

# Formal Methods to Comply with Rules of the Road in Autonomous Driving: State of the Art and Grand Challenges <sup>★</sup>

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## Abstract

We provide a review of recent work on formal methods for autonomous driving. Formal methods have been traditionally used to specify and verify the behavior of computer programs and digital circuits. Enabled by abstraction techniques for dynamical systems and the availability of verification and synthesis tools for finite systems, they have been adopted by the control and robotics communities. In particular, in autonomous driving, recent research proposes formal languages such as temporal logics to specify driving behaviors ranging from safety, such as collision avoidance, to compliance with complex rules of the road. Our review focuses on formal verification, monitoring, and synthesis techniques enabling autonomous vehicles to adhere to such specifications. We only consider works about system-level methods that have an ego-centric perspective, i.e., we focus on the behavior of an autonomous vehicle in its entirety, rather than specific software code within the vehicle or traffic networks consisting of multiple vehicles. This paper also identifies the main remaining challenges.

*Key words:* autonomous driving; formal methods; temporal logic; formal verification; formal synthesis; falsification; monitoring; machine learning.

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## 1 Introduction

The development and integration of cyber-physical and safety-critical systems in various engineering disciplines requires their verification and control with respect to rich specifications. A prominent example is autonomous driving, which received a lot of attention during the last decade. Autonomous vehicles (AVs) aim to optimize common control objectives, such as minimizing the energy consumption and travel time, and satisfy constraints on control variables, such as maximum acceleration. In addition, AVs aim to drive safely and follow the

rules of the road (ROTRs), which include traffic laws and other informal rules or cultural expectations of reasonable driving behavior. For example, an AV tries to avoid collisions with other road users, avoid obstructing traffic, maintain longitudinal clearance with the lead vehicle, yield when required, and stop at red lights and stop signs. These rules could be prioritized, e.g., by specifying that maintaining clearance to pedestrians is more important than staying in lane, which, in turn, takes precedence over observing the maximum speed limit. There currently exists no consensus on how and to what extent AVs should follow such complex (possibly prioritized) driving specifications.

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Formal methods is an area of computer science, traditionally focused on checking the correctness of digital circuits and computer programs. Correctness can pertain to safety (something bad should never happen), liveness (something good should eventually happen), or general statements expressed as formulas of Temporal Logics

(TL), such as Linear Temporal Logic (LTL), Computation Tree Logic (CTL) [1, 2], or Signal Temporal Logic (STL) [3]. Due to the high expressivity of these specification languages, the existence of verification, monitoring, and control synthesis tools for finite systems, and recent developments on abstractions for systems with infinite state spaces, formal methods have been adopted by the control community, and successfully used for dynamical [4–6] and autonomous systems [7, 8].

In particular, there is a growing body of work on the use of formal methods for autonomous driving. TMs have been proposed for the formal specification of safety requirements and complex ROTRs. Formal verification, monitoring, and synthesis techniques have been used for analysis and control of autonomous driving behaviour. Machine learning algorithms have been employed to infer formal rules describing ROTRs and desired behaviors from data, to assess the relative importance of different formal rules from data, and to generate driving strategies.

In this paper, we review the literature on formal methods used for autonomous driving. The focus is on studies that consider ROTRs from the ego vehicle’s point of view. We *do not include* traffic networks, e.g., traffic light control, conflict resolution at intersections, congestion control, and merging control. We also focus on formal methods at the system level. We do not consider formal methods for the software running on the autonomous cars.

Recent review papers covering the fast growing field of autonomous driving include [9–16]. A comprehensive review of a broad range of topics, including system architectures, localization, mapping, perception, planning, and human machine interfaces is provided in [11]. The work in [12] also provides a general overview of the field, with particular emphasis on planning, but does not survey the state of the art in formal methods for autonomous driving. With particular relevance to our review, [12] includes a discussion on formal methods for planning. A comprehensive review of the state of the art in software verification and validation of AVs is provided in [13]. Another comprehensive review, which includes a discussion on safety verification of controllers for AVs, is provided in [15].

Among more focused reviews, [9] surveys scenario-based approaches, in which individual traffic situations are tested through simulation. The focus is on safety assessment. A literature review and analysis of threat-assessment methods used for collision avoidance is presented in [16], including the use of formal methods. The focus of [10], which reviews a limited number of papers, is on human cyber-physical systems, with particular focus on semi-autonomous driving, and formal methods. Finally, [14] introduces a control and planning architecture for connected vehicles and AVs and surveys the state of the art on each functional block therein.

Compared to the survey papers covering general topics in autonomous driving mentioned above, this review focuses solely on formal methods. It provides a comprehensive overview of the state of the art that is more in-depth and detailed than the reviews referenced above, which only contain sections of formal methods. Finally, it covers very recent papers in the fast growing field of autonomous driving, with specific emphasis on formalization of ROTRs.

The remainder of this paper is organized as follows. In Sec. 2, we review the methods used to formalize ROTRs and other driving behaviors. We discuss the formal verification approaches used to analyze vehicle models and behaviors from such formal specifications in Sec. 3. We review monitoring algorithms and formal synthesis strategies in Secs. 4 and 5, respectively. We discuss remaining challenges in the field in Sec. 6 and conclude with final remarks in Sec. 7.

For quick reference, each section concludes with a table that lists all the papers cited in that section. Its columns represent the particular categories covered in that section. For example, Table 4 from Sec. 4 that covers monitoring, has two columns “Offline” and “Online”, corresponding to the two monitoring techniques from the reviewed papers. Each table has three rows, which correspond to the three application areas: “Vehicle following”, “Lane keeping / changing”, and “Other”. Papers listed under “Vehicle following” and “Lane keeping / changing” focus on the respective application only. The “Other” category corresponds to papers that discuss at least one application area different from the ones listed above (e.g., pedestrian clearance or speed limit), or that includes discussion of both “Vehicle following” and “Lane keeping / changing”. We believe that these tables makes it easy for the reader to find a paper using a specific technique in a particular application area. For example, if she wants to find a paper that uses online monitoring from STL specifications with applications to lane keeping, then she would use Table 4 to find the papers that apply online monitoring to lane keeping: [17] and [18]. She would then check which of these papers appear at the intersection of the “STL” column and the “lane keeping / changing” row in Table 2. In this example, it turns out that [18] is the only paper that meets these specific criteria. Note that, given the organization of the review, a paper can appear in several sections and tables.

## 2 Formal specifications

Most ROTRs are stated in natural language in traffic legislations or driving manuals, which can differ among countries and localities within countries. Some ROTRs can be formalized as simple *safety specifications* that guarantee safety when satisfied. For example, a safety specification might be a formal rule that the ego vehicle

Table 1  
List of frequently used abbreviations.

