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ORIGINAL RESEARCH

Optimal water tariffs for domestic, agricultural and industrial use

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Abstract
Consider a water supplier who determines sales rates with the goals of maximizing profits, protecting consumer welfare, and ensuring adequate future water supplies. Buyers are differentiated and can use the water for domestic, agricultural, and industrial purposes. We propose a leader-follower finite-horizon differential game. The leader (the water supplier) determines the selling price and the followers (consumers) react by requesting their optimal amount of water. We calculate a feedback Stackelberg equilibrium assuming that all user demand is satisfied (interior equilibrium). We compare two different tariff schemes: linear tariffs (the price paid is a multiple of the volume of water purchased), and increasing block tariffs (the unit price is lower for quantities of water that do not exceed a fixed threshold). We show that block pricing is never optimal and linear pricing is always preferred.

Keywords Water pricing · Block tariffs · Differential games · Stackelberg equilibrium · Corner solutions

Mathematics Subject Classfication 91A23 · 91B76 · 91B15

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1 Introduction

In 1992 the Dublin Water Principles claimed "water as an economic good" for the first time in a UN setting. In the late 1980s the World Bank and other multilateral and bilateral

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institutions discovered the virtues of “privatization” in the provision of public services and with privatization all of the attendant problems of setting tariffs and prices (Rogers et al., 2002).

Water is often a scarce commodity and at the same time a basic necessity. Policymakers need to set rules for the sale and distribution of water that allow both the coverage of costs (extraction, maintenance of systems, etc.) and access to the asset by a large number of users and, in any case, the satisfaction of basic needs. There are many tools to achieve these goals. Pricing can be an allocation mechanism, directing water to where it is more valuable to use (OECD, 2010).

According to Leflaive and Hjort (2020) “Tariffs for water supply (WS) constitute an important instrument for economic, financial, social, and environmental policy objectives, potentially reflecting costs of service provision. The issue of WS pricing is all the more important as developed and developing countries face severe challenges in financing the operation and maintenance, renewal or extension of WS infrastructure and services. In this context, it is critical that governments, national and local, develop adequate financing strategies and make the best use of tariffs to provide and sustain WS services.”

Tariff structures can combine various elements in different ways. Linear (or volumetric) tariffs require customers to pay a fixed per-unit price, multiplied by the volume of water consumed in a charging period; each additional unit of water costs the same. If an increasing block tariff (IBT) applies, the volumetric charge increases in steps with volumes consumed. The IBTs are applied in many countries mainly to domestic users. South Africa introduced a first block that provides basic water volumes for free, funded by tax revenue. In some countries (Belgium, Canada, France) decreasing block tariffs are applied for large industrial users. IBTs ensure inexpensive, or even free, access to a given amount of water for low-income households. The higher prices for subsequent blocks are intended to subsidize water use of poorer households and strengthen overall cost recovery by charging more to households using more water. The higher prices in the upper blocks should also discourage inappropriate water use (Grafton et al., 2014; OECD, 2016). However, in practice, the implementation of IBTs may be challenging, and can potentially have some adverse social consequences, if such variables as household size—which can drive domestic water consumption—are not taken into account (Leflaive & Hjort, 2020).

Nonetheless, the question of whether block tariffs are preferred to linear tariffs remains controversial. This is demonstrated by the fact that tariff structures for water supply vary within and across OECD countries. Some countries implement linear tariffs (Denmark, Germany, UK), others implement block tariffs (Italy, Japan, Spain) and others implement both (Australia, Canada, USA) (OECD, 2010).

There is a large literature on mathematical models for water resource management. The main strand deals with groundwater management and focuses on issues such as the type of access to the resource (open access or restricted access), and the type of interaction between potential users (competition, cooperation). In many of these models, the price is not critical and sometimes it is an exogenous variable.

