Optimal data gathering paths and energy balance mechanisms in wireless networks

Jarry, Aubin; Leone, Pierre; Nikoletseas, Sotiris; Rolim, Jose

How to cite

JARRY, Aubin et al. Optimal data gathering paths and energy balance mechanisms in wireless networks. In: Ad hoc networks, 2011, vol. 9, n° 6, p. 1036–1048. doi: 10.1016/j.adhoc.2010.11.003

This publication URL: https://archive-ouverte.unige.ch/unige:32446

Publication DOI: 10.1016/j.adhoc.2010.11.003

Optimal Data Gathering Paths and Energy Balance Mechanisms in Wireless Networks

Aubin Jarry[1], Pierre Leone[1], Sotiris Nikoletseas[2], and Jose Rolim[1]

[1] Computer Science Department, University of Geneva, Battelle Batiment A, route de Drize 7, 1227 Geneva , Switzerland
[2] Computer Technology Institute N. Kazantzaki Str 1, Patras University Campus, 26504 Rion, Patras Greece

Abstract. This paper studies the data gathering problem in wireless networks, where data generated at the nodes has to be collected at a single sink. We investigate the relationship between routing optimality and fair resource management. In particular, we prove that for energy balanced data propagation, Pareto optimal routing and flow maximization are equivalent, and also prove that flow maximization is equivalent to maximizing the network lifetime. We algebraically characterize the network structures in which energy balanced data flows are maximal. Moreover, we algebraically characterize communication links which are not used by an optimal flow. This leads to the characterization of minimal network structures supporting the maximal flows.
We note that energy balance, although implying global optimality, is a local property that can be computed efficiently and in a distributed manner. We suggest online distributed algorithms for energy balance in different optimal network structures and numerically show their stability in particular setting. We remark that although the results obtained in this paper have a direct consequence in energy saving for wireless networks they do not limit themselves to this type of networks neither to energy as a resource. As a matter of fact, the results are much more general and can be used for any type of network and different types of resources.[3]

1 Introduction, our contribution and related work

In full generality, this paper addresses the question and impact of fairly allocating resources while routing messages in networks. Resources belong to the nodes composing the network and the constraints emerging from resource limitation concern the traffic handled by nodes. By fairly allocating the resources we mean that their use must be proportionally distributed among the nodes in accordance to the node's available resources. To exemplify, we consider the particular, important case where the resource is the total energy available at the nodes for transmitting data. We consider the *data gathering* problem, where the nodes generate data that has to be collected by a unique sink. In this setting, nodes have generally many choices for routing the data to the sink following a multiple-hop pattern. The energy consumption of a node depends on the particular costs of the links chosen for transmitting the data. Classically, we are interested in Pareto optimal routing schemes which are such that no node can decrease its energy consumption without increasing the energy consumption of others. Although Pareto optimality is classically used to solve multiobjective optimization problems, this criterion usually does not define a routing scheme uniquely. However, we show that if we consider energy-balanced routing schemes then Pareto optimal and maximal flows are equivalent. This result is relevant because energy-balance is a local characteristic of flows and is suitable to be efficiently and distributively computed. Moreover, we show that maximizing the flow of data is equivalent to maximizing the lifetime of the network.

[3] Contact author: Pierre Leone, E-mail: pierre.leone@unige.ch. Partially supported by the ICT Program of the European Union under contract number ICT-2008-215270 (FRONTS).

Another novelty of the paper is to algebraically characterize network's structure such that energy-balanced flows of data are maximal. We call such networks *energy-balance optimal*. This result is based on the equivalence between maximal flow and Pareto optimal solution provided that the flow is energy-balanced. Moreover, we also consider communication graphs Γ which contain an energy-balance optimal subgraph Λ and which are such that the maximal energy-balance flow in Λ cannot be increased by adding edges in Γ to the communication graph. In this case, we define that Λ is *energy-balance optimal in* Γ. As an application we investigate a particular simple topology which is energy-balance optimal in the complete graph and two realistic energy-balance optimal network structures.

To conclude the paper, we suggest an algorithm to online and distributively balance the energy-consumption of the nodes on the top of energy-balance optimal network structures. Numerical validations show that the algorithm is stable in the sense that the difference between the maximal and minimal energy consumption is bounded. Although theoretical works have still to be conducted to theoretically understand the conditions ensuring the existence of energy-balance flows, this (partly) validates our assertion that the local character of energy-balance flows is suitable for distributed online algorithms.

An important application of our work is the ability to maximize the flow (and also the network lifetime) in any particular communication graph by generating an energy balanced flow. This is an important generalization over previous work ([19, 23]). Interestingly, our results here imply (a) energy balanced data propagation using only two transmission levels (i.e. either to one hop neighbors or to the sink directly) is optimal, since they maximize the flow (b) and, we show the conditions under which we can compute such an energy balanced data propagation pattern.

To conclude this section, we review some relevant works dedicated to energy aware mechanisms, maximization of the network's functioning time and appropriate network structures.

A general framework is proposed in [11] to define the data traffic flows that maximize the lifetime of sensor networks. The authors provide explicit expressions of the lifetimes of the sensors by considering a particular model of the energy consumption. They formulate the problem as a multiobjective optimization problem. The optimal solutions are weak Pareto optimum and are computed in a centralized way. The paper contains the result that energy-balance consumption maximizes the lifetime of sensor networks.

Another general line of research consists in applying reinforcement learning theory to develop energy-aware routing protocols [15, 16]. This approach is relevant in the situations where no information is known about the structure of the network because the protocol learns dynamically the structure. The development of efficient protocols depends on the use of a suitable cost function. A natural extension of our present work would be to learn dynamically the network structure and find optimal paths.

In [20] the authors consider the problem of gathering data to a single sink and the search for flows that balance the energy consumption between sensors. This work is extended by the same authors in [27] in which they divide the network into slices, composed of sensors located at nearly equal distances to the sink and able to send the data to sensors belonging to the next slice (toward the sink) or directly to the sink. They assume that the energy used to send data directly to the sink is proportional to the square of the distance of transmission. To balance the energy consumption between the sensors, they define two periodic epochs: during the first the sensors send data to the sink and during the second the sensors send the data to the next slice. The optimal ratios must be determined in order to balance the energy consumptions of sensors. In their work, simulations are required to compute the optimal ratios. Independently, in [14] a very similar setting is considered. However, to balance the energy consumption the sensors transmit the data either directly to the sink or to the next slice. The sensors choose randomly between the two routes. The authors provide an offline algorithm to compute the optimal probabilities. We also mention [38] that is very close in spirit to the articles already mentioned. They formulate the problem of balancing the energy consumption as an allocation problem that is solved in a centralized way. Numerical validations show that the algorithm performs well, in particular if the network is dense.

We have extended this line of research by establishing formally the conditions ensuring that such energy-balance strategy is realistic in [34]. By establishing the formal link between the lifetime of the network and energy-balance mechanism, we provide a formal definition of optimal routing strategies in [34]. Moreover, we provide a decentralized algorithm to compute the probabilities with minimal assumptions and we establish formally that energy-balance mechanisms are efficient. Indeed, such a strategy maximizes the flow in the network [23, 26]. The numerical simulations show that the division of the network in slices is not regular and some sensors contribute more to the routing process than some others do. In [34], we propose and we validate numerically an inter-slice spreading mechanism to overcome the impact of these irregularities to the energy consumption. Our present work generalizes these results.

In [33], the authors consider the problem of network design and they assume that sensor parameters can be set accordingly to the position of the sensors. More precisely, the ranges of emissions vary depending on the slice in which the sensors belong. The network is decomposed in slices of varying size and the aim is to find the appropriate values of the ranges of emission in order to maximize the lifetime of the network. With this model they prove that in order to minimize the total energy spent on routing the slices must have the same size, and they point out that this result in not satisfactory since some critical nodes run out of energy and they prevent the continuation of the global process. Hence, they specifically consider the problem of balancing the energy among the slices. However, it turns out that in the situation where the power attenuation factor is 2, energy depletion is intrinsic to the system and cannot be avoided with this strategy. The impossibility result shows that energy-balance mechanisms necessitate some kind of heterogeneity, i.e. different power transmissions, nodes with more energy available, non-uniform distribution of the nodes, and so on

In [2], the authors consider a problem similar than in [33]. They show that a suitable deployment of heterogeneous nodes can guarantee energy-balance consumption. The deployment strategy locates the nodes with the larger amount of energy close to the sink to support the heavy data traffic. They also suggest a dissemination protocol that exploit the mobility of the sink. Moreover, they show that by suitably choosing the ranges of the communications, energy-balance depletion of energy is possible if the sensors are uniformly distributed.

