniously integrated to develop an effective benchmarking and performance assessment method.

Actually, different EIS techniques, such as TOPSIS (Namazi and Mohammadi 2018), Social network analysis (Lozano and Calzada-Infante 2017), Decision trees (Samoilenko and Osei-Bryson 2008), Cooperative game theory (Hinojosa et al. 2017) or Self-organizing maps (Sharma and Yu 2009) have already been used for benchmarking and efficiency assessment. In particular, as regards DMU ranking, Hosseinzadeh Lofti et al. (2011) and Jahantighi et al. (2013) use TOPSIS to integrate the results from different conventional DEA ranking methods. Jahanshahloo et al. (2011) also use TOPSIS but applied to the cross-efficiency matrix computed using interval DEA. Social network analysis (SNA) has also been applied to DMU ranking by Liu et al. (2009, 2010) and Leem and Chun (2015). In these papers Eigenvector centrality and PageRank are used to rank the efficient DMUs. The corresponding network is built using the optimal values of the lambda variables computed by an envelopment DEA model. The approaches differ in that Liu et al. (2009, 2010) consider a denser network by solving DEA models with all possible input/output specifications, while Leem and Chun (2015) consider only the full input/output DEA model. More recently, Simon de Blas et al. (2018) use the authority and hub indexes computed by a modified HITS algorithm to rank efficient and inefficient DMUs. Finally, Multiobjective optimization (MO) has also been used for DMU ranking (e.g. Carrillo and Jorge 2016).

This paper proposes an integrated approach that combines different EIS methods in a novel way for the purpose of ranking efficient DMUs. Thus, SNA and MO are used to provide information on the role and significance of the input-output patterns of the observed DMUs. This information is then effectively integrated using TOPSIS (Hwang and Yoon 1981). The proposed approach shows how different EIS tools can be used synergistically to generate and process a richer information set instead of using the conventional optimization-based DEA approach. In particular, SNA and MO are used to extract second-order features from the raw input-output data by using concepts such as Dominance Networks (DN, Calzada-Infante and Lozano 2016) and Pareto Front. Although some of the SNA and MO performance indicators used (e.g. PageRank, Hypervolume, Crowding distance, etc) are well known, their joint use to provide a multidimensional perspective is innovative as it is also original the use of DN for DMU ranking. The use of TOPSIS in the second step is not original but perfectly fits its purpose as integrator of the proposed SNA and MO indicators.

The structure of the paper is the following. In section 2 the different tools used in this paper are briefly reviewed. Section 3 presents the proposed EIS-based DMU ranking approach. Section 4 applies this approach to the major US Airlines, discussing the results and comparing them with other DEA ranking methods. Finally, Section 5 summarizes and concludes.

## 2. Review of techniques used

In this section the different techniques that will take part in the proposed approach are presented separately. This will facilitate the understanding of the proposed EIS-based DMU ranking approach presented in the next section.

### *2.1. Data Envelopment Analysis*

Consider a set of DMUs whose inputs consumption and outputs productions are known. Let

$j$ index on DMUs (varying from 1 to n)

$i$ index for inputs (varying from 1 to m)

$k$ index for outputs (varying from 1 to s)

$x_{ij}$ amount of input i consumed by DMU j

$y_{kj}$ amount of output k produced by DMU j

The Variable Returns to Scale (VRS) DEA technology corresponds to the set of feasible operating points

$$T(\hat{x},\hat{y}) = \left\{ (\hat{x},\hat{y}) : \exists\, \lambda = (\lambda_1,\lambda_2,\ldots,\lambda_n) \geq 0 \;\; \sum_{j=1}^{n}\lambda_j = 1 \;\; \sum_{j=1}^{n}\lambda_j x_{ij} \leq \hat{x}_i \;\; \forall i \;\; \sum_{j=1}^{n}\lambda_j y_{kj} \geq \hat{y}_k \;\; \forall k \right\} \tag{1}$$

Given a DMU 0, it can be projected onto the EF using any of a number of DEA models (input or output-oriented, radial, non-radial, slack-based, etc.). In particular, consider the following Slacks-Based Inefficiency (SBI) DEA model (Fukuyama and Weber 2009):

$$\text{Max} \quad \frac{1}{m+s} \cdot \left( \sum_{i=1}^{m} \frac{s_i^-}{c_i^x} + \sum_{k=1}^{s} \frac{s_k^+}{c_k^y} \right) \tag{2}$$

s.t.

$$\sum_{j=1}^{n} \lambda_j x_{ij} = x_{i0} - s_i^- \qquad \forall i \tag{3}$$

$$\sum_{j=1}^{n} \lambda_j y_{kj} = y_{k0} + s_k^+ \qquad \forall k \tag{4}$$

$$\sum_{j=1}^{n} \lambda_j = 1 \tag{5}$$

$$\lambda_j \geq 0 \ \ \forall j \quad s_i^- \geq 0 \ \ \forall i \quad s_k^+ \geq 0 \ \ \forall k \tag{6}$$

The optimal solution of the above DEA model provides an efficiency score

$$SBI_0 = 1 - \frac{1}{m+s} \cdot \left( \sum_{i=1}^{m} \frac{s_i^-}{c_i^x} + \sum_{k=1}^{s} \frac{s_k^+}{c_k^y} \right) \tag{7}$$

and corresponding efficient targets

$$\hat{x}_{i0} = \sum_{j=1}^{n} \lambda_j x_{ij} \quad \forall i \qquad \hat{y}_{k0} = \sum_{j=1}^{n} \lambda_j y_{kj} \quad \forall k \tag{8}$$

The efficiency scores can be used to rank the inefficient DMUs. However, conventional DEA models cannot discriminate between the efficient DMUs as all of them are assigned unity efficiency scores. As indicated in the introduction, this is the justification for the need of ranking methods in DEA.

*2.2. Dominance network analysis*

A DN is a directed weighted network (D,E) where D is the set of nodes and E is the set of arcs between them. Each node $j \in D$ corresponds to a feasible operating point with input and output vectors $\left(x_j, y_j\right)$. The arcs correspond to the dominance relationships between the nodes. Thus, an arc between a node r and a node j exists if j dominates r, i.e. $x_{ij} \le x_{ir}\ \forall i \ \wedge \ y_{kj} \ge y_{kr}\ \forall k \ \wedge \ \left(x_j, y_j\right) \ne \left(x_r, y_r\right)$. Such an arc has an associated weight

$$e_{rj} = \frac{1}{m+s} \cdot \left( \sum_{i=1}^{m} \frac{x_{ir} - x_{ij}}{c_i^x} + \sum_{k=1}^{s} \frac{y_{kj} - y_{kr}}{c_k^y} \right) \tag{9}$$

where $c_i^x$ and $c_k^y$ are slacks normalizing constants. Let $D(r)$ be the set of nodes that dominate node r. As shown in Lozano and Calzada-Infante (2018b), the arc weights have the following additive property $r \in D(p) \ \wedge \ j \in D(r) \Rightarrow \ e_{pj} = e_{pr} + e_{rj}$. When visualizing the network it is convenient, in order to reduce the clutter, not to draw the transitive arcs. This type of filter is called skeletonization (Lozano and Calzada-Infante 2018b).