A seminal paper in this stream is Gisser and Sánchez (1980). Here, the economic aspects of a model for the problems of farmers pumping groundwater out of an aquifer are discussed. The authors assume that the demand and the extraction cost are linear decreasing functions of the water’s price and of the level of water in the aquifer above the level of the sea, respectively. The farmer determines the water’s demand maximizing the present value of his future profits (income minus costs). The authors compare the optimal strategies with the perfect competition strategies and they find no relevant differences. The same topics are addressed in Allen and Gisser (1984) but with a nonlinear demand function.

More recently, a large number of papers have addressed the problem of how a public agency allocates water from one or more underground aquifers to different types of users (e.g. farmers and municipalities) to ensure collective welfare for users and sufficient drainage to preserve the ecosystem (Pereau et al., 2019; Pereau, 2020; Augeraud-Veron & Pereau, 2022). The authors analyze both viable solutions and the social optimum in a discrete and continuous dynamic model.

In Biancardi et al. (2020), the authors address the issue of ensuring the use of the groundwater resource for future generations and develop a differential game to determine the efficient extraction among overlapping generations. The effects of legal and illegal firms' actions and the contribution of taxes and penalties imposed by public authorities are analyzed, using the framework of differential and evolutionary games, in Biancardi et al. (2021, 2022a, b, 2023).

An important and widely discussed topic in the literature is the allocation of a scarce resource such as water among a multiplicity of users who normally have a higher water requirement than the available quantity. Among others, we cite Du et al. (2018) who analyze a static leader-follower game in which two competitive water resources supply chains derive their optimal decision strategies under different power structures, Zomorodian et al. (2017) who use a dynamic Nash bargaining approach, Xiao et al. (2016) who address the issue in a coalitional game through core-based and non-core-based solution concepts, Sechi and Zucca (2015) and Zheng et al. (2022) who use the Bankruptcy Games techniques.

In Europe, the Water Framework Directive (WFD) 2000/60/EC[1] recommends that the pricing politics in a river basin take into account the cost recovery and the economic sustainability of the water use. Sechi et al. (2013) present, in the framework of cooperative game theory, a methodology to allocate water service costs in a water resource system among different users that attempts to fulfill the WFD requirements. Sadegh et al. (2010) investigate the problem of optimal allocation of shared water resources in water transfer projects through a methodology based on crisp and fuzzy Shapley games.

In Erdlenbruch et al. (2014), a group of farmers overexploits a groundwater stock and causes excessive pollution. Specifically, the authors study a differential game in which (i) a water agency decides how to tax the use of water and fertilizers and how much to invest in policies to reduce pollution caused by the use of fertilizers, and (ii) companies compete with each other, after observing the choices of the social planner. Moreover, the payoff of the social planner and that of companies are assumed to be linear quadratic. Over-exploitation of groundwater resources is also investigated in Esteban et al. (2021). The possibility of regime shifts in freshwater ecosystems is included in the model when a critical water level is reached.

Rubio and Casino (2001) compare socially optimal and private extraction of a common property aquifer. Open-loop and feedback equilibria in linear strategies have been computed to characterize private extraction. The results show that strategic behavior increases the over-exploitation of the aquifer compared to the open-loop solution.

de Frutos Cachorro et al. (2019, 2021) study groundwater management under a regime shock affecting water availability, using differential games. Water users have water demand quadratic functions. Players are symmetric in de Frutos Cachorro et al. (2019) while in de Frutos Cachorro et al. (2021) the different players correspond to different groundwater uses (irrigation or urban water supply) and have different demand functions and different discount rates. Cooperative and non-cooperative solutions are compared using linear strategies with respect to the water stock.

[1] http://eur-lex.europa.eu/resource.html?uri=cellar:5c835afb-2ec6-4577-bdf8-756d3d694eeb.0004.02/DOC_1&format=PDF.

In Provencher and Burt (1993), the rate of groundwater extraction under the common property arrangement is the outcome of a dynamic game played with feedback strategies. The analysis identifies risk externalities that arise when firms are risk-averse. Negri (1989) compares open-loop and feedback equilibria using a common property aquifer model finding two sources of dynamic inefficiency: a pumping cost externality and a "strategic externality" that arises from the competition among users to capture the groundwater reserves.

