# Investigating the implementation of restricted sets of multiqubit operations on distant qubits: a communication complexity perspective

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**Abstract** We propose a protocol for Alice to implement a multiqubit quantum operation from the restricted sets on distant qubits possessed by Bob, and then we investigate the communication complexity of the task in different communication scenarios. By comparing with the previous work, our protocol works without prior sharing of entanglement, and requires less communication resources than the previous protocol in the qubit-transmission scenario. Furthermore, we generalize our protocol to *d*-dimensional operations.

**Keywords** Quantum communication  $\cdot$  Nonlocal operations  $\cdot$  Remote implementation  $\cdot$  Communication complexity

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#### 1 Introduction

Quantum information processing (QIP) usually involves the implementation of quantum operations between spatially separated qubits. For example, distributed quantum computation requires implementing nonlocal operations on the qubits at distant nodes. This task can always be completed via the so-called bidirectional quantum state teleportation (BQST) [1]. In BQST, all the involved qubits are first teleported to Alice, and then Alice applies the operation and teleports them back to other parties. Thus far, many authors [1,2,3,4,5,6,7,8,9,10,11,12,13,14] have studied how to implement nonlocal operations using prior sharing of entanglement, local operations, and classical communication (LOCC), and these researches [1,2,3,4,5,6,7,8,9,10,11,12,13,14] have shown that some special operations on distant

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qubits can be implemented by LOCC using less communication resources than they are implemented in the BQST scheme.

Eisert *et al* [2] investigated the minimal communication resources that are required in the local implementation of nonlocal quantum gates, and presented optimal protocols for a number of important gates using prior sharing of entanglement and LOCC. For example, 1 shared ebit and communication of 1 cbit in each direction are both necessary and sufficient for the nonlocal implementation of a controlled-*U* gate; an *N*-qubit controlled-*U* gate with N - 1 control qubits possessed by Bob and 1 target qubit possessed by Alice (denoted as CU(N-1,1)in the remainder of this paper) can also be implemented using 1 shared ebit and communication of 1 cbit in each direction; an *N*-qubit controlled-*U* gate with N-1control qubits and 1 target qubit possessed by *N* spatially separated parties can be implemented using N - 1 shared ebits and communication of 2(N - 1) cbits.

From another point of view, the task can be completed without prior sharing of entanglement as follows: Bob sends his qubits to Alice, and then Alice applies the operation and sends them back to Bob. In the case of CU(N-1, 1), a total of 2(N-1) qubits need to be communicated (we assume that U is only known to Alice). Can Alice and Bob communicate fewer qubits to complete this task? The answer is positive. Yang [15] proposed a protocol for implementing CU(N-1, 1)without prior sharing of entanglement. The required communication resources are 1 qubit transmitted from Bob to Alice and 1 cbit transmitted from Alice to Bob.

Huelga *et al* [5] showed that there are two restricted sets of one-qubit operations that can be implemented remotely using 1 shared ebit and communication of 1 cbit in each direction. One of these two restricted sets consists of diagonal operations and the other one consists of antidiagonal operations:

$$U_{diag} = \begin{pmatrix} e^{i\phi} & 0\\ 0 & e^{-i\phi} \end{pmatrix}, U_{anti} = \begin{pmatrix} 0 & e^{i\phi}\\ -e^{-i\phi} & 0 \end{pmatrix}.$$
 (1)

Wang [9] generalized these restricted sets to the case of multiqubits, and proposed a protocol for Alice to implement restrict sets of N-qubit operations on Bob's qubits. Operations belonging to the restricted sets have just one nonzero element in any column or any row. Each restricted set is characterized by a permutation f, and thus an N-qubit operation in the restricted set f can be expressed as:

$$U(f,\phi) = \sum_{x=0}^{2^{N}-1} e^{i\phi(x)} |f(x)\rangle \langle x|.$$
 (2)

In this paper, we assume that Alice knows  $U(f, \phi)$  but Bob only knows which restricted set f the operation belongs to (i.e. Alice has the device to implement  $U(f, \phi)$  and Bob knows the type of Alice's device. If Bob does not know which one of the  $2^{N}$ ! restricted sets Alice's device belongs to, Alice should first tell Bob this information through classical communication). The required communication resources are N shared ebits and N cbits in each direction.

