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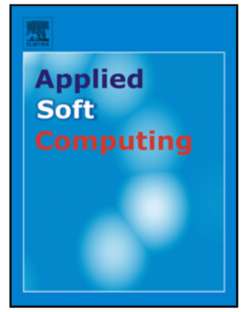

Title: Composite adaptive locally weighted learning control for multi-constraint nonlinear systems

Author: Tairen Sun Yongping Pan Chenguang Yang

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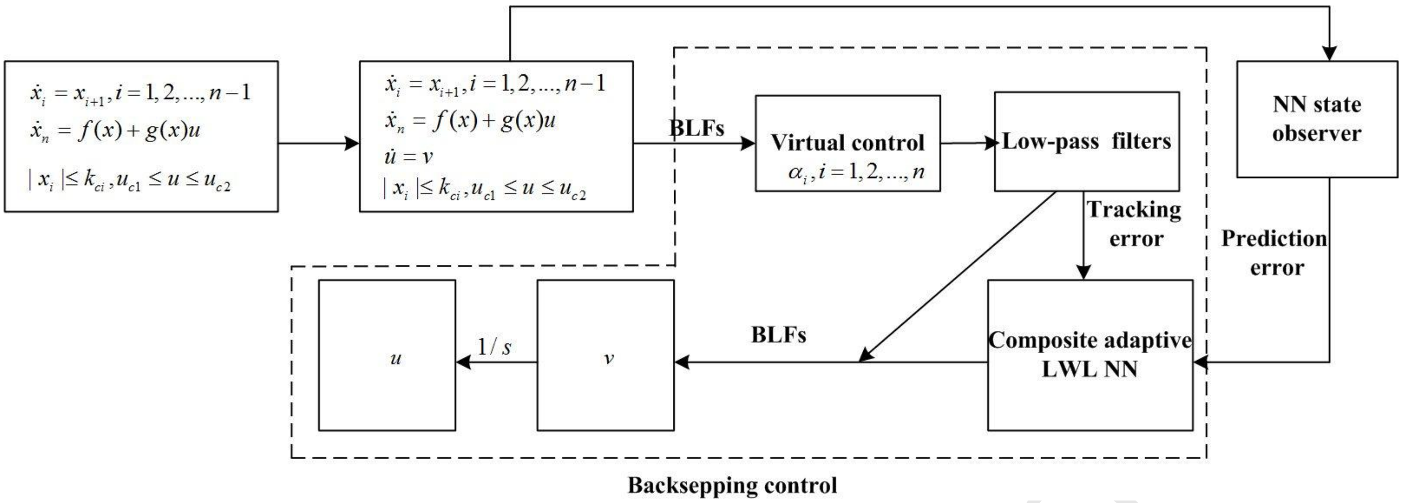
Highlights

1. Barrier functions are introduced into the backstepping procedure to tackle the state constraints and the asymmetric control saturation.

2. A serial-parallel estimation model is designed to construct the prediction errors.

3. A composite adaptive locally weighted learning NN that improves approximation and tracking accuracy is designed.

4. A dynamic surface control technique is used to decrease computational complexity of the backstepping control.


$\dot{x}_i = x_{i+1}, i = 1,2,...,n-1$
$\dot{x}_n = f(x) + g(x)u$
$|x_i| \le k_{ci}, u_{c1} \le u \le u_{c2}$
$\dot{x}_i = x_{i+1}, i = 1,2,...,n-1$
$\dot{x}_n = f(x) + g(x)u$
$\dot{u} = v$
$|x_i| \le k_{ci}, u_{c1} \le u \le u_{c2}$
BLFs
Virtual control
$\alpha_i, i = 1,2,...,n$
Low-pass filters
NN state observer
Tracking error
Prediction error
Composite adaptive LWL NN
BLFs
v
$1/s$
u
Backsepping control

Composite adaptive locally weighted learning control for multi-constraint nonlinear systems

Tairen Sun[a], Yongping Pan[b*], and Chenguang Yang[c]

[a] *Department of Electrical and Information Engineering, Jiangsu University, Zhenjiang 212013, China*

[b] *Department of Biomedical Engineering, National University of Singapore, Singapore 117583, Singapore*

[c] *Zienkiewicz Centre for Computational Engineering, Swansea University, Swansea SA1 8EN, UK*

**Corresponding author, Email: biepany@nus.edu.sg*

Abstract

A composite adaptive locally weighted learning (LWL) control approach is proposed for a class of uncertain nonlinear systems with system constraints, including state constraints and asymmetric control saturation in this paper. The system constraints are tackled by considering the control input as an extended state variable and introducing barrier Lyapunov functions (BLFs) into the backstepping procedure. The system uncertainty is approximated by a composite adaptive LWL neural networks (NNs), where a prediction error is constructed via a series-parallel identification model, and NN weights are updated by both the tracking error and the prediction error. The update law with composite error feedback improves uncertainty approximation accuracy and trajectory tracking accuracy. The feasibility and effectiveness of the proposed approach have been demonstrated by formal proof and simulation results.

Key words: Barrier Lyapunov function; neural network; control saturation; state constraint; locally weighted learning.

1. Introduction

State and control constraints exist in many mechanical systems and industrial processes due to safety and performance consideration. Almost all realworld systems have nonlinear dynamics and model uncertainties. Accordingly, control of uncertain nonlinear constrained systems has become a challenging topic and gained more and more research attention [1-4].

Control approaches for constrained systems include model predictive control (MPC) [5-7], reference governors (RGs) [8], and barrier Lyapunov functions (BLFs) [9-15]. In MPC, system constraints are explicitly considered and the control law is obtained by solving online receding horizon optimizations. In RGs, system constraints are guaranteed by the redesign of reference signals obtained by solving online optimizations. MPC and RGs have been considered as effective ways to tackle state constraints. However, high computational complexity and the requirement on high system modeling accuracy bring difficulties in applications of MPC and RGs to real-time control of uncertain nonlinear systems. Recently, barrier Lyapunov function (BLF)-based control for constrained nonlinear systems has gained more and more attention [9-13]. The function values of BLFs will grow to infinity if the arguments approach the constraints boundary resulting in constraints violation. The avoidance of constraints violation can be reached by bounding the BLFs [9]. BLF-based controllers have been designed for the nonlinear systems with time-invariant output constraints [9-11], time-varying output constraints [12], and full state constraints [13].

Neural network (NN) control has been widely developed for uncertainties nonlinear systems due to the inherent approximation abilities of NNs [14-16]. The newly developed locally weighted learning (LWL) NNs apply independently adjusted local models to approximate nonlinear uncertainties [17-19]. The advantages of LWL approximation include [20]: 1) Easy learning from the continuous stream of training data in real time; 2) negative interference avoidance for their abilities in retaining all training data; 3) allowance of quick identification due to simple learning rules with a single optimum for building a local model. Conventional adaptive NN control is directed towards achieving stability of the closed-loop system by updating NN weights through only tracking errors. By updating NN weights through both prediction errors and tracking errors, composite adaptive control has been proposed for uncertain nonlinear systems to improve both identification accuracy and tracking accuracy [21-30].

