Investigating the implementation of restricted sets of multiqubit operations on distant qubits: a communication complexity perspective

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Abstract We propose a protocol for Alice to implement a multiqubit quantum operation from the restricted sets on distant qubits possessed by Bob, and then we investigate the communication complexity of the task in different communication scenarios. By comparing with the previous work, our protocol works without prior sharing of entanglement, and requires less communication resources than the previous protocol in the qubit-transmission scenario. Furthermore, we generalize our protocol to d-dimensional operations.

Keywords Quantum communication · Nonlocal operations · Remote implementation · Communication complexity

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1 Introduction

Quantum information processing (QIP) usually involves the implementation of quantum operations between spatially separated qubits. For example, distributed quantum computation requires implementing nonlocal operations on the qubits at distant nodes. This task can always be completed via the so-called bidirectional quantum state teleportation (BQST) [\[1\]](#page-8-0). In BQST, all the involved qubits are first teleported to Alice, and then Alice applies the operation and teleports them back to other parties. Thus far, many authors $[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]$ $[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]$ $[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]$ $[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]$ $[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]$ $[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]$ $[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]$ $[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]$ $[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]$ $[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]$ $[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]$ $[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]$ $[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]$ $[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]$ $[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]$ have studied how to implement nonlocal operations using prior sharing of entanglement, local operations, and classical communication (LOCC), and these researches [\[1,](#page-8-0)2, [3,](#page-8-2) [4,](#page-8-3) [5,](#page-8-4) [6,](#page-8-5) [7,](#page-8-6) [8,](#page-8-7) [9,](#page-8-8) [10,](#page-8-9) [11,](#page-8-10) [12,](#page-8-11) [13,](#page-8-12) [14\]](#page-8-13) have shown that some special operations on distant

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qubits can be implemented by LOCC using less communication resources than they are implemented in the BQST scheme.

Eisert et al [\[2\]](#page-8-1) investigated the minimal communication resources that are required in the local implementation of nonlocal quantum gates, and presented optimal protocols for a number of important gates using prior sharing of entanglement and LOCC. For example, 1 shared ebit and communication of 1 cbit in each direction are both necessary and sufficient for the nonlocal implementation of a controlled-U gate; an N-qubit controlled-U gate with $N-1$ control qubits possessed by Bob and 1 target qubit possessed by Alice (denoted as $CU(N-1, 1)$) in the remainder of this paper) can also be implemented using 1 shared ebit and communication of 1 cbit in each direction; an N-qubit controlled-U gate with $N-1$ control qubits and 1 target qubit possessed by N spatially separated parties can be implemented using $N-1$ shared ebits and communication of $2(N-1)$ cbits.

From another point of view, the task can be completed without prior sharing of entanglement as follows: Bob sends his qubits to Alice, and then Alice applies the operation and sends them back to Bob. In the case of $CU(N-1,1)$, a total of $2(N-1)$ qubits need to be communicated (we assume that U is only known to Alice). Can Alice and Bob communicate fewer qubits to complete this task? The answer is positive. Yang [\[15\]](#page-8-14) proposed a protocol for implementing $CU(N-1, 1)$ without prior sharing of entanglement. The required communication resources are 1 qubit transmitted from Bob to Alice and 1 cbit transmitted from Alice to Bob.

Huelga et al [\[5\]](#page-8-4) showed that there are two restricted sets of one-qubit operations that can be implemented remotely using 1 shared ebit and communication of 1 cbit in each direction. One of these two restricted sets consists of diagonal operations and the other one consists of antidiagonal operations:

$$
U_{diag} = \begin{pmatrix} e^{i\phi} & 0\\ 0 & e^{-i\phi} \end{pmatrix}, U_{anti} = \begin{pmatrix} 0 & e^{i\phi}\\ -e^{-i\phi} & 0 \end{pmatrix}.
$$
 (1)

Wang [\[9\]](#page-8-8) generalized these restricted sets to the case of multiqubits, and proposed a protocol for Alice to implement restrict sets of N-qubit operations on Bob's qubits. Operations belonging to the restricted sets have just one nonzero element in any column or any row. Each restricted set is characterized by a permutation f, and thus an N -qubit operation in the restricted set f can be expressed as:

$$
U(f,\phi) = \sum_{x=0}^{2^N - 1} e^{i\phi(x)} |f(x)\rangle \langle x|.
$$
 (2)

In this paper, we assume that Alice knows $U(f, \phi)$ but Bob only knows which restricted set f the operation belongs to (i.e. Alice has the device to implement $U(f, \phi)$ and Bob knows the type of Alice's device. If Bob does not know which one of the 2^N ! restricted sets Alice's device belongs to, Alice should first tell Bob this information through classical communication). The required communication resources are N shared ebits and N cbits in each direction.