By hybridizing the protocol in Ref. [9] and BQST, Zhao *et al* [12] proposed a protocol for Alice to implement restricted sets of (N + M)-qubit operations on Bob's qubits. Operations belonging to the restricted sets are  $2^N \times 2^N$  block matrices with just one nonzero block in any column or any row, and every block is a  $2^M \times 2^M$  unitary matrix. Each restricted set is characterized by a permutation f, and thus an (N + M)-qubit operation in the restricted set f can be expressed as:

$$U(f,G) = \sum_{x=0}^{2^N-1} |f(x)\rangle\langle x| \otimes G(x),$$
(3)

where G(x)'s are arbitrary  $2^M \times 2^M$  unitary matrices. In this paper, we assume that Alice knows U(f, G) but Bob only knows which restricted set f the operation belongs to (i.e. Alice has the device to implement U(f, G) and Bob knows the type of Alice's device. If Bob does not know which one of the  $2^N$ ! restricted sets Alice's device belongs to, Alice should first tell Bob this information through classical communication). To implement U(f, G) on distant N + M qubits possessed by Bob, M qubits need to be teleported back and forth between the two parties, and the required communication resources are N + 2M shared ebits plus N + 2M cbits in each direction [12]. If the M qubits that need to be teleported in this hybrid protocol are initially possessed by Alice, BQST is unnecessary and the required communication resources are N shared ebits plus N cbits in each direction [13]. If M = 0, this hybrid protocol is reduced to the protocol in Ref. [9].

Inspired by Yang's work [15], we study the possibility of implementing U(f, G)on distant qubits without prior sharing of entanglement, and consider the required communication resources in different communication scenarios. Actually, our protocol can complete the same tasks as those in Ref. [5,9,12,13] without using prior sharing of entanglement. Our protocol requires less communication resources than the previous protocol [5,9,12,13] in the qubit-transmission scenario.

The remainder of the paper is organized as follows. In Sect. 2, we propose a protocol for implementing U(f,G) on distant qubits. Sect. 3 contains a communication complexity analysis of the task in different communication scenarios. In Sect. 4, we generalize our protocol to *d*-dimensional operations. A brief conclusion follows in Sect. 5.

### 2 Our protocol

We first explain how to implement U(f, G) on N qubits possessed by Bob and M qubits possessed by Alice, and then we explain another two cases.

In the first case, the initial state of the N + M qubits can be written as

$$|\varphi\rangle_{B_1\dots B_N A_1\dots A_M} = \sum_{j=0}^{2^N-1} \alpha_j |j\rangle_{B_1\dots B_N} |\xi_j\rangle_{A_1\dots A_M},\tag{4}$$

where subscripts  $A_1 \ldots A_M$   $(B_1 \ldots B_N)$  denotes Alice's (Bob's) M (N) qubits. Our protocol consists of six steps:

(1) Bob introduces N ancilla qubits  $C_1 \ldots C_N$  initially prepared in the state  $|0\rangle^{\otimes N}$ , and then performs N controlled-NOT (CNOT) gates with  $B_i$  as the control qubit and  $C_i$  the target qubit. After this step, the state of the composite system becomes

$$\sum_{j=0}^{2^N-1} \alpha_j |j\rangle_{B_1\dots B_N} |j\rangle_{C_1\dots C_N} |\xi_j\rangle_{A_1\dots A_M}.$$
(5)

(2) Bob sends N qubits  $C_1 \ldots C_N$  to Alice.

