# Robust Quadrotor Control: Attitude and Altitude Real-Time Result

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# Robust Quadrotor Control: Attitude and Altitude Real-Time Results

**Ricardo López-Gutiérrez · Abraham Efraim Rodriguez-Mata · Sergio Salazar · Ivan González-Hernández · Rogelio Lozano**

**Abstract** This paper addresses the problem of designing and experimentally validating a nonlinear robust control to attitude and altitude of a quadrotor unmanned flying vehicle (UAV). First a disturbance observer is proposed, focus on the attitude regulation control problem of a quadrotor in presence of external disturbances based on the angular velocity measurements and the control inputs. The stability analysis of the nonlinear observer scheme is proven via the use of Lyapunov theory. Later, we focus on the altitude dynamics of a quadrotor in the presence of uncertainty like wind gust are presented. A sliding mode control was proposed, the gain of control can be decreased and, as a result, the chattering amplitude is reduced. The objective is to introduce an adaptation in the control law in order to decrease the gain to the minimal value preserving the sliding mode control and keeping his property of a finite-time convergence. Finally, simulation and experimental results in a quadrotor are presented to show the effectiveness of the proposed nonlinear algorithm in presence of external disturbances.

**Keywords** Nonlinear observer · Parameter/disturbance estimation · Sliding mode/adaptive control · Quadrotor unmanned flying vehicle

## 1 Introduction

Nowadays, flight control and applications of UAV is an active and challenging topic of research. For instance, in this context the most popular UAV system has been the quadrotor helicopter, if we compare one of these multirotor vehicles with a conventional helicopter it has many advantages such as greater trust-weight ratio and better maneuverability. It is well known that the interest in the investigation of UAVs is growing due to their different applications. Now

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today remains very important the research on the design of autonomous flight control systems in the presence of uncertainty, but this is not an easy work, actually is very difficult task due to its inherent nonlinearity associated with the dynamical model, under actuated property and external disturbances associated with uncertain flying environment. The quadrotors, has been studied recently by some authors using different control techniques such as [1] backsteping, [2] robust nonlinear PI, Robust neural network [3]. Dual camera visual feedback [4], Fault tolerant margins [5], Backsteping/Nonlinear $H_\infty$, backstepping and sliding-mode [6] and [7]. These systems as many other dynamic systems, present constant or slowly varying uncertain parameters. In UAV performance, the presence of wind gust disturbances, model uncertainties, and nonlinear parts is inevitable, for example, when an aerial vehicle robot grasp an unknown payload, it is affected by unknown inertia variation and gravity force but these changes are rarely considered in the models. It is most desirable that the controller be insensitive to these uncertainties. Moreover this method considers the upper and lower bounds of the disturbance as a priori information where the main objective is to estimate the disturbance using a higher order filter. Afterwards, in [8] the author present a robust observer-based linear output feedback control scheme utilizing a Generalized Proportional Integral (GPI) observer in order to estimating the external disturbances on quadrotor.

For other hand it is known the adaptive control provides many tools to tackle such systems with unknown parameters including UAVs since the main idea in adaptive control is to estimate the uncertain plant parameters in a on-line estimation in order to use the estimated parameters in the control input computation [9–13] and [14].

Even more, there are reports on the use of the adaptive sliding mode control in quadrotors, in [15] use the adaptive SMC to stabilize both bank and pitch angles while tracking heading and altitude trajectories and to compensate additive perturbation and parameter uncertainties related to the mass and inertia matrix of the quadrotor. In [16] the adaptation laws are used in both position and attitude dynamics to learn and compensate uncertainty associated with the variation of the payload mass, inertia, aerodynamic and gyroscopic force, external disturbances and unpredictable change in outdoor flying environments.