This paper considers the adaptive control design for a class of high-order uncertain nonlinear systems with system constraints, including state constraints and asymmetric control saturation. The system constraints are tackled by considering the control input as an extended state and introducing symmetric/asymmetric BLFs into the backstepping procudure. Computational complexity of the backstepping design is highly decreased by using a dynamic surface control technology [31-32]. The system uncertainty is approximated by a composite adaptive LWL NN, where the prediction error is constructed by a serial-parallel estimation model through designing a NN state observer. Compared with existing works, the main contributions of this study include:

1. By considering the control input as an extended state and introducing BLFs into the backstepping procedure, the state constraints and the asymmetric control saturation are tackled, which extends current research on BLF-based control for nonlinear systems with state/output constraints to state constraints and asymmetric control saturation;
2. By designing a serial-parallel estimation model and feeding the prediction error back the update law, the composite adaptive LWL NN is designed to approximate the system uncertainty, which improves approximation accuracy and further improves tracking accuracy.

2. Problem construction and preliminaries

2.1. Problem Formulation

Consider the following nth order SISO nonlinear system:

$$\dot{x}_i = x_{i+1},\ \ i = 1, 2, \cdots, n-1 \tag{1}$$

$$\dot{x}_n = f(\mathbf{x}) + g(\mathbf{x})u \tag{2}$$

where $x_i \in R$ and $u \in R$ are the state variable and the control input, respectively, and $f(\mathbf{x})$ and $g(\mathbf{x})$ are unknown nonlinear functions with $\mathbf{x} = [x_1, \cdots, x_n]^T$.

The system (1)-(2) is constrained by the state constraint and control saturation:

$$|x_i| \leq k_{c,i}, -u_{c1} \leq u \leq u_{c2},\ i = 1, 2, .., n \tag{3}$$

where $k_{c,i}$, u_{c1} and u_{c2} are known positive constants. We call the control input constraint as symmetric control saturation if $u_{c1} = u_{c2}$ and asymmetric control saturation if $u_{c1} \neq u_{c2}$. The objective of this paper is to design a BLF-based LWL NN control u such that $x_1(t)$ tracks a desired trajectory $y_d(t) \in R$ without violation of the system constraints described by (3).

Assumption 1. The function $f(\mathbf{x})$ is locally Lipschits continuous.

Assumption 2. The function $g(\mathbf{x})$ is locally Lipschits continuous and $0 < g_0 \leq g(\mathbf{x}) \leq g_1$ for $\forall \mathbf{x} \in D \triangleq \{\mathbf{x} \in R^n : |x_i| \leq k_{c,i},\ i = 1, 2, .., n\}$, where $g_0 > 0, g_1 > 0$ are positive constants with g_0 being known.

Assumption 3. The reference signal $y_d(t)$ and the jth-order time derivatives $y_d^{(j)}(t),\ j = 1, 2, \cdots, n$ are known and satisfy $|y_d| \leq A_0 < k_{c1}$ and $|y_d^{(j)}| \leq Y_j$, where $A_0, Y_1, \cdots, Y_n$ are positive constants.

2.2. LWL NN Approximation

To facilitate control design, the uncertain nonlinear function $f(\mathbf{x})$ is estimated by the following LWL NN:

$$\hat{f}(\mathbf{x}) = \frac{\sum_{k=1}^{N} w_k(\mathbf{x})\hat{f}_k(\mathbf{x})}{\sum_{k=1}^{N} w_k(\mathbf{x})}, \tag{4}$$

where $w_k(\mathbf{x}), k = 1, \cdots, N$ are weighted functions, and $\hat{f}_k(\mathbf{x}), k = 1, \cdots, N$ are given by

$$\hat{f}_k(\mathbf{x}) = \hat{\theta}_k^T \phi_k(\mathbf{x}),\ \phi_k(\mathbf{x}) = [1, (\mathbf{x} - c_k)^T]^T \tag{5}$$

with $\hat{\theta}_k$ and c_k the weight and center of the k-th local estimator, respectively.

Assume $S_k = \{\mathbf{x} : w_k \neq 0\}, k = 1, 2, \cdots, N$ are a series of compact sets, which satisfy $D \subseteq \cup_{k=1}^{N} S_k$. Then, for any $\mathbf{x} \in D$, there exists at least one k such that $w_k \neq 0$.

Define the weighted functions $w_k(\mathbf{x})$ as

$$w_k(\mathbf{x}) = \begin{cases} (1 - (||\mathbf{x} - c_k||/\mu_k)^2)^2, & \text{if } ||\mathbf{x} - c_k|| \leq \mu_k \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

where μ_k is the radius of the k-th local estimator. Let $\bar{w}_k(\mathbf{x}) = w_k(\mathbf{x}) / \sum_k w_k(\mathbf{x})$ which satisfies $\sum_{k=1}^{N} \bar{w}_k = 1$. For ease of notation, the symbols ϕ_k, $\bar{w}_k$, f and $\hat{f}$ are used to represent the functions $\phi_k(\mathbf{x})$, $\bar{w}_k(\mathbf{x})$, $f(\mathbf{x})$ and $\hat{f}(\mathbf{x})$, respectively. Then, the locally weighted approximation (4) can be expressed as

$$\hat{f}(\mathbf{x}) = \sum_{k=1}^{N} \bar{w}_k \hat{f}_k(\mathbf{x}). \tag{7}$$

Define the optimal weight θ_k for $\mathbf{x} \in S_k$ as

$$\theta_k = \arg\min_{\theta_k} \left(\int_{\mathbf{x} \in D} w_k(\mathbf{x}) ||f(\mathbf{x}) - \hat{f}_k(\mathbf{x})||^2 dX \right) \tag{8}$$

and the local estimation error ϵ_k as

$$\epsilon_k = \begin{cases} f(\mathbf{x}) - \hat{f}_k(\mathbf{x}), & \text{on } \bar{S}_k \\ 0, & \text{on } D - \bar{S}_k \end{cases}$$

where $\bar{S}_k$ is the minimum compact set that containing S_k as a subset. Then, $f(\mathbf{x})$ and its NN estimator can be represented as

$$f = \sum_{k=1}^{N} \bar{w}_k \theta_k^T \phi_k + \sum_{k=1}^{N} \bar{w}_k \epsilon_k, \tag{9}$$

$$\hat{f} = \sum_{k=1}^{N} \bar{w}_k \hat{\theta}_k^T \phi_k \tag{10}$$

Then, the estimation error $\tilde{f} \triangleq f - \hat{f}$ can be expressed as

$$\tilde{f} = \sum_{k=1}^{N} \bar{w}_k \tilde{\theta}_k^T \phi_k + \sum_{k=1}^{N} \bar{w}_k \epsilon_k \tag{11}$$

with $\tilde{\theta}_k = \theta_k - \hat{\theta}_k$. Assume that $|\epsilon_k| \leq \epsilon$ and $||\theta_k|| \leq \theta_{max}$ with ϵ and θ_{max} being positive constants. It is obvious that $|\sum_{k=1}^{N} \bar{w}_k \epsilon_k| \leq \max(|\epsilon_k|) \sum_{k=1}^{N_i} \bar{w}_k \leq \epsilon$.