By hybridizing the protocol in Ref. [\[9\]](#page-8-8) and BQST, Zhao et al [\[12\]](#page-8-11) proposed a protocol for Alice to implement restricted sets of $(N + M)$ -qubit operations on Bob's qubits. Operations belonging to the restricted sets are $2^N \times 2^N$ block matrices with just one nonzero block in any column or any row, and every block is

a $2^M \times 2^M$ unitary matrix. Each restricted set is characterized by a permutation f, and thus an $(N + M)$ -qubit operation in the restricted set f can be expressed as:

$$
U(f, G) = \sum_{x=0}^{2^N - 1} |f(x)\rangle \langle x| \otimes G(x), \qquad (3)
$$

where $G(x)$'s are arbitrary $2^M \times 2^M$ unitary matrices. In this paper, we assume that Alice knows $U(f, G)$ but Bob only knows which restricted set f the operation belongs to (i.e. Alice has the device to implement $U(f, G)$ and Bob knows the type of Alice's device. If Bob does not know which one of the $2^N!$ restricted sets Alice's device belongs to, Alice should first tell Bob this information through classical communication). To implement $U(f, G)$ on distant $N + M$ qubits possessed by Bob, M qubits need to be teleported back and forth between the two parties, and the required communication resources are $N+2M$ shared ebits plus $N+2M$ cbits in each direction $[12]$. If the M qubits that need to be teleported in this hybrid protocol are initially possessed by Alice, BQST is unnecessary and the required communication resources are N shared ebits plus N cbits in each direction [\[13\]](#page-8-12). If $M = 0$, this hybrid protocol is reduced to the protocol in Ref. [\[9\]](#page-8-8).

Inspired by Yang's work [\[15\]](#page-8-14), we study the possibility of implementing $U(f, G)$ on distant qubits without prior sharing of entanglement, and consider the required communication resources in different communication scenarios. Actually, our protocol can complete the same tasks as those in Ref. [\[5,](#page-8-4) [9,](#page-8-8) [12,](#page-8-11) [13\]](#page-8-12) without using prior sharing of entanglement. Our protocol requires less communication resources than the previous protocol [\[5,](#page-8-4) [9,](#page-8-8) [12,](#page-8-11) [13\]](#page-8-12) in the qubit-transmission scenario.

The remainder of the paper is organized as follows. In Sect. [2,](#page-2-0) we propose a protocol for implementing $U(f, G)$ on distant qubits. Sect. [3](#page-4-0) contains a communication complexity analysis of the task in different communication scenarios. In Sect. [4,](#page-6-0) we generalize our protocol to d-dimensional operations. A brief conclusion follows in Sect. [5.](#page-7-0)

2 Our protocol

We first explain how to implement $U(f, G)$ on N qubits possessed by Bob and M qubits possessed by Alice, and then we explain another two cases.

In the first case, the initial state of the $N + M$ qubits can be written as

$$
|\varphi\rangle_{B_1...B_N A_1...A_M} = \sum_{j=0}^{2^N-1} \alpha_j |j\rangle_{B_1...B_N} |\xi_j\rangle_{A_1...A_M},
$$
\n(4)

where subscripts $A_1 \ldots A_M$ $(B_1 \ldots B_N)$ denotes Alice's (Bob's) M (N) qubits. Our protocol consists of six steps:

(1) Bob introduces N ancilla qubits $C_1 \ldots C_N$ initially prepared in the state $|0\rangle^{\otimes N}$, and then performs N controlled-NOT (CNOT) gates with B_i as the control qubit and C_i the target qubit. After this step, the state of the composite system becomes

$$
\sum_{j=0}^{2^N-1} \alpha_j |j\rangle_{B_1...B_N} |j\rangle_{C_1...C_N} |\xi_j\rangle_{A_1...A_M}.
$$
 (5)

(2) Bob sends N qubits $C_1 \ldots C_N$ to Alice.