Abbreviation	Description
ACC	Adaptive Cruise Control
AV	Autonomous Vehicle
CBF	Control Barrier Function
CLF	Control Lyapunov Function
CTL	Computation Tree Logic
ICS	Inevitable Collision State
LTL	Linear Temporal Logic
MI(L)P	Mixed-integer (Linear) Programming
MPC	Model Predictive Control
MTL	Metric Temporal Logic
QP	Quadratic Program
ROTR(s)	Rule(s) of the road
RRT	Rapidly-exploring Random Tree
RSS	Responsibility-Sensitive Safety
STL	Signal Temporal Logic
TL	Temporal Logic

should maintain a given minimum clearance from pedestrians on the road for all times. Safety specifications are sometimes given equivalently as reachability specifications. In the above example, the reachability specification is that the ego vehicle can only reach distances to pedestrians that are larger than the minimum clearance.

Safety specifications are a particular case of TL specifications, which specify formal rules that can also express eventuality (e.g., “reach destination in at most 10 min”, “maintain a speed less than 25 mph until the end work zone sign is reached”), logical conditions (“use the left lane only when passing”), and combinations of the above. Formal rules are sometimes prioritized. For example, a safety specification such as “maintain clearance from pedestrians at all times” might have precedence over “reach destination in at most 10 min”. In Sec. 2.1, we briefly discuss safety specifications. Richer, TL formalisms are covered in Sec. 2.2. Papers dealing with rule priorities are reviewed in Sec. 2.3.

It is important to note that, throughout the paper, “safety specification” refers to a formal specification of the form: “for all times, an undesired outcome never happens”. In other words, as stated above, it is just a particular type of a TL formula. Safety in this context is not necessarily a stringent requirement such as collision avoidance. For example, “Stay in lane for all times” is a safety specification in formal methods (and in this survey) but not a safety requirement since changing lanes is a perfectly acceptable behavior in many driving situations (e.g., when preparing for a left turn).

## 2.1 Safety specifications

In most of the papers included in this review, the formal specification is given simply as a safety specification only (see Table 2). Therefore, even though safety is just a specific kind of temporal logic formula, we dedicate Section 2.1 and the first column of Table 2 to such papers. The most predominant safety specifications are collision avoidance, maintaining a minimum clearance from the preceding car, staying in lane and / or on the road. In the rest of this section, we briefly discuss works that provide safety specifications using control theoretic or motion planning concepts, or combine safety specifications with other specifications, such as lawfulness.

The safety specifications in [65–67] refer to avoiding collisions with static and dynamic obstacles, and are formalized using Inevitable Collision States (ICS) (i.e., states for which, no matter what the future trajectory followed by the ego vehicle is, a collision with an obstacle eventually occurs [131]). ICS are used to enforce safety during motion planning. Predictions of future occupancies for surrounding traffic participants are used for safety specifications in [68–73, 105], and applied to the influential Responsibility-Sensitive Safety (RSS) modeling framework [24] in [74].

Positive invariant sets (e.g., sets that are guaranteed to contain all trajectories of the vehicle for all times) are used to formalize safety in some works. In [41], these are used to ensure that the ego vehicle stays on the road. Positive invariant sets are also used in [106] to prove safety as defined through velocity and obstacle collision constraints. Control invariant sets (i.e., sets that are made positive invariant using control) are used in [35, 47, 75, 77]. The more recent, closely related concept of control barrier functions (CBF) is used in [25, 26, 30]. Compositional and contract-based principles are used for formal verification of safety specifications [78, 79].

Finally, safety specifications are combined with lawfulness and liabilities of traffic participants in [27, 42, 80, 81]. The authors of [27, 42, 81] focus on liabilities of traffic participants if a collision occurs using formal rules based on the Vienna Convention on Road Traffic. The concept of legal safety is defined in [80] as a set of rules that could safely and efficiently manage mixed traffic of human drivers and AVs, and illustrated for automated driving on highways with distance keeping, speed adaptation, and lane-changing. Requirements induced by legal safety on perception and control components are also presented. Legal safety is used in [94] for critical urban scenarios, which have been recorded in real traffic.

## 2.2 TL specifications

Most of the reviewed papers use standard TLs, such as LTL and a common fragment called syntactically

Table 2

Papers organized by type of formal specification and application area. **Safety only** refers to papers that only specify safety.

	Safety only	LTL	STL	MTL	Other TLs	Priorities
Vehicle following	[19–35]	[36]		[37, 38]		[23]
Lane keeping / changing	[17, 39–48]		[18, 49]			
Other	[50–107]	[108–113]	[114–121]	[122, 123]	[124–126]	[79, 82, 110, 115, 126–130]

co-safe LTL (scLTL) [132], STL [3], and MTL [133]. Others propose new logics, specifically tailored for formalizing ROTRs [124–126]. With few exceptions (e.g., [81, 112, 124, 134]), which use high-order temporal logic (i.e., logics that allow for universal and existential quantifiers), all the reviewed works focus on propositional and predicate temporal logics.

Informally, LTL formulas are made of three ingredients: (1) atomic propositions (e.g.,  $por =$  “pedestrian on the road”) or predicates (e.g.,  $v_{ego} < 30 =$  “the ego vehicle’s speed is less than 30 miles per hour”); (2) Boolean operators  $\vee$  (disjunction),  $\wedge$  (conjunction),  $\neg$  (negation), etc.; and (3) temporal operators, such as  $\mathbf{G}$  (globally, or always),  $\mathbf{F}$  (in the future, or eventually),  $\mathbf{X}$  (next), and  $\mathbf{U}$  (until). For example, the LTL formula  $\mathbf{G}(por \rightarrow (sd \mathbf{U} pos))$ , reads “for all times, if a pedestrian is on the road, slow down until she reaches the sidewalk” ( $por$ ,  $sd$ , and  $pos$  are propositions that are true when the pedestrian is on the road, the ego vehicle slows down, and the pedestrian is on the sidewalk, respectively). LTL formulas are interpreted over infinite executions. scLTL is a strict fragment of LTL, in which the satisfaction of formulas can be decided in finite time [132]. For example, formula  $por \rightarrow (sd \mathbf{U} pos)$  is in scLTL, while  $\mathbf{G}(por \rightarrow (sd \mathbf{U} pos))$  is in LTL but not in scLTL. For both LTL and scLTL, time is abstract, i.e., only the order of the events matter.