3. Control design and stability analysis

In this section, the state constraints and the asymmetric control saturation are tackled by considering control as an extended state and introducing a BLF in each step of the backstepping procedure. The system uncertainty $f(\mathbf{x})$ is approximated by a LWL approximator with weights updated by a composite error.

3.1. Locally weighted learning control

Step 1: Define $z_1 = x_1 - y_d$ as the trajectory tracking error, whose dynamics can be written as

$$\dot{z}_1 = \dot{x}_1 - \dot{y}_d = x_2 - \dot{y}_d. \tag{12}$$

Consider the BLF

$$V_1 = \frac{1}{2} \ln \frac{k_{b,1}^2}{k_{b,1}^2 - z_1^2} \tag{13}$$

where $k_{b,1}$ is a positive design parameter. Taking time derivative of V_1 and substituting (12), yields

$$\dot{V}_1 = \frac{z_1}{k_{b,1}^2 - z_1^2}(x_2 - \dot{y}_d) \tag{14}$$

Design a reference signal for x_2 as

$$\alpha_1 = -\lambda_1 z_1 + \dot{y}_d - \frac{1}{2} \frac{z_1}{k_{b,1}^2 - z_1^2}, \tag{15}$$

with λ_1 being a positive design parameter. Passing α_1 through the following low-pass filter

$$\tau_1 \dot{\alpha}_{1,c} = -\alpha_{1,c} + \alpha_1 \tag{16}$$

with $\tau_1 > 0$ being a positive design parameter. Define $z_2 = x_2 - \alpha_{1,c}$ as the second tracking error, then, from (14-16) we can obtain

$$\begin{aligned}
\dot{V}_1 &= -\lambda_1 \frac{z_1^2}{k_{b,1}^2 - z_1^2} + \frac{z_1 z_2}{k_{b,1}^2 - z_1^2} - \frac{z_1^2}{2(k_{b,1}^2 - z_1^2)^2} + \frac{z_1 \tilde{\alpha}_1}{k_{b,1}^2 - z_1^2} \\
&\leq -\lambda_1 \frac{z_1^2}{k_{b,1}^2 - z_1^2} + \frac{z_1 z_2}{k_{b,1}^2 - z_1^2} + \frac{1}{2}\tilde{\alpha}_1^2
\end{aligned} \tag{17}$$

where $\tilde{\alpha}_1 = \alpha_{1,c} - \alpha_1$.

Step i($i = 2, 3, \cdots, n-1$): Define the tracking errors $z_i = x_i - \alpha_{i-1,c}$. Taking time derivative of z_i, yields

$$\dot{z}_i = \dot{x}_i - \dot{\alpha}_{i-1,c} = x_{i+1} - \dot{\alpha}_{i-1,c} \tag{18}$$

Design the reference signal α_i for x_{i+1} as

$$\alpha_i = -\lambda_i z_i - \frac{k_{b,i}^2 - z_i^2}{k_{b,i-1}^2 - z_{i-1}^2} z_{i-1} - \frac{1}{2} \frac{z_i}{k_{b,i}^2 - z_i^2} + \dot{\alpha}_{i-1,c}, \tag{19}$$

where λ_i and $k_{b,i}$ are positive design parameters. Passing α_i through the following low-pass filter

$$\tau_i \dot{\alpha}_{ic} = -\alpha_{ic} + \alpha_i \tag{20}$$

with τ_i being a positive design parameter.

Define the candidate Lyapunov functions $V_i, i = 2, \cdots, n$ as

$$V_i = V_{i-1} + \frac{1}{2} \ln \frac{k_{b,i}^2}{k_{b,i}^2 - z_i^2} \tag{21}$$

whose time derivative satisfies

$$\begin{aligned} \dot{V}_i &= \dot{V}_{i-1} - \lambda_i \frac{z_i^2}{k_{b,i}^2 - z_i^2} + \frac{z_i z_{i+1}}{k_{b,i}^2 - z_i^2} - \frac{1}{2} \frac{z_i^2}{(k_{c,i}^2 - z_i^2)^2} + \frac{z_i \tilde{\alpha}_i}{k_{b,i}^2 - z_i^2} \\ &\leq -\sum_{j=1}^{i} \lambda_j \frac{z_j^2}{k_{b,j}^2 - z_j^2} + \frac{z_i z_{i+1}}{k_{b,i}^2 - z_i^2} + \frac{1}{2} \sum_{j=1}^{i} \tilde{\alpha}_j^2 \end{aligned} \tag{22}$$

with $\tilde{\alpha}_i = \alpha_{i,c} - \alpha_i$

Step n: Define $z_n = x_n - \alpha_{n-1,c}$. Taking time derivative of z_n, yields

$$\dot{z}_n = f(x) + g(x)u - \dot{\alpha}_{n-1,c} \tag{23}$$

Design the reference signal α_n for the control law u as

$$\alpha_n = g_0^{-1} \left(-\lambda_n z_n - \hat{f} + \dot{\alpha}_{n-1,c} - \frac{k_{b,n}^2 - z_n^2}{k_{b,n-1}^2 - z_{n-1}^2} z_{n-1} - \frac{1}{2} \frac{z_n}{k_{b,n}^2 - z_n^2} \right) \tag{24}$$

where λ_n is a designed positive parameter. Passing α_n through the following low-pass filter

$$\tau_n \dot{\alpha}_{n,c} = -\alpha_{n,c} + \alpha_n. \tag{25}$$

Then,

$$\dot{z}_n = -\lambda_n z_n - z_{n-1}\frac{k_{b,n}^2 - z_n^2}{k_{b,n-1}^2 - z_{n-1}^2} - \frac{1}{2}\frac{z_n}{k_{b,n}^2 - z_n^2} + \sum_{k=1}^{N}\varpi_k\tilde{\theta}_k^T\phi_k$$
$$+ \sum_{k=1}^{N}\varpi_k\epsilon_k + g(x)(z_{n+1} + \tilde{\alpha}_n) + \tilde{g}\alpha_n + \frac{1}{2}\sum_{j=1}^{i}\tilde{\alpha}_j^2 \tag{26}$$

where $\tilde{g} = g(x) - g_0$, $\tilde{\alpha}_n = \alpha_{n,c} - \alpha_n$ and $z_{n+1} = u - \alpha_{n,c}$.