This paper is a continuation of the work presented in [17] and [18] which presents a real-time robust control implementation of the altitude and attitude of in an outdoor quadrotor aircraft task. The main advantage of these techniques is that it is possible to eliminate disturbances unwanted such as wind gusts or disturbances which may exist even in measurements sensor, we combined the benefices of this both techniques. The main obstacles for application of Sliding Mode Control are two interconnected phenomena: chattering and high activity of control action. It is well known that the amplitude of chattering is proportional to the magnitude of discontinuous control. These two problems can be handled simultaneously if the magnitude is reduced to a minimal admissible level defined by the conditions for the sliding mode to exist. The altitude in a quadrotor helicopter is a great issue due to their dynamic include the mass and the roll and pitch angles. The perturbation in altitude could be provided from the many factors, the rotational dynamics, change of mass, wind gusts, etc. The paper presents an altitude and attitude control for a quadrotor and a robust high gain observer, the main contribution of this paper is on the adaptation of the sliding mode controller gain the objective of the adaptation process looks transparent: The gain is considered a variable and cominated with a robust control. As a result, the minimal possible value of the discontinuity magnitude is found for the current value of disturbances to reduce the amplitude of chattering [19].

A quadrotor mini aircraft dynamical model is presented in Sections 2, Sections 3 presents the development of the control law strategies of attitude, the Section 4 presents the development of the control law strategies of altitude. Both sections have subsections within the simulation and experimental results obtained Finally the Section 5 presents the conclusion.

## 2 Quadrotor Mathematical Model

Quad-rotor unmanned aircraft, consisting of four individual rotors in "+" arrangement set. This quadrotor is an attractive rotary-wing vertical take-off and landing (VTOL) Unmanned Aerial Vehicle (UAV) for both military and civilian usages. In this type of vehicles, vertical motion is created by collectively increasing and decreasing the speed of all four rotors; pitch or roll motion is achieved by the differential speed of the front-rear set or the left-right set of rotors, coupled with lateral motion; yaw motion is realized by the different reaction torques between the (1,3) and (2,4) rotors. The main thrust is the sum of the thrusts of each motor, as shown in Fig. 1.

Let the inertial frame and the fixed to the rigid aircraft frame respectively as:

$$I = i_I, j_I, k_I$$
$$B = i_B, j_B, k_B$$

The generalized coordinates vector $q$ defined as:

$$q = (x, y, z, \phi, \theta, \psi)^T \in \mathbb{R}^6 = (\xi, \eta)^T$$

describe the position and orientation of the flying machine, the model could be separated in two coordinate subsystems: translational and rotational. They are defined respectively by $\xi = (x, y, z)^T \in \mathbb{R}^3$: denotes the position of the vehicle's mass center relative to the inertial frame $I$ and $\eta = (\phi, \theta, \psi)^T \in \mathbb{R}^3$: describe the attitude of the aerial vehicle, i.e. roll, pitch and yaw angles respectively.

For modeling for a quadrotor aircraft, a few assumptions are made:

**Assumption 1** Quad-rotor fuselage is a rigid body.

**Assumption 2** The structure is symmetrical and

**Assumption 3** The center of gravity (CoG) and the body fixed frame origin are assumed to coinciding. The dynamic model is obtained via Lagrange approach [13].

The equation:

$$R^{B\to I} = \begin{bmatrix} C_\theta C_\psi & S_\psi C_\theta & -S_\theta \\ C_\psi S_\theta S_\phi - S_\psi C_\phi & S_\psi S_\theta S\phi + C_\psi C_\phi & C_\theta S_\phi \\ C_\psi S_\theta S_\phi + S_\psi C_\phi & S_\psi S_\theta S\phi - C_\psi C_\phi & C_\theta S_\phi \end{bmatrix}$$