A topic of considerable practical interest which, as far as we know, has so far found inadequate space in the literature is the following: how to fix the price of water in the presence of users capable of regulating the demand for water as a result of the supplier's decision?

An interesting contribution in this direction is given by Kogan and Tapiero (2010). The authors consider a vertical supply chain consisting of a water provider and a consumer (municipality). The inherent conflicts over stocks and supply costs that emerge among the parties in the water supply chain are modeled as a zero-sum stochastic differential game. Kogan (2021) and Kogan et al. (2022) address the problem of dynamic interaction between two firms committing to provide water supply within a limited time horizon. In Kogan (2021) the author finds that competition does not necessarily reduce product scarcity compared to the monopolistic industry while a longer contract results in lower scarcity of the products. In Kogan et al. (2022), the authors compare the spot-market-based competitive supply model for water with a supply chain approach, in which a non-profit public entity encourages competition between private water providers within the framework of a regulated, time-invariant price. They find that the public-private partnership can have an advantage in the form of both higher consumption and higher consumer welfare.

In the European Union, water supply services are referred to as 'services of general interest', meaning that they are subject to multiple, potentially conflicting, public service obligations. Martins et al. (2013) consider empirical data for Portuguese municipalities and provide a comprehensive approach to evaluate whether the concerns of universal access to water services for basic needs, affordability, and equity are embodied in the corresponding water supply block's tariff. In Brill et al. (1997), efficient water pricing schemes are introduced for non-profit water agencies, where members have property rights based on historical usage. The authors analyze three policy options of water agencies to reduce water supply: average cost pricing with the administration of quota allocation; block rate pricing; a transferable water rights regime. Fridman (2015) compares alternative transition paths to efficient water pricing. The analysis is based on the representative agent model, where two sources of water supply exist: exhaustible groundwater stock and a renewable substitute. Two alternative water pricing reforms are considered: gradual tariff increase and block pricing. The results of comparative analysis prove that under the same reform time horizon block pricing is preferred to the gradual tariff increase. Elnaboulsi (2001) determines the optimal nonlinear pricing rules for water services. The optimization process of a welfare utility function subject to different kinds of constraints provides the optimal pricing rules for water. These prices reflect efficiently the costs of systems constraints, the cyclicity of demands, the time-of-use, heterogeneity types of consumers, and the real value of water resources scarcity. An interesting review on the applications of game theory to water management is given by Dinar and Hogarth (2015).

In the context of competitive games that model the withdrawal of water by a multiplicity of users, the Nash equilibria of the game are normally studied in the literature if the players are only the users. The case is different when, in addition to the users, a player is also a supplier (typically an agency) who makes some decisions (such as price, taxation, etc.) that the users observe before in turn deciding their strategies. In this case, it is natural to look for Stackelberg equilibria by assuming the supplier is the leader and the users are followers. See, for example, Erdlenbruch et al. (2014), Du et al. (2018), Biancardi et al. (2022a), Biancardi et al. (2022b) and Biancardi et al. (2023). Moreover in Kogan et al. (2022) the authors say "Another important variation that requires investigation is to assume that the state agent is no longer a non-profit entity and acts instead as a Stackelberg leader who sets the water price optimally at the initial period under duopoly competition."

In this paper, we consider an agency (public or private) that sells and distributes water to a group of users (domestic, agricultural, industrial) of a given territory. The seller has several goals: to maximize revenue, minimize costs, take into account the well-being of users, and preserve the necessary availability of the resource for the future. The seller determines the price to apply to each user. It can vary over time and depends on the availability of water. Users adjust their demand for water in response to its price and their current (possibly seasonal) needs.