The out-degree of a node is the number of arcs that leave that node and corresponds to $\left|D(r)\right|$. Similarly, the in-degree of a node is the number of arcs that enter a node, i.e. the number of nodes that dominate it. The asymmetry and transitivity of the dominance relationships mean that DNs have a layered structure with some nodes (labelled layer 0) being non-dominated and hence having zero out-degree. The layer of all the nodes can be computed recursively as

$$\lambda(r) = \begin{cases} 0 & \text{if } D(r) = \varnothing \\ 1 + \max\limits_{j \in D(r)} \lambda(j) & \text{otherwise} \end{cases} \tag{10}$$

Many SNA indexes can be used to characterize a DN, both at the local and global levels, but for the purpose of this research, i.e. ranking the efficient DMUs, we will only need the following three:

(i) In-strength: This is the sum of the weights of the arcs that enter a node $s_j^{in} = \sum_{\{r: j \in D(r)\}} e_{rj}$. If j is an efficient DMU then this SNA index measures the total increase in efficiency if all the inefficient DMUs dominated by j reduce their inputs and increase their outputs to the corresponding levels of DMU j.

(ii) Inefficiency radius: This is the maximum of the weights of the arcs that enter a node $\rho_j = \max_{\{r:j \in D(r)\}} e_{rj}$. It corresponds to the length of the longest arc entering that node.

(iii) PageRank: This is a node centrality index that is commonly used in directed networks. It was originally proposed to rank the relevance of the web pages returned by Google (Brin and Page 1998). Basically, this index measures the relative frequency with which a random surfer that, starting from any node and randomly following one outgoing link (using the weights of those outgoing links to compute the probability of choosing each of them), visits each node of the network. This measure has two refinements, one to allow nodes with a zero in-degree to have a certain probability of being visited and another to escape from nodes with a zero out-degree that otherwise would be like dead ends. The parameter $\alpha$ is called the damping factor and usually takes a value of 0.85.

$$\text{PageRank}_j = \alpha \cdot \sum_{\{r:s_r^{out}>0\}} \frac{e_{rj}}{s_r^{out}} \cdot \text{PageRank}_r + \alpha \cdot \frac{1}{n} \cdot \sum_{\{r:s_r^{out}=0\}} \text{PR}_r + (1-\alpha) \cdot \frac{1}{n} \tag{11}$$

*2.3. Multiobjective Optimization performance indicators*

MO problems appear when there are multiple conflicting objective functions, so there is no single solution which could simultaneously optimize all of them. In MO, the solutions of interest are, as the operating points in DEA, those that are not dominated. The equivalent to the DEA EF is called, in MO, the Pareto Frontier (PF, a.k.a. Pareto Optimal Set). Many different MO methods have been proposed in the literature (see Marler and Arora 2004) with population-based methods, such as Multi-Objective Evolutionary Algorithms (Nedjah and de Macedo Mourelle 2015; Zhou et al. 2011), among the most commonly used. Actually, population-based MO methods do not compute a single solution but try to estimate the whole PF. Since the solution method computes a discrete approximation of the true Pareto Front there exist a number of so-called performance indicators that measure how good those approximations are and how much better one approximation is with respect to another. There are many performance indicators measuring the separation/diversity of the solutions in the PF, the ratio of non-dominated solutions, etc. Other performance indicators frequently used go by the names of hypervolume (a.k.a. size of space covered), spread, spacing, Zitzler measure, etc (see, e.g., Zitzler et al. 2003, Zhou et al. 2011). Although, as indicated above, these performance indicators aim at assessing the whole PF, they are generally computed as the average or the sum of measures computed for each individual solution. This means that we can also use those measures at the solution level. Specifically, we are interested in the following three MO indicators:

(i) Spacing: This indicator measures the minimum distance between a point and all the others. The idea is that, since the PF is a discrete approximation to a possible infinite set, having two solutions that are very similar implies an undesirable redundancy. In the case of the DEA EF, this indicator measures the degree of similarity of two operating points. Thus, two efficient DMUs that represent close operating points are somewhat redundant as benchmarks and it would be more useful if they represented differentiated efficient operating points.

$$\text{Spacing}_j = \min_{j' \in EF} \frac{1}{m+s} \cdot \left( \sum_{i=1}^{m} \frac{\left| x_{ij} - x_{ij'} \right|}{c_i^x} + \sum_{k=1}^{s} \frac{\left| y_{kj} - y_{kj'} \right|}{c_k^y} \right) \tag{12}$$

(ii) Hypervolume: This indicator measures the size of the space covered by each non-dominated solution. Each dimension is normalized using the optimal objective function value, which, in the case of the DEA EF, would be the maximum value if it is an output dimension and the minimum value if it is an input dimension

$$\text{Hypervolume}_j = \prod_k \frac{y_{kj}}{\max_r \{y_{kr}\}} \cdot \prod_i \frac{\min_r \{x_{ir}\}}{x_{ij}} \tag{13}$$

(iii) Crowding distance: This indicator was proposed in the popular NSGAII MO evolutionary algorithm (Deb et al. 2002) and its aim is also to determine how separate the non-dominated solutions are from one another. It is calculated as the rectangular distance between the two PF solutions that are closest to a given non-dominated solution. It is computed sorting the non-dominated solutions for each objective function and summing the difference, in each dimension, between the neighbouring upper and lower values of the given solution. For the solutions which are on the border of the frontier, the crowding distance is generally assigned an infinite value but in the case of the proposed approach we can make it equal to two times the distance to the next upper or lower value (depending on whether it is the maximum or minimum value of the corresponding input or output dimension). Defining the neighbouring upper and lower values of a given DMU j for each input and output dimension as $u(x_{ij}) = \min_{\{r \neq j : x_{ir} \geq x_{ij}\}} x_{ir}$, $l(x_{ij}) = \max_{\{r \neq j : x_{ir} \leq x_{ij}\}} x_{ir}$, $u(y_{kj}) = \min_{\{r \neq j : y_{kr} \geq y_{kj}\}_k} y_{ir}$ and $l(y_{kj}) = \max_{\{r \neq j : y_{kr} \leq y_{kj}\}_k} y_{ir}$, its crowding distance can be expressed as

$$\text{CrowdingDistance}_j = \sum_{i=1}^{m} \frac{u(x_{ij}) - l(x_{ij})}{c_i^x} + \sum_{k=1}^{s} \frac{u(y_{kj}) - l(y_{kj})}{c_k^y} \tag{14}$$

### *2.4. TOPSIS*

TOPSIS is a popular multicriteria decision making technique and can thus rank a finite number of alternatives using multiple attributes or criteria (see, e.g., Behzadian et al. 2012). Its name is an acronym of Technique for Order Preference by Similarity to Ideal Solution which refers to its use of the concepts of Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS, a.k.a. Anti-Ideal Solution). The former corresponds to a virtual alternative having the best value for each of the attributes while the latter is the opposite, i.e. a virtual alternative having the worst value for each of the attributes. For each alternative, its Euclidean distance to the PIS and NIS are computed and alternatives that are closer to PIS and farther from NIS are preferred. There are different TOPSIS variants (see, e.g., Zavadskas et al. 2016). The one that is proposed in this paper is Modified TOPSIS (Deng et al. 2000) which uses objective weights directly derived from the data. This approach is the most adequate in our case since there is no specific Decision Maker that could supply the importance weights for the different criteria.