The LEACH protocol (Low Energy Adaptive Clustering Hierarchy) is proposed in [21] as a protocol to achieve good performances in terms of sensors' lifetime. The constraints imposed to the protocol are that the deployment of the network must be easy; the protocol must increase the lifetime of the network and being respectful of the latency. Actually, the protocol self-organizes the network into clusters with local cluster head election. In order to consume energy evenly the cluster head changes with time. Recently, [32] uses clustering techniques to balance energy as well as tasks in the network. In a sense the scope of applications of protocols like LEACH is broader than what we suggest in this paper. However, this generality requires much more complex operations and more collaboration between the nodes. Moreover, optimality results are hardly possible because of the generality of the applications. Nevertheless, broaden the scope of applications of our results is relevant and we expect that our present work can be extended to more general situations like the optimal routing of multicommodity flows for instance.

The mobility of the sink [29, 30, 28, 13] is another alternative to increase the lifetime of sensor networks when gathering data is the issue. In [30], the authors consider different approaches to mobility and they suggest a theoretical framework suitable to the formal analysis of the protocol performances. Given the data flow, they formulate the problem as a linear program. In [28] it is proved that the problem is NP-hard and they investigate approximation algorithms in [30]. Notice that the optimal and decentralized coordination of the motion of the sink is still a challenging problem.

The mobility of the sensors to convey data toward the sink received some attention. In [5], a mobility index is proposed and used to evaluate the chance that a mobile sensor passes close to the sink in the near future. They propose different mobility patterns that extend the usual ray waypoint model and an index that measures the amount of distance covered by the mobile sensor. Sensors with small mobility index try to send data to sensors with higher mobility index. Heuristically, this allows the reduction of the latency of delivering data. Numerical validation of the protocol shows that the approach is relevant. In [3] we improve the performances of the algorithm by recursively estimating the probability of delivering the data to the sink.

There are various possible definition of the network lifetime. Usually, one assume that all the sensors should function to implement the global function. However, one might also assume that only part of the network should function. This leads to the concept of alpha-lifetime which is the time duration of the network during which at least α percent of the network functions. For instance, for region monitoring applications, solving this problem amounts to schedule appropriately the sensors activity in order to maximize the lifetime of the network and monitor at least α percent of the required area at any time [36, 37]. In [36], the authors provide upper bound of alpha-lifetime for large sensor networks. In [37], a centralized algorithm is suggested and numerically validated. In the same setting, in [1], the authors suggest taking advantage of the overlap between the sensor monitored regions. They propose an algorithm to schedule active and inactive periods while ensuring that sensors monitor the entire region at any time. Alternate inactive and active period is a classic way of reducing the energy consumption.

Another new metric accounting for network lifetime is the lifetime per unit cost [10] which is the average network lifetime divided by the number of sensors enabled. The observation that the lifetime of networks increases monotonically with the number of sensors involved in the network justifies this definition. However, the lifetime per unit cost decreases for large value of the total number of sensor after reaching a maximal value. This implies that efficient network design must consider the optimal number of sensors to deploy. The optimal placement of nodes is also discussed in the paper. The problem is to reduce the transmission energy consumption to report data to running sensors.

Classical optimization methods are relevant for lifetime maximization. We mention here an application of linear programming. In [18], it is assumed that sensors have to send data periodically toward the sink and accordingly a linear program is solved in order to minimize the energy required for each period. The constraints express the minimization of the energy consumption and the preservation of the flow. They formulate solutions in closed form of the linear program for various network topologies. In particular, they show that for the line model, the routing strategy ensuring maximum lifetime consists only in choosing between sending a data directly to the next node or directly to the sink (a result that we obtain independently with different technical details). In [22], a similar linear programming approach is proposed. The authors show that solving the original linear program is equivalent to solve a maximum-flow problem. To proceed to the reduction, the authors assume that sensors transmit with only one level of energy transmission. They propose and they validate numerically a heuristic distributed solution to the maximum-flow problem.

Simultaneous transmissions lead to collisions, i.e. data are lost. An appropriate scheduling of the transmissions avoids collision. In [35, 6, 8, 7], the authors discuss the problem of determining the optimal scheduling. In [35], the quantity that is optimized is the energy consumption. Energy-latency tradeoffs are formulated and an off-line algorithm that computes the optimal solution to the problem is proposed. A heuristic on-line algorithm is proposed and the simulations show the good performances of the algorithm. In [6, 8, 7], the quantity that is optimized is the data latency. The authors compute tight bounds of the minimal number of rounds that are required for solving the problem. They propose an efficient algorithm that computes the optimal scheduling in tree-like networks.

These considerations are relevant for real applications and should be part of the extensions of our present work. In this paper, we do not consider interferences and focus on energy management. This is possible in our setting. Indeed, we assume that we solve the data gathering problem without aggregation. In this setting the rate of data generated by the nodes is not necessarily large and it is likely that at most one node transmits at any time.

2 Balancing the flow and maximizing the lifetime of a network

In this section, we first prove that maximizing the lifetime of a sensor network is equivalent to solving a max-flow problem (Proposition 1). We then define energy-balanced flows (Definition 2) and give sufficient conditions under which energy-balanced flows are optimal (Proposition 2).

We do not consider interferences between nodes that transmit simultaneously. This is justified with the argument that in the data gathering problem that we consider the rate of data generated by the nodes is not necessarily large. Then, we can assume that at any time at most one node transmits and focus on energy aware mechanisms. Further works should include the consideration of the interferences due to simultaneous transmissions. For real applications it is relevant to know if energy-balance mechanisms are compatible with lowering the interferences.

We consider a finite set of nodes and label them $i = 1, \ldots, N$. Nodes are able to communicate between each other only if they share a communication link. The set of nodes and communication links has the structure of a graph, called the communication graph, and they compose the network under study. In order to communicate, the nodes need to spend resources and the total amount of available resources is a local property of the nodes. Specifically, a node i may have to spend c_{ij} units of energy in order to transmit a message to node j. In this instance, assuming that the total energy available per node is limited, the nodes have to wisely use the available communications links in order to maximize the functional lifetime of the network. When a wireless sensor network monitors an area, the events that are detected near a sensor i must continuously be reported to the base station or sink, and this generates a fraction g_i of the total flow of information f. In other words, $g_i \cdot f$ messages per second are generated by sensor i and $\sum_{i=1}^{N} g_i = 1$. We denote f_{ij} the flow of messages from node i to node j with the convention that the sink is numbered 0, i.e. f_{i0} is the flow from node i to the sink. By convention, we define $f_{ii} = 0,\ i = 1, \ldots, N$ and we do not repeat this while we consider equations for flows. Finding the flow $\{f_{ij}\}$ which maximizes the lifetime of the sensor network amounts to solving the following problem:

Problem 1.

$$\text{maximize } T \text{ such that} \tag{1}$$

$$g_i \cdot f + \sum_{j=1}^{N} f_{ji} = \sum_{j=0}^{N} f_{ij}, \quad i = 1, \ldots, N \tag{2}$$

$$T \sum_{j=0}^{N} f_{ij} c_{ij} \le b_i, \quad i = 1, \ldots, N \tag{3}$$

$$T \ge 0,\ f_{ij} \ge 0, \quad i, j = 1, \ldots, N \tag{4}$$

Equation (2) represents the constraints ensuring that $\{f_{ij}\}$ is a flow and Equation (3) ensures that in the duration T no sensor consumes more than its available energy b_i.