The weights of composite adaptive LWL NN are updated by the tracking error z_n and the prediction error $\tilde{r} = r - \hat{r}$, where

$$r = x_n + l_{n-2}x_{n-1} + \cdots + l_0 x_1 \tag{27}$$

with $l_i, i = 0, \cdots, n-1$ being chosen parameters such that the roots of $s^{n-1} + \cdots + l_1 s + l_0 = 0$ are all negative, and $\hat{r}$ is the estimation of r. The dynamics of r and $\hat{r}$ are described as follows

$$\dot{r} = f(x) + g(x)u + \gamma, \tag{28}$$
$$\dot{\hat{r}} = \sum_{k=1}^{N}\bar{w}_k\hat{\theta}_k^T\phi_k + g_0 u + \gamma + l\tilde{r} \tag{29}$$

where $\gamma = \sum_{i=1}^{n-1} l_{i-1}x_i$ and l is a positive design parameter. Then, we can obtain the following dynamics of $\tilde{r}$

$$\dot{\tilde{r}} = \sum_{k=1}^{N}\bar{w}_k\tilde{\theta}_k^T\phi_k + \sum_{k=1}^{N}\bar{w}_k\epsilon_k + \tilde{g}u - l\tilde{r}. \tag{30}$$

Consider the following candidate Lyapunov function V_n:

$$V_n = V_{n-1} + \frac{1}{2}\ln\frac{k_{b,n}^2}{k_{b,n}^2 - z_n^2} + \frac{1}{2q}\sum_{i=1}^{N}\tilde{\theta}_k^T\tilde{\theta}_k + \frac{\chi}{2}\tilde{r}^2 \tag{31}$$

with χ and q being positive constants. Taking time derivative of V_n and substituting (22), (26) and (30) into it, yields

$$
\begin{aligned}
\dot{V}_n &= \dot{V}_{n-1} + \frac{z_n \dot{z}_n}{k_{b,n}^2 - z_n^2} - \frac{1}{q}\sum_{k=1}^{N} \tilde{\theta}_k^T \dot{\hat{\theta}}_k + \chi \tilde{r}\dot{\tilde{r}} \\
&\leq -\sum_{j=1}^{n} \lambda_j \frac{z_j^2}{k_{b,j}^2 - z_j^2} - \sum_{k=1}^{N} \tilde{\theta}_k^T \left(\frac{\dot{\hat{\theta}}_k}{q} - \varpi_k \phi_k \frac{z_n}{k_{b,n}^2 - z_n^2} - \chi \tilde{w}_k \phi_k \tilde{r} \right) - l\chi \tilde{r}^2 \\
&\quad - \frac{1}{2}\frac{z_n^2}{(k_{b,n}^2 - z_n^2)^2} + \frac{z_n z_{n+1}}{k_{b,n}^2 - z_n^2} g(x) + \frac{z_n}{k_{bn}^2 - z_n^2}\left(\sum_{k=1}^{N} \varpi_k \epsilon_k + \tilde{g}\alpha_n + g(x)\tilde{\alpha}_n\right) \\
&\quad + \chi \tilde{r}\left(\sum_{k=1}^{N} \bar{w}_k \epsilon_k + \tilde{g}u\right) + \frac{1}{2}\sum_{j=1}^{i} \tilde{\alpha}_j^2
\end{aligned} \tag{32}
$$

Design the adaptive law of $\hat{\theta}_k$ as

$$
\dot{\hat{\theta}}_k = -\sigma_k \hat{\theta}_k + q\varpi_k \phi_k \frac{z_n}{k_{b,n}^2 - z_n^2} + q\chi \tilde{w}_k \phi_k \tilde{r} \tag{33}
$$

with σ_k being positive design parameters. Since $|\sum_{k=1}^{N} \bar{w}_k \epsilon_k| \leq \epsilon$, $|\tilde{g}\alpha_n| \leq g_{10}|\alpha_n|$, $|g(x)\tilde{\alpha}_n| \leq g_1|\tilde{\alpha}_n|$ and $|\tilde{g}u| \leq g_{10}u_{max}$ with $g_{10} \triangleq g_1 - g_0$ and $u_{\max} = \{u_{c1}, u_{c2}\}$, then

$$
\frac{z_n}{k_{bn}^2 - z_n^2}\left(\sum_{k=1}^{N} \varpi_k \epsilon_k + \tilde{g}\alpha_n + g(x)\tilde{\alpha}_n\right) \leq \frac{z_n^2}{2(k_{bn}^2 - z_n^2)^2} + \frac{1}{2}(\epsilon + g_{10}|\alpha_n| + g_1|\tilde{\alpha}_n|)^2, \tag{34}
$$

$$
\chi \tilde{r}\left(\sum_{k=1}^{N} \bar{w}_k \epsilon_k + \tilde{g}u\right) \leq \frac{\chi \tilde{r}^2}{2} + \frac{\chi(\epsilon + g_{10}u_{max})^2}{2}. \tag{35}
$$

Since $\hat{\theta}_k = \theta_k - \tilde{\theta}_k$ and $|\theta_k| \leq \theta_{max}$, then

$$
\tilde{\theta}_k^T \hat{\theta}_k \leq -\frac{1}{2}\tilde{\theta}_k^2 + \frac{1}{2}\theta_{max}^2. \tag{36}
$$

Substituting (33-36) into (32), we can obtain

$$
\dot{V}_n \leq -\sum_{j=1}^{n} \lambda_j \frac{z_j^2}{k_{b,j}^2 - z_j^2} - \frac{1}{2}\sum_{j=1}^{N} \sigma_k \tilde{\theta}_k \tilde{\theta}_k - (l - 0.5)\chi \tilde{r}^2 + \frac{z_n z_{n+1}}{k_{b,n}^2 - z_n^2} g(x) + \rho_1 \tag{37}
$$

where $\rho_1 = \frac{1}{2}(\epsilon + g_{10}|\alpha_n| + g_1|\tilde{\alpha}_n|)^2 + \frac{\chi(\epsilon + g_{10}u_{max})^2}{2} + \frac{1}{2}\sum_{j=1}^{i} \tilde{\alpha}_j^2 + \frac{N}{2}\theta_{max}^2$.

Step n+1: Consider the following candidate Lyapunov function:

$$
V_{n+1} = V_n + \frac{1}{2}q(z_{n+1}) \ln \frac{k_{u2}^2}{k_{u2}^2 - z_{n+1}^2} + \frac{1}{2}(1 - q(z_{n+1})) \ln \frac{k_{u1}^2}{k_{u1}^2 - z_{n+1}^2} \tag{38}
$$

where k_{u1}, k_{u2} are parameters to be specified latter, and $q(z_{n+1}) = 1$ if $z_{n+1} \geq 0$ and $q(z_{n+1}) = 0$ if $z_{n+1} < 0$.