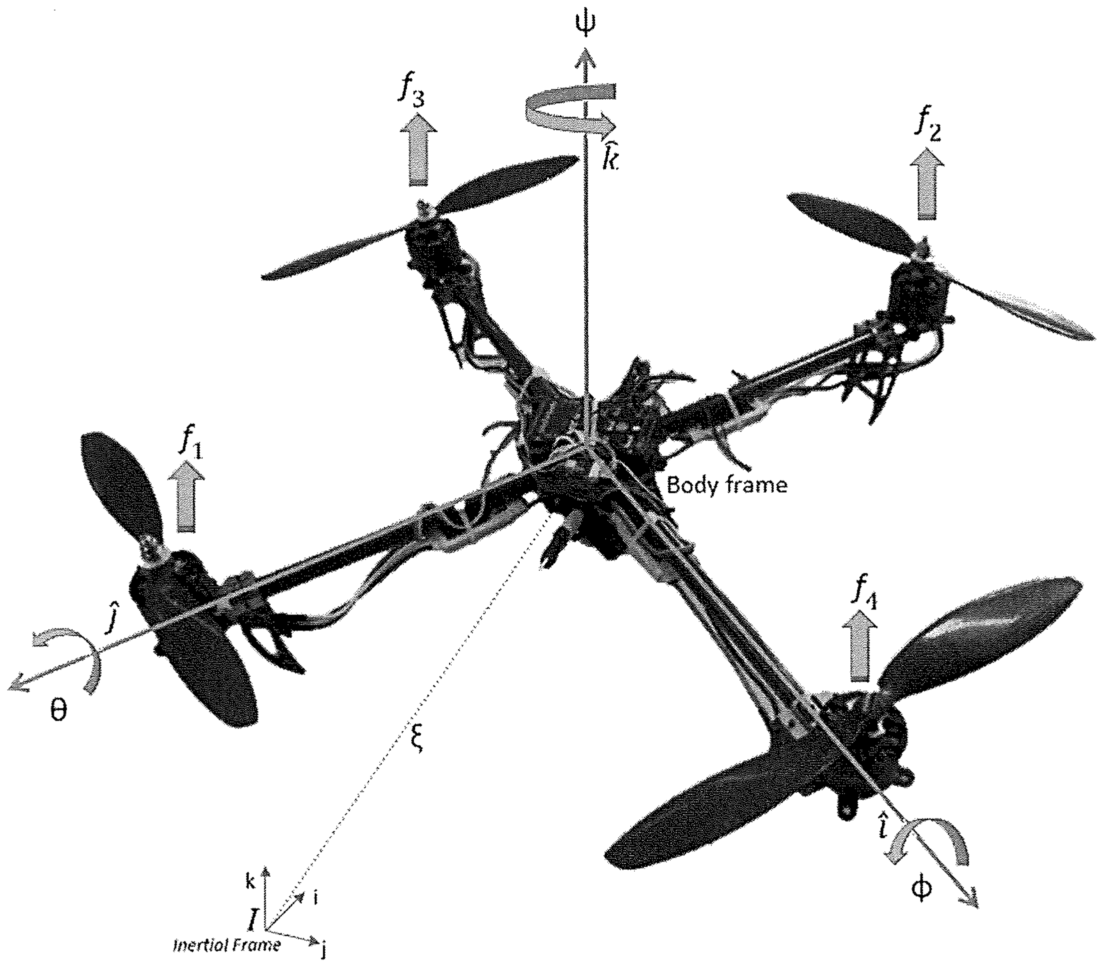


**Fig. 1** Scheme of forces produced by rotors, and torques generated in attitude (roll, pitch and yaw angles) acting on the quadrotor

is the rotational matrix which is defined by three Euler angles $\eta = (\phi, \theta, \psi)^T \in \mathbb{R}^3$ and $R \in SO(3)$ where $S(\cdot)$ and $C(\cdot)$ are $Sin(\cdot)$ and $Cos(\cdot)$ respectively. Then the control torques generated by the four rotors are:

$$\tau_\eta = \begin{pmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{pmatrix} = \begin{pmatrix} (f_3 - f_1)l \\ (f_2 - f_4)l \\ ((f_1 + f_2) - (f_3 - f_1))d \end{pmatrix} \quad (1)$$

Let $l$ denote the distance from the rotors to the center of mass and $d$ is the drag coefficient produced by coordinated reactive torque involving the four rotors because of the geometry of the quadrotor UAV. Since the lagrangian contains no cross terms in the kinetic energy, combining $\dot{\xi}$ and $\dot{\eta}$ vectors in the Euler Lagrange equation can be partitioned into the dynamics for the $\xi$ coordinates and the $\eta$ dynamics. So, we obtain

$$F_I = m\ddot{\xi} + mg \quad (2)$$

$$\tau_\eta = \mathbb{J}\ddot{\eta} + \dot{\mathbb{J}}\dot{\eta} - \frac{1}{2}\frac{\partial}{\partial\eta}(\dot{\eta}^T \mathbb{J}\dot{\eta}) \quad (3)$$

Defining the Coriolis terms and gyroscopic and centrifugal terms as:

$$C(\eta, \dot{\eta})\dot{\eta} = \dot{\mathbb{J}}\dot{\eta} - \frac{1}{2}\frac{\partial}{\partial\eta}(\dot{\eta}^T \mathbb{J}\dot{\eta}) \quad (4)$$