(3) After receiving the ancilla qubits $C_1 \ldots C_N$, Alice performs $U(f, G)$ on qubits $C_1 \ldots C_N A_1 \ldots A_M$. The state of the composite system becomes

$$
\sum_{j=0}^{2^N-1} \alpha_j |j\rangle_{B_1...B_N} |f(j)\rangle_{C_1...C_N} G(j)|\xi_j\rangle_{A_1...A_M}.
$$
 (6)

(4) Alice performs a Hadamard transform on each qubit C_i , and then measures C_i in the computational basis. After the Hadamard transform, the state of the composite system becomes

$$
\frac{1}{\sqrt{2^N}} \sum_{k=0}^{2^N-1} |k\rangle_{C_1...C_N} \sum_{j=0}^{2^N-1} (-1)^{f(j)\cdot k} \alpha_j |j\rangle_{B_1...B_N} G(j) |\xi_j\rangle_{A_1...A_M},\tag{7}
$$

where $f(j) \cdot k$ means the inner product modulo 2 of bit vectors $f(j)$ and k.

If the measurement result of qubits $C_1 \ldots C_N$ is $k = k_1...k_N$, the state of $B_1 \ldots B_N A_1 \ldots A_M$ becomes

$$
\sum_{j=0}^{2^N-1} (-1)^{f(j)\cdot k} \alpha_j |j\rangle_{B_1...B_N} G(j) | \xi_j \rangle_{A_1...A_M}.
$$
 (8)

(5) Alice informs Bob of the measurement result k by sending N cbits.

(6) Since Bob knows which restricted set f the operation $U(f, G)$ belongs to, he can construct a corresponding N-qubit unitary operation

$$
V(f) = \sum_{x=0}^{2^N - 1} |f(x)\rangle \langle x|.
$$
 (9)

Bob first performs $V(f)$ on qubits $B_1 \dots B_N$, and then performs $\sigma_z = |0\rangle\langle0| - |1\rangle\langle1|$ on qubit B_i if and only if $k_i = 1$. The state of qubits $B_1 \dots B_N A_1 \dots A_M$ becomes

$$
\sum_{j=0}^{2^N-1} \alpha_j |f(j)\rangle_{B_1...B_N} G(j) |\xi_j\rangle_{A_1...A_M} = U(f, G) |\varphi\rangle_{B_1...B_N A_1...A_M}.
$$
 (10)

Thus, $U(f, G)$ has been successfully implemented on qubits $B_1 \dots B_N A_1 \dots A_M$. The required communication resources are N qubits transmitted from Bob to Alice in step 2 and N cbits transmitted from Alice to Bob in step 5. The protocol in Ref. $[13]$ which implements the same operations requires N shared ebits and communication of N cbits in each direction.

In the second case, if all $N + M$ qubits $B_1 \dots B_N A_1 \dots A_M$ are initially possessed by Bob, he has to send qubits $A_1 \ldots A_M$ together with qubits $C_1 \ldots C_N$ to Alice in step 2. After performing $U(f, G)$ on qubits $C_1 \ldots C_N A_1 \ldots A_M$, Alice has to send qubits $A_1 \ldots A_M$ back to Bob in step 5. In this case, the required communication resources are $N + M$ qubits transmitted from Bob to Alice plus M qubits transmitted from Alice to Bob plus N cbits transmitted from Alice to Bob. The protocol in Ref. [\[12\]](#page-8-11) which deals with this case requires $N + 2M$ shared ebits and communication of $N + 2M$ cbits in each direction.

In the third case, if $M = 0$, the restricted sets are reduced to $U(f, \phi)$. In this case, Alice implements an N-qubit operation on N qubits possessed by Bob. The required communication resources are N qubits transmitted from Bob to Alice and N cbits transmitted from Alice to Bob. The protocol in Ref. [\[9\]](#page-8-8) which deals with this case requires N shared ebits and communication of N cbits in each direction.