(3) After receiving the ancilla qubits  $C_1 \ldots C_N$ , Alice performs U(f,G) on qubits  $C_1 \ldots C_N A_1 \ldots A_M$ . The state of the composite system becomes

$$\sum_{j=0}^{2^N-1} \alpha_j |j\rangle_{B_1\dots B_N} |f(j)\rangle_{C_1\dots C_N} G(j)|\xi_j\rangle_{A_1\dots A_M}.$$
(6)

(4) Alice performs a Hadamard transform on each qubit  $C_i$ , and then measures  $C_i$  in the computational basis. After the Hadamard transform, the state of the composite system becomes

$$\frac{1}{\sqrt{2^N}} \sum_{k=0}^{2^N-1} |k\rangle_{C_1...C_N} \sum_{j=0}^{2^N-1} (-1)^{f(j)\cdot k} \alpha_j |j\rangle_{B_1...B_N} G(j) |\xi_j\rangle_{A_1...A_M},$$
(7)

where  $f(j) \cdot k$  means the inner product modulo 2 of bit vectors f(j) and k.

If the measurement result of qubits  $C_1 \ldots C_N$  is  $k = k_1 \ldots k_N$ , the state of  $B_1 \ldots B_N A_1 \ldots A_M$  becomes

$$\sum_{j=0}^{2^{N}-1} (-1)^{f(j) \cdot k} \alpha_{j} |j\rangle_{B_{1} \dots B_{N}} G(j) |\xi_{j}\rangle_{A_{1} \dots A_{M}}.$$
(8)

(5) Alice informs Bob of the measurement result k by sending N cbits.

(6) Since Bob knows which restricted set f the operation U(f,G) belongs to, he can construct a corresponding N-qubit unitary operation

$$V(f) = \sum_{x=0}^{2^{N}-1} |f(x)\rangle\langle x|.$$
 (9)

Bob first performs V(f) on qubits  $B_1 \dots B_N$ , and then performs  $\sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|$ on qubit  $B_i$  if and only if  $k_i = 1$ . The state of qubits  $B_1 \dots B_N A_1 \dots A_M$  becomes

$$\sum_{j=0}^{2^N-1} \alpha_j |f(j)\rangle_{B_1\dots B_N} G(j)|\xi_j\rangle_{A_1\dots A_M} = U(f,G)|\varphi\rangle_{B_1\dots B_N A_1\dots A_M}.$$
 (10)

Thus, U(f, G) has been successfully implemented on qubits  $B_1 \ldots B_N A_1 \ldots A_M$ . The required communication resources are N qubits transmitted from Bob to Alice in step 2 and N cbits transmitted from Alice to Bob in step 5. The protocol in Ref. [13] which implements the same operations requires N shared ebits and communication of N cbits in each direction.

In the second case, if all N + M qubits  $B_1 \dots B_N A_1 \dots A_M$  are initially possessed by Bob, he has to send qubits  $A_1 \dots A_M$  together with qubits  $C_1 \dots C_N$  to Alice in step 2. After performing U(f, G) on qubits  $C_1 \dots C_N A_1 \dots A_M$ , Alice has to send qubits  $A_1 \dots A_M$  back to Bob in step 5. In this case, the required communication resources are N + M qubits transmitted from Bob to Alice plus M qubits transmitted from Alice to Bob plus N cbits transmitted from Alice to Bob. The protocol in Ref. [12] which deals with this case requires N + 2M shared ebits and communication of N + 2M cbits in each direction.

In the third case, if M = 0, the restricted sets are reduced to  $U(f, \phi)$ . In this case, Alice implements an N-qubit operation on N qubits possessed by Bob. The required communication resources are N qubits transmitted from Bob to Alice and N cbits transmitted from Alice to Bob. The protocol in Ref. [9] which deals with this case requires N shared ebits and communication of N cbits in each direction.