Given the decision matrix U (whose rows correspond to the alternatives and whose columns correspond to the criteria), the steps of the Modified TOPSIS method are the following:

1. Normalize the decision matrix so that the sum of each column is unity $v_{jt} = \dfrac{u_{jt}}{\sum_r u_{rt}}$

2. Compute the criteria weights from the normalized decision matrix. This can be done using, for example, the standard deviation of the normalized attribute values of each criterion, i.e.

$$w_t = \frac{\sigma_t}{\sum_q \sigma_q}$$

3. Compute PIS and NIS vectors with the best and worst values, respectively, for each of the c criteria $v^+ = \left(v_1^+, v_2^+, ..., v_c^+\right)$ and $v^- = \left(v_1^-, v_2^-, ..., v_c^-\right)$

$$v_t^+ = \begin{cases} \max\limits_j v_{jt} & \text{if } t \text{ is a positive attribute} \\ \min\limits_j v_{jt} & \text{if } t \text{ is a negative attribute} \end{cases} \qquad v_t^- = \begin{cases} \min\limits_j v_{jt} & \text{if } t \text{ is a negative attribute} \\ \qquad j & \\ \max\limits_j v_{jt} & \text{if } t \text{ is a positive attribute} \end{cases} \tag{15}$$

4. Compute the weighted Euclidean distance from each alternative to PIS and NIS

$$d_j^+ = \sqrt{\sum_{t=1}^{c} w_t \cdot \left(v_t^+ - v_{jt}\right)^2} \qquad d_j^- = \sqrt{\sum_{t=1}^{c} w_t \cdot \left(v_t^- - v_{jt}\right)^2} \tag{16}$$

5. Compute the closeness/farness ratio and rank the alternatives in decreasing order of this ratio

$$T_j = \frac{d_j^-}{d_j^- + d_j^+} \tag{17}$$

**3. Proposed EIS-based DMU ranking approach**

The first step of the proposed approach consists of solving DEA models (2)-(6) and computing SBI efficiency scores (7) and, for the inefficient DMUs, corresponding efficient targets (8). Inefficient DMUs can be ranked using their SBI efficiency scores. In order to rank the efficient DMUs, which have their SBI efficiency score equal to one, we propose:

1. Build a DN whose nodes are the observed DMUs plus the efficient targets computed for the inefficient DMUs.

2. Compute the in-strength, inefficiency radius and PageRank of the DN nodes. In the case of the in-strength and the inefficiency radius, they need to be computed only for the efficient DMUs. In the case of PageRank, although only the values for the efficient DMUs will be used later on, the recursive nature of the PageRank calculation (11) requires that it be computed for all the nodes of the network.

3. Compute the Spacing, Hypervolume and Crowding Distance of each efficient DMU.

4. Form a decision matrix where the alternatives are the efficient DMUs to be ranked and the attributes are the three SNA DN measures plus the three MO indicators computed in steps 2 and 3 above.

5. Apply Modified TOPSIS considering that all the attributes are positive, i.e. the larger the better, i.e. an efficient DMU having more in-strength, a larger inefficiency radius, higher PageRank, more Spacing, more Crowding Distance and a larger Hypervolume is preferred.

In summary, the proposed approach makes use and takes advantage of the strength of different techniques. First of all is DEA, which is the basic Frontier Analysis methodology. DEA, however, has limitations one of which is its inability to discriminate between the efficient DMUs. To that end, six important indicators are used. Three of them correspond to SNA measures of the nodes of a DN that takes into account the dominance relationships between the DMUs. Thus, if an efficient DMU dominates many inefficient DMUs and by a large margin (i.e. it consumes much less inputs and much more outputs) then that efficient DMU is more "important" in the sense of being a relevant and useful benchmark. Similarly, the PageRank of an efficient DMU is a measure of how likely it is to be selected as a benchmark if the inefficient DMUs choose their benchmarks proportional to the corresponding efficiency differences between the dominating and dominated nodes. Therefore, these three SNA measures allow the gauging of the relative benchmark importance of each efficient DMU.

In order to enrich the analysis, and instead of including more SNA measures, we also propose to use information from an MO perspective. Given the closeness between the EF of DEA and the PF of MO, we have been able to adapt some of the multiple MO performance indicators to the DEA ranking context. Thus, the separation of the efficient DMUs is seen as a positive attribute as it provides non-redundant information about the EF, thus defining distinct efficient operating points and hence increasing the overall benchmarking possibilities. Also, the hypervolume of the region of the PPS covered by each efficient DMU can be used to gauge the benchmarking capacity of each efficient DMU.

As already mentioned, although to keep it simple and parsimonious we have considered only a limited number of SNA and MO measures, additional indicators can be included if desired. One of the advantages of using TOPSIS is that it can accommodate multiple and varied types of attribute. That is one of its strengths, together with its ease of calculation and clear graphical interpretation.

## 4. Efficiency assessment and Ranking of US airlines

In this section, the usefulness of the proposed approach is illustrated, applying it to assessing the efficiency and ranking the passenger services of major US airlines. Before we describe this application, we should mention that DEA has been extensively applied to assess the efficiency of airlines using conventional DEA (e.g. Barros and Dieke 2007; Lozano and Gutiérrez 2011; Cui and Li 2015; Merkert and Pearson 2015) as well as network DEA models (e.g. Zhu 2011; Tavassoli et al. 2014; Lozano and Gutiérrez 2014; Omrani and Soltanzadeh 2016). It is also worth mentioning that TOPSIS has been used by some researchers, as an alternative to DEA, for airlines performance assessment and ranking (Feng and Wang 2000; Wang 2008; Barros and Wanke 2015; Wanke et al. 2015).

The data used for this research correspond to year 2015 and were obtained from the Bureau of Transportation Statistics website (https://www.bts.gov/). Discarding those airlines for which some data was missing, a total of 27 observed DMUs were obtained. The inputs considered are aircraft hours (which is considered a non-discretionary input, in thousands), number of employees (Full Time Equivalent (FTE), in thousands) and fuel consumption (in million litres) while the output considered is Revenue Passenger Kilometres (RPK, in millions). Table 1 shows the values of the corresponding variables for the different DMUs.