MTL is an extension of propositional LTL, in which time is concrete, and formulas can refer to both past and future times. Informally, the main difference is that the temporal operators are timed. For example, the requirement that the ego vehicle slows down and comes to a complete stop within 5 seconds translates to the MTL formula  $sd \mathbf{U}_{[0,5]} stop$ , where  $sd$  is the same as above and  $stop$  is a proposition that is true when the ego vehicle stops. STL is an extension of LTL with real-time and real-valued constraints, and its formulas are usually over predicates. For example, a formal specification to comply with the maximum speed limit is written in STL as  $\mathbf{G}_{[0,T]}(v(t) < v_{max})$ , where  $v(t)$  is the ego vehicle’s speed at time  $t$ ,  $v_{max}$  is the maximum speed limit, and  $T$  is the total duration of the scenario during which compliance with this specification is evaluated. In addition to Boolean semantics, in which a word or signal satisfies or violates a formula, MTL and STL have quantitative semantics. This is defined using a robustness function that

gives the degree of satisfaction of a formula by a word or signal. Many papers reviewed below use the robustness function for monitoring and / or controller synthesis.

**LTL** ROTRs based on the German concretization of the Vienna Convention on Road Traffic are formalized using LTL in [108]. The focus is on dual carriageways, such as highways, and the formulas are restricted to the particular form “premise implies conclusion”, or  $\mathbf{G}(\phi^p \rightarrow \phi^c)$ , where  $\phi^p$  is the premise and  $\phi^c$  is the conclusion. An LTL formula of this form states that “at all times, if  $\phi^p$  is True, then  $\phi^c$  must be True”. The authors provide algorithms for constructing such formulas from ROTRs with the help of graphical representations. To define semantics for the LTL formulas over vehicle trajectories, the atomic propositions are concretized to predicates, e.g., atomic proposition  $acc^{(i)}$  corresponds to predicate “ $i$  accelerates with  $a > a_{lim}$ ”. A related approach, for a related set of German ROTRs, is proposed in [109], where the focus is on overtaking. In this work, the LTL formulas are more general and the predicates are concretized through legal and engineering analyses.

LTL specifications obtained by interpreting relevant Adaptive Cruise Control (ACC) standards are considered in [36] and used to produce correct-by-construction controllers. LTL formulas over a semantic abstraction obtained by partitioning the continuous state space corresponding to a traffic scenario are considered in [111], where the authors consider a subset of the Vienna Convention of Road Traffic that cover the interaction of the ego vehicle with only one other traffic participant. In the related work [112], the authors perform AV motion planning from ROTRs expressed as LTL formulas, which are interpreted over maneuver automata, and allow for automatic satisfiability checking. LTL is proposed in [113] to specify a small set of ROTRs, together with a quantitative semantics used for reinforcement learning. scLTL is used in [110] to formalize a set of ROTRs that need to be satisfied, while customer demands (e.g., pick ups, drop offs) are met within desired deadlines.

**STL** Our review shows that STL is the preferred logic for specifying ROTRs. One of the main advantages of

STL is its quantitative semantics, which allows for monitoring, and also to map verification and control synthesis problems to optimization problems [114, 121]. Recent work also points to an interesting connection between the quantitative semantics of STL and deep learning for autonomous driving. The work in [115] represents ROTRs as STL formulas and uses its quantitative semantics and a deep learning framework to predict future behavior of nearby vehicles and to recognize the importance of predefined formal rules. Using a parameterized version of STL (pSTL), [116] proposes a method for integrating TL formulas into a neural network, which allows incorporating ROTRs into deep learning-based trajectory prediction approaches. This framework is extended in [117], where ROTRs expressed as STL formulas are integrated as inductive biases into deep learning-based prediction models.

A requirements-driven approach for test case generation is proposed in [118, 119], which covers both component-level and system-level behaviors for an AV. Test cases are evaluated against STL formulas and the requirements are used to automatically discover test cases that fail to satisfy the requirements. The related work [120] describes an approach for finding interpretable failures of an AV system. The failures are described as STL formulas and optimization is used to produce likely failures.

Recent works showed that STL can be efficiently used to formalize assume-guarantee conditions. In particular, in [18], the author develops a set of contracts for control software for AVs ensuring that if all traffic participants follow the contracts (i.e., the assumption), then the overall traffic system is collision-free (i.e., the guarantee). In [49], it is shown that RSS assumptions can be encoded in assume-guarantee logical conditions in STL, which enables the use of verification and testing tools to verify and validate AV compliance with RSS.

**MTL** MTL is used in [37] to formalize ROTRs for interstates based on the German Road Traffic Regulation, the Vienna Convention on Road Traffic, and legal decisions from courts. In this paper, the authors also use first-order logic to define the predicates and functions used in the formulas. Specifications for a case study using the RSS model are given as MTL formulas in [38]. The work in [122] proposes a scenario description language to create driving scenarios with different numbers of agents and on different road topologies, which also enables the specification of formal correctness specifications in MTL. A future-bounded, propositional MTL is used in [123] to specify correctness properties for components of an AV.

**Other TMs** Three out of the 20 reviewed papers that use TMs for formalizing ROTRs propose new log-

ics specifically tailored to autonomous driving. The work in [124] introduces Timed Quality Temporal Logic (TQTL), an extension of STL, to monitor and test the performance of object detection and situation awareness algorithms. An example of a vision quality requirement in this framework is “at every time step, for all the objects  $id$  in the frame, if the object class is cyclist with probability more than 0.7, then in the next 5 frames the object  $id$  should still be classified as a cyclist with probability more than 0.6”. A probabilistic TL, called Chance Constrained Temporal Logic (C2TL), is proposed in [125] to specify correctness requirements in the presence of probabilistic uncertainty. The main addition of C2TL over STL is the inclusion of chance constraints as predicates. A chance constraint is a probabilistic extension of deterministic predicates and is of the form  $Pr(\phi_{det}) \geq 1 - \delta$ , where  $0 \leq \delta \leq 1$  represents uncertainty about whether the inequality holds and  $\phi_{det}$  is a Boolean combination of linear predicates, where the coefficients are random variables with Gaussian probability distributions. Finally, a version of LTL, called stutter-invariant Finite Linear Temporal Logic (si-FLTLGX), is introduced in [126]. While sufficient to describe many ROTRs, si-FLTLGX also allows for prioritized ROTRs and for efficient computation of optimal motion plans through sampling.

### 2.3 Rule priorities

Only a few studies deal with the formal specification of multiple potentially competing driving objectives. The work in [23] encodes a specification of an adaptive cruise controller that ensures compliance with safety specifications while maintaining comfortable control actions. In [80], the authors discuss the need to not *a priori* exclude trajectories that violate a formal rule like staying in the driving lanes, because in an emergency such a trajectory might be necessary. However, they stop short of specifying rule priorities. In [79], the authors ensure the satisfaction of safety specifications and eight formalized ROTRs. They encode an implicit notion of rule priorities by relaxing a subset of the rules in some experiments. The work in [127] considers a set of safety specifications as formal rules and defines a notion of global minimization of rule violation based on discrete priority levels for each formal rule. While all of these studies consider multiple formal ROTRs, none of them explicitly captures priorities.