In Problem 1 we use implicitly that

Definition 1. *The lifetime of the network is the minimal lifetime of the nodes composing the network, i.e.*

$$\min_{i=1,\ldots,N} \frac{b_i}{\sum_{j=0}^{N} f_{ij} c_{ij}}. \tag{5}$$

To be precise, we mention that the units of b_i are energy $[J]$, the units of flows f_{ij} are $[messages/second]$, the units of c_{ij} are $[J/message]$. With this choice of units, the lifetime is given in seconds. If we proceed to the change of variables $\tilde{f_{ij}} = T f_{ij}$ ($\tilde{f_{ij}}$ is the total amount of messages sent from i to j in the duration T), and $\tilde{f} = T f$, we get the following linear program:

Problem 2.

$$\text{maximize } \tilde{f} \text{ such that} \tag{6}$$

$$g_i \cdot \tilde{f} + \sum_{j=1}^{N} \tilde{f_{ji}} = \sum_{j=0}^{N} \tilde{f_{ij}} \quad i = 1, \ldots, N \tag{7}$$

$$\sum_{j=0}^{N} \tilde{f_{ij}} c_{ij} \leq b_i \quad i = 1, \ldots, N \tag{8}$$

$$\tilde{f} \geq 0, \ \tilde{f_{ij}} \geq 0, \ i, j = 1, \ldots, N \tag{9}$$

We emphasize the fact that Problem 1 considers the rate of data measured in $[messages/second]$ while Problem 2 considers the total amount of data collected by the sink measured in $[messages]$. Another difference is that in Problem 1 the total flow f is fixed while in Problem 2 $\tilde{f}$ is variable. This distinction appears again when we discuss a weak optimal Pareto approach to the problem. We will now formally prove that maximizing T amounts to maximizing $\tilde{f}$.

Proposition 1. *Problem 1 is equivalent to Problem 2, which means that* $\{f_{ij}\}, T$ *is an optimal solution to the first if and only if* $\{\tilde{f_{ij}} = Tf_{ij}\}, \tilde{f} = Tf$ *is an optimal solution to the second.*

Proof. We proceed by contradiction. Let us assume that $\{f_{ij}\}, T$ is an optimal solution to Problem 1 and that $\{\tilde{f_{ij}} = Tf_{ij}\}, \tilde{f} = Tf$ is not an optimal solution to Problem 2. Then, there is solution $\{\tilde{g_{ij}}\}, \tilde{g}$ to Problem 2 such that $\tilde{g} > \tilde{f}$. We consider $T' > T$ such that $\tilde{g} = T'f > \tilde{f}$ and define $\{g_{ij}\} = \{\frac{1}{T'}\tilde{g_{ij}}\}$. We check directly that $\{g_{ij}\}, T'$ is a feasible solution to Problem 1 with $T' > T$ which contradicts the optimality of T. We proceed similarly to show the other direction of the equivalence.

This proposition shows that maximizing the lifetime of the network is equivalent to maximizing the total number of messages gathered by the sink. Problem 2 corresponds to a max-flow problem where constraints are placed on the nodes. We now define what an energy-balanced flow is and propose a sufficient set of conditions ensuring that an energy-balanced flow maximizes the lifetime of the network.

Definition 2. *A flow* $\{f_{ij}\}$ *is called energy-balanced if there is a constant* k *such that for all* $i = 1, \ldots, N$ *we have* $\sum_{j=0}^{N} f_{ij} c_{ij} = kb_i$.

Proposition 2. *If for all* $i = 1, \ldots, N$ *there is* λ_i *such that*

$$\lambda_i \geq 0 \quad \textit{and}$$

$$-\lambda_i c_{i0} + \lambda_i c_{ij} + \lambda_j c_{j0} \geq 0,$$

and if there is an energy-balanced flow $\{f_{ij}\}$ *such that*

$$f_{ij} \cdot \left(-\lambda_i c_{i0} + \lambda_i c_{ij} + \lambda_j c_{j0}\right) = 0 \tag{10}$$

then there is a duration T *such that* $\{f_{ij}\}, T$ *is a solution to Problem 1.*

Proof. We consider a path decomposition P of the flow $\{f_{ij}\}$, and a path $i_1, i_2, \ldots, i_k$ of this decomposition from a sensor $i = i_1$ towards the sink $i_k = 0$, we can see from Equation (10) that

$$\lambda_{i_1} c_{i_1 0} = \lambda_{i_1} c_{i_1 i_2} + \lambda_{i_2} c_{i_2 i_3} + \ldots + \lambda_{i_{k-1}} c_{i_{k-1} i_k} \tag{11}$$

We want to compute the sum $\sum_i \lambda_i \sum_j f_{ij} c_{ij}$ by decomposing the flow into the paths followed by messages to the sink. For each such path p from i to the sink, Equation (11) shows that the contribution to the sum is $\lambda_i f_p c_{i0}$ where f_p is the number of messages per second flowing through the path p in the decomposition P. Since $g_i \cdot f$ is equal to the sum $\sum_{p \text{ from } i \text{ to } 0} f_p$, we get

$$\sum_i \lambda_i \sum_j f_{ij} c_{ij} = \sum_i \lambda_i g_i f c_{i0}.$$

On the other hand, any other solution $\{f'_{ij}\}, T'$ to Equations (2) and (3) uses paths satisfying the equation

$$\lambda_{i_1} c_{i_1 0} \leq \lambda_{i_1} c_{i_1 i_2} + \lambda_{i_2 i_3} c_{i_2 i_3} + \ldots + \lambda_{i_{k-1}} c_{i_{k-1} i_k}$$

which leads to

$$\sum_i \lambda_i \sum_j f'_{ij} c_{ij} \geq \sum_i \lambda_i g_i f c_{i0}$$

We conclude by using the assumption that the flow $\{f_{ij}\}$ is energy-balanced and by calling $T = \frac{1}{b_i} \sum_{j=0}^{N} f_{ij} c_{ij}$. Indeed,

$$f \cdot T \sum_i \lambda_i g_i c_{i0} = T \sum_i \lambda_i \sum_j f_{ij} c_{ij} = \sum_i \lambda_i b_i \geq T' \sum_i \lambda_i \sum_j f'_{ij} c_{ij} \geq f' \cdot T' \sum_i \lambda_i g_i c_{i0}$$

which shows that $T \geq T'$.

Actually, what we have shown is that if a communication graph contains only edges (i, j) such that $-\lambda_i c_{i0} + \lambda_i c_{ij} + \lambda_j c_{j0} = 0$, then any energy-balanced flow will maximize its lifetime. In other words, our result shows that edges (i, j) such that $-\lambda_i c_{i0} + \lambda_i c_{ij} + \lambda_j c_{j0} > 0$ are useless to increase the efficiency of the network. In a former work [23], appropriate λ_i values were computed in an ad-hoc way under the specific condition that the energy consumption grows quadratically with the range of emission.

3 Weak Pareto optimality and energy-balanced flows

In this section, we introduce weak Pareto optimality (Definition 3) and prove that for energy-balanced flows in sensor networks, weak Pareto optimality is equivalent to maximizing the lifetime of the network (Proposition 3).

Assuming a flow $\{f_{ij}\}$ of information towards the sink of a sensor network, the rate of energy consumption of a sensor i is given by $\sum_j f_{ij} c_{ij}$ and the lifetime of this sensor is inversely proportional to this rate, i.e. given by $b_i / \sum_j f_{ij} c_{ij}$. To maximize the lifetime of the sensors we have to maximize all the lifetime simultaneously. This leads to study the following multiobjective optimization problem:

Problem 3. Minimize $\{i : \sum_j f_{ij} c_{ij}\}$ such that $\forall i = 1, \ldots, N$

$$f \cdot g_i + \sum_{j=1}^{N} f_{ji} = \sum_{j=0}^{N} f_{ij}$$

The weak Pareto approach, proposed in [12], is particularly well suited to study such multi-objective optimization problems.

Definition 3 (weak Pareto optimal flow). *A flow $\{f_{ij}\}$ is weak Pareto optimal if and only if there does not exist any flow $\{f'_{ij}\}$ such that $\sum_i f'_{i0} = \sum_i f_{i0}$ and*

$$\sum_{j=0}^{N} f'_{ij} c_{ij} < \sum_{j=0}^{N} f_{ij} c_{ij}, \quad \forall i = 1, \ldots, N \tag{12}$$

Intuitively this means that given a weak Pareto optimal solution it is not possible to increase the lifetime of a node without decreasing the lifetime of another. In [12], an algorithm is suggested to compute a flow such that the lifetime of all the sensors is the same and which produces a best approximation if no solution exists.

In the following, we prove that by looking only at energy-balanced flows, maximizing the lifetime of the network (Problem 1) is equivalent to finding a weak Pareto optimal solution to Problem 2. We emphasize that the equivalence is proved for energy-balanced propagation scheme.