============================= Table 1 ==================================

The first step was to solve the SBI VRS DEA model (2)-(6) using the variables standard deviation as slacks-normalizing constants. Note that the non-discretionary input has not been included in the objective function as per Banker and Morey (1986). Eleven DMUs were found to be efficient. For the other 16, the corresponding efficiency scores and targets were computed. The optimal input and output slacks and corresponding objective function value (2) are shown in Table 2.

============================= Table 2 ==================================

The second step is building the DN formed by the original 27 DMUs, plus the 16 efficient targets computed in step one for the 16 inefficient DMUs. Again, when computing the arcs weights, the non-discretionary input is not considered. That input is, however, taken into account when determining whether a DMU dominates another. Figure 1 shows the visualization of this DN using a layered layout. The width of the edges is proportional to their weights. Note that of the total 43 nodes, 27 belong to layer 0 (the original 11 efficient DMUs plus the 16 efficient targets). Most of the 16 inefficient DMUs belong to layer 1.

============================== Figure 1 ===================================

The next step is to compute the SNA measures, i.e. in-strength, inefficiency radius and PageRank of the nodes corresponding to the efficient DMU. These are shown in Table 3. The table also shows the MO indicators computed for each of the efficient DMUs. As before, the non-discretionary input is not considered in the computation of these indicators.

============================== Table 3 ===================================

The final step is to apply TOPSIS using the data in Table 3 as the decision matrix. The normalized decision matrix and PIS and NIS vectors are shown in Table 4, which also shows the "objective" criteria weights computed using the standard deviation method (see Deng et al. 2000)

============================== Table 4 ===================================

Table 5 shows the distance of each efficient DMU to PIS and NIS as well as the closeness/farness ratio and the resulting ranking. The highest rank is assigned to Southwest Airlines (WN), which does not dominate any other (it has zero in-strength and inefficiency radius) but operates in a niche of the PPS where there are no other similar operating points (it has high spacing and crowding distance).

============================== Table 5 ===================================

The results of the proposed approach can be compared with those of the SNA DEA approaches of Liu et al. (2009, 2010) and Leem and Chun (2015). Figures 2 and 3 show the weighted network considered in each case. Note that the Liu et al. (2009, 2010) networks are denser than that of Leem and Chun (2015) as the former considers many different input/output specifications. Note that none of these networks is based on the concept of dominance and hence are completely different from the proposed DN.

============================== Figure 2 ===================================

============================== Figure 3 ===================================

Table 6 shows the Eigenvector and PageRank centrality computed for the networks of Liu et al. (2009, 2010) and Leem and Chun (2015), respectively, and the resulting ranking of the efficient DMUs. As a representative of conventional (in the sense of not using SNA) DEA ranking methods, in Table 6, the super-SBM scores (Tone 2002) and the corresponding ranking are also shown. Note that although the second and third ranked DMUs in the proposed approach (NK and F9, respectively) occupy similar positions in the other methods, that does not happen in the case of WN, which only in super-SBM occupies a highly ranked position.

============================== Table 6 ===================================

Finally, Table 7 shows the Spearman's rank order correlation between the four methods. The different methods rank the DMUs from different points of view and thus it is normal that their rankings differ. In particular, it can be seen that rankings computed by the SNA DEA approaches are not correlated with that of super-SBM.

============================== Table 7 ===================================

## 5. Conclusions

This paper proposes a new EIS-based ranking approach to discriminate between efficient DMUs. The gist of the approach is the extraction of second-order features from the observed input-output data using DN, SNA and MO concepts, integrating them, in a second step, using TOPSIS. The proposed approach, thus, generate and process a richer information set on which to base the ranking decisions, which enhances its discrimating power compared with the existing optimization-based approaches. It is also more effective than existing DMU ranking approaches than only use SNA but not DN or MO.

Specifically, the proposed approach computes three DN indicators that measure the in-strength, inefficiency radius and PageRank centrality of the efficient DMUs and can be used to determine the centrality and benchmarking importance of the DMUs in the network. This is complemented with another three MO performance indicators measuring the spacing and Crowding distance, i.e. the uniformity of the spread of the Pareto Front formed by the efficient DMUs, as well as the size of the hypervolume dominated by each efficient DMU.

Thus, the proposed approach uses an innovative perspective that integrates information about the extent of the dominance and the degree of spacing of the efficient DMUs. The criteria considered favour those efficient DMUs that dominate (and hence can be used as benchmarks) many other

DMUs. Also, the larger the extent of that dominance, the larger the efficiency improvements that can be credited to the DMU. The size of the hypervolume (in the input-output space) associated with the efficient DMU operating point is also taken into account. And finally, the degree of specialization of the DMU is also considered, with DMUs occupying a sparse region of the PPS being preferred. This is because efficient DMUs that are close within the input-output space are, to some extent, redundant in the definition of the EF while specialized DMUs make a larger incremental contribution to enlarging the PPS.

The proposed approach has been applied to a sample of 27 major US airlines. A conventional VRS DEA approach determines that 11 of the airlines are efficient. In order to rank them, the DN of the sample has been built and the in-strength, inefficiency radius and PageRank measures of the different DMUs have been computed. This information has been complemented with that provided by the hypervolume, spacing and crowding distance computed for each efficient DMU. The results put Southwest Airlines, the world's largest low-cost carrier and a success story in the sector, at the top of the efficient DMUs. None of the methods with which it has been compared ranks this airline on top. Because it processes the input-output data to derive a richer set of performance indicators the proposed approach provides an enhanced method of discriminating between the efficient DMUs. It is also an example of the synergistic use of multiple EIS techniques and shows their potential for extracting higher-level information from raw data and process it in a more effective way. Of course, using multiple techniques instead of just a conventional optimization approach requires more expertise and a broader, more integrative perspective but this increase in complexity pays off in terms of the quality of the results.

As topics for further research, one should be to extend the proposed approach to rank the DMUs when data on multiple time periods are available. For multiperiod data, either a contemporaneous, a sequential or an intertemporal approach (Tulkens and Vanden Eeckaut 1995) can be adopted. The ability to also handle uncertainty in the form of imprecise data is more challenging but would lead to a more robust method. Another interesting line of research is integrating information provided by other EIS techniques (such as Cooperative Game Theory or Self-Organizing Maps) as well as from other DEA ranking methods (e.g. cross-efficiency, super-efficiency, etc.) keeping TOPSIS as top-level integrator. Finally, a more sophisticated approach would involve substituting TOPSIS by a mixture of experts method.

**Acknowledgements**

This research was carried out with the financial support of the Spanish Ministry of Science and the European Regional Development Fund (ERDF), grant DPI2017-85343-P.