If $z_{n+1} \geq 0$, then $q(z_{n+1}) = 1$ and $V_{n+1} = V_n + \frac{1}{2}\ln(k_{u2}^2/(k_{u2}^2 - z_{n+1}^2))$. Taking time derivative of V_{n+1}, yields

$$\dot{V}_{n+1} = \dot{V}_n + \frac{z_{n+1}}{k_{u2}^2 - z_{n+1}^2}(\dot{u} - \dot{\alpha}_{n,c}) \tag{39}$$

Using Young's inequality,

$$\frac{z_n z_{n+1}}{k_{b,n}^2 - z_n^2} g(x) \leq \frac{\xi}{2}[z_n z_{n+1}/(k_{b,n}^2 - z_n^2)]^2 + \frac{g_1^2}{2\xi} \tag{40}$$

where ξ is a positive constant. If $\dot{u}$ satisfies

$$\dot{u} = -\lambda_{n+1} z_{n+1} - \frac{\xi}{2}\frac{k_{u2}^2 - z_{n+1}^2}{(k_{b,n}^2 - z_n^2)^2}(z_n z_{n+1})^2 + \dot{\alpha}_{n,c} \tag{41}$$

with λ_{n+1} being a designed positive parameter, then

$$\begin{aligned}\dot{V}_{n+1} \leq & -\sum_{j=1}^{n}\lambda_j \frac{z_j^2}{k_{b,j}^2 - z_j^2} - \lambda_{n+1}\frac{z_{n+1}^2}{k_{u2}^2 - z_{n+1}^2} - \frac{1}{2}\sum_{j=1}^{N}\sigma_k \tilde{\theta}_k \tilde{\theta}_k \\ & -(l-0.5)\chi\tilde{r}^2 + \rho_2 \end{aligned} \tag{42}$$

with $\rho_2 = \frac{g_1^2}{2\xi} + \rho_1$.

If $z_{n+1} < 0$, then $q(z_{n+1}) = 0$ and $V_{n+1} = V_n + \ln\frac{k_{u1}^2}{k_{u1}^2 - z_{n+1}^2}$. By derivation similar to the case $z_{n+1} \geq 0$, if $u(t)$ satisfies

$$\dot{u} = -\lambda_{n+1} z_{n+1} - \frac{\xi}{2}\frac{k_{u1}^2 - z_{n+1}^2}{(k_{b,n}^2 - z_n^2)^2}(z_n z_{n+1})^2 + \dot{\alpha}_{n,c}, \tag{43}$$

then

$$\begin{aligned}\dot{V}_{n+1} \leq & -\sum_{j=1}^{n}\lambda_j \frac{z_j^2}{k_{b,j}^2 - z_j^2} - \lambda_{n+1}\frac{z_{n+1}^2}{k_{u1}^2 - z_{n+1}^2} - \frac{1}{2}\sum_{j=1}^{N}\sigma_k \tilde{\theta}_k \tilde{\theta}_k \\ & -(l-0.5)\chi\tilde{r}^2 + \rho_2. \end{aligned} \tag{44}$$

Based on the above analysis, one can see that if designing the control law $u(t)$ as

$$\begin{aligned} u(t) = & \ \alpha_{n,c} - \lambda_{n+1}\int_0^t z_{n+1}(\sigma)d\sigma \\ & + \int_0^t q(z_{n+1})[-\frac{1}{2}\frac{k_{u2}^2 - z_{n+1}^2}{(k_{b,n}^2 - z_n^2)^2}(z_n z_{n+1})^2](\sigma)d\sigma \\ & + \int_0^t (1 - q(z_{n+1}))[-\frac{1}{2}\frac{k_{u1}^2 - z_{n+1}^2}{(k_{b,n}^2 - z_n^2)^2}(z_n z_{n+1})^2](\sigma)d\sigma, \end{aligned} \tag{45}$$

then

$$\dot{u} = -\lambda_{n+1} z_{n+1}(t) + \dot{\alpha}_{n,c}(t)$$
$$+q(z_{n+1})[-\frac{1}{2}\frac{k_{u2}^2 - z_{n+1}^2}{(k_{b,n}^2 - z_n^2)^2}(z_n z_{n+1})^2](t)$$
$$+(1-q(z_{n+1}))[-\frac{1}{2}\frac{k_{u1}^2 - z_{n+1}^2}{(k_{b,n}^2 - z_n^2)^2}(z_n z_{n+1})^2](t) \tag{46}$$

and

$$\dot{V}_{n+1} \le -\sum_{j=1}^{n} \lambda_j \frac{z_j^2}{k_{b,j}^2 - z_j^2} - q(z_{n+1})\frac{\lambda_{n+1} z_{n+1}^2}{k_{u2}^2 - z_{n+1}^2} - (1-q(z_{n+1}))\frac{\lambda_{n+1} z_{n+1}^2}{k_{u1}^2 - z_{n+1}^2}$$
$$-\frac{1}{2}\sum_{j=1}^{N}\sigma_k \tilde{\theta}_k \tilde{\theta}_k - (l-0.5)\chi \tilde{r}^2 + \rho_2. \tag{47}$$

3.2. Stability analysis

Lemma 1 [31]. For the filters (16), (20) and (25), if $\tilde{\alpha}_i(0) = 0$ and $x(t) \in D_n, \forall t \in [0, T_f]$, then given $\mu \in R^+$, there exist $\tau_i > 0$ such that $|\tilde{\alpha}_i(t)| \le \mu, \forall t \in [0, T_f], i = 1, 2, \cdots, n$.

Theorem. Consider the system (1-2) with system constraints (3) under Assumption 1-3, initial condition $x(0) \in D$, and control law (45). Let

$$A_i = \max_{[\bar{z}_i^T, \bar{y}_{di}^T]^T \in \Omega_i} |\alpha_i(\bar{z}_i, \bar{y}_{di})|, \ i = 1, 2, \cdots, n \tag{48}$$

where $\bar{z}_i = [z_1, \cdots, z_i]^T, \bar{y}_{di} = [y_d, y_d^{(1)}, \cdots, y_d^{(i-1)}]^T$, and

$$\Omega_i = \{[\bar{z}_i^T, \bar{y}_{di}^T]^T \in R^{2i} : |z_i| \le k_{ci}, |y_d| \le A_0, |y_d^{(i-1)}| \le Y_{i-1}, i = 1, \cdots, n\}. \tag{49}$$

If there exist positive parameters $\lambda_1, \cdots, \lambda_{n+1}$ such that

$$k_{c,i} \ge A_{i-1} + k_{b,i}, i = 0, 1, \cdots, n-1, \tag{50}$$
$$u_{c1} \le k_{u1} + A_n, u_{c2} \ge k_{u2} + A_n \tag{51}$$

then, (i) the state constraints and the asymmetric control saturation are not violated; (ii) the tracking errors and the NN weights estimation errors in the closed-loop system are uniformly ultimately bounded and the tracking error z_1 converges to a small neighborhood of zero.