The equations of motion of the quadrotor can be expressed as:

$$\begin{pmatrix} m\ddot{x} \\ m\ddot{y} \\ m\ddot{z} \end{pmatrix} = \begin{pmatrix} -uS_\theta \\ uS_\phi C_\theta \\ uC_\phi C_\theta \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -mg \end{pmatrix} \quad (5)$$

The equations for the rotational motion are:

$$\mathbb{J}\ddot{\eta} = \tau_\eta - C(\eta, \dot{\eta})\dot{\eta} \quad (6)$$

where $x$ and $y$ are the coordinates in the horizontal plane and $z$ is the vertical position, $\phi$ is the roll angle around the $x$-axis, $\theta$ is the pitch angle around the $y$-axis and $\psi$ is the yaw angle around the $z$-axis for the vector $\eta = (\psi, \theta, \phi)^T$. Knowing that:

$$\mathbb{J} = W_\eta^T I W_\eta. \quad (7)$$

where $W_\eta$ is a transformation matrix and is given by

$$W_\eta = \begin{bmatrix} -\sin(\theta) & 0 & 1 \\ \cos(\theta)\sin(\theta) & \cos(\phi) & 0 \\ \cos(\theta)\cos(\phi) & -\sin(\phi) & 0 \end{bmatrix} \quad (8)$$

Can be written the $\eta$-dynamic in the general form as:

$$M(\eta)\ddot{\eta} + C(\eta, \dot{\eta})\dot{\eta} = \tau_\eta + w \quad (9)$$

where

$$M(\eta) = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}; \; C(\eta, \dot{\eta}) = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}.$$

with

$$\begin{aligned}
c_{11} &= I_{xx}\dot{\theta}s_\theta c_\theta + I_{yy}(-\dot{\theta}s_\theta c_\theta s_\theta^2 + \dot{\phi}c_\theta^2 s_\phi c_\theta) \\
&\quad - I_{zz}(\dot{\theta}s_\theta c_\theta c_\phi^2 + \dot{\phi}c_\theta^2 s_\phi c_\phi) \\
c_{12} &= I_{xx}\dot{\psi}s_\theta c_\theta - I_{yy}(\dot{\theta}s_\theta s_\phi c_\phi + \dot{\phi}c_\theta s_\phi^2 \\
&\quad - \dot{\phi}c_\theta c_\phi^2 + \dot{\psi}s_\theta c_\theta s_\phi^2) + I_{zz}(\dot{\phi}c_\theta s_\phi^2 - \dot{\phi}c_\theta c_\phi^2 \\
&\quad - \dot{\psi}s_\theta c_\theta c_\phi^2 + \dot{\theta}s_\theta s_\phi c_\phi) \\
c_{13} &= -I_{xx}\dot{\theta}c_\theta + I_{yy}\dot{\psi}c_\theta^2 s_\phi c_\phi - I_{zz}\dot{\psi}c_\theta^2 s_\phi c_\phi \\
c_{21} &= -I_{xx}\dot{\psi}s_\theta c_\theta + I_{yy}\dot{\psi}s_\theta c_\theta s_\phi^2 + I_{zz}\dot{\psi}s_\theta c_\theta c_\phi^2 \\
c_{22} &= -I_{yy}\dot{\phi}s_\phi c_\phi + I_{zz}\dot{\phi}s_\phi c_\phi \\
c_{23} &= I_{xx}\dot{\psi}c_\theta + I_{yy}(-\dot{\theta}s_\phi c_\phi + \dot{\psi}c_\theta c_\phi^2 - \dot{\psi}c_\theta s_\phi^2) \\
&\quad + I_{zz}(\dot{\psi}c_\theta s_\phi^2 - \dot{\psi}c_\theta c_\phi^2 + \dot{\theta}s_\phi c_\phi) \\
c_{31} &= -I_{yy}\dot{\psi}c_\theta^2 s_\phi c_\phi + I_{zz}\dot{\psi}c_\theta^2 s_\phi c_\phi \\
c_{32} &= -I_{xx}\dot{\psi}c_\theta + I_{yy}(\dot{\theta}s_\phi c_\phi + \dot{\psi}c_\theta s_\phi^2 - \dot{\psi}c_\theta c_\phi^2) \\
&\quad - I_{zz}(\dot{\psi}c_\theta s_\phi^2 - \dot{\psi}c_\theta c_\phi^2 + \dot{\theta}s_\phi c_\phi) \\
c_{33} &= 0
\end{aligned}$$