We address the following research questions: what is the water demand of different users in response to the tariffs applied; is the block pricing scheme preferable to the linear one? We characterize, through the Hamilton-Jacobi-Bellman (HJB) equations, an interior leader-follower feedback equilibrium of the game (Stackelberg equilibrium). Furthermore, supported by a series of numerical simulations, we analyze the interesting properties of this solution. Finally, the problem of the possible depletion of the water resource and the impossibility of fully satisfying the water demand is discussed. In this case, it is also necessary to study corner solutions. The latter analysis is limited to an essentially static version of the game.

To the best of our knowledge, this is the first article that explicitly addresses the problem of optimal water pricing by a "socially aware" seller towards an audience of different types of buyers and where the water demand is determined by buyers who react strategically to the prices set by the seller. All the papers cited previously, in fact, focus on different problems and essentially on the best allocation of groundwater or desalinated water owned by buyers (generally farmers).

The rest of the paper is organized as follows. Section 2 presents the model. In Sect. 3, two different tariff schemes are discussed. In Sect. 4, an interior feedback equilibrium of the game is characterized using the HJB equations. In Sect. 5, numerical simulations are provided and some properties of the solution and policy implications are discussed. In Sect. 6, corner solutions are examined, but the analysis is restricted to an essentially static version of the game. Section 7 concludes. All proofs are provided in the appendix.

2 The model

Consider an agency that sells water from one or more aquifers to customers which can be farms that use it for irrigation of farmland, industrial companies or private individuals that use it for domestic needs. Unlike what is usually done in groundwater management models, here the users do not observe the water reserve available to the seller. They purchase water based on their needs, which can be seasonal (this is plausible both for irrigation of agricultural land and for domestic use: there is greater consumption in the hottest periods of the year and with less rainfall). Users also adjust their water purchase volume based on price—the higher the price, the less they buy. The available water reserve depends on consumption and natural recharge which depends on the flow rate of the tributaries which, in turn, depends on rainfall and also has a seasonal pattern. The costs incurred by the seller are of two types:

fixed costs and costs that may depend, as in the case of groundwater management models, on the available water reserve.

Water sales contracts usually have a fixed term, often one year. For this, we consider a finite time horizon equal to one year. We assume that there are n buyers, $n \in \mathbb{N}$; $n \geq 1$. Let:

- $S(t)$ be the stock of the aquifer at time t, $0 \leq S(t) \leq S_{max}$; $S_{max} > 0$ is the capacity of the aquifer.
- $C(S(t)) = c_0 + c_1 (S_{max} - S(t))$ be the pumping cost of water per unit of volume at time t, $c_0 > 0$, $c_1 \geq 0$. The unitary cost is c_0 if the aquifer is completely full and it increases as the water stock decreases reaching the maximum $c_0 + c_1 S_{max}$ when the aquifer is empty.
- $D_i(t)$ be the amount of water demanded by user $i = 1, ..., n$ at time t, $D_i(t) \geq 0$.
- $W_i(t)$ be the amount of water supplied to the user $i = 1, ..., n$ at time t, $0 \leq W_i(t) \leq D_i(t)$.
- $R(t)$ be the natural recharge of the aquifer at time t, $R(t) \geq 0$. $R(\cdot)$ is a continuous function.

User i pays to the agency the amount

$$Z_i(t, W_i) \geq 0,$$

to buy the quantity of water W_i at the time $t \in [0, 1]$. The function Z_i is assumed to be continuous with respect to t and W_i and increasing in W_i.

The net income of the seller agency at time t is

$$\pi(t) = \sum_{i=1}^{n} \left[Z_i(t, W_i(t)) - C(S(t)) W_i(t) \right]. \tag{1}$$

The buyer i gets a utility ψ_i from the water purchase. The utility function ψ_i must be increasing with respect to W_i: the more water the user i has available, the greater his well-being; however, it is reasonable to assume that marginal utility is decreasing instead. We will therefore assume that ψ_i is a strictly increasing and concave function with respect to W_i. To obtain partially explicit results and/or implement numerical simulations, it is appropriate to use an explicit form for the ψ_i functions. We will use quadratic utility functions, in agreement with a significant part of the literature in this field. See e. g. Rubio and Casino (2001), Roseta-Palma (2003), Erdlenbruch et al. (2014), de Frutos Cachorro et al. (2019, 2021), Esteban et al. (2021), Biancardi et al. (2021, 2022a, b, 2023).