Proposition 3. *An energy balanced flow (Definition 2) maximizes the lifetime of the network (Problem 1) if and only if it is a weak Pareto optimal solution (Definition 3) to the multiobjective optimization problem (Problem 3).*

Proof. We first prove that an energy balanced flow $\{f_{ij}\}$ that maximizes the lifetime T of the network is a weak Pareto optimal solution to (3). Assume that $\{f_{ij}\}$ is not weak Pareto optimal: then there is a flow $\{f'_{ij}\}$, such that $\sum_i f'_{i0} = \sum_i f_{i0}$ and

$$\sum_{j=0}^{N} f'_{ij} c_{ij} < \sum_{j=0}^{N} f_{ij} c_{ij}, \quad \forall i = 1, \ldots, N \tag{13}$$

The flow $\{f_{ij}\}$ satisfies the energy constraints (3), hence using (13) we have

$$T \sum_{j} f'_{ij} c_{ij} < b_i, i = 1, \ldots, N.$$

Therefore there is $T' > T$ such that $\{f'_{ij}\}, T'$ is a solution to Problem 1. This contradicts the optimality of the flow $\{f_{ij}\}$.

We next show that an energy-balanced flow which is a weak Pareto optimal solution solves Problem 1. We proceed by contradiction and assume that the flow $\{f_{ij}\}$ is energy-balanced and weak Pareto optimal but does not maximize the lifetime of the network. Then, there is a solution $\{f'_{ij}\}, T'$ to Problem 1 such that $T' > T$. The energy constraints satisfied are

$$T \sum_{j=0}^{N} f_{ij} c_{ij} = b_i, \quad \text{and} \quad T' \sum_{j=0}^{N} f'_{ij} c_{ij} \leq b_i, \quad \forall i = 1, \ldots, N.$$

Therefore, we have

$$\sum_{j=0}^{N} f'_{ij} c_{ij} \leq \frac{T}{T'} \sum_{j=0}^{N} f_{ij} c_{ij}$$

which shows that the flow $\{f_{ij}\}$ is not weak Pareto optimal.

4 Optimal communication graphs

We have seen that the good structures of the communication graphs on which optimal energy-balanced flows exist can be characterized (Proposition 2). In this section, we broaden this characterization by using the fact that optimal energy-balanced flows are also weak Pareto solutions to the multiobjective optimization problem (Problem 3.

Given a weak Pareto optimal solution of a multiobjective optimization problem, there is a way to formulate the problem as a classical linear program such that the optimal solution is the weak Pareto optimal solution [17, 4]. Precisely, given a weak Pareto optimal solution to Problem 3 there is $\{\lambda_i\}$, such that $\sum_{i=1}^{N} \lambda_i = 1$, such that $\forall i = 1, \ldots, N$, $\lambda_i \geq 0$, and such that the weak Pareto solution coincides with the optimal solution of the following linear problem:

Problem 4. Minimize $\sum_{i=1}^{N} \lambda_i \sum_{j=0}^{N} f_{ij} c_{ij}$ such that $\forall i = 1, \ldots, N$

$$f \cdot g_i + \sum_{j=1}^{N} f_{ji} = \sum_{j=0}^{N} f_{ij} \tag{14}$$

$$f_{ij} \geq 0 \quad \forall j \tag{15}$$

We can readily prove that any solution to Problem 4 is a weak optimal solution to Problem 3 for any $\lambda_i \geq 0$. This linear problem is easier to study by making the $\{f_{ij}\}$ independent. We remove their dependency by replacing f_{i0} with $f \cdot g_i + \sum_j f_{ji} - \sum_j f_{ij} \geq 0$. We can then consider the following linear program:

Problem 5. Minimize $\sum_{i=1}^{N} \lambda_i \big[\big(f \cdot g_i + \sum_{j=1}^{N} f_{ji} - \sum_{j=1}^{N} f_{ij} \big) c_{i0} + \sum_{j=1}^{N} f_{ij} c_{ij} \big]$ such that

$$f \cdot g_i + \sum_{j=1}^{N} f_{ji} - \sum_{j=1}^{N} f_{ij} \geq 0, \quad \forall i = 1, \ldots, N.$$

The dual of Problem 5 is

Problem 6. Maximize $\sum_{i=1}^{N} \big(\lambda_i g_i c_{i0} - \beta_i g_i \big)$ such that

$$-\lambda_i c_{i0} + \lambda_i c_{ij} + \lambda_j c_{j0} - \beta_j + \beta_i \geq 0, \; i, j = 1, \ldots, N.$$

By classical duality theory [24], we have the following equation:

$$\sum_i \lambda_i \big[\big(f \cdot g_i + \sum_j f_{ji} - \sum_j f_{ij} \big) c_{i0} + \sum_j f_{ij} c_{ij} \big] \geq f \sum_i (\lambda_i g_i c_{i0} - \beta_i g_i) \tag{16}$$

and the complementary slackness conditions are given by

$$f_{ij} \big(-\lambda_i c_{i0} + \lambda_i c_{ij} + \lambda_j c_{j0} - \beta_j + \beta_i \big) = 0, \; i = 1, \ldots, N. \tag{17}$$

$$\beta_i \big(f \cdot g_i + \sum_j f_{ji} - \sum_j f_{ij} \big) = 0, \quad i = 1, \ldots, N \tag{18}$$

Definition 4. *A communication graph Λ is energy-optimal if there exists constants $\lambda_i \geq 0, \; i = 1, \ldots, N$ and $\beta_i \geq 0, \; i = 1, \ldots, N$ such that $-\lambda_i c_{i0} + \lambda_i c_{ij} + \lambda_j c_{j0} - \beta_j + \beta_i = 0$.*

A subgraph Λ of Γ is energy-optimal in Γ if Λ is an energy-optimal path and $-\lambda_i c_{i0} + \lambda_i c_{ij} + \lambda_j c_{j0} - \beta_j + \beta_i > 0$ for all edges $(i, j) \in \Gamma \setminus \Lambda$.

Precisely, the complementary slackness conditions (17) and (18) say that

- An energy-balance flow on the top of an energy-optimal communication graph Λ is maximal.
- If Λ is energy-balance optimal in Γ then using edges in $\Gamma \setminus \Lambda$ cannot improve the flow of data (optimal subgraph property).
- In any case, direct transmissions to the sink are possible and optimal only if $\beta_i = 0$.

Proposition 4. *The set of conditions defining an energy-balanced optimal communication graph is a convex set.*

Proof. Given $\{\lambda_i\}$, $\{\beta_i\}$ and $\{\lambda'_i\}$, $\{\beta'_i\}$ satisfying the equations

$$-\lambda_i c_{i0} + \lambda_i c_{ij} + \lambda_j c_{j0} - \beta_j + \beta_i = 0$$

and

$$-\lambda'_i c_{i0} + \lambda'_i c_{ij} + \lambda'_j c_{j0} - \beta'_j + \beta'_i = 0$$

it is readily checked that $\{\bar{\lambda}_i = p\lambda_i + (1-p)\lambda'_i\}$, $\{\bar{\beta}_i = p\beta_i + (1-p)\beta'_i\}$ with $0 < p < 1$ satisfies the equation

$$-\bar{\lambda}_i c_{i0} + \bar{\lambda}_i c_{ij} + \bar{\lambda}_j c_{j0} - \bar{\beta}_j + \bar{\beta}_i = 0.$$

Energy-balance optimal communication graphs are algebraically characterized and have the important property that energy-balance flows are also maximal. This property is necessary and sufficient, as stated in the following proposition.

Proposition 5. *An energy-balanced optimal flow determines an energy-balanced optimal communication graph.*

Proof. An energy-balanced optimal flow is equivalent to a Pareto optimal solution to Problem 3 or Problem 4 by Proposition 3. The complementary slackness conditions (17) and (18) are then satisfied (for suitable λ_i, β_i) and characterize an energy-balance optimal communication graph.

Proposition 6. *If the optimal flow satisfies* $f_{i0} = f \cdot g_i + \sum_j f_{ji} - \sum_j f_{ij} > 0$, *then* $\beta_i = 0,\ i = 1, \ldots, N$.

Proof. This is due to the slackness condition expressed in Equation (18).

Two remarks can be made to conclude this section. First, the conditions stated in Equation (17) restrict the set of non-vanishing f_{ij} in the optimal solutions. Then, the λ_i values depend on the particular Pareto optimal solution we search for.

5 Examples of energy-balance optimal communication graphs

In this section, we present two examples of structures that allow for the existence of energy-balance optimal flows.