3 Communication Complexity Analysis

In this section, we go on to discuss the communication complexity of implementing restricted sets of multiqubit operations on distant qubits. In the theory of quantum communication complexity, two communication scenarios are often compared. In the qubit-transmission scenario, introduced by Yao [\[16\]](#page-8-15), the parties can communicate qubits but are not allowed to share prior entanglement in the initialization phase. In the shared-entanglement scenario, introduced by Cleve and Buhrman [\[17\]](#page-9-0), the parties have an initial supply of shared entanglement but they can only communicate classical bits. In this paper, we refer to a protocol as a qubit-transmission protocol if it requires (and only requires) transmission of qubits and cbits, and refer to a protocol as a shared-entanglement protocol if it requires (and only requires) prior sharing of entanglement and transmission of cbits.

The protocols in Refs. [\[2,](#page-8-1) [9,](#page-8-8) [12,](#page-8-11) [13\]](#page-8-12) are shared-entanglement protocols, and the required communication resources of these protocols are ebits and cbits, whereas the protocol in Ref. [\[15\]](#page-8-14) and our protocols are qubit-transmission protocols, and the required communication resources of these protocols are qubits and cbits. The required communication resources of these two kinds of protocols can not be compared directly. They can only be compared in the same communication scenario. Any qubit-transmission protocol that requires the transmission of N qubits and M cbits can be simulated in the shared-entanglement scenario through quantum teleportation [\[18\]](#page-9-1) at the cost of N shared ebits and transmission of $2N + M$ cbits. On the other hand, any shared-entanglement protocol that requires N shared ebits and communication of M cbits can be implemented in the qubittransmission scenario at the cost of communication of N qubits and M cbits, because one party can prepare a pair of entangled qubits and then transmit one of them to distribute 1 shared ebit. Tables [1,](#page-5-0) [2,](#page-5-1) [3](#page-6-1) and [4](#page-6-2) summarize the required communication resources of these protocols in both scenarios. The term gap is defined as the communication resources of the upper protocol minus that of the lower protocol.

Table [1](#page-5-0) shows the required communication resources of implementing $CU(N -$ 1, 1) which performs an arbitrary unitary operation on a target qubit possessed by Alice with distant $N-1$ control qubits possessed by Bob. In the qubit-transmission scenario, the protocol in Ref. [\[15\]](#page-8-14) can save 1 cbit of communication compared to the protocol in Ref. [\[2\]](#page-8-1). In the shared-entanglement scenario, the simulation of the protocol in Ref. [\[15\]](#page-8-14) requires 1 more cbit of communication compared to the protocol in Ref. [\[2\]](#page-8-1).

Table [2](#page-5-1) shows the required communication resources of implementing Alice's $(N + M)$ -qubit operation on Bob's N qubits and Alice's M qubits. Our protocol can save N cbits of communication compared to the protocol in Ref. [\[13\]](#page-8-12) in the qubit-transmission scenario. Table [3](#page-6-1) shows the required communication resources of implementing Alice's $(N + M)$ -qubit operation on Bob's $N + M$ qubits. Our

| | Qubit-transmission scenario | | Shared-entanglement scenario | |
|-----------------------|-----------------------------|-------|------------------------------|-------|
| | qubits | chits | ebits | chits |
| Protocol in Ref. [2] | | | | |
| Protocol in Ref. [15] | | | | |
| Gap | | | | |

Table 1 N-qubit controlled-U operation with Bob's $N-1$ control qubits and Alice's 1 target qubit

Table 2 Alice's $(N + M)$ -qubit operation on Bob's N qubits and Alice's M qubits

| | Qubit-transmission scenario | | Shared-entanglement scenario | |
|-----------------------|-----------------------------|-------|------------------------------|-------|
| | qubits | chits | ebits | cbits |
| Protocol in Ref. [13] | | 2.N | | 2 N |
| Our protocol | | | | 3N |
| Gap | | | | |

protocol can save $N + 4M$ cbits of communication compared to the protocol in Ref. [\[12\]](#page-8-11) in the qubit-transmission scenario. Table [4](#page-6-2) shows the required communication resources of implementing Alice's N-qubit operation on Bob's N qubits. Our protocol can save N cbits of communication compared to the protocol in Ref. [\[9\]](#page-8-8) in the qubit-transmission scenario. In summary, our protocols requires less communication resources than the protocols in Ref. [\[9,](#page-8-8) [12,](#page-8-11) [13\]](#page-8-12) implemented in the qubit-transmission scenario.