## 3 Atitude Control Design

Starting from the following system:

$$M(\eta)\ddot{\eta} + C(\eta, \dot{\eta})\dot{\eta} = \tau_\eta + w(t) \quad (10)$$

where $\eta, w(t), u \in \mathbb{R}$ and $M(\eta)$, $C(\eta, \dot{\eta}) \in \mathbb{R}^{nxn}$. The states vector named $\eta$ describe the attitude of the aerial vehicle, $w(t)$ represents a disturbance vector due the modeling errors and wind perturbation, $M(\eta)$ is the inertial matrix and $C$ is the Coriolis matrix. The system (10) must fulfill the next assumptions:

**Assumption 4** The nonlinear Coriolis matrix $C(\eta, \dot{\eta})$ is a Lipschitz matrix function and it satisfies (to abound more see [20]):

$$\|C(\eta', \dot{\eta}') - C(\eta, \dot{\eta})\| \leq l_c \|\eta' - \eta\| \; \forall \eta' \neq \eta, \, l_c > 0 \quad (11)$$

**Assumption 5** The wind $w(t)$ is a bounded disturbance vector that satisfies: $\|w(t)\| \leq w_{\max}$, where $w_{\max}$ is a nonzero positive constant.

**Assumption 6** Exists an unknown bounded nonlinear function such that: $\delta(t) = w(t) - C(\eta, \dot{\eta})\dot{\eta}$. The unknown function $\delta(t)$ describes a type of disturbance variable on the system (10).

Through to above assumptions it is possible to built the next mathematical reduced model, in order to describe the orientation (attitude) of the UAV. Therefore it due to invertible property of $M(\eta)$ it is obtained the next model:

$$\ddot{\eta} = M(\eta)^{-1}(\tau_\eta + \delta(t)) \tag{12}$$

This model can be seen as a sum of a linear and a nonlinear part of atitiude general system shown in Eq. 10. This characteristic allows to use control techniques such as active perturbations rejection together with robust high gain observers. The Fig. 2 shows a diagram of the nonlinear robust control strategy is applied to the UAV. This algorithm will be discussed in the next section.

### 3.1 Design Robust Observer Algorithm

Representing the system (12) as:

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 \\ \dot{\eta}_2 &= M(\eta)^{-1}(\tau_\eta + \delta(t)) \end{aligned} \tag{13}$$

To represent the system (13) in the classical canonical observability form, we propose a change of variables where $\eta_1 = x_1$ and $\eta_2 = x_2$. The transformed system is represented as:

$$\begin{aligned} \dot{x} &= A_0 x + G(\tau_\eta + \delta(t)) \\ y &= C_y x \end{aligned} \tag{14}$$

where $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $G = \begin{bmatrix} 0 \\ M(\eta)^{-1} \end{bmatrix}$, $A_o = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $C_y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is the output matrix. We propose a modification of classical high gain observer in order to solve a wind disturbance or perturbation rejection in the UVA problem, where the matrix $S_\infty$ is a non-negative and symmetric high gain matrix [21]:

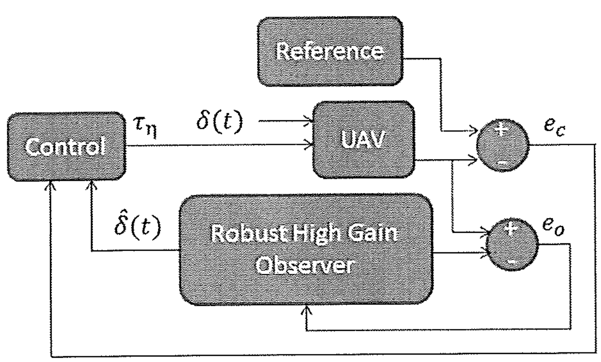


**Fig. 2** Diagram of the nonlinear robust control strategy using a robust high gain observer for atitude control of UAV

$$S_\infty = \begin{bmatrix} h^{-1} & -h^{-2} \\ -h^{-2} & 2h^{-3} \end{bmatrix} \tag{15}$$