Proof.

From Lemma 1, we can see that given $t_f > 0$ and $\mu > 0$, there exist $\tau_i > 0$ such that $|\tilde{\alpha}_i| \leq \mu, i = 1, 2, \cdots, n$. Then, $\rho_2 \leq \rho_{max} \triangleq \frac{1}{2}(\epsilon + g_{10}A_n + g_1\mu)^2 + \frac{\chi(\epsilon+g_{10}u_{max})^2}{2} + \frac{1}{2}\sum_{j=1}^{n}\mu^2 + \frac{N}{2}\theta_{max}^2 + \frac{g_1^2}{2\xi}$. Therefore,

$$\dot{V}_{n+1} \leq -\sum_{j=1}^{n}\lambda_j\frac{z_j^2}{k_{b,j}^2 - z_j^2} - q(z_{n+1})\frac{\lambda_{n+1}z_{n+1}^2}{k_{u2}^2 - z_{n+1}^2} - (1 - q(z_{n+1}))\frac{\lambda_{n+1}z_{n+1}^2}{k_{u1}^2 - z_{n+1}^2}$$
$$-\frac{1}{2}\sum_{j=1}^{N}\sigma_k\tilde{\theta}_k\tilde{\theta}_k - (l - 0.5)\chi\tilde{r}^2 + \rho_{max}, \ \forall t \in [0, t_f]. \quad (52)$$

It has been proved in [9-13] that $\ln[k_{b,i}^2/(k_{b,i}^2 - z_i^2)] \leq z_i^2/(k_{b,i}^2 - z_i^2)$ for $i = 1, 2, \cdots, n$ and

$$q(z_{n+1})\ln\frac{k_{u2}^2}{k_{u2}^2 - z_{n+1}^2} + (1 - q(z_{n+1}))\ln\frac{k_{u1}^2}{k_{u1}^2 - z_{n+1}^2}$$
$$\leq q(z_{n+1})\frac{z_{n+1}^2}{k_{u2}^2 - z_{n+1}^2} + (1 - q(z_{n+1}))\frac{z_{n+1}^2}{k_{u1}^2 - z_{n+1}^2} \quad (53)$$

Based on (52) and (53), we can obtain

$$\dot{V}_{n+1} \leq -\frac{1}{2}\lambda V_{n+1} - (\frac{1}{2}\lambda V_{n+1} - \rho_{max}), \quad \forall t \in [0, t_f] \quad (54)$$

where $\lambda = \min\{2\lambda_i, i = 1, \ldots, n+1, q\sigma_k, k = 1, 2, \cdots, N, 2(l - 0.5)\}$. Then,

$$\dot{V}_{n+1} \leq -\frac{1}{2}\lambda V_{n+1}, \text{ if } \frac{1}{2}\lambda V_{n+1} \geq \rho_{max}, \quad \forall t \in [0, t_f] \quad (55)$$

Therefore, V_{n+1} and the signals of the closed-loop system are bounded over any finite time interval, by [33, Lemma A.3.2], the solution exists for $t \in [0, \infty)(i.e. T_f = \infty)$.

(i) From boundedness of the BLFs V_i, $i = 1, 2, \cdots, n+1$, we can see the boundedness of $\ln\frac{k_{bi}^2}{k_{bi}^2 - z_i^2}, i = 1, 2, \cdots, n$ and $q(z_{n+1})\ln\frac{k_{u2}^2}{k_{u2}^2 - z_{n+1}^2} + (1 - q(z_{n+1}))\ln\frac{k_{u1}^2}{k_{u1}^2 - z_{n+1}^2}$. Since $x(0) \in D$, we can conclude that $|z_i(t)| < k_{b,i}, i = 1, 2, \cdots, n$ and $-k_{u1} < z_{n+1}(t) < k_{u2}$ for $t \in [0, \infty)$. Otherwise, $log\frac{k_{bi}^2}{k_{bi}^2 - z_i^2}, i =$

$1, 2, \cdots, n$ or $q(z_{n+1}) \ln \frac{k_{u2}^2}{k_{u2}^2 - z_{n+1}^2} + (1 - q(z_{n+1})) \ln \frac{k_{u1}^2}{k_{u1}^2 - z_{n+1}^2}$ will go to infty, which conflicts their boundedness. Since $|\alpha_i| \leq A_i, i = 1, 2, \cdots, n$, from construction of the low-pass filters, we can obtain $|\alpha_{i,c}| \leq A_i, i = 1, 2, \cdots, n$. Since $|x_1| \leq |z_1| + |y_d|$, $|x_i| \leq |z_i| + |\alpha_{i-1,c}|$, $u = z_{n+1} + \alpha_{n,c}$ and the inequalities (50), (51) hold, constraints satisfaction $|x_i| \leq k_{c,i}$ and $-u_{c1} \leq u \leq u_{c2}$ can be concluded.

(ii) Solving the inequality (54), one can further obtain

$$V_{n+1}(t) \leq e^{-\lambda t}(V_{n+1}(0) - \frac{\rho_{max}}{\lambda}) + \frac{\rho_{max}}{\lambda} \tag{56}$$

Since $log \frac{k_{b,1}^2}{k_{b,1}^2 - z_1^2}/2 \leq V_{n+1}(t)$, then

$$\frac{1}{2} \ln \frac{k_{b,1}^2}{k_{b,1}^2 - z_1^2} \leq (V_{n+1}(0) - \frac{\rho_{max}}{\lambda})e^{-\lambda t} + \frac{\rho_{max}}{\lambda} \tag{57}$$

and

$$\frac{k_{b,1}^2}{k_{b,1}^2 - z_1^2} \leq \exp\left[2e^{-\lambda t}(V_{n+1}(0) - \frac{\rho_{max}}{\lambda}) + 2\frac{\rho_{max}}{\lambda}\right] \tag{58}$$

$$\limsup_{t \to \infty} \frac{k_{b,1}^2}{k_{b,1}^2 - z_1^2} \leq \exp(2\rho_{max}/\lambda) \tag{59}$$

from which we can further obtain

$$\limsup_{t \to \infty} |z_1| \leq k_{b,1}\sqrt{1 - \exp(-2\rho_{max}/\lambda)}. \tag{60}$$

Therefore, the trajectory tracking error z_1 converges to a small neighborhood of zero by properly selecting the design parameters.

Remark 1. In the proposed NN control, the uncertainty function $f(\mathbf{x})$ is approximated by the composite adaptive LWL NN (CALWLNN) (10) with NN weights updated in (33) by composite errors composed of estimation errors $\tilde{r}$ and tracking errors z_n. Compared with the conventional adaptive LWL NN (ALWLNN) with weights updated only by tracking errors, the composite errors-based update law improves smoothness of control responses resulting in the possibility of using high adaptation gain, and thus, improves uncertainty approximation accuracy and trajectory tracking accuracy [27]. This claim will be verified by simulation results in the next section. The combination of emerging learning techniques [29]-[31] with the current design to enhance the ability of NN modeling during control is interesting for further studies.