We first expose simple sufficient conditions ensuring the optimality of energy-balance flows. If we set $\beta_i = 0$ in Equations (17) and (18), the optimal solution to both Problems 4 and 6 is equal to $f \cdot \sum_i \lambda_i g_i c_{i0}$ (found by replacing f_{ij} with 0 in Problem 5) and the constraints set in Problem 6 read

$$-\lambda_i c_{i0} + \lambda_i c_{ij} + \lambda_j c_{j0} \geq 0,\ i = 1, \ldots, N. \tag{19}$$

The complementary slackness conditions set in Equation (17) now read

$$f_{ij}\left(-\lambda_i c_{i0} + \lambda_i c_{ij} + \lambda_j c_{j0}\right) = 0, \tag{20}$$

Thus, f_{ij} can be non-vanishing only if $-\lambda_i c_{i0} + \lambda_i c_{ij} + \lambda_j c_{ij} = 0$. We summarize this in the following proposition.

Proposition 7. *Let us assume that there exist constants* λ_i *with* $\sum_i \lambda_i = 1$ *satisfying Equation* (19) *and a flow* $\{f_{ij}\}$, f *satisfying the complementary slackness conditions* (20). *Then, the flow is a weak Pareto optimal solution to Problem 3.*

Proof. Conditions set in Equation (20) imply that the value function of Problems 5 and 6 is equal. Given any set of λ_i values, the solution to Problem 5 is always a weak Pareto optimal solution to Problem 3.

5.1 A first energy-balance optimal topology

We now turn to the application of Proposition 7. We consider the complete graph on the set of vertices. Each edge is assigned a weight c_{ij} which corresponds to the energy cost of transmitting through that edge. We assume that we are able to compute λ_i constants such that Equation (19) is satisfied. If we can generate an energy-balanced flow $\{f_{ij}\}$ such that the conditions (20) are satisfied then by Proposition 3 this flow is maximal. Moreover, by the discussion of Section 2 this flow also maximizes the lifetime of the network.

This is used in an ad-hoc way in the papers [19, 23] where it is assumed that sensors can transmit data to their nearest neighbor or directly to the sink. In these papers, the energy required to send data to neighbors at distance ≤ 1 is constant and the same for all the sensors. Moreover, it is assumed that the energy needed for transmissions otherwise grows like the square of the transmission distance. A directed acyclic graph is built to transmit data from any sensor to a unique sink by decomposing the network in slices corresponding to nodes distance from the sink to an equal number of hops, as shown in the left of Figure 1. The nodes belonging all to any particular slice are identified as a *super* node. This leads to the topology shown in the right of Figure 1. Normalizing the distances such that the transmission power to the nearest neighbors is one, explicit values for the λ_i constants are provided, i.e. $\lambda_i = \frac{1}{i(i+1)}$. Proposition 7 (and the results contained in the cited papers) shows that an energy-balanced flow using only one hop communications or direct transmissions to the sink maximizes the flow of data.

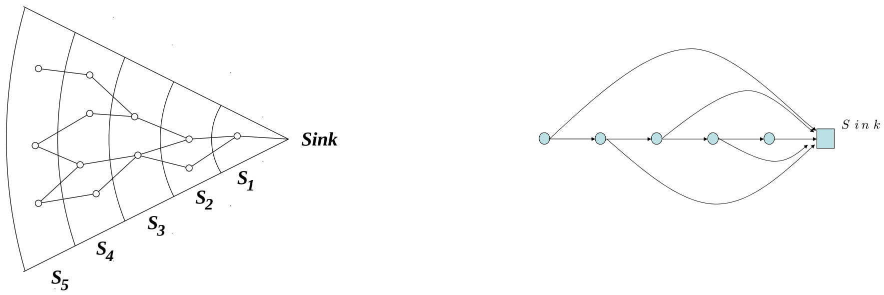

Fig. 1. An energy-balance optimal topology on the left, sensors transmit data to the next slice or directly to the sink (not represented). A simple model of it on the right.

In fact, assuming that sensors use only two levels of energy in their transmissions, corresponding to short range transmissions to close neighbors and to long range transmissions to the sink, we prove that we can always compute λ_i satisfying the hypothesis on proposition 7.

Proposition 8. *We assume that the sensors composing the network transmit with two levels of energy, i.e. sensor i transmits with energy level c_i to a close neighbor or with c_{i0} to the sink with $c_i = c_j$ and $c_{i0} > c_i$, for all $i = 2, \ldots, N$ and $j = 1, \ldots, N$ (for sensors belonging to the first slice $c_i = c_{i0}$). Then, there exists λ_i satisfying the hypothesis of proposition 7. The transmission graph is directed to the sink and any sensors transmit either to the sink or to sensors belonging to the next slice (see the left side of Figure 1). The λ_i's values are equal for sensors belonging to the same slice.*

Proof. We first set arbitrarily $\lambda_i = 1$ for sensors belonging to the first slice. Using Equation (19) we recursively compute the values λ_j for sensors that are one more hop away from the sink. Because $c_{i0} > c_i$, we have $\lambda_i > 0$. At the end, we normalize to $\sum_{i=1}^{N} \lambda_i = 1$.

5.2 A second energy-balance optimal topology

We now present another energy-balance optimal topology. The network is again divided into slices of width r. Sensors can transmit either to sensors belonging to the next slice or to sensors belonging to the next slice following the next slice (two slices away towards the sink). The energy costs are respectively c_1 and c_2, $c_2 > c_1$, independently of the slice the sensor belongs to. Sensor belonging to the second slice away from the sink transmit to the first slice with energy consumption c_1 or directly to the sink with energy c_2. Sensors belonging to the first slice have no other choice than to transmit to the sink with energy consumption c_1. Figure 2 illustrates this energy-balance optimal topology.

Proposition 9. *The topology described above and depicted on Figure 2 is energy-balance optimal if $c_{i0} \geq c_1 + c_{i-1,0}(c_2 - c_1)/c_1$*

Proof. The coefficients λ and β depend only on the slice number S_i; we use λ_i and β_i to denote the coefficients of sensors belonging to the $i-$th slice. We have to prove that there exist coefficients λ_i and β_i such that the conditions stated in Definition 4 are satisfied. To each sensor belonging to the first slice we assign a value $\lambda_1 \neq 0$ and $\beta_1 = 0$. The value λ_1 will be defined by normalizing the λ_i such that $\sum \lambda_1 = 1$. The value $\beta_1 = 0$ is necessary since sensors transmit data to the sink as stated by Proposition 6.

For sensors belonging to the second slice, we assign a value λ_2 solution to the equation $-\lambda_2 c_2 + \lambda_2 c_1 + \lambda_1 c_1 = 0$ and $\beta_2 = 0$. We have $\lambda_2 \geq 0$ since $c_2 > c_1$.

For sensors belonging to another slice we compute the parameters λ_i and β_i recursively. Let us assume that the coefficients are computed for all slices $1, 2, \ldots, i-1$. We determine λ_i and β_i as solution to the system of equations

$$-\lambda_i c_{i0} + \lambda_i c_1 + \lambda_{i-1} c_{(i-1)0} - \beta_{i-1} + \beta_i = 0 \quad (21)$$

$$-\lambda_i c_{i0} + \lambda_i c_2 + \lambda_{i-2} c_{(i-2)0} - \beta_{i-2} + \beta_i = 0 \quad (22)$$

By subtracting equation (21) to (22) we get

$$\lambda_i \underbrace{(c_2 - c_1)}_{>0} + \underbrace{\lambda_{i-2} c_{i-2,0} - \lambda_{i-1} c_{i-1,0} + \beta_{i-1} - \beta_{i-2} = 0}_{=-\lambda_{i-1} c_1 < 0},$$

where the second underbraced term is $-\lambda_{i-1} c_1$ by recurrence hypothesis (see Equation (21) where $i-1$ is substituted by $i-2$ and i by $i-1$) and leads to $\lambda_i > 0$. We then have $\lambda_i = c_1/(c_2 - c_1)\lambda_{i-1}$.

To see that $\beta_i \geq 0$ we use Equation (21) recursively. We know that $\beta_1 = 0$ for sensors in the first slice. We assume that $\beta_j \geq 0$ for $j = 1, \ldots, i-1$. Equation (21) states that $\beta_i \geq \lambda_i c_{i0} - \lambda_i c_1 - \lambda_{i-1} c_{i-1,0}$ which is positive by the assumption on c_{i0}.