#### **3** Communication Complexity Analysis

In this section, we go on to discuss the communication complexity of implementing restricted sets of multiqubit operations on distant qubits. In the theory of quantum communication complexity, two communication scenarios are often compared. In the *qubit-transmission scenario*, introduced by Yao [16], the parties can communicate qubits but are not allowed to share prior entanglement in the initialization phase. In the *shared-entanglement scenario*, introduced by Cleve and Buhrman [17], the parties have an initial supply of shared entanglement but they can only communicate classical bits. In this paper, we refer to a protocol as a *qubit-transmission protocol* if it requires (and only requires) transmission of qubits and cbits, and refer to a protocol as a *shared-entanglement protocol* if it requires (and only requires) prior sharing of entanglement and transmission of cbits.

The protocols in Refs. [2,9,12,13] are shared-entanglement protocols, and the required communication resources of these protocols are ebits and cbits, whereas the protocol in Ref. [15] and our protocols are qubit-transmission protocols, and the required communication resources of these protocols are qubits and cbits. The required communication resources of these two kinds of protocols can not be compared directly. They can only be compared in the same communication scenario. Any qubit-transmission protocol that requires the transmission of Nqubits and M cbits can be simulated in the shared-entanglement scenario through quantum teleportation [18] at the cost of N shared ebits and transmission of 2N + M cbits. On the other hand, any shared-entanglement protocol that requires N shared ebits and communication of M cbits can be implemented in the qubittransmission scenario at the cost of communication of N qubits and M cbits, because one party can prepare a pair of entangled qubits and then transmit one of them to distribute 1 shared ebit. Tables 1, 2, 3 and 4 summarize the required communication resources of these protocols in both scenarios. The term gap is defined as the communication resources of the upper protocol minus that of the lower protocol.

Table 1 shows the required communication resources of implementing CU(N-1,1) which performs an arbitrary unitary operation on a target qubit possessed by Alice with distant N-1 control qubits possessed by Bob. In the qubit-transmission scenario, the protocol in Ref. [15] can save 1 cbit of communication compared to the protocol in Ref. [2]. In the shared-entanglement scenario, the simulation of the protocol in Ref. [15] requires 1 more cbit of communication compared to the protocol in Ref. [2].

Table 2 shows the required communication resources of implementing Alice's (N + M)-qubit operation on Bob's N qubits and Alice's M qubits. Our protocol can save N cbits of communication compared to the protocol in Ref. [13] in the qubit-transmission scenario. Table 3 shows the required communication resources of implementing Alice's (N + M)-qubit operation on Bob's N + M qubits. Our

	Qubit-transmission scenario		Shared-entanglement scenario	
	qubits	cbits	ebits	cbits
Protocol in Ref. [2]	1	2	1	2
Protocol in Ref. [15]	1	1	1	3
Gap		1		-1

**Table 1** N-qubit controlled-U operation with Bob's N-1 control qubits and Alice's 1 target qubit

**Table 2** Alice's (N + M)-qubit operation on Bob's N qubits and Alice's M qubits

	Qubit-transmission scenario		Shared-entanglement scenario	
	qubits	cbits	ebits	cbits
Protocol in Ref. [13]	N	2N	N	2N
Our protocol	N	N	N	3N
Gap		N		-N

protocol can save N + 4M cbits of communication compared to the protocol in Ref. [12] in the qubit-transmission scenario. Table 4 shows the required communication resources of implementing Alice's N-qubit operation on Bob's N qubits. Our protocol can save N cbits of communication compared to the protocol in Ref. [9] in the qubit-transmission scenario. In summary, our protocols requires less communication resources than the protocols in Ref. [9, 12, 13] implemented in the qubit-transmission scenario.

In the shared-entanglement scenario, the right parts of tables 2, 3 and 4 show that the simulation of our protocols requires N more cbits of communication compared to the protocols in Ref. [9,12,13].