**References**

Adler, N., Friedman, L. and Sinuany-Stern, Z., "Review of ranking methods in the data envelopment analysis context", *European Journal of Operational Research*, 140, 2 (2002) 249–265

Aldamak, A. and Zolfaghari, S., "Review of efficiency ranking methods in data envelopment analysis", *Measurement*, 106 (2017) 161–172

Banker, R.D. and Morey, R., "Efficiency analysis for exogenously fixed inputs and outputs", *Operations Research*, 34 (1986) 513–521

Barros, C.P. and Dieke, P.U.C., "Performance Evaluation of Italian Airports: A Data Envelopment Analysis", *Journal of Air Transport Management*, 13, 4 (2007) 184–191

Barros, C.P. and Wanke, P., "An analysis of African airlines efficiency with two-stage TOPSIS and neural networks", *Journal of Air Transport Management*, 44-45 (2015) 90–102

Behzadian, M., Khanmohammadi Otaghsara, S., Yazdani, M., and Ignatius, J., "A state-of the-art survey of TOPSIS applications", *Expert Systems with Applications*, 39, 17 (2012) 13051–13069

Brin, S. and Page, L., "The Anatomy of a Large-Scale Hypertextual Web Search Engine", *Computer Networks and ISDN Systems*, 30 (1998) 107–117

Calzada-Infante, L., and Lozano, S., "Analysing Olympic Games through dominance networks", *Physica A: Statistical Mechanics and Its Applications*, 462 (2016) 1215–1230

Carrillo, M. and Jorge, J.M., " A multiobjective DEA approach to ranking alternatives", *Expert Systems With Applications*, 50 (2016) 130–139

Cui, Q. and Li, Y., " Evaluating energy efficiency for airlines: An application of VFB-DEA", *Journal of Air Transport Management*, 44–45 (2015) 34–41

Deb, K., Pratap, A., Agarwal, S. and Meyarivan, T., "A fast and elitist multiobjective genetic algorithm: NSGA-II", *IEEE Transactions on Evolutionary Computation*, 6, 2 (2002) 182–197

Deng, H., Yeh, C.H. and Willis, R.J., "Inter-company comparison using modified TOPSIS with objective weights", *Computers & Operations Research*, 27, 10 (2000) 963–973

Feng, C.M. and Wang, R.T., "Performance evaluation for airlines including the consideration of financial ratios", *Journal of Air Transport Management*, 6 (2000) 133–142

Fukuyama, H. and Weber, W.L.,"A directional slacks-based measure of technical inefficiency", *Socio-Economic Planning Sciences*, 43, 4 (2009) 274–287

Hladík, M., "Universal efficiency scores in data envelopment analysis based on a robust approach", *Expert Systems With Applications*, (2019), doi: 10.1016/j.eswa.2019.01.019

Hinojosa, M.A., Lozano, S., Borrero, D.V. and Mármol, A.M., "Ranking efficient DMUs using cooperative game theory", *Expert Systems With Applications*, 80 (2017) 273–283

Hosseinzadeh Lotfi, F., Fallahnejad, R. and Navidi, N., "Ranking Efficient Units in DEA by Using TOPSIS Method", *Applied Mathematical Sciences*, 5, 17 (2011) 805–15

Hosseinzadeh Lotfi, F., Jahanshahloo, G.R., Khodabakhshi, M., Rosdtamy-Malkhlifeh, M., Moghaddas, Z. and Vaez-Ghasemi, M., "A Review of Ranking Models in Data Envelopment Analysis", *Journal of Applied Mathematics*, 2013 (2013) Article ID 492421

Hwang, C.L. and Yoon, K.P., *Multiple attribute decision making: Methods and applications*, Springer-Verlag, New York, (1981)

Jahanshahloo, G.R., Hosseinzadeh Lotfi, F., Sanei, M. and Fallah Jelodar, M., "Review of Ranking Models in Data Envelopment Analysis", *Applied Mathematical Sciences*, 2, 29 (2008) 1431–1448

Jahanshahloo, G.R., Khodabakhshi, M., Hosseinzadeh Lotfi, F. and Moazami Goudarzi, M.R., "A cross-efficiency model based on super-efficiency for ranking units through the TOPSIS approach and its extension to the interval case", *Mathematical and Computer Modelling*, 53 (2011) 1496–1955

Jahantigh, M., Hosseinzadeh Lotfi, F. and Z. Moghaddas, Z., "Ranking of DMUs by Using TOPSIS and Different Ranking Models in DEA", *International Journal of Industrial Mathematics*, 5, 3 (2013) 217–225

Leem, B.H. and Chun, H., "Measuring the Influence of Efficient Ports using Social Network Metrics", *International Journal of Engineering Business Management*, 7, 1 (2015) 1–8

Liu, J.S., Lu, L.Y.Y., Lu, W.M. and Lin, B.J.Y., "Data Envelopment Analysis 1978-2010: A Citation-Based Literature Survey", *Omega*, 41, 1 (2013) 3–15

Liu, J.S., Lu, W.M., Yang, C. and Chuang, M., "A network-based approach for increasing discrimination in data envelopment analysis", *Journal of the Operational Research Society*, 60 (2009) 1502–1510

Liu, J.S. and Lu, W.M., "DEA and ranking with the network-based approach: a case of R&D performance", *Omega*, 38 (2010) 453–464

Lozano, S. and Calzada-Infante, L., "Dominance network analysis of economic efficiency", *Expert Systems With Applications*, 82 (2017) 53–66

Lozano, S. and Calzada-Infante, L., “Computing gradient-based stepwise benchmarking paths”, *Omega*, 81 (2018a) 195–207

Lozano, S. and Calzada-Infante, L., “Efficiency assessment using network analysis tools”, *Journal of the Operational Research Society*, 69, 11 (2018b) 1803–1818

Lozano, S. and Gutiérrez, E., "A multiobjective approach to fleet, fuel and operating cost efficiency of European airlines", *Computers & Industrial Engineering*, 61, 3 (2011) 473–481

Lozano, S. and Gutiérrez, E., "A slacks-based network DEA efficiency analysis of European airlines", *Transportation Planning and Technology*, 37, 7 (2014) 623–637

Lozano, S. and Soltani, N., “Efficiency assessment using a multidirectional DDF approach”, *International Transactions in Operational Research*, (2018) (doi: 10.1111/itor.12617)

Marler, R. T. and Arora, J. S., "Survey of Multi-Objective Optimization Methods for Engineering," *Structural and Multidisciplinary Optimization*, 26, 6 (2004) 369–395

Merkert, R. and Pearson, J., "A Non-parametric Efficiency Measure Incorporating Perceived Airline Service Levels and Profitability", *Journal of Transport Economics and Policy*, 49, 2 (2015) 261–275

Nedjah, N. and de Macedo Mourelle, M., "Evolutionary multi-objective optimisation: a survey". *International Journal of Bio-Inspired Computation*, 7, 1 (2015) 1–25

Namazi, M., and Mohammadi, E., " Natural resource dependence and economic growth: A TOPSIS/DEA analysis of innovation efficiency", *Resources Policy*, 59 (2018) 544–552