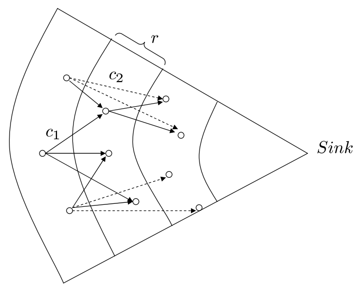

Fig. 2. An energy-balance optimal topology on the left, sensors transmit data to the next slice with energy consumption c_1 or two slices away with energy consumption c_2.

6 On the existence of energy-balanced probabilistic mechanisms

We proposed in the previous section reasonable network structures which are energy-balance optimal (Propositions 8 and 9). In this section, we discuss the existence of distributed energy-balance routing schemes on these topologies. We take advantage of the fact that energy-balance is a local property. Indeed, any node composing the network can check the balance of energy by comparing its energy consumption to that of its neighboring nodes. We exploit this local property to construct an energy-balanced flow while routing data in a distributed manner.

6.1 A first online distributed algorithm

We first consider the energy-balance optimal network structure defined in Proposition 8 with the additional constraint that the energy to transmit to the next slice is the same for any slice, i.e. in terms of Proposition 8 we have $c_i = c$, $\forall i$. We also assume that the communication channels are bidirectional and that while transmitting a message the nodes add information about their current level of energy consumption to it. This mechanism ensures that any node is aware of the level of energy consumption of its neighboring nodes. Moreover, we assume that the initial level of available energy is the same for all the nodes.

Upon reception of a message, node i forwards the message to the neighboring node j ($i \to j$) with probability p_{ij} or directly to the sink with probability p_{ii}. The probabilities are computed online by using the following rule: *If the energy consumption of node j is larger than the average energy consumption of the neighboring nodes then node i decreases p_{ij}, else p_{ij} is increased*[4], see Figure 3.

variables:
i: the identifier of the current node
p_{ij}: the probability of transmitting to node j, p_{ii} the probability of transmitting to the sink.
x_i: the level of energy consumption of the current node.
x_j: the level of energy consumption of a neighboring node j.
t_i: counts the number of messages sent by the current node
Initialize $p_{ij} = 1/deg_i$, where deg_i is the degree of the current node and includes the node i itself.
upon reception of a message
  $p_{ij} \leftarrow p_{ij} + \frac{1}{t_i}(\frac{1}{deg_i} \sum_{i \to k} x_k - x_j)$
  Normalize the p_{ij} so that $0 \leq p_{ij} \leq 1$ and $\sum_j p_{ij} = 1$
  $t_i \leftarrow t_i + 1$
  select a node j such that $i \to j$ with probability p_{ij}
  forward the message to the selected node or transmit directly to the sink if the selected node is i
end upon

Fig. 3. Pseudo-code of the program executed by the nodes with the topology define by Proposition 8

This algorithm is an application of stochastic approximation where we compute online the probability that the energy consumption of the neighboring nodes is higher than the average. As such, it can be straightforwardly adapted to more general situations. A formal analysis of the properties of the algorithm might be done in this framework and is left for further work. Background material can be found in [9, 31, 25].

The numerical validation of the algorithm is presented on Figure 4. Messages are generated successively and once routed to the sink we record the maximal and the minimal levels of energy consumption among all the nodes. The plots show clearly that the energy growth is linear and that the difference between the maximal and the minimal energy consumption is bounded. The difference is also plotted. The plots represent the routing of $50'000$ messages.

The particular communication graphs on the top of which the protocol was applied and produce the numerical results discussed above are plotted on Figure 5.

6.2 A second online distributed algorithm

The second energy-balance optimal topology that we consider is the one defined by Proposition 9. The strategy to balance the energy consumption is the following: *If the current level of energy consumption of a node is larger than the average energy of its neighbors then it decreases its energy consumption by increasing the number of messages sent to the next slice (cost c_1) and decreasing the number of messages sent two slices away (cost $c_2 > c_1$). In the*

[4] This is implemented by computing $p_{ij} \leftarrow p_{ij} + \frac{1}{t_i}(\frac{1}{deg_i} \sum_{i \to k} x_k - x_j)$

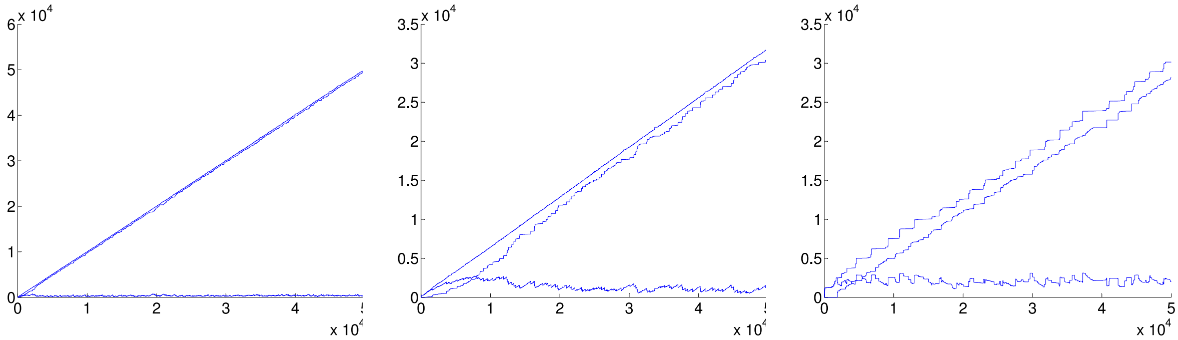

Fig. 4. Min-max energy consumption plots. From the left to the right the number of nodes is $30, 80, 130$ (the maximal distance from a node to the sink is 1), the communication slice size are $0.3, 0.2, 0.1$ and the nodes are scattered randomly.

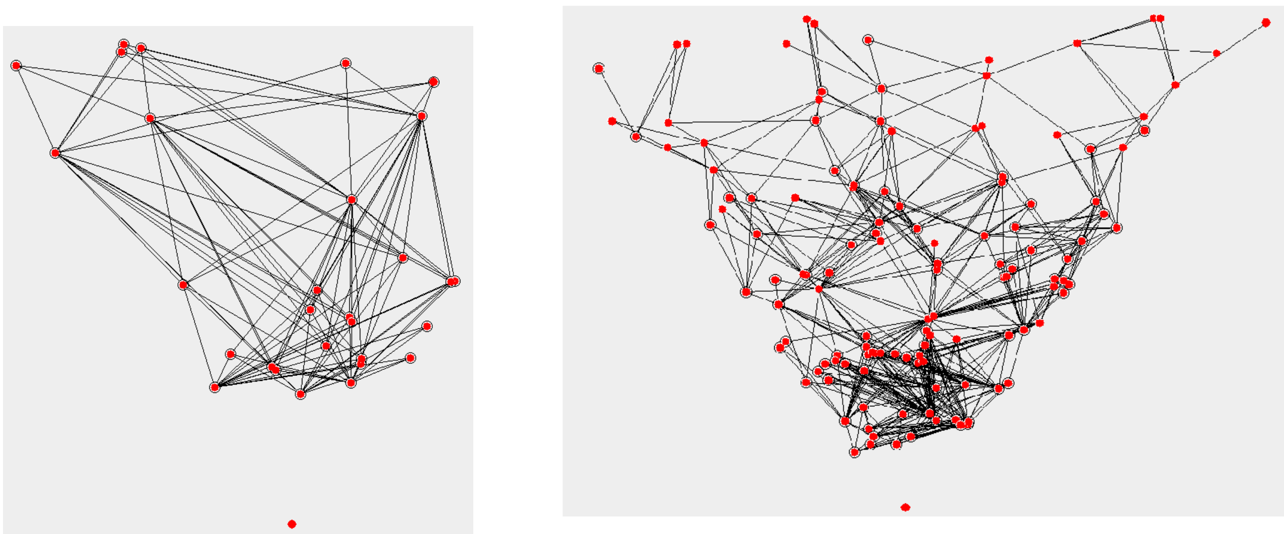

Fig. 5. The communication graphs of the two networks on the top of which were conducted the experiments. Only edges conveying more than 5% of the total traffic going out from a node are represented. On the left there are 80 nodes with size slice of 0.3 and on the left 130 nodes with size slice 0.1 (the maximal distance from the sink to a node is 1).