In the shared-entanglement scenario, the right parts of tables [2,](#page-5-1) [3](#page-6-1) and [4](#page-6-2) show that the simulation of our protocols requires N more cbits of communication compared to the protocols in Ref. [\[9,](#page-8-8) [12,](#page-8-11) [13\]](#page-8-12).

Therefore the saving of communicated cbits in our protocols is at least N cbits and goes up to $N + 4M$ cbits when Alice's $(N + M)$ -qubit operation needs to be implemented on Bob's $N + M$ qubits. More essentially, our protocol has advantage because it is a lot easier to transmit qubits than distribute and store entanglement pairs.

Alice's multiqubit operations on Bob's qubits can be implemented without prior sharing of entanglement as follows: Bob sends his qubits to Alice, and then Alice applies the operation and sends them back to Bob. By using this simple method, no auxiliary qubits are used, no additional CNOT and Hadamard operations are required, and no classical communications between Alice and Bob are needed. However, our method requires fewer qubits to be communicated than this simple method. This trade-off between computation and communication in our method is analogous to the trade-off between time and space in the field of algorithm design. Different benefits are required in different situations. Sometimes we need an easy-to-implement protocol, and sometimes we need to communicate as few qubits as possible. The shared-entanglement protocol in Ref. [\[13\]](#page-8-12) is fit for the parties who already have shared entanglement. The simple method is fit for the case where additional quantum operations are undesirable. However, our method is fit for the case where we place a high price on communication.

| | Qubit-transmission scenario | | Shared-entanglement scenario | |
|-----------------------|-----------------------------|---------|------------------------------|---------|
| | qubits | cbits | ebits | cbits |
| Protocol in Ref. [12] | $N+2M$ | $2N+4M$ | $N+2M$ | $2N+4M$ |
| Our protocol | $N+2M$ | | $N+2M$ | $3N+4M$ |
| Gap | | $N+4M$ | | $-N$ |

Table 3 Alice's $(N + M)$ -qubit operation on Bob's $N + M$ qubits

Table 4 Alice's N -qubit operation on Bob's N qubits

| | Qubit-transmission scenario | | Shared-entanglement scenario | |
|----------------------|-----------------------------|-------|------------------------------|-------|
| | qubits | chits | ebits | cbits |
| Protocol in Ref. [9] | | 2N | | 2N |
| Our protocol | | | | 3Ν |
| \mathbf{Gap} | | | | |

4 Generalization to d-dimensional operations

In this section, we generalize the protocol proposed in section 2 to d -dimensional operations. The $(N + M)$ -qudit quantum operation $U_d(f, G)$ can be expressed as:

$$
U_d(f, G) = \sum_{x=0}^{d^N - 1} |f(x)\rangle \langle x| \otimes G(x), \qquad (11)
$$

where $G(x)$'s are arbitrary $d^M \times d^M$ unitary matrices.

Suppose that Alice wants to implement $U_d(f, G)$ on N qudits possessed by Bob and M qudits possessed by herself. The initial state of the $N + M$ qudits can be written as:

$$
|\varphi\rangle_{B_1...B_N A_1...A_M} = \sum_{j=0}^{d^N-1} \alpha_j |j\rangle_{B_1...B_N} |\xi_j\rangle_{A_1...A_M},
$$
\n(12)

where subscripts $A_1 \ldots A_M$ $(B_1 \ldots B_N)$ denotes Alice's (Bob's) M (N) qudits. The protocol consists of six steps:

(1) Bob introduces N ancilla qudits $C_1 \ldots C_N$ initially prepared in the state $|0\rangle^{\otimes N}$, and then performs N generalized controlled-NOT (CNOT) gates [\[19\]](#page-9-2)

$$
|x\rangle_{B_j}|y\rangle_{A_j} \to |x\rangle_{B_j}|x-y \mod d\rangle_{A_j} \tag{13}
$$

with B_i as the control qudit and C_i the target qudit. After this step, the state of the composite system becomes

$$
\sum_{j=0}^{d^N-1} \alpha_j |j\rangle_{B_1...B_N} |j\rangle_{C_1...C_N} |\xi_j\rangle_{A_1...A_M}.\tag{14}
$$

(2) Bob sends N qudits $C_1 \ldots C_N$ to Alice.