The work in [128] provides a general framework to specify how an AV can transparently resolve conflicts between formal rules using a priority structure. The proposed priority structure is a pre-ordered set of formal rules, which induces a pre-order on any set of potential trajectories in a scenario. Several studies build on this framework to develop algorithms for planning [126] or control [82, 110]. In [82], the authors propose an offline methodology for pass/fail evaluation of AV behavior

to determine whether a given AV trajectory complied with a priority structure of formal rules. They do so by defining a candidate AV trajectory as non-compliant if another trajectory exists that violates only lower priority rules than the candidate AV trajectory, which they determine through iterative relaxation of the rules.

Some studies explore the use of a learned priority structure among various formal rules. For example, [115] learns the margins of satisfaction for formal rules and then applies them in Model Predictive Control (MPC) of the ego vehicle and surrounding vehicles. Another study [129] queries pairwise preferences between trajectories to learn the weights that can be viewed as quantitative measures of how well a trajectory satisfies rules for staying on the road and avoiding collisions. The work in [130] creates a dataset consisting of 92 traffic scenarios and used crowd-sourced annotations to compare an instance of the rulebook pre-ordered priority structure from [128] with models obtained using machine learning with varying degree of interpretability, such as Bayesian networks, decision trees, and logistic regression.

### 3 Formal verification

Formal verification is the process of verifying that all the possible executions of a system satisfy a formal specification, such as safety or a TL formula. While autonomous system verification can proceed with incomplete or gray-box models by combining statistics with structural reasoning (see [99, 100] for treatments of such approaches), in this review we focus on a formal, traditional approach to verification that requires a model of the system. The model typically consists of the ego vehicle and its environment in autonomous driving. Online verification is performed during the execution of the system and it only requires checking the satisfaction of a specification against all possible behaviors originating at the current time.

Since finding a suitable non-deterministic model of the environment as well as formalizing all ROTRs are challenging, most formal verification methods focus on vehicle following (which includes ACC, emergency braking systems, and platooning) and lane keeping / changing. In the first part of this section, we focus on these types of maneuvers. Afterwards, we discuss more general methods for arbitrary traffic situations. A summary of the reviewed papers is listed in Table 3. All reviewed papers performing formal verification use safety as specifications. Consequently, we do not list the considered type of specification in Table 3. Instead, we list whether the method is applied offline (during design time) or online (during vehicle operation) and whether the approach can be applied to mixed traffic, i.e., traffic with autonomous vehicles, manually-driven vehicles, and other forms of non-automated movements, such as riding a bicycle or walking. Some approaches require that all vehicles are

autonomous or that the behavior of other traffic participants is known. For instance, some papers assume that a leading vehicle moves with constant velocity. These would not necessarily prove safety in mixed traffic.

**Theorem proving** The first formally-correct controllers have been developed for vehicle following and verified using handwritten proofs, see e.g., [19, 32, 33]; an extension to game-theoretic techniques for cooperative controlled vehicles is presented in [31]. Lane following is especially amenable for handwritten proofs since it only requires one-dimensional movement along a lane, and the corresponding dynamics are monotone [135]. To avoid human error in proofs, a theorem prover is used in [20, 83], which, however, assumes that all vehicles are automated. Theorem proving has also been extended to prove the safety of lane changes by reserving space for vehicles [40, 62]. An advantage of theorem proving is that the number of traffic participants is unbounded, however, it typically cannot be used for online verification, because most theorem provers are not fully automatic.

**Barrier certificates** Barrier certificates verify systems by proving that a barrier between the set of initial states and unsafe states always exists. This idea was applied to ACC [25] and was experimentally validated in [30]. An extension for varying velocities of the leading vehicle and lane keeping is presented in [36] and [26], respectively. Barrier certificates are particularly useful for proving the correctness of specific controllers, such as controllers for following vehicles and staying within a lane (the construction of the controllers is discussed in Sec. 5). So far, no approach has been presented to automatically create barrier certificates for a given traffic situation so that no universal online verification scheme has yet been realized.

**Worst-case behaviors** Due to the previously-mentioned monotone dynamics of vehicles staying within the same lane, vehicle-following problems can be verified through worst-case behaviors. Those are used to safeguard exchangeable nominal controllers, by embedding them in an emergency controller that only engages if the nominal controller performs an unsafe action [22, 84]. This idea is also applied to vehicle platooning [29] and was later extended to handle cut-in vehicles and also lane changes of the leading vehicle [23]. A lane change of the leading vehicle can suddenly reveal an occluded obstacle, which either requires detecting further vehicles ahead or assuming a standing obstacle within the occluded region.

By conservatively separating the dynamics into a lateral and longitudinal dynamics, one can also use worst-case behaviors for lane changes. The work in [42] uses safe

distances to ensure that lane changes are safe, despite the uncertainty in the movement of other traffic participants. The special case of a fixed sinusoidal lane change and given accelerations of surrounding traffic participants is shown in [43]. The approach in [42] is also used to safeguard reinforcement learning for lane changes [44]. Additional formal rules are added in [45] under which a lane change is deemed to comply with ROTRs. As a special case, swerve maneuvers are verified in [85].

**Reachability analysis** Currently, the most popular verification technique for AVs is reachability analysis. Reachability analysis automatically verifies systems by computing the set of reachable states. If no reachable state enters an unsafe region, safe behavior is proven [28,34]. Invariant sets are a special case of reachable sets in which the state of a system stays indefinitely. Thus, if the invariant set does not contain any unsafe states, correctness can be proven analogously to reachability analysis. Most of the reviewed papers that use reachability analysis and invariant sets perform both verification and control synthesis. Here, we focus on verification. We revisit some of these papers and discuss their control strategies in Section 5.

Safety is proven for ACC in [35, 39, 60, 77] using invariant sets. Since the relatively simple task of safe vehicle following can be verified by handwritten proofs, only [21] verified this problem using reachability analysis. Another sub-problem we consider is the problem of verifying whether a safe solution still exists. This can be used to prove that an aggressive evasive maneuver has to be executed or that a collision can no longer be avoided and a collision mitigation procedure needs to be initiated. The work in [86] computed the reachable set of the ego vehicle to check whether it becomes empty—in this event, the ego vehicle is in an inevitable collision state [67]. To reduce the conservatism of that work, the velocity information within the reachable set and road geometry are explicitly considered in [73]. This work was later extended to compute the time to react in a formal way, i.e., the remaining time to avoid a potential collision [87]. Instead of determining whether the current state is an inevitable collision state, one can also compute the set of inevitable collision states [66]; however, this is computationally expensive and thus currently not real-time capable. The work in [59] does not only compute the first point in time when a collision is possible, but also the last point in time.