4. Simulation results

To illustrate the effectiveness of the proposed BLF-based localized adaptive NN control, simulations are carried out for the following nonlinear system

$$\dot{x}_1 = x_2 \tag{61}$$

$$\dot{x}_2 = x_1 x_2 + 2x_2 + (1.5 + 0.3\cos(x_1))u \tag{62}$$

with the constraints

$$|x_1| \le 1, |x_2| \le 1, -3 \le u \le 2. \tag{63}$$

In the simulation, the initial system states are $x_1(0) = 0.2, x_2(0) = 0$, the reference trajectory is $y_d = 0.5\sin(0.5t)$, and $z_1 = x_1 - y_d, z_2 = x_2 - \alpha_1, z_3 = u - \alpha_2$ which are constrained by $|z_1| \le 0.5, |z_2| \le 0.5, -2 \le z_3 \le 1$.

The control is designed to satisfy:

$$\begin{aligned} \dot{u} = & -6z_3 + \dot{\alpha}_{2,c} + q(z_3)(-\frac{1^2 - z_3^2}{2(0.5^2 - z_2^2)^2} z_2^2 z_3^2) \\ & +(1 - q(z_3))(-\frac{2^2 - z_3^2}{2(0.5^2 - z_2^2)^2} z_2^2 z_3^2) \end{aligned} \tag{64}$$

where the virtual control α_1, α_2 are described as

$$\begin{aligned} \alpha_2 &= \frac{1}{1.5}(-2z_2 - \frac{z_2}{2(0.5^2 - z_2^2)} - \frac{0.5^2 - z_2^2}{0.5^2 - z_1^2} z_1 - \hat{f} + \dot{\alpha}_{1,c}), \\ \alpha_1 &= -z_1 + \dot{y}_d - \frac{z_1}{2(0.5^2 - z_1^2)} \end{aligned}$$

with $\hat{f}$ being the LWL NN approximation of $f = x_1 x_2 + 2x_2$. The low-pass filters are designed as $\dot{\alpha}_{1,c} = -10(\alpha_{1,c} - \alpha_1)$ and $\dot{\alpha}_{2,c} = -20(\alpha_{2,c} - \alpha_2)$.

In the NN approximation, the centers location are chosen as $c_1 = [-0.6, 0.8]^T, c_2 = [-0.3, 0.8]^T, c_3 = [0, 0.8]^T, c_4 = [0.3, 0.8]^T, c_5 = [0.6, 0.8]^T, c_6 = [-0.6, 0.5]^T, c_7 = [-0.3, 0.5]^T, c_8 = [0, 0.5]^T, c_9 = [0.3, 0.5]^T, c_{10} = [0.6, 0.5]^T,\ c_{11} = [-0.6, 0.2]^T,$

$c_{12} = [-0.3, 0.2]^T$, $c_{13} = [0, 0.2]^T$, $c_{14} = [0.3, 0.2]^T$, $c_{15} = [0.6, 0.2]^T$, $c_{16} = [-0.6, -0.2]^T$, $c_{17} = [-0.3, -0.2]^T$, $c_{18} = [0, -0.2]^T$, $c_{19} = [0.3, -0.2]^T$, $c_{20} = [0.6, -0.2]^T$, $c_{21} = [-0.6, -0.6]^T$, $c_{22} = [-0.3, -0.6]^T$, $c_{23} = [0, -0.6]^T$, $c_{24} = [0.3, -0.6]^T$, $c_{25} = [0.6, -0.6]^T$, $c_{26} = [1, -1]^T$, and $c_{27} = [-1, -1]^T$, the basis functions are chosen as $\phi_i = [1, x_1, x_2]^T - [0, c_i^T]^T, i = 1, \cdots, 27$, and the weighted functions are chosen as

$$w_i(x) = \begin{cases} (1 - (||\mathbf{x} - c_i||/0.5)^2)^2, & \text{if } ||\mathbf{x} - c_i|| \leq 0.5 \\ 0, & \text{otherwise.} \end{cases} \tag{65}$$

From Figure 1, we can see that the circles with centers being $c_i, i = 1, \cdots, 27$ and radiuses being 0.5 cover the constrained state space $D = \{\mathbf{x} = [x_1, x_2]^T \in R^2 : |x_1| \leq 1, |x_2| \leq 1\}$. In the NN observer (29), $r = x_2 + 5x_1$ and $l = 10$. In the weights update, $\sigma_k = 1,\ q = 25$ and $q\chi = 35$.

In Figures. 2-3, we present the simulation results of the proposed BLF-based composite adaptive LWL control (CALWLC) and the BLF-based adaptive LWL control (ALWLC), where Figure 1 presents comparison of the tracking performance of the two controllers and the NN approximation performance of the proposed composite adaptive LWL NN (CALWLNN) and the conventional adaptive LWL NN (ALWLNN). From Figure 2(a) and Figure 3, we can see that the constraints $|z_1| \leq 0.5, |z_2| \leq 0.5, -2 \leq z_3 \leq 1$ and $|x_1| \leq 1, |x_2| \leq 1, -3 \leq u \leq 2$ are not violated, which illustrates effectiveness of the BLFs in tackling the state and control constraints for the nonlinear system is obtained by bounding the designed BLFs. From the comparison in Figure 2, we can see that the CALWLNN approximates the system uncertainty $f(x)$ more accurately than the conventional ALWLNN. Compared with the BLFs-based ALWLC, the proposed BLF-based CALWLC has better tracking performance due to the composite adaptive LWL NN's more accurate approximation.

5. Conclusions

This paper presents a BLF-based adaptive LWL NN control law for a class of nonlinear systems with state and asymmetric control constraints. Our study extends current results on BLF-based control for nonlinear systems with state and output constraints to systems with state and asymmetric control constraints, by considering the control input as an extended state. In the control law, the system uncertainty is estimated and compensated for by a composite adaptive LWL NN. The use of the prediction error in the weights update law improves the approximation accuracy and, in turn, improves the tracking accuracy. From simulation results, we observe that both the tracking error and the NN approximation error of the constrained system converge to a small neighborhood of zero. The effectiveness of the proposed control scheme has been verified based on theoretical analysis and simulation results.