Omrani, H. and Soltanzadeh, E., " Dynamic DEA models with network structure: An application for Iranian airlines", *Journal of Air Transport Management*, 57 (2016) 52–61

Shrama, M.J. and Yu, S.J., "Performance based stratification and clustering for benchmarking of container terminals", *Expert Systems With Applications*, 36 (2009) 5016–5022

Simon de Blas, C., Simon Martin, J. and Gomez Gonzalez, D. "Combined social networks and data envelopment analysis for ranking", European Journal of Operational Research, 266, 3 (2018) 990–999

Samoilenko, S. and Osei-Bryson, K.M., "Increasing the discriminatory power of DEA in the presence of the sample heterogeneity with cluster analysis and decision trees", *Expert Systems with Applications*, 34 (2008) 1568–1581

Tavassoli, M., Faramarzi, G.R. and Farzipoor Saen, R., "Efficiency and effectiveness in airline performance using a SBM-NDEA model in the presence of shared input", *Journal of Air Transport Management*, 34 (2014) Pages 146–153

Tone, K., "A slacks-based measure of super-efficiency in data envelopment analysis", *European Journal of Operational Research*, 143, 1 (2002) 32–41

Tulkens, H. and Vanden Eeckaut, P., "Non-parametric efficiency, progress and regress measures for panel data: Methodological aspects", *European Journal of Operational Research*, 80 (1995) 474–499

Wang, Y.J., "Applying FMCDM to evaluate financial performance of domestic airlines in Taiwan", *Expert Systems With Applications*, 34 (2008) 1837–1845

Wanke, P., Barros, C.P. and Chen, Z., "An analysis of Asian airlines efficiency with two-stage TOPSIS and MCMC generalized linear mixed models", *International Journal of Production Economics*, 169 (2015) 110–126

Zavadskas, E.K., Mardani, A., Turskis, Z., Jusoh, A., and Nor, K.M., "Development of TOPSIS Method to Solve Complicated Decision-Making Problems: An Overview on Developments from 2000 to 2015", *International Journal of Information Technology & Decision Making*, 15, 3 (2016) 645–682

Zhou, A., Qu, B., Li, H., Zhao, S. and Nagaratnam, P., "Multiobjective evolutionary algorithms : A survey of the state of the art", *Swarm and Evolutionary Computation*, 1, 1 (2011) 32–49

Zhu, J., "Airlines Performance via Two-Stage Network DEA Approach", *Journal of CENTRUM Cathedra: The Business and Economics Research Journal*, 4, 2 (2011) 260–269

Zitzler, E., Thiele, L., Laumanns, M., Fonseca, C.M. and Fonseca, V.G., "Performance assessment of multiobjective optimizers: an analysis and review", *IEEE Transactions on Evolutionary Computation*, 7, 2 (2003) 117–131

**List of figures and table captions**

Table 1. Inputs and output variables for major US airlines (year 2015)

| DMU | | Inputs | | | Output |
|---|---|---|---|---|---|
| | | Non-discretionary | Discretionary | | |
| IATA | Airline | Hours | Fuel | FTE | RPK |
| WN | Southwest Airlines Co. | 2,200.09 | 7,208.17 | 47.98 | 189,284.16 |
| QX | Horizon Air | 125.09 | 91.76 | 3.10 | 3,711.39 |
| HA | Hawaiian Airlines Inc. | 144.72 | 883.38 | 4.92 | 23,193.80 |
| DL | Delta Air Lines Inc. | 2,533.17 | 12,805.40 | 78.35 | 303,159.02 |
| AA | American Airlines Inc. | 2,459.27 | 11,521.81 | 79.65 | 268,866.21 |
| 8C | Air Transport Int'l | 16.10 | 84.21 | 0.28 | 81.92 |
| AS | Alaska Airlines Inc. | 491.23 | 1,662.26 | 10.73 | 48,835.47 |
| UA | United Air Lines Inc. | 2,689.09 | 12,130.42 | 78.65 | 295,514.83 |
| 5Y | Atlas Air Inc. | 119.60 | 1,317.24 | 1.86 | 1,516.89 |
| ZW | Air Wisconsin Airlines | 167.78 | 303.57 | 1.89 | 3,681.31 |
| OO | SkyWest Airlines Inc. | 835.55 | 1,602.27 | 10.00 | 27,319.30 |
| 9E | Endeavor Air Inc. | 245.23 | 578.41 | 3.28 | 8,902.24 |
| EV | ExpressJet Airlines Inc. | 719.03 | 2,568.26 | 7.97 | 20,415.27 |
| G4 | Allegiant Air | 139.05 | 567.63 | 2.58 | 14,473.70 |
| X9 | Omni Air Int'l LLC | 23.59 | 159.36 | 0.84 | 2,767.26 |
| YV | Mesa Airlines Inc. | 244.47 | 600.68 | 2.56 | 10,319.77 |
| MQ | Envoy Air | 340.29 | 678.61 | 10.53 | 8,681.39 |
| GL | Miami Air International | 7.58 | 26.27 | 0.34 | 435.75 |
| B6 | JetBlue Airways | 776.24 | 2,649.25 | 14.61 | 67,191.19 |
| NK | Spirit Air Lines | 277.34 | 931.68 | 4.34 | 28,961.11 |
| SY | MN Airlines LLC | 64.86 | 201.47 | 1.37 | 5,439.34 |
| F9 | Frontier Airlines Inc. | 218.93 | 737.49 | 2.98 | 21,557.34 |
| S5 | Shuttle America Corp. | 220.12 | 501.22 | 2.69 | 6,980.89 |
| YX | Republic Airlines | 301.48 | 717.06 | 3.14 | 10,912.09 |
| CP | Compass Airlines | 136.12 | 331.80 | 1.54 | 5,650.11 |
| VX | Virgin America | 192.40 | 639.47 | 2.64 | 16,796.38 |
| G7 | GoJet Airlines LLC | 112.38 | 260.48 | 1.18 | 3,937.35 |

Table 2. Optimal input/output slacks and objective function value (2) of the inefficient DMUs

| | Input slacks | | Output slack | Obj. function (2) |
|---|---|---|---|---|
| DMU | Fuel | FTE | RPK | |
| AA | 401.99 | 10.45 | 0.00 | 0.174 |
| 5Y | 1,254.57 | 1.38 | 0.00 | 0.128 |
| ZW | 168.01 | 1.14 | 0.00 | 0.030 |
| OO | 713.65 | 5.96 | 0.00 | 0.141 |
| 9E | 267.05 | 1.88 | 0.00 | 0.048 |
| EV | 1,869.23 | 5.13 | 0.00 | 0.231 |
| G4 | 93.57 | 0.25 | 0.00 | 0.011 |
| X9 | 48.66 | 0.06 | 0.00 | 0.005 |
| YV | 241.59 | 0.98 | 0.00 | 0.034 |
| MQ | 374.69 | 9.15 | 0.00 | 0.154 |
| SY | 6.71 | 0.40 | 0.00 | 0.006 |
| S5 | 254.56 | 1.53 | 0.00 | 0.043 |
| YX | 338.02 | 1.49 | 0.00 | 0.049 |
| CP | 129.95 | 0.55 | 0.00 | 0.019 |
| VX | 62.30 | 0.26 | 0.00 | 0.009 |
| G7 | 116.30 | 0.40 | 0.00 | 0.016 |