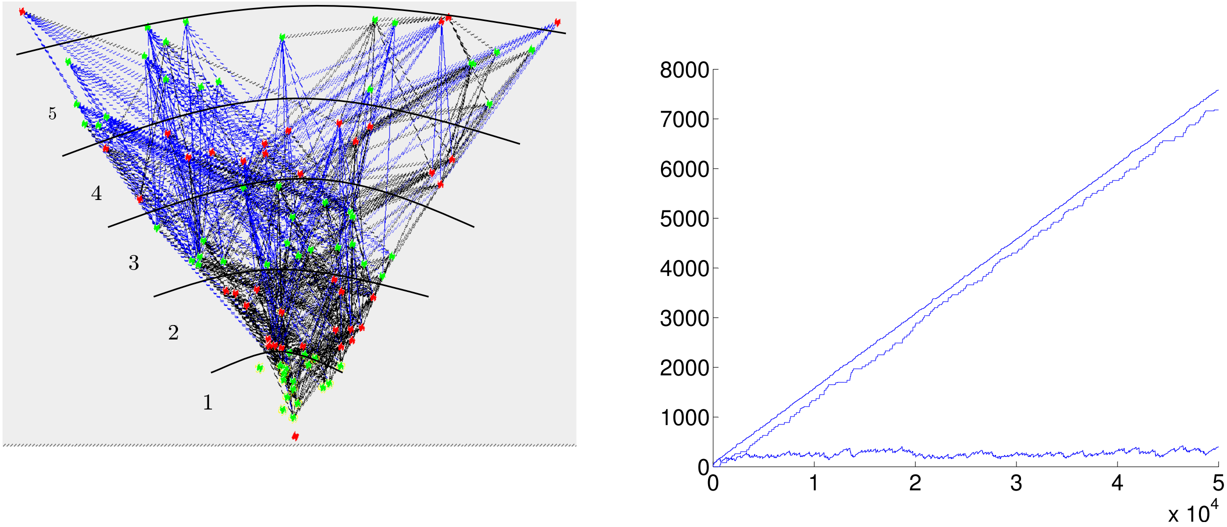

Fig. 6. Min-max energy consumption plots. From the left to the right the number of nodes is $30, 80, 130$ (the maximal distance from a node to the sink is 1), the communication slice size are $0.3, 0.2, 0.1$ and the nodes are scattered randomly.

other case it proceeds by increasing the two slice away transmission (cost c_2) and decrease the transmission to the next slice (cost c_1).

This mechanism is similar to the mechanism applied in the first proposed algorithm. However, nodes now have to balance the energy among nodes belonging to a same slice. For this purpose, each node computes the average energy consumption of the neighboring nodes belonging to the next slice, $mean_1$, and two slices away $mean_2$. The inter-slice energy consumption is balanced with a similar strategy, i.e. each node transmits more messages to nodes whose energy consumption is smaller than the slice average ($mean_1$ or $mean_2$ depending on the position of the receiving node). More precisely, the probability that node i forward a message to node j is updated in the following way if we assume that node j belongs to the first slice

$$p_{ij} \leftarrow p_{ij} + \frac{1}{t_i}\big(\underbrace{(mean_1 - x_j)}_{\text{inter-slice}} + \underbrace{(x_i - \frac{1}{deg_i}\sum_{i\to k} x_k)}_{neighbor-balance}\big).$$

The first underbraced term balances the energy consumption of node j with respect to the others nodes belonging to the same slice. This is done by increasing/decreasing the probability of transmission to j if its energy consumption is below/above the mean ($mean_1$). The second underbraced term balances the energy consumption of node i with its neighboring nodes by increasing/decreasing the transmission to nodes belonging to the first slice (like j) if the energy consumption of i is above/below the neighbor's average energy consumption. It is straightforward that increasing/decreasing the probability of transmission to nodes belonging to the first slice implies that we decrease/increase the probability of transmission to nodes belonging two slices away.

We point out that in this case it is no longer evident that a node can know at each time the current energy consumption of nodes which are two slices away without requiring extra transmissions. Indeed, in our scenario, the current level of energy consumption is attached to messages sent by adding an extra field. Thus, a node can update the current level of energy consumption of a two-slices away neighboring node only when it transmits with maximal power. This corresponds to the case where the scheme is an asynchronous scheme as discussed in [9].

The second set of numerical validation considers a network composed of 100 nodes and sliced with slice size 0.2[5].Nodes transmit with two levels of energy consumption. Transmission to nodes belonging to the next slice costs $c_1 = 10$ and to two slices away costs $c_2 = 40$. Only nodes belonging to the last three slices (number $6, 5, 4$) generate messages, whereas the others nodes only act as relays. The quantity of generated messages is uniform among nodes of the last three slices.

The pseudo-code of the program executed by the nodes is represented on Figure 7. We denote $i \rightarrow_1 j$ and $i \rightarrow_2 k$ the transmissions such that node j belongs to the next slice from i and k two slices away. $N_i^1 = \sum_{i \rightarrow_1 j} 1$, $N_i^2 = \sum_{i \rightarrow_2} 1$ are the corresponding total number of nodes. Notice that nodes belonging to the first slice have no choice and transmit directly to the sink with energy cost c_1 while the nodes belonging to the second slice use the program on Figure 3 since they can transmit to the sink directly.

The results of the numerical validation are plotted on Figure 6. On the left side, the communication graph is plotted and communications links which are not effectively used are not displayed. We observe that nodes belonging to the last slices favor long range transmission while nodes closer to the sink favor small range transmission. This is due to the fact that the nodes closer to the sink have a larger number of messages to handle than the nodes belonging to the last slices. In the right side of Figure 6 we observe that the energy consumption grows linearly with time and that at any time the difference between the maximal and minimal levels of energy consumed remains bounded. This numerically validates the stability of the algorithm under the given conditions.

variables:
i: the identifier of the current node
p_{ij}: the probability of transmitting to node j.
x_i: the level of energy consumption of the current node.
x_j: the level of energy consumption of a neighboring node j.
t_i: the number of message sent by the current node
Initialize $p_{ij} = 1/deg_i$, deg_i is the degree of the current node and includes the node i itself.
upon reception of a message
  compute $mean_1 = \frac{1}{N_i^1} \sum_{i \rightarrow_1 j} x_j$, $mean_2 = \frac{1}{N_i^2} \sum_{i \rightarrow_2 j} x_j$
  if j belongs to the next slice ($i \rightarrow_1 j$)
    $p_{ij} \leftarrow p_{ij} + \frac{1}{t_i}((mean_1 - x_j) + (x_i - \frac{1}{deg_i} \sum_{i \rightarrow k} x_k))$
  else
    $p_{ij} \leftarrow p_{ij} + \frac{1}{t_i}((mean_2 - x_j) - (x_i - \frac{1}{deg_i} \sum_{i \rightarrow k} x_k))$
  Normalize the p_{ij} so that $0 \leq p_{ij} \leq 1$
  $t_i \leftarrow t_i + 1$
  select a node j such that $i \rightarrow j$ with probability p_{ij} and forward the message
end upon

Fig. 7. Pseudo-code of the program executed by the nodes with the topology defined by Proposition 9

7 Conclusion

In this paper we have considered network lifetime maximization problems in the setting of data gathering without aggregation. In this setting, we prove that balancing the energy consumption among the nodes is a suitable strategy that maximizes both the network lifetime and the flow of data that is transmitted to the sink. We have proved that this result is valid if

[5] The maximal distance between two nodes is 1.

the flow of data is constrained to network structures that are algebraically described, and have provided examples of such network structures. Furthermore, considering energy-balanced flows of data leads to the design of optimal distributed routing strategies. Indeed, the property that a flow is energy-balanced is local and does not require any communication between the nodes to be enforced. We have provided two examples of such distributed mechanisms and the numerical validations of these mechanisms.

A natural extension of the results presented in this paper would consist in considering multicommodity flows. In such a setting, the lifetime of the network should appropriately be redefined because a subset of the network rather than the whole network would likely be routing the data. This would lead to study of which nodes should participate in routing the data. Another relevant direction would be to consider interference minimization. Indeed energy-balanced mechanisms naturally tend to reduce the collisions caused by simultaneous transmissions because such mechanisms favor different routes at different instants. Implementing spatial awareness in the algorithms would further limit the interferences. To generalize the results of the present paper to such a situation the description of the optimal communication graphs would thus include spatial information.