The set of possible scenarios for fully autonomous driving cannot be constrained in the same way as it is done for vehicle following or lane changing. Thus, most approaches compute reachable sets online for fully autonomous driving so that all occurring situations are considered—an offline procedure might have missed potentially dangerous situations. To the best knowledge of

the authors, the first work using online reachability analysis for autonomous driving is [51]. The disadvantage of that work is that it requires that vehicles communicate with each other and that they have to travel with constant velocity. These restrictions were later removed in [88]; however, the used vehicle model is just a single-track model. A method to consider high-dimensional models through non-deterministic low-dimensional models is presented in [46,89]; this approach is extended in [58] to show conformance with real vehicles. The first work that applied online reachability analysis to a real vehicle is [70]; later works can be found in [90–94]. Although this approach works in principle for all kinds of traffic situations, it does not contain an algorithm for computing the reachable set of other traffic participants on arbitrary road networks—this is addressed in [68] and implemented by the tool SPOT [71]. Further developments, in particular with respect to handling occlusions, are presented, e.g., in [69,95–97]. Online reachability analysis was recently used to safeguard reinforcement learning for AVs [48]. To engage safe but aggressive maneuvers more comfortably, the work in [98] additionally considers probabilistic information to slow down the AV early when a dangerous situation is likely to occur. An approach that combines ideas from contract-based verification with reachability analysis is presented in [78,79]; however, this approach is not yet real-time capable. Other approaches, such as [52,99,122], are primarily designed for formal offline verification for specific scenarios. To reduce computation times for online use, some approaches consider exemplary traces instead of the set of possible solutions [111].

Table 3

Papers covering verification techniques organized by application areas, suitability for online use, and applicability in mixed traffic.

	Online	Mixed traffic	References
<b>Vehicle following</b>	✗	✗	[20, 21, 25, 35, 83]
	✗	✓	[19, 30–34, 36]
	✓	✗	[28]
	✓	✓	[22, 23, 29, 84]
<b>Lane keeping / changing</b>	✗	✗	[40, 62]
	✓	✗	[43, 48]
	✓	✓	[42, 44, 45, 85]
<b>Other</b>	✗	✗	[39, 66, 77]
	✓	✗	[51]
	✗	✓	[26, 52, 60, 99, 122]
	✓	✓	[46, 58, 59, 67–69, 69–71, 73, 78, 79, 86–97, 101, 102]

## 4 Monitoring

Monitoring (or runtime verification) refers to lightweight formal verification methods designed to check system executions against formal requirements. The main difference from the verification approaches discussed in Sec.

3 is that the latter reason over all possible system executions and uncertainties. Online monitoring refers to checking the current execution of a system, while offline monitoring is the process of checking a (finite set of) recorded execution(s). In most monitoring applications, including autonomous driving, execution traces are long, and are only available incrementally. Waiting for and storing an entire execution trace and then performing offline monitoring can be expensive. Moreover, in offline monitoring, verification might occur too late to allow the system to recover or take a shutdown action. For this reason, online monitoring is the prevalent technique in autonomous driving.

**Safety specifications** Monitoring for compliance with safety specifications is presented in [17, 63, 64, 134]. The authors of [17] use backward reachability analysis to construct the monitor. Monitoring for Multi-Lane Spatial Logic is considered in [134], where the authors show that formula satisfaction can be mapped to feasibility of formulas in the first-order theory of real-closed fields. Runtime monitoring techniques based on predictions of future behaviors of traffic participants are developed for safety specifications in [63, 64]. The authors of [63] present a pedestrian intent estimation framework that can predict future pedestrian trajectories, and integrate it into a reachability-based online monitoring and decision making scheme. A predictive runtime monitoring method for estimating future vehicle positions and the probability of collisions with obstacles is presented in [64]. Their approach combines Bayesian inference techniques and set-valued reachability analysis to approximate future positions of a vehicle.

**LTL** Similar to verification and synthesis, most monitoring techniques against LTL formulas require converting the LTL formula to an automaton. Depending on the structure of the formula, this automaton can be a finite state automaton, a Büchi automaton, or a Rabin automaton. Monitoring against ROTRs expressed as LTL formulas is performed in [108, 109, 111]. The authors of [109] focus on overtaking and safe distance keeping. In [108], ROTRs are modeled as objects called Rule-Monitors, which are then used to monitor rule compliance through simulation and comparison against a public dataset. In [111], the authors develop an LTL offline monitoring method that does not require the computation of a complete automaton from the specification and the partition of the ego vehicle’s continuous environment, but rather constructs a smaller automaton corresponding to a specific traversal of the quotient graph.

**STL and MTL** As mentioned previously, STL and MTL are particularly fit for monitoring due to their quantitative semantics (i.e., robustness functions that

Table 4

Papers organized by type of monitoring and application area (there are no works in the “Vehicle following” application area).

	Offline	Online
<b>Lane keeping / changing</b>	[49]	[17]
<b>Other</b>	[109, 111, 134]	[63, 64, 108, 114, 121, 123]

quantify the degree of satisfaction or violation with respect to a formal specification). In [49], the authors encode the RSS model in STL, and perform monitoring of two RSS specifications (i.e., keeping a safe distance to front and side vehicles) on traffic scenarios from CommonRoad [136] using S-TALIRO [137]. STL specifications for vehicle following are encoded using a special, block-sparse Mixed Integer Programming (MIP) problem structure in [121], which is exploited to increase the efficiency of the computation involved in monitoring.

The authors of [114] propose STL monitoring to compute corrections in a two-level AV control architecture. At the top level, simple representations of the environment and vehicle dynamics are used to derive controllers using an MPC approach. At the bottom level, STL runtime monitoring techniques, together with detailed representations of the environment and vehicle dynamics, are used to compensate for the mismatch between the simple models used in the MPC and the real complex models.

A runtime monitoring algorithm that checks for violations of properties written in a future-bounded, propositional MTL by an experimental AV is presented in [123]. The algorithm incrementally takes as input a system state, which maps propositions to either true or false, and a MTL formula, and eagerly checks the state trace for violations. It uses an iteration based on dynamic programming to reduce the input formula as soon as possible using history - summarizing structures and formula-rewriting-based simplifications.