(3) After receiving the ancilla qudits $C_1 \ldots C_N$, Alice performs $U_d(f, G)$ on qudits $C_1 \ldots C_N A_1 \ldots A_M$. The state of the composite system becomes

$$
\sum_{j=0}^{d^N-1} \alpha_j |j\rangle_{B_1...B_N} |f(j)\rangle_{C_1...C_N} G(j)|\xi_j\rangle_{A_1...A_M}.\tag{15}
$$

(4) Alice performs the one-qudit quantum Fourier transform

$$
|x\rangle \to \frac{1}{\sqrt{d}} \sum_{y=0}^{d-1} e^{2\pi i xy/d} |y\rangle \tag{16}
$$

on each qudit C_i , and then measures C_i in the computational basis. After the quantum Fourier transform, the state of the composite system becomes

$$
\frac{1}{\sqrt{d^N}} \sum_{k=0}^{d^N-1} |k\rangle_{C_1...C_N} \sum_{j=0}^{d^N-1} exp\left[\frac{2\pi i}{d} \sum_{l=1}^N f(j)_l k_l \right] \alpha_j |j\rangle_{B_1...B_N} G(j) |\xi_j\rangle_{A_1...A_M}.
$$
\n(17)

If the measurement result of qudits $C_1 \ldots C_N$ is $k = k_1...k_N$, the state of qudits $B_1 \ldots B_N A_1 \ldots A_M$ becomes

$$
\sum_{j=0}^{d^N-1} \exp\left[\frac{2\pi i}{d} \sum_{l=1}^N f(j)_l k_l\right] \alpha_j |j\rangle_{B_1...B_N} G(j) |\xi_j\rangle_{A_1...A_M}.\tag{18}
$$

(5) Alice informs Bob of the measurement result k by sending $[N \log_2 d]$ cbits. (6) Since Bob knows which restricted set f the operation $U_d(f, G)$ belongs to, he can construct a corresponding N-qudit unitary operation

$$
V_d(f) = \sum_{x=0}^{d^N - 1} |f(x)\rangle \langle x|.
$$
\n(19)

Bob first performs $V_d(f)$ on qudits $B_1 \dots B_N$, and then performs S^{k_i} on qudit B_i , where

$$
S = \sum_{x=0}^{d-1} e^{-2\pi ix/d} |x\rangle\langle x|.
$$
 (20)

The state of qudits $B_1 \ldots B_N A_1 \ldots A_M$ becomes

$$
\sum_{j=0}^{d^N-1} \alpha_j |f(j)\rangle_{B_1...B_N} G(j)|\xi_j\rangle_{A_1...A_M} = U_d(f,G)|\varphi\rangle_{B_1...B_NA_1...A_M}.\tag{21}
$$

Thus, $U_d(f, G)$ has been successfully implemented on qudits $B_1 \dots B_N A_1 \dots A_M$. The required communication resources are N qudits transmitted from Bob to Alice in step 2 and $[N \log_2 d]$ cbits transmitted from Alice to Bob in step 5.

5 Conclusion

We have considered the implementation of Alice's multiqubit operation from the restricted sets [\[12,](#page-8-11) [13\]](#page-8-12) on distant qubits possessed by Bob from a communication complexity perspective. The restricted sets are $2^N \times 2^N$ block matrices with just one nonzero block in any column or any row, every block of which is a $2^M \times 2^M$ unitary matrix. Protocols for implementing these restricted sets of multiqubit operations on distant qubits using prior sharing of entanglement have been proposed in Ref. [\[12,](#page-8-11) [13\]](#page-8-12). Inspired by Yang's work [\[15\]](#page-8-14) for constructing a nonlocal N-qubit controlled-U gate without prior sharing of entanglement, we have proposed a protocol to complete the same tasks as those in Ref. [\[5,](#page-8-4) [9,](#page-8-8) [12,](#page-8-11) [13\]](#page-8-12) without prior sharing of entanglement. We have shown that our qubit-transmission protocol requires less communication resources than the previous shared-entanglement protocols [\[9,](#page-8-8) [12,](#page-8-11) [13\]](#page-8-12) in the qubit-transmission scenario. Because it is a lot easier to transmit qubits than distribute and store entanglement pairs, our protocol has advantage in the case that the parties have no prior sharing of entanglement. Furthermore, we have generalized our protocol to d-dimensional operations.

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