The variable $h \gg 0$ is a high gain of observer. Knowing that $\hat{x}$ is the estimate state and $x$ is the real state of attitude system, the observation error is defined as $e_o = \hat{x} - x$. Therefore it is proposed the next robust high gain observer:

$$\begin{aligned} \dot{\hat{x}} &= A_0\hat{x} + G(\tau_\eta + \hat{\delta}) - S_\infty^{-1} C_y^T C_y e_o \\ \dot{\hat{\delta}} &= -F e_o \\ F &= \Gamma^{-1} G^T S_\infty \end{aligned} \tag{16}$$

with $\Gamma = \Gamma^T > 0$

The previous observer (16) has main goal to estimated unknown function $\delta(t)$, it is necessary that some conditions are fulfilled on the system and the type of wind gust:

**Assumption 7**

1. - First derivative of $\delta(t)$ is bounded such that: $\left\|\dot{\delta}(t)\right\| \leq b S_\infty \|e_o\| \leq b \|e_o\|_\infty$ for $b > 0$
   This above condition shows that the superior value of the wind rate will be not higher the state estimator detection capability, this reflected on the observation error induced by the $S_\infty$ high gain matrix.
2. - The matrix $S_\infty$ is a solution of the following matrix equation: $0 = -hS_\infty - A_o S_\infty - S_\infty A_o + C_y^T C_y$ for any $h$ positive high gain (see proof in [21]).
3. -The estimate unknown nonlinear function fulfill with: $\|\delta(t)\| \leq \sup \delta(t)$
4. - The estimate error of the disturbance $\delta(t)$ is defined as: $\tilde{\delta}(t) = \hat{\delta}(t) - \delta(t)$ with $\hat{\delta}(t)$ as estimate disturbance and $\delta(t)$ as the real unknown wind on attitude system.

Based on the previous assumptions we can propose the following theorem where it is demonstrated the performance of a robust high gain observer in the estimation of unknown disturbance function $\delta(t)$.

**Theorem 1** (Wind Robust High Gain Observer) *Let* Eq. 16 *be a robust high gain observer of the* Eq. 14, *then the error $e_o$ is ultimate bounded with practical stability in presence disturbance wind such that it is obtained the best wind approximation $\delta(t) \approx \hat{\delta}(t)$ when the high gain $h \gg 0$ and it holds with Assumption 7.*

*Proof* The observation error and its first derivative are:

$$e_o = \hat{x} - x \tag{17}$$

$$\dot{e}_o = \dot{\hat{x}} - \dot{x} \tag{18}$$

Defining $A_c = (A_0 - S_\infty^{-1} C_y^T C_y)$ and substituting the systems (14) and (16) in Eq. 18 is obtained:

$$\dot{e}_o = A_c e_o + G\tilde{\delta}(t) \tag{19}$$

It is proposed the next Lyapunov function and its first trajectories derivative is calculated:

$$V(t) = e_o^T S_\infty e_o + \tilde{\delta}(t)^T \Gamma \tilde{\delta}(t)$$

$$\begin{aligned}\dot{V}(t) = &\ \dot{e}_o^T S_\infty e_o + e_o^T S_\infty \dot{e}_o + 2\tilde{\delta}(t)^T \Gamma \dot{\hat{\delta}}(t) \\ &-2\tilde{\delta}(t)^T \Gamma \dot{\delta}(t)\end{aligned}$$

replacing the observation error (18) and using the *Assumption 7.2*, is obtained:

$$\begin{aligned}\dot{V}(t) = &-he^T S_\infty e_o + 2\tilde{\delta}(t)^T \Gamma \dot{\hat{\delta}}(t) - 2\tilde{\delta}(t)^T \Gamma \dot{\delta}(t) \\ &+ e_o^T C_y^T C_y e_o + 2\tilde{\delta}(t)^T G^T S_\infty e_o \end{aligned} \tag{20}$$

in order to $2\tilde{\delta}(t)^T \Gamma \dot{\hat{\delta}}(t) + 2\tilde{\delta}(t)^T G^T S_\infty e_o = 0$, it should to hold the next array :

$$2\tilde{\delta}(t)^T (\Gamma \dot{\hat{\delta}}(t) + 2G^T S_\infty e_o) = 0$$

Such that it is obtained the same disturbance estimation as Eq. 16 :