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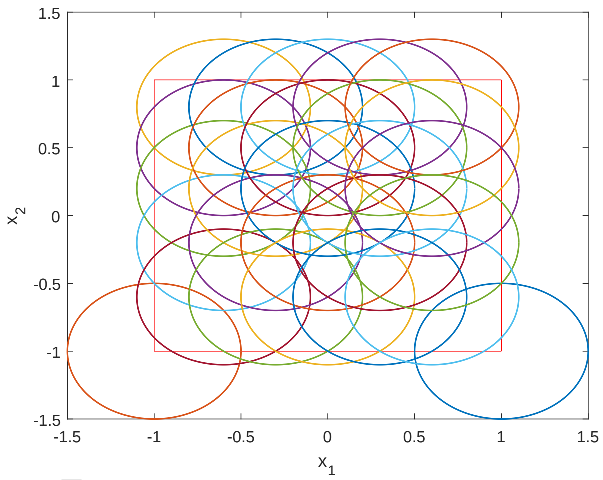
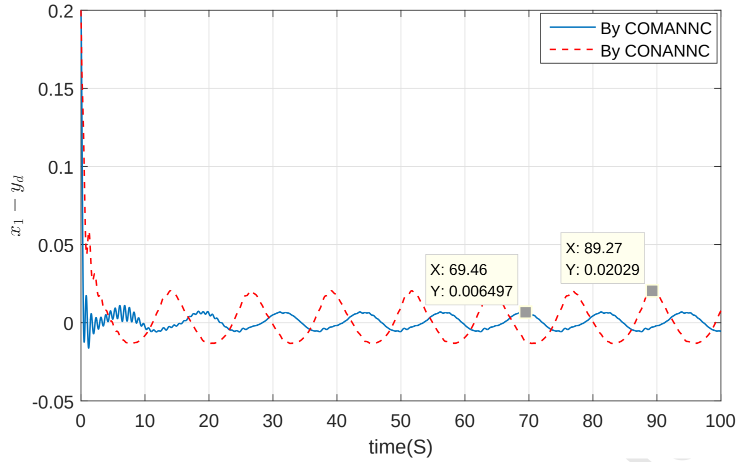


Figure 1: The circles centered at $c_i, i = 1, \cdots, 27$ and the constrained state space D.



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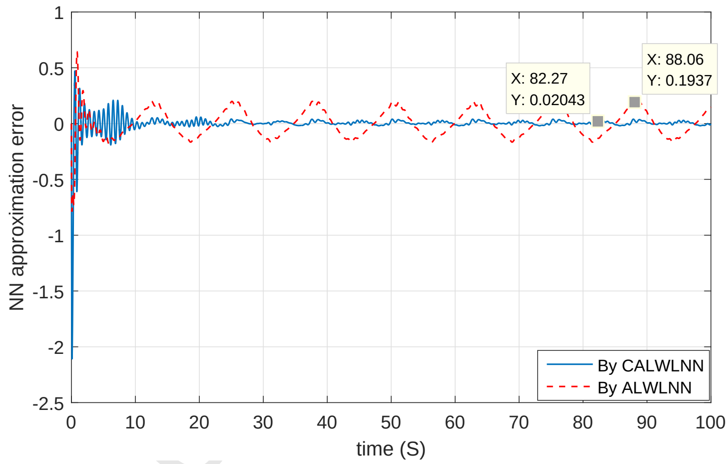

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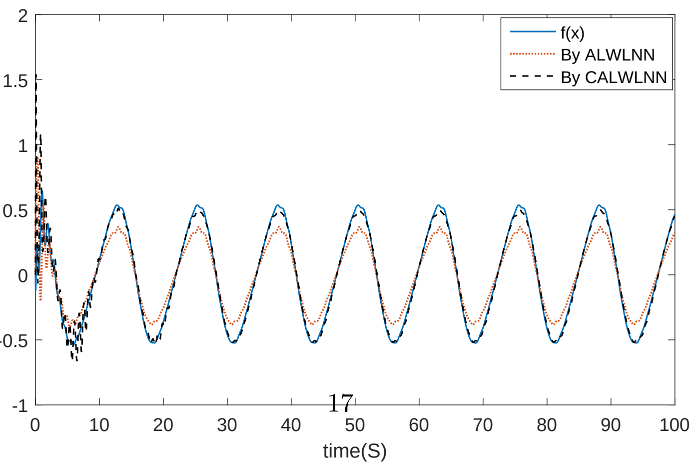
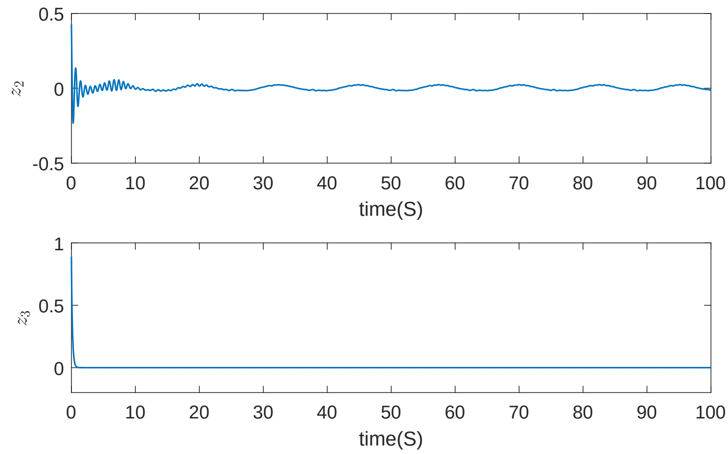


(c)

Figure 2: Comparison of the BLF-based CALWLC and the BLF-based ALWLC: (a) Comparison of the tracking performance; (b) Comparison of the NN approximation errors; (c) Comparison of the NN approximation performance.



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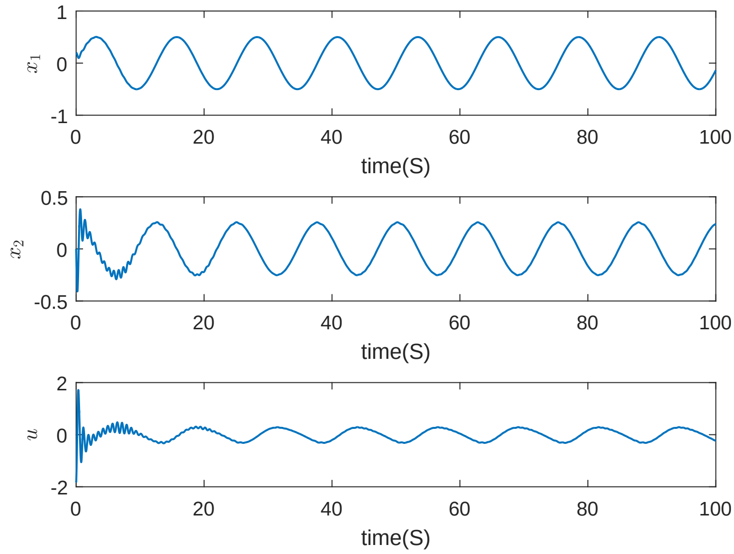

(b)

Figure 3: The tracking errors z_2, z_3, the states and the control input by the BLF-based CALWLC: (a) The tracking errors z_2 and z_3; (b) The states x_1, x_2 and the control input u.