## 5 Control synthesis

The control synthesis problem is to find controllers for AVs that minimize a cost, while satisfying physical constraints and formal rules. This section presents commonly used approaches for formal control synthesis for AVs. We first review papers that use automata-based techniques for control from specifications given in LTL or fragments of LTL. We then focus on optimization-based approaches that exploit the quantitative semantics (robustness) of concrete-time temporal logics such as STL (see [138] for a review and comparison of automata-based and optimization-based approaches to formal synthesis) and on papers that use Control Barrier Functions (CBF)

and Control Lyapunov Functions (CLF). The most popular formal synthesis techniques for AV control involve reachability analysis and invariant sets, and most of this section reviews such papers. Finally, we review papers using falsification techniques and machine learning. A summary of the reviewed papers is listed in Table 5.

**Automata-based synthesis** For formal rules written in LTL and fragments of LTL, the formal synthesis problem can be mapped to solving an automaton game. In short, this method is based on translating the specification to an automaton, such as a Finite State Automaton (FSA), Büchi automaton, or Rabin automaton, and then combining this with a finite abstraction of the dynamics of the system. The control strategy is generated by graph analysis, or by solving an automaton (Büchi or Rabin) game [5].

The authors of [110] propose a receding-horizon approach to synthesize controllers in static environments without any other traffic participants by solving a minimum-violation motion planning problem. This problem is formulated given a conflicting set of customer demands (e.g., pick up or drop off a customer at certain locations within desired deadlines) and ROTRs specified in scLTL. A delay penalty is associated with meeting customer demands and is minimized in the global long-term routing, while a lower-level RRT\* (see [139, 140]) planner is used to compromise between delay penalty and violating the ROTRs, while guaranteeing safety. The scLTL specifications are converted to deterministic automata with weighted transitions that are used to capture the level of violation based on the priorities assigned to the ROTRs. The related work in [126] develops an incremental sampling-based approach to solve minimum-violation planning problems for static environments considering multiple, potentially conflicting, ROTRs specified in si-FLTLGX that have different priorities.

**Optimization-based synthesis** For formal rules written in logics with real-time and real-valued specifications such as STL and MTL, the control synthesis problem can be formulated as an optimal control problem, where the cost captures traditional objectives, such as energy spent and / or distance travelled. Vehicle limitations, such as acceleration and turning radius, are modeled as constraints. Boolean rule satisfactions can also be imposed as constraints. Alternatively, rule satisfactions can be maximized by adding weighted aggregations of their robustness values to the cost. An example of this approach can be found in [121], where the authors translate selected ROTRs formulated as STL specifications into a set of mixed-integer and linear constraints and solve the synthesis problem for a

simplified vehicle motion model with bounded additive uncertainty using MIP techniques.

A two-level control architecture for a fully autonomous system in a deterministic environment with real-time performance is proposed in [114]. At the top level, ROTRs formulated as STL specifications are translated into MIP constraints and imposed in a linear MPC problem defined over a simple representation of the environment and vehicle dynamics. At the bottom level, specification-based run-time monitoring techniques, together with detailed representations of the environment and vehicle dynamics, are used to compensate for the mismatch between the simple models used in the MPC and the real complex models. The authors of [125] propose a correct-by-construction algorithm to control AVs under perception uncertainty with probabilistic correctness guarantees specified as C2TL formulas. By approximating C2TL constraints with a set of mixed-integer constraints, the synthesis problem is formulated as a scalable second-order cone program that can be solved using off-the-shelf optimization tools.

**Synthesis through CBFs and CLFs** The control synthesis problem has also been formulated as an optimal control problem in which the satisfaction of the rule(s) and the vehicle’s state limitations are enforced by CBFs, and convergence to desired states (e.g., vehicle following) is achieved through CLFs. These ideas are proposed in [25] and used for ACC, in which CBFs are associated with safe sets, and the inequality constraints that ensure forward invariance of the set are imposed over the control strategies as optimization constraints. The unified optimal control problem with simultaneous safety (CBFs) and ACC objectives (CLFs) is solved through a sequence of computationally efficient Quadratic Programs (QPs). Building on this approach, the authors of [26] develop controllers with probabilistic correctness guarantees for simultaneous lane keeping and ACC obtained using fast QPs. The work in [82] uses high-order CBFs (i.e., CBFs that can accommodate constraints with high relative degree) to guarantee satisfaction of a set of prioritized formal rules including lane keeping, following speed limits, and maintaining clearance with other traffic participants.

**Reachability techniques, invariant sets, and ICSs** Many recent works combine reachability analysis and control techniques in hierarchical planning architectures. These are usually composed of a high-level route planner and a low-level controller used to follow the constructed trajectory, while guaranteeing the satisfaction of ROTRs and physical constraints of AVs. As already stated, the reachability analysis of most papers reviewed here is discussed in Sec. 3. Here, we focus on the planning and control aspects.

*Reachability techniques* have been extensively investigated for vehicle platooning [28, 34], ACC [55], and path planning [56] for AVs in complex, safety-critical situations. In [105], the authors model obstacles as single speed, maximum turn-rate unicycle robots and define velocity obstacle occupancy sets as unions of the sets of all reachable points by the obstacles. This study proves that subject to certain initial conditions, an infinite-horizon iterative planner guarantees collision avoidance for all times with respect to moving obstacles that have constrained dynamics. The work in [78] uses reachability analysis to concurrently solve compositional verification for the local road model and synthesizes assume-guarantee contracts to certify the safety of the given controllers. To improve the computation time for online applications, multiple works study offline pre-computations of reachable sets [54, 58].

*Control invariant sets* are mainly used to guarantee the indefinite feasibility of MPC frameworks despite limited prediction horizons [75]. Safety constraints normally make the admissible domain of the MPC optimization problem non-convex. Convexification of the safety constraint presented in [75] makes the computation of the control invariant sets fast for real-time applications. However, it reduces the set of feasible solutions. The work in [141] identifies collision-free driving corridors that represent spatio-temporal constraints for motion planning using set-based reachability analysis. In [75], look-up tables are generated offline to determine the control invariant sets in real-time [75]. The work in [36] formulates ACC specifications as LTL formulas assuming varying velocities of the leading vehicle, and designs two synthesis methods: one based on control invariant set computations on the continuous state space domain, and one using set computations on a non-deterministic finite state abstraction of the system. In [103], the authors use smooth over-approximations of the collision avoidance constraints and present a tube-based robust MPC framework with formal guarantees on recursive feasibility and satisfaction of the constraints. Synthesis of correct-by-construction centralized and distributed ACC policies for vehicle platooning with infinite-time collision avoidance guarantees in the presence of bounded additive disturbances are investigated in [35] using robust control invariant sets and QP optimization. In [77], the authors use control invariant sets for synthesizing controllers for an AV with linear parameter-varying dynamics, ACC, and lane keeping subsystems, which are robust against additive parametric uncertainties. The work in [47] designs a safety supervisor for lane departure assist systems to keep a semi-autonomous vehicle in a lane using control invariance techniques.