Therefore the saving of communicated cbits in our protocols is at least N cbits and goes up to N + 4M cbits when Alice's (N + M)-qubit operation needs to be implemented on Bob's N + M qubits. More essentially, our protocol has advantage because it is a lot easier to transmit qubits than distribute and store entanglement pairs.

Alice's multiqubit operations on Bob's qubits can be implemented without prior sharing of entanglement as follows: Bob sends his qubits to Alice, and then Alice applies the operation and sends them back to Bob. By using this simple method, no auxiliary qubits are used, no additional CNOT and Hadamard operations are required, and no classical communications between Alice and Bob are needed. However, our method requires fewer qubits to be communicated than this simple method. This trade-off between computation and communication in our method is analogous to the trade-off between time and space in the field of algorithm design. Different benefits are required in different situations. Sometimes we need an easy-to-implement protocol, and sometimes we need to communicate as few qubits as possible. The shared-entanglement protocol in Ref. [13] is fit for the parties who already have shared entanglement. The simple method is fit for the case where additional quantum operations are undesirable. However, our method is fit for the case where we place a high price on communication.

	Qubit-transmission scenario		Shared-entanglement scenario	
	qubits	cbits	ebits	cbits
Protocol in Ref. [12]	N + 2M	2N + 4M	N + 2M	2N + 4M
Our protocol	N + 2M	N	N + 2M	3N + 4M
Gap		N + 4M		-N

**Table 3** Alice's (N + M)-qubit operation on Bob's N + M qubits

**Table 4**Alice's N-qubit operation on Bob's N qubits

	Qubit-transmission scenario		Shared-entanglement scenario	
	qubits	cbits	ebits	cbits
Protocol in Ref. [9]	N	2N	N	2N
Our protocol	N	N	N	3N
Gap		N		-N

### 4 Generalization to *d*-dimensional operations

In this section, we generalize the protocol proposed in section 2 to *d*-dimensional operations. The (N + M)-qudit quantum operation  $U_d(f, G)$  can be expressed as:

$$U_d(f,G) = \sum_{x=0}^{d^N - 1} |f(x)\rangle \langle x| \otimes G(x),$$
(11)

where G(x)'s are arbitrary  $d^M \times d^M$  unitary matrices.

Suppose that Alice wants to implement  $U_d(f, G)$  on N qudits possessed by Bob and M qudits possessed by herself. The initial state of the N + M qudits can be written as:

$$|\varphi\rangle_{B_1\dots B_N A_1\dots A_M} = \sum_{j=0}^{d^N-1} \alpha_j |j\rangle_{B_1\dots B_N} |\xi_j\rangle_{A_1\dots A_M},\tag{12}$$

where subscripts  $A_1 \ldots A_M$   $(B_1 \ldots B_N)$  denotes Alice's (Bob's) M (N) qudits. The protocol consists of six steps:

(1) Bob introduces N ancilla qu*d*its  $C_1 \ldots C_N$  initially prepared in the state  $|0\rangle^{\otimes N}$ , and then performs N generalized controlled-NOT (CNOT) gates [19]

$$|x\rangle_{B_j}|y\rangle_{A_j} \to |x\rangle_{B_j}|x - y \mod d\rangle_{A_j} \tag{13}$$

with  $B_i$  as the control qudit and  $C_i$  the target qudit. After this step, the state of the composite system becomes

$$\sum_{j=0}^{d^N-1} \alpha_j |j\rangle_{B_1...B_N} |j\rangle_{C_1...C_N} |\xi_j\rangle_{A_1...A_M}.$$
(14)

(2) Bob sends N qudits  $C_1 \ldots C_N$  to Alice.