$$\dot{\hat{\delta}}(t) = -Fe_o$$

with $F = \Gamma^{-1} G^T S_\infty$. If this above array is hold, the Eq. 20 is modified as follows:

$$\dot{V}(t) = -he_o^T S_\infty e_o - 2\tilde{\delta}(t)^T \Gamma \dot{\delta}(t) + e_o^T C_y^T C_y e_o \tag{21}$$

$$\dot{V}(t) \le -h \|e_o\|_\infty^2 + 2\lambda_{\max}(\Gamma) \left\|\dot{\delta}(t)\right\|$$

$$\dot{V}(t) \le -h \|e_o\|_\infty^2 + 2\lambda_{\max}(\Gamma) b \|e_o\|_\infty$$

$$\dot{V}(t) \le -h \|e_o\|_\infty \left( \|e_o\|_\infty - \frac{2\lambda_{\max}(\Gamma) b}{h} \right)$$

For any $h \gg 2\lambda_{\max}(\Gamma) b$ the system has a convergence ball as follows:

$$B_{e_o} = \|e_o\|_\infty \in \mathbb{R}^n \ \|e_o\|_\infty < \frac{2\lambda_{\max}(\Gamma) b}{h} \tag{22}$$

then the error is ultimate bounded. □

### 3.2 Attitude Control Plus Wind Disturbance Rejection

From Eq. 14 we have:

$$\dot{x}_2 = M(\eta)^{-1} (\tau_\eta - \delta(t)) \tag{23}$$

where the $\dot{x}_2$ represents the angular velocity. It is proposed a vector $l = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$ as a reference vector in order to solve a tracking control problem. We defined the tracking control error and its derivative as:

$$e_c = l - x_2 \tag{24}$$

$$\dot{e}_c = -\dot{x}_2 \tag{25}$$

In order to have an asymptotic stability property it is necessary that $\dot{e}_c = Ke_c$, where $K$ is a Hurwitz gain matrix, therefore

$$Ke_c = -\dot{x}_2$$

Replacing $\dot{x}_2$ of Eq. 23 in the previous equation:

$$Ke_c = M(\eta)^{-1} (\delta(t) - \tau_\eta)$$

$$\tau_\eta = -M(\eta) Ke_c + \delta(t) \tag{26}$$

We replace the nonlinear disturbance $\delta(t)$ for their estimate $\hat{\delta}(t)$ obtained by the robust high gain observer (16) in order to obtain the robust feedback nonlinear control as:

$$\tau_\eta = -M(\eta) Ke_c + \hat{\delta}(t) \tag{27}$$

**Theorem 2** (Attitude control plus wind disturbance rejection) *Be $\tau_\eta$ in* Eq. 27 *the robust control of* Eq. 23 *with* Eq. 16 *as a robust high gain observer in sense of Theorem 1, the tracking error control* (24) *will be converge to zero if K is a Hurwitz matrix.*

*Proof* Substituting the control law (27) in Eq. 24:

$$\dot{e}_c = M(\eta)^{-1} (-M(\eta) Ke_c + \hat{\delta}(t) - \delta(t))$$

$$\dot{e}_c = Ke_c$$

It given that $K$ is Hurwitz, it will be $V(t) = e_c^T Pe_c$, it is hold $P^T K + KP = -Q$, such that $\dot{V} \le -\lambda_{\max}(Q) \|e_c\|^2$. □

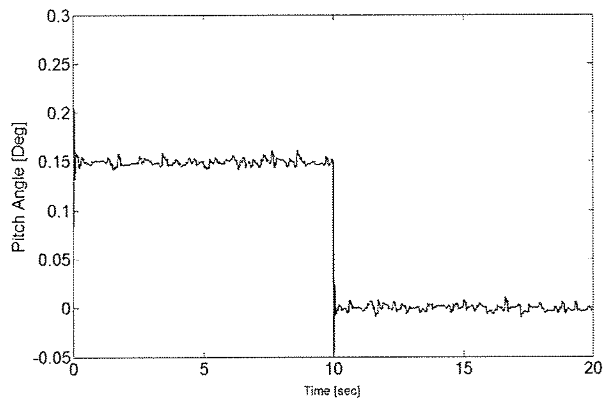


**Fig. 3** Pitch angle position, with feedback of disturbances estimation in $t = 10s$

### 3.3 Numerical Results in Attitude Control

In this subsection is present a simulation study on orientation UAV stabilization (10). The attitude of UAV is perturbed with a disturbance.The objective is only disturbance in pitch angle. The complete feedback control simulated is:

$$\begin{aligned} \dot{\hat{x}} &= A_0\hat{x} + G(u + \hat{\delta}(t)) - S_{\infty}^{-1} C_y^T C_y e_o \\ \dot{\hat{\delta}}(t) &= -F e_o \\ F &= \Gamma^{-1} G^T S_{\infty} \\ \tau_\eta &= -M(\eta) K e_c + \hat{\delta}(t) \end{aligned} \tag{28}$$

where the function $\delta(t)$ is the disturbance generated by a step function with white noise. The high gain is $h = 30$ with $S_\infty$ as Eq. 15 and the next matrices are used:

$$M(\eta) = \begin{bmatrix} 0.018 & 0 & 0 \\ 0 & 0.018 & 0 \\ 0 & 0 & 0.036 \end{bmatrix};$$

$$K = \begin{bmatrix} -120 & 0 & 0 \\ 0 & -50 & 0 \\ 0 & 0 & -20 \end{bmatrix}; \Gamma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix}.$$

The Fig. 3 shown the pitch angular position over the time, where we can observe convergence in the $t = 10s$ time, when the compensation $\hat{\delta}(t)$ obtained from the observer is feedback applied, that is, in the previous time there was a signal of control but without the estimation, later in $t = 10s$ the estimation is added and this compensates the disturbances and bring the vehicle to the hover position (angular references $\theta = 0$). The left side of Fig. 4 shows the funcion $\delta(t)$ with the blue line and the estimated disturbance $\hat{\delta}(t)$ with red dotted line. The control law is shown in rigth side of Fig. 4.

### 3.4 Experimental Results in Attitude Control

This subsection shown a real-time results with the control law Eq. 28. The Fig. 5 shows an estimate of a disturbance in real-time, it is noticed that the disturbance approximately 0.25Nm converge to zero over the time. For the experimental setup, we use a small quadrotor aircraft see Fig. 6, In the first test (left side of Fig. 7) was applied a constant disturbance to maintain the vehicle with a slope over the pitch angle (15 degrees), then in a second test (see the rigth side

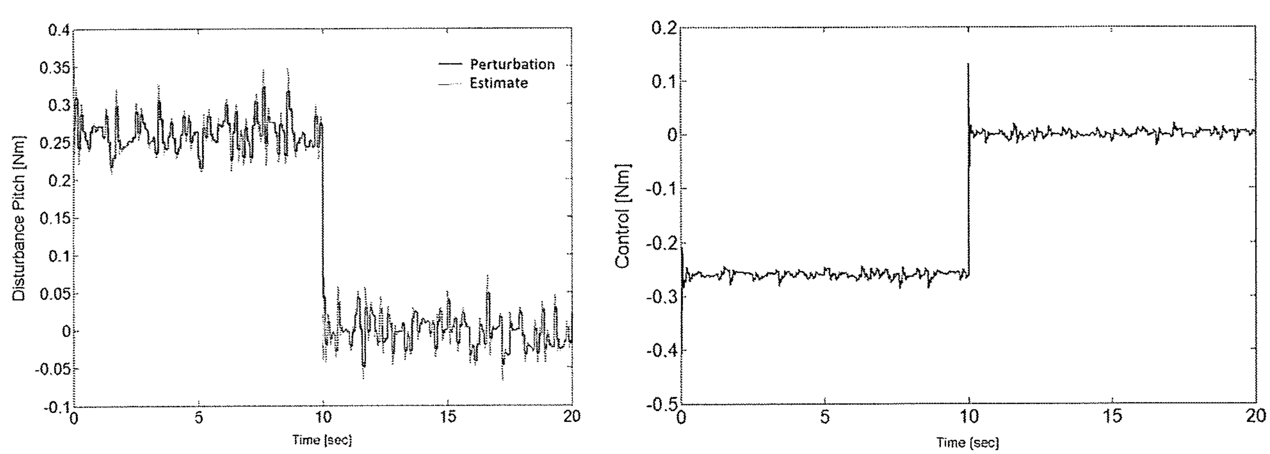


**Fig. 4** Disturbance identification in pitch angle through robust observer and the control output