*Positive invariant sets* are augmented with state-feedback control in [41] to guarantee collision-free closed-loop trajectory tracking under modeling errors in overtaking and lane-change maneuvers. The framework proposed in this study relies on graph search to find in-

variant sets over a finite set of lateral displacements on the road. A similar idea is proposed in [106], which integrates motion planning and state-feedback control. The authors of [104] investigate robust positive invariant set motion planners for systems with persistently varying disturbances and parametric model uncertainty. The invariant sets are parameterized using a pre-computed input-to-state Lyapunov function.

*ICS* are employed in safe motion planning where a safety checker determines whether a system motion could lead to an ICS [131]. To efficiently build a conservative approximation of the ICS set rather than checking for collisions for all possible future trajectories of infinite duration, the authors of [65] propose a principle to select a finite subset of the possible future trajectories through *imitating maneuvers*, in which the AV tries to duplicate the object's behaviour. The works in [57, 67] propose a less conservative version of ICS, called braking ICS, which is used to guarantee that a collision could occur only when the AV was at rest.

*Fail-safe maneuvers* have been proposed to reach time-invariant safe states such that safety for an infinite time horizon can be ensured. The works in [22, 50] consider the most likely trajectory of other traffic participants for ACC control design, and maintain an emergency maneuver based on an over-approximation of the predicted occupancy set. Cooperative ACC is investigated in [29], in which a pre-defined gradual braking strategy overrides the nominal controller to guarantee collision avoidance. In a more recent study, the authors of [23] investigate fail-safe controllers for ACC in various driving conditions studied in the literature, such as full braking of the lead vehicle as well as more complex cut-in scenarios while taking into account uncertainties.

Considering all dynamically feasible behaviours of other traffic participants may over-conservatively limit the maneuverability of the AV. The work in [80] proposes a nominal control framework for highly automated driving on highways (e.g., with ACC and lane-changing functionalities), which considers legal and reasonably foreseeable nonlegal behavior of other traffic participants, and designs failure functioning trajectories for critical situations. Legal safety is guaranteed in [94], which finds legal and dynamically feasible behaviors of traffic participants using online reachability, and proposes a fail-safe trajectory to a standstill state in designated safe areas. In [107], the authors use Stochastic Model Predictive Control (SMPC) to reformulate hard constraints (e.g., for lane change and collision avoidance) in uncertain environments into probabilistic chance constraints. A fail-safe trajectory is planned using reachability analysis, which overrides the nominal SMPC controller.

**Falsification** Falsification can be used to validate safety requirements [120] and to provide guidance on control design [119]. A simulation-based adversarial test generation framework for AVs to check closed-loop properties of autonomous driving systems has been studied in [118]. The authors of [122] propose a hierarchical control stack (including an ACC planner, trajectory planner and trajectory tracker), to reach a goal within a fixed time and meet selected ROTRs formulated in MTL. In that work, S-TALIRO [137] and dReach [142] are used to evaluate the existence of a falsifying trajectory under different types of uncertainty, including uncertainty in AV’s perception and non-determinism in the dynamics of other vehicles. In [79], safety contracts are constructed by alternatively using falsification to create counterexamples for collision-free specifications, and employing them as obstacles in a reach-avoid problem solved through reachability analysis.

**Machine learning and control** The quantitative semantics (robustness function) of formal rules has enabled a growing interest in combining machine learning techniques with TL-guided control for autonomous driving. The authors of [115] formulate urban driving ROTRs in STL and combine the benefits of deep learning and MPC to propose controllers that can reason about the future behavior of nearby vehicles and behave close to human experts. Allowing relaxation of the rule constraints up to the predicted margin to satisfaction of each rule provides insight on the importance of rules as described in Sec. 2.3. The related work [116] integrates a set of parametric STL rules into a neural network for trajectory prediction. A differentiable STL robustness of the rules is optimized using gradient techniques.

An increasing number of papers propose formal methods for reinforcement learning in AV control. The work in [113] proposes a hierarchical structure with a high-level deep reinforcement learning model and a low-level (adapted RRT\*) motion planner. The reinforcement learning reward function and the motion planner cost function are formulated using quantitative robustness of LTL specifications that represent ROTRs. Learning the suitable reward function based on human’s preferences is studied in [129]. Safeguarded reinforcement learning for lane change AV control is proposed in [44].

## 6 Discussion and remaining challenges

In this section, we discuss remaining challenges and directions for future work.

**Formal specifications** Even though, as shown above, formally specifying ROTRs has received a lot of attention recently, it is still one of the main challenges facing the AV community. Ideally, we would like to have

a computational framework allowing to automatically map sets of traffic laws written in plain English, such as state driving laws in the US, or the Vienna Convention on Road Traffic, or the German Road Traffic Regulation, to sets of formal rules, such as the TLs reviewed in Sec. 2. This is a very daunting task.

First, a specification language should be chosen. As already noted, most existing works use LTL-type languages, as opposed to branching time logics such as CTL, which are popular in the formal methods community. The explanation for this is most probably that translation from natural language to LTL is easier and less prone to error than CTL [143]. However, the few existing works that propose frameworks to translate natural language to formulas of logics such as LTL [144, 145] (see also [146] for a review) in related fields are restrictive and difficult to automate. Second, off-the-shelf TLs might be unnecessarily expressive, and as a result, the corresponding verification and synthesis algorithms too expensive for AVs. Defining languages that are specifically tailored to autonomous driving is a current direction of research. Third, based on our own experience and of others, most probably there exists a rather small set of formal rules (primitives) in a properly chosen logic, such that any traffic law can be written as a (temporal, Boolean) combination of these primitives. Choosing the primitives, the composition rules, and the logic, is a challenging and open problem.

As reviewed in Sec. 2, recent works propose (pre-, partial, total) orders and / or weights to model rule relative importance, or priorities. The problems of trajectory selection and control synthesis while satisfying rule priority structures are not well understood. In particular, dealing with priorities under uncertainty is a widely open problem. Consider, for example, the problem of trajectory selection under classification uncertainty. Assume there are two rules: pedestrian and parked vehicle clearance, with the first being more important. If pedestrian classification is less reliable than parked vehicle classification, it is not clear how to perform trajectory selection when both pedestrians and parked vehicles are detected.

In most traffic scenarios, parameterizing formal specifications is challenging. For example, in pedestrian clearance, it is not obvious what combination of vehicle distance to pedestrian and approach speed would not look dangerous to the pedestrian. Some of the reviewed papers show that, when the specification is safety, such parameters can be learned from data. When the specifications are formulas of LTL, STL, or MTL, the problem is more complicated. Works from the controls and formal methods communities suggest that STL robustness can be used to find parameters in rules with given structures by solving optimization problems [147]. More recent works [148] show that the formula structures can be learnt from data as well, which can prove useful for AV applications. For example, safely engaging a curve

Table 5

Papers organized by control approaches and application areas (Reachability\* refers to works on reachability analysis, control and positive invariant sets and ICS).