Table 3. SNA and MO indicators for efficient DMUs

| DMU | Crowding Distance | Spacing | Hypervolume | In-strength | PageRank | Inefficiency Radius |
|---|---|---|---|---|---|---|
| WN | 6.7125 | 0.9286 | 1.3E-05 | 0.0000 | 0.7455 | 0.0000 |
| QX | 0.0537 | 0.0366 | 3.1E-04 | 0.0000 | 0.7455 | 0.0000 |
| HA | 0.3689 | 0.0278 | 1.3E-04 | 0.1653 | 0.8876 | 0.1653 |
| DL | 0.1672 | 0.0914 | 7.2E-06 | 0.0000 | 0.7455 | 0.0000 |
| 8C | 0.0077 | 0.0073 | 8.4E-05 | 0.0000 | 0.7455 | 0.0000 |
| AS | 1.2742 | 0.2075 | 6.5E-05 | 0.0000 | 0.7455 | 0.0000 |
| UA | 0.8447 | 0.0914 | 7.4E-06 | 0.0000 | 0.7455 | 0.0000 |
| GL | 0.0349 | 0.0073 | 1.2E-03 | 0.0000 | 0.7455 | 0.0000 |
| B6 | 4.5491 | 0.2075 | 4.2E-05 | 0.0000 | 0.7455 | 0.0000 |
| NK | 0.4764 | 0.0139 | 1.7E-04 | 0.3030 | 1.2199 | 0.1857 |
| F9 | 0.0899 | 0.0095 | 2.3E-04 | 0.1928 | 0.9112 | 0.1928 |

Table 4. Normalized decision matrix, criteria weights and PIS and NIS vectors

| DMU | Crowding Distance | Spacing | Hypervolume | In-strength | PageRank | Inefficiency Radius |
|---|---|---|---|---|---|---|
| WN | 0.4604 | 0.5701 | 0.0059 | 0.0000 | 0.0830 | 0.0000 |
| QX | 0.0037 | 0.0225 | 0.1401 | 0.0000 | 0.0830 | 0.0000 |
| HA | 0.0253 | 0.0171 | 0.0572 | 0.2501 | 0.0988 | 0.3040 |
| DL | 0.0115 | 0.0561 | 0.0032 | 0.0000 | 0.0830 | 0.0000 |
| 8C | 0.0005 | 0.0045 | 0.0379 | 0.0000 | 0.0830 | 0.0000 |
| AS | 0.0874 | 0.1274 | 0.0294 | 0.0000 | 0.0830 | 0.0000 |
| UA | 0.0579 | 0.0561 | 0.0033 | 0.0000 | 0.0830 | 0.0000 |
| GL | 0.0024 | 0.0045 | 0.5222 | 0.0000 | 0.0830 | 0.0000 |
| B6 | 0.3120 | 0.1274 | 0.0186 | 0.0000 | 0.0830 | 0.0000 |
| NK | 0.0327 | 0.0086 | 0.0769 | 0.4583 | 0.1358 | 0.3415 |
| F9 | 0.0062 | 0.0058 | 0.1052 | 0.2916 | 0.1014 | 0.3545 |
| **Weight** | 0.1894 | 0.2060 | 0.1865 | 0.2033 | 0.0204 | 0.1944 |
| **PIS** | 0.4604 | 0.5701 | 0.5222 | 0.4583 | 0.1358 | 0.3545 |
| **NIS** | 0.0005 | 0.0045 | 0.0032 | 0.0000 | 0.0830 | 0.0000 |

Table 5. Distance to PIS and NIS, closeness/farness ratio and ranking obtained by proposed approach

| DMU | $d_j^+$ | $d_j^-$ | $T_j$ | Rank |
|---|---|---|---|---|
| WN | 0.3419 | 0.3255 | 0.4877 | 1 |
| QX | 0.4424 | 0.0597 | 0.1188 | 8 |
| HA | 0.3854 | 0.1771 | 0.3149 | 5 |
| DL | 0.4583 | 0.0239 | 0.0496 | 10 |
| 8C | 0.4657 | 0.0149 | 0.0311 | 11 |
| AS | 0.4233 | 0.0683 | 0.1390 | 7 |
| UA | 0.4500 | 0.0343 | 0.0707 | 9 |
| GL | 0.4157 | 0.2241 | 0.3503 | 4 |
| B6 | 0.3988 | 0.1467 | 0.2690 | 6 |
| NK | 0.3696 | 0.2582 | 0.4112 | 2 |
| F9 | 0.3778 | 0.2090 | 0.3561 | 3 |

Table 6. Ranking of Liu et al. (2009, 2010), Leem and Chun (2015) and Super-SBM (Tone 2002)

| | Liu et al. approach | | Leem and Chun approach | | Super-SBM | |
|---|---|---|---|---|---|---|
| **DMU** | **Eigenvector centr.** | **Rank** | **PageRank centr.** | **Rank** | **Score** | **Rank** |
| WN | 0.3708 | 10 | 1.2560 | 5 | 1.0417 | 3 |
| QX | 0.2214 | 5 | 1.0000 | 9 | 1.1506 | 1 |
| HA | 0.1264 | 4 | 1.0797 | 6 | 1.0000 | 10 |
| DL | 0.0652 | 9 | 1.5940 | 4 | 1.0278 | 5 |
| 8C | 0.9491 | 6 | 1.0000 | 9 | 1.0000 | 10 |
| AS | 1.0000 | 7 | 1.0000 | 9 | 1.0143 | 6 |
| UA | 0.6302 | 11 | 1.0000 | 9 | 1.0123 | 7 |
| GL | 0.8658 | 1 | 8.4521 | 1 | 1.0000 | 10 |
| B6 | 0.5244 | 8 | 1.0000 | 9 | 1.0017 | 8 |
| NK | 0.0196 | 3 | 2.0857 | 3 | 1.0879 | 2 |
| F9 | 0.0349 | 2 | 5.1325 | 2 | 1.0415 | 4 |

Table 7. Spearman's rank correlation coefficients

| | **Proposed approach** | **Liu et al.** | **Leem and Chun** | **Super-SBM** |
|---|---|---|---|---|
| **Proposed approach** | 1.000 | 0.382 | 0.610 | 0.312 |
| **Liu et al.** | - | 1.000 | 0.582 | -0.083 |
| **Leem and Chun** | - | -- | 1.000 | 0.106 |
| **Super-SBM** | | - | | 1.000 |