References

1. A. Alfieri, A. Bianco, P. Brandimarte, and C.F. Chiasserini. Maximizing system lifetime in wireless sensor networks. *European Journal of Operational Research*, 181:307–402, 2007.
2. H.M. Ammari and S.K. das. Promoting heterogeneity, mobility, and energy-aware voronoi diagram in wireless sensor networks. *IEEE transactions on parallel and Distributed Systems*, 19(17):995–1008, 2008.
3. H. Aslanyan, P. Leone, and J. Rolim. Data propagation with guaranteed delivery for mobile networks. In *The proceedings of the 9th Syposium onExperimental Algorithm (SEA)*, pages 386–397, 2010.
4. J.-P. Aubin. *Applied Functional Analysis.* WILEY-IEEE, 2000.
5. A. Bamis, A. Boukerche, I. Chatzigiannakis, and S. Nikoletseas. A mobility aware protocol synthesis for efficient routing in ad hoc mobile networks. *The Computer Networks Journal*, 52:130–154, 2008.
6. J.-C. Bermond, R. Correa, and M.-L. Yu. Optimal gathering protocols on paths under interference constraints. *Discrete Mathematics*, 309(18):5574–5587, 2008.
7. J-C. Bermond, L. Gargano, and A.A. Rescigno. Gathering with minimum delay in tree sensor networks. In *SIROCCO 2008*, volume 5058 of *Lecture Notes in Computer Science*, pages 262–276, Villars-sur-Ollon, Switzerland, June 2008. Springer-Verlag.
8. J.-C. Bermond and M-L. Yu. Optimal gathering algorithms in multi-hop radio tree networks with interferences. In *ADHOC-NOW 2008*, volume 5198 of *Lecture Notes in Computer Science*, pages 204–217. Springer-Verlag, september 2008.
9. V.S. Borkar. *Stochastic Approximation: A Dynamical Systems ViewPoint.* Cambridge University Press, 2008.
10. Y. Chen, C-N. Chuah, and Q. Zhao. Sensor placement for maximizing lifetime per unit cost in wireless sensor networks. volume 2, pages 1097–1102, October 2005.
11. J.C. Dagher, M.W. Marcellin, and M.A. Neifeld. A theory for maximizing the lifetime of sensor networks. *IEEE transactions on communications*, 55(2), 2007.
12. J.C. Dagher, M.W. Marcellin, and M.A. Neifeld. A theory for maximizing the lifetime of sensor networks. *IEEE Transactions on Communications*, 55(2):323–332, 2007.
13. D.Vass, Z.Vincze, R.Vida, and AttilaVidcs. Energy efficiency in wireless sensor networks using mobile base station. In *EUNICE 2005: Networks and Applications Towards a Ubiquitously Connected World*, 2005.
14. C. Efthymiou, S. Nikoletseas, and J. Rolim. Energy balanced data propagation in wireless sensor network. In IEEE Computer Society Press, editor, *The Proceedings of the 4th IEEE Workshop on Ad-hoc Mobile and Sensor Networks (WMAN)*, 2004.
15. A. Forster and A.L. Murphy. Froms: Feedback routing for optimizing multiple sinks in wsn with reinforcement learning. In *The Proceedings of the 3rd International Conference on Intelligent Sensors, Sensor Networks and Information*, pages 371 – 376, 2007.

16. A. Forster and A.L. Murphy. Balancing energy expenditure in wsns through reinforcement learning: A study. In *Workshop on Energy in Wireless Sensor Networks, in conjunction with DCOSS 2008*, Santorini Island, Greece, 2008.
17. A.M. Geoffrion. Proper efficiency and the theory of vector maximization. *Journal of Mathematical Analysis and Applications*, 22(3):618–630, 1968.
18. A. Giridar and P.R. Kumar. Maximizing the functional lifetime of sensor networks. In *The Proceedings of the fourth International Conference on Information Processing in Sensor Networks, IPSN'05*, UCLA Los-Angeles, 2005.
19. A. Giridhar and P.R. Kumar. Maximizing the functional lifetime of sensor networks. Los Angeles, California, 2005. Proceedings of the 4th international symposium on Information processing in sensor networks.
20. W. Guo, Z. Liu, and G. Wu. An energy-balanced transmission scheme for sensor networks. In *In the First ACM International Conference on Embedded Networked Sensor Systems (ACM SenSys 2003), Poster Session*, Los Angeles, CA, November 2003.
21. W.B. Heinzelman, A.P. Chandrakasan, and H. Balakrishnan. An application-specific protocol architecture for wireless microsensor networks. *IEEE transactions on Wireless Communications*, 1(4), October 2002.
22. B. Hong and V.K. Prasanna. Maximum lifetime data sensing and extraction in energy constrained networked sensor systems. *Journal of Parallel and Distributed Computing*, 66:566–577, 2006.
23. A. Jarry, P. Leone, O. Powell, and J. Rolim. An optimal data propagation algorithm for maximizing the lifespan of sensor networks. In Lecture Notes in Computer Science, editor, *The Proceedings of the IEEE International Conference on Distributed Computing in Sensor Systems*, San Francisco, CA, USA, June 18-20 2006.
24. S. Karlin. *Mathematical methods and theory in games, programming, and economics.* Dover Publications, 2003.
25. H.J. Kushner and G.G. Yin. *Stochastic Approximation and Recursive Algorithms and Applications.* Springer-Verlag, 2003.
26. P. Leone, A. Jarry, S. E. Nikoletseas, and J. D. P. Rolim. Optimal data gathering paths and energy balance mechanisms in wireless networks. In *The proceedings of the 6th IEEE International Conference on Distributed Computing in Sensor Systems (DCOSS)*, pages 288–305, Santa Barbara, California, 2010. Best Paper Award.
27. Z. Liu, D. Xiu, and Weihua Guo. An energy-balanced model for data transmission in sensor networks. In *The Proceedings of The IEEE 62nd Semiannual Vehicular Technology Conference*, Dallas, Texas, USA, September 2005.
28. J. Luo. *Mobility in Wireless networks: friend or Foe - network design and Control in the Age of Mobile Computing.* PhD thesis, PhD thesis, School of Computer and Communications Science, EPFL, Switzerland, 2006.
29. J. Luo and J.-P. Hubaux. Joint mobility and routing for lifetime elongation in wireless sensor networks. In *The Proceedings of the 24th IEEE INFOCOM*, 2005.
30. J. Luo and J.-P. Hubaux. Mobility to improve the lifetime of wireless sensor networks: A theoretical framework. In *The Proceedings of the Workshops of the Second International Conference on Distributed Computing in Sensor Systems (DCOSS)*, 2006.
31. S. Meyn. *Control Techniques for Complex Networks.* Cambridge University Press, 2008.
32. S.K. Muheymin, Z. Tari, and I. Khalil. Energy balanced topology for the sensor networks. In *The Proceedings of the Post Graduate Research Students Conference - 2005 (PRSC 2005)*, RMIT University, November 2005.
33. S. Olariu and I. Stojmenovic. Design guidelines for maximizing lifetime and avoiding energy holes in sensor networks with uniform distribution and uniform reporting. In *IEEE INFOCOM*, Barcelona, Spain, April 2006.
34. O. Powell, P. Leone, and J. Rolim. Energy optimal data propagation in sensor networks. *Journal on Parallel and Distributed Computing*, 67(3):302–317, 2007.
35. Y. Yu and V.K. Prasanna. Exploring energy-latency tradeoffs for data gathering in wireless sensor networks. *IEEE Transaction on Wireless Communications*, 5(11):7–11, November 2006.
36. H. Zhang and J.C. Hou. On deriving the upper bound of alpha-lifetime for large sensor network. In *The Proceedings of the 5th ACM international symposium on Mobile ad hoc networking and computing*, pages 121–132, 2004.
37. H. Zhang and J.C. Hou. Maximizing alpha-lifetime for wireless sensor networks. In *The Proceedings of Third International Workshop on Measurement, Modelling, and Performance Analysis of Wireless Sensor Networks (SenMetrics 2005)*, July 2005.

38. H. Zhang, H. Shen, and Y. Tan. Optimal energy balanced data gathering in wireless sensor networks. In *Proc. IEEE IPDPS*, pages 1–10, 2007.