(3) After receiving the ancilla qudits  $C_1 \ldots C_N$ , Alice performs  $U_d(f, G)$  on qudits  $C_1 \ldots C_N A_1 \ldots A_M$ . The state of the composite system becomes

$$\sum_{j=0}^{d^N-1} \alpha_j |j\rangle_{B_1\dots B_N} |f(j)\rangle_{C_1\dots C_N} G(j) |\xi_j\rangle_{A_1\dots A_M}.$$
(15)

(4) Alice performs the one-qudit quantum Fourier transform

$$|x\rangle \to \frac{1}{\sqrt{d}} \sum_{y=0}^{d-1} e^{2\pi i x y/d} |y\rangle \tag{16}$$

on each qudit  $C_i$ , and then measures  $C_i$  in the computational basis. After the quantum Fourier transform, the state of the composite system becomes

$$\frac{1}{\sqrt{d^N}} \sum_{k=0}^{d^N-1} |k\rangle_{C_1...C_N} \sum_{j=0}^{d^N-1} exp\Big[\frac{2\pi i}{d} \sum_{l=1}^N f(j)_l k_l\Big] \alpha_j |j\rangle_{B_1...B_N} G(j) |\xi_j\rangle_{A_1...A_M}.$$
(17)

If the measurement result of qudits  $C_1 \dots C_N$  is  $k = k_1 \dots k_N$ , the state of qudits  $B_1 \dots B_N A_1 \dots A_M$  becomes

$$\sum_{j=0}^{d^{N}-1} exp \Big[ \frac{2\pi i}{d} \sum_{l=1}^{N} f(j)_{l} k_{l} \Big] \alpha_{j} |j\rangle_{B_{1}...B_{N}} G(j) |\xi_{j}\rangle_{A_{1}...A_{M}}.$$
 (18)

(5) Alice informs Bob of the measurement result k by sending  $\lceil N \log_2 d \rceil$  cbits. (6) Since Bob knows which restricted set f the operation  $U_d(f, G)$  belongs to, he can construct a corresponding N-qudit unitary operation

$$V_d(f) = \sum_{x=0}^{d^N - 1} |f(x)\rangle \langle x|.$$
 (19)

Bob first performs  $V_d(f)$  on qudits  $B_1 \dots B_N$ , and then performs  $S^{k_i}$  on qudit  $B_i$ , where

$$S = \sum_{x=0}^{d-1} e^{-2\pi i x/d} |x\rangle \langle x|.$$

$$\tag{20}$$

The state of qudits  $B_1 \ldots B_N A_1 \ldots A_M$  becomes

$$\sum_{j=0}^{d^N-1} \alpha_j |f(j)\rangle_{B_1\dots B_N} G(j) |\xi_j\rangle_{A_1\dots A_M} = U_d(f,G) |\varphi\rangle_{B_1\dots B_N A_1\dots A_M}.$$
 (21)

Thus,  $U_d(f, G)$  has been successfully implemented on qudits  $B_1 \dots B_N A_1 \dots A_M$ . The required communication resources are N qudits transmitted from Bob to Alice in step 2 and  $[N \log_2 d]$  cbits transmitted from Alice to Bob in step 5.

## **5** Conclusion

We have considered the implementation of Alice's multiqubit operation from the restricted sets [12,13] on distant qubits possessed by Bob from a communication complexity perspective. The restricted sets are  $2^N \times 2^N$  block matrices with just one nonzero block in any column or any row, every block of which is a  $2^M \times 2^M$  unitary matrix. Protocols for implementing these restricted sets of multiqubit operations on distant qubits using prior sharing of entanglement have been proposed

in Ref. [12,13]. Inspired by Yang's work [15] for constructing a nonlocal N-qubit controlled-U gate without prior sharing of entanglement, we have proposed a protocol to complete the same tasks as those in Ref. [5,9,12,13] without prior sharing of entanglement. We have shown that our qubit-transmission protocol requires less communication resources than the previous shared-entanglement protocols [9,12, 13] in the qubit-transmission scenario. Because it is a lot easier to transmit qubits than distribute and store entanglement pairs, our protocol has advantage in the case that the parties have no prior sharing of entanglement. Furthermore, we have generalized our protocol to d-dimensional operations.

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