Figure 1. Layered layout of proposed DN

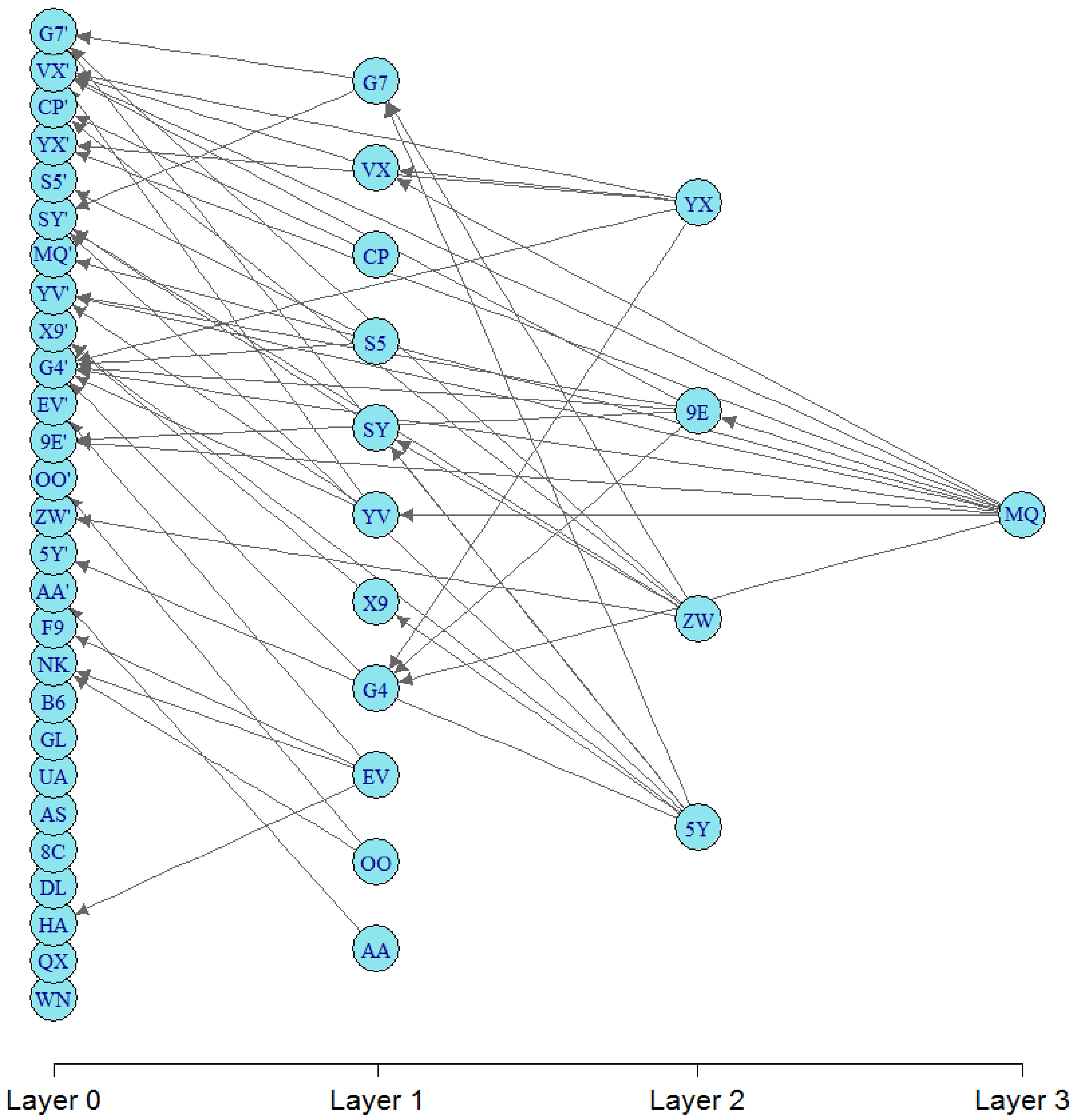

Figure 2. Liu et al. (2009, 2010) network

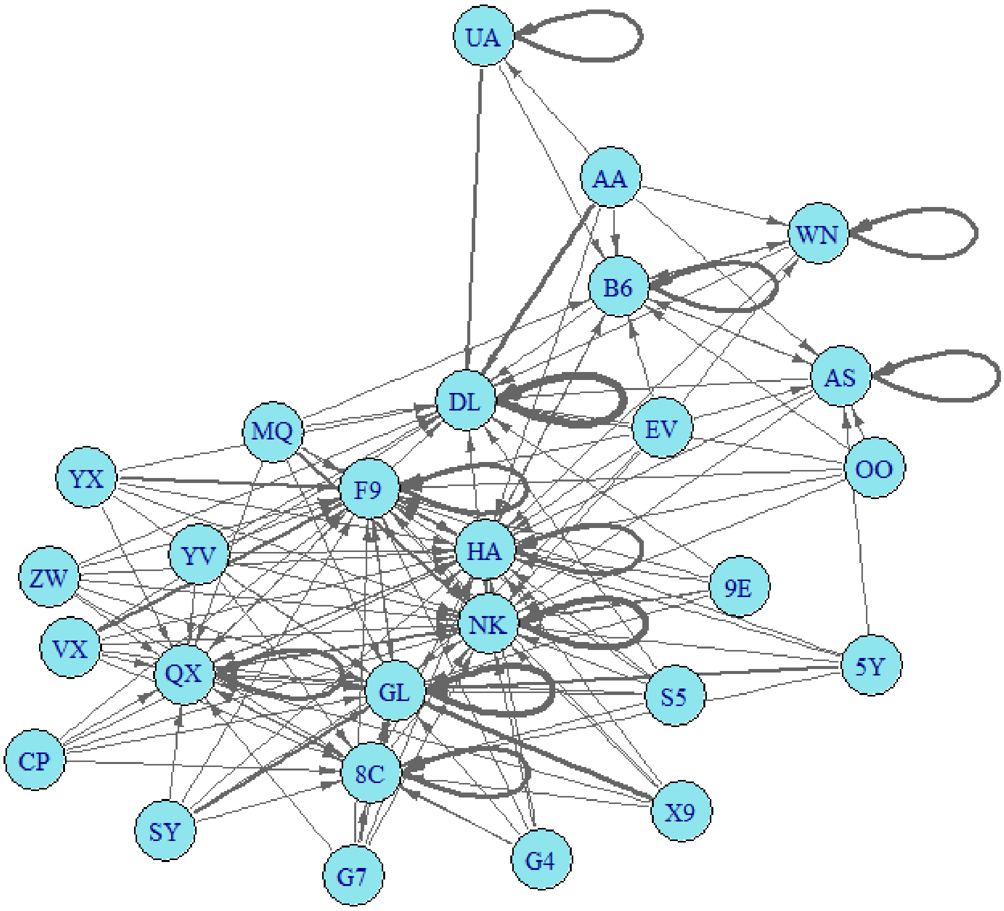

Figure 3. Leem and Chun (2015) network