	Automata	Optimization	CBFs& CLFs	Reachability*	Machine learning	Falsification
Vehicle following			[25]	[22, 23, 28, 29, 34–36]		
Lane keeping / changing				[41, 47]		
Other	[110, 126]	[114, 121, 125]	[26, 82]	[50, 53–57, 65, 67, 75, 77, 79, 80, 94, 103–107, 141]	[44, 48, 113, 115, 116, 129]	[79, 118–120, 122]

might require a TL combination of positions, speeds, and accelerations that is not easy to formulate, but can be learnt from good driving behavior. Finally, even with known rules, specifying their relative importance (priorities) is a challenging problem. An encouraging direction, supported by very recent preliminary results [130], is to learn them from data.

Sets of ROTR formalized by translating traffic rules or by learning from data can be inconsistent and / or incomplete. The first can be dealt with using priority structures, as reviewed in Sec. 2.3, and corresponding iterative control schemes. An interesting alternative is to generate rules that are consistent and complete by construction. Very recently, the authors of [149] addressed this problem using a distributed assume-guarantee structure. However, characterizing consistency and completeness of sets of ROTRs is an open and challenging problem.

**Verification** Most of the verification approaches reviewed here focus on well-defined use cases for autonomous driving, such as safe vehicle following. In such a well-defined scenario, one can verify the system offline by making certain assumptions; e.g., the leading vehicle is not allowed to perform a lane change. However, when removing this assumption, a standing vehicle could suddenly be revealed so that the ego vehicle is in an inevitable collision state. It is necessary to be able to continuously monitor such situations and assume that obstacles can be present in occluded spaces. This requires to develop online verification methods that can react to each situation appropriately—offline verification methods are infeasible for fully autonomous driving at the system level due to the large amount of test cases required to obtain a meaningful coverage. Online verification methods have to be able to consider all possible legal behaviors of surrounding traffic participants to verify that the planned action is not causing an accident. This is one of the main remaining challenges in this area. In order to avoid requiring that online approaches are real-time capable and that a new safe solution can always be found, most current approaches

use fail-safe maneuvers that are executed in case no safe maneuver can be computed on time. Another remaining challenge is that verification methods currently focus on safety specifications, while methods for verifying more complicated rules are in their infancy. This is because in contrast to monitoring approaches, verification methods have to verify a system given all uncertainties. The combined challenge of verifying complicated specifications for all possible executions of the system and its environment is still an unsolved problem.

**Synthesis** There is an increasing number of works that propose temporal logics as formal specification languages for autonomous driving. The methods for synthesis of control strategies from temporal logic specifications can be roughly grouped into two categories: automata-based and optimization-based approaches. The first group is mostly used for LTL-type specifications, and synthesis maps to solving an automaton game (e.g., Büchi or Rabin games). This is usually expensive, and, as a result, not suited to real-time control of autonomous vehicles. Most of the reviewed papers in formal synthesis belong to the second category, for which the specifications are given in concrete-time TLs, such as STL and MTL, which have quantitative semantics. Optimization methods can be subdivided into MIP-based and gradient-based approaches. Computational complexity is still a limitation for both methods. Current research is aimed at defining meaningful and smooth convex robustness functions that can be efficiently used in optimization. Most of existing approaches are based on MILPs, which only apply to linear dynamics. A current research direction is their extension to realistic vehicle dynamics. CBF-based methods are fast but myopic, and the corresponding QPs can easily become infeasible, which is one of the main challenges in this approach. Another challenge and direction of future work is designing frameworks for automatic construction of barrier functions for a given formal rule or state constraint.

Reachability analysis remains the most used technique

in AV control. As shown in this review, various tools are available for computing the reachable sets on arbitrary road networks, which allows such techniques to be used for online control. However, reducing conservativeness caused by over-approximation of reachable states, while maintaining guarantees on safety, remains a challenge.

### Uncertainty, misclassification, and sensor noise

While formal methods for discrete systems often do not require to consider uncertainties, this is of paramount importance for physical systems, such as AVs. Undoubtedly, the main uncertainty arises due to the unknown future behavior of surrounding traffic participants—even if a traffic participant performs full braking, the ego vehicle has to ensure safety. A second major source of uncertainty originates from the sensing of surrounding traffic participants whose positions, velocities, and headings are subject to measurement uncertainties. This also applies to the proprioceptive sensors of the ego vehicle, whose noise has to be considered when predicting the maximum deviation from a planned trajectory. Besides uncertainty in physical measurements, another challenge is dealing with misclassifications of traffic participants.

Formal verification and synthesis methods have to consider the above-mentioned uncertainties in their entirety. In contrast, monitoring approaches only have to evaluate a concrete evolution of a traffic scene. For instance, when the classification of a traffic participant is uncertain, formal approaches require to compute with all remaining classification hypotheses. As a consequence, most formal verification and synthesis approaches compute with sets to ensure that disturbances and sensor noise are appropriately considered. The challenge here is to consider all sources of uncertainties. While some papers focus on the uncertain future movements of other traffic participants, others only focus on the tracking error of the ego vehicle when following a planned trajectory, yet others only focus on the unknown number and states of occluded traffic participants. Obviously, formal verification can only be accomplished for the real system—and not just its mathematical model—if all sources of uncertainty of the real-world are considered. To address the potential model mismatch, several works have developed conformance checking techniques for autonomous vehicles (see [150] for a recent review). Nevertheless, this area of research is underrepresented in our point of view.

## 7 Conclusion

In this paper, we reviewed recent works that use formal methods for autonomous driving. We covered formal specifications for rules of the road, with particular emphasis on temporal logics. Verification, monitoring, and control synthesis techniques from such specifications were reviewed. We restricted our attention to ego-centric

approaches and system-level methods that focus on the behavior of an autonomous vehicle in its entirety, rather than specific software code within the vehicle. In addition, we included a critical discussion on the field and discussed remaining challenges and directions for future research.

We believe this paper will be of interest to a large audience, which includes academia and the rapidly growing AV industry. Control theorists will learn how control techniques and basic stability concepts are used in autonomous driving. They will also get exposure to formal methods techniques and their connection to dynamical systems. Computer scientists working in formal methods will see how the expressivity of temporal logic formulas can be exploited to formalize traffic laws. Last but not least, this paper will be of interest to engineers working on developing autonomous cars. We also hope that this paper will help form a community of researchers and educators interested in using tools and concepts from formal methods in the rapidly increasing area of autonomous driving.

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