We assume that

$$\psi_i(t, W_i(t)) := \phi_i(t) (\alpha_i - \beta_i W_i(t)) W_i(t);$$

where $\phi_i(t) > 0 \; \forall t \in [0, 1]$, $\alpha_i > 0$, $\beta_i > 0$.

The parameter $\phi_i(t)$ in the utility function ψ_i emphasizes the common notion that the need for water is greater at some seasons of the year than in others. This is certainly true for agricultural and domestic users; it may also be true for some industrial users. For these reasons, the parameter $\phi_i(t)$ can be interpreted as the seasonal factor of the buyer's utility. Furthermore, considering that the concavity of the utility function represents decreasing marginal utility, we can say that the larger β_i is, the more accentuated this "demand saturation" effect is.

The payoff of the buyer i at time t is

$$\pi_i(t) = \psi_i(t, W_i(t)) - Z_i(t, W_i(t)); \quad i = 1, ..., n. \tag{2}$$

The dynamics of the water stock is given by

$$\frac{dS(t)}{dt} = R(t) - W(t); \quad S(0) = S_0 > 0, \tag{3}$$

where

$$W(t) = \sum_{i=1}^{n} W_i(t),$$

and

$$0 \leq S(t) \leq S_{max}; \quad \forall t \in [0, 1]. \tag{4}$$

The game is played as follows:

1. The agency discloses the water tariff regimes $Z_i(t, \cdot)$, $i = 1, ..., n$.
2. The buyer, having observed the tariffs for all buyers but not $S(t)$, determines the quantity of water $D_i(t)$ to request, maximizing the present value of his net income.
3. The agency provides the buyer with the amount of water $W_i(t) \leq D_i(t)$.

The agency makes its choices to maximize the present value of its overall net income. It can also take into account the welfare of the buyers (which is reasonable when the seller is a public agency). Since the time horizon of the model is short, we neglect any discount factors.

3 Tariff schemes and water demands

The seller chooses from several pricing schemes. There is substantial literature on the subject and a vast typology of schemes adopted in practice. The simplest is to charge each user a fixed price per unit of volume purchased. However, to discourage the excessive and sometimes inappropriate use of a scarce commodity such as water, tariff schemes in which the user pays a marginal price that increases with the volume of water consumed have been proposed. The best known of these schemes is the so-called block tariffs: the unit price of water is established for blocks of volumes of water purchased and increases as consumption increases. Finally, the tariff scheme can be different according to the type of user (agricultural, industrial, private) (*price discrimination*).

Assume that, for any $i = 1, ..., n$ and $t \in [0, 1]$, the function $Z_i(t, x)$ is strictly increasing with respect to x and $Z_i(t, 0) = 0$.

First-order conditions applied to the payoff (2) give

$$\phi_i(t)(\alpha_i - 2\beta_i D_i(t)) - \frac{\partial Z_i(t, D_i(t))}{\partial D_i} = 0.$$

3.1 Pricing schemes

1. *Linear pricing scheme*. If water scarcity is not taken into account, the most reasonable choice seems to be to charge the same price for each unit volume of water purchased. In this case

$$Z_i(t, D_i(t)) := p_i(t)D_i(t); \quad i = 1, ..., n,$$

where $p_i(t) > 0$ is the unitary price paid by the i-th user. The optimal demand $\hat{D}_i(t)$ is

$$\hat{D}_i(t) = \frac{\alpha_i \phi_i(t) - p_i(t)}{2\beta_i \phi_i(t)}.$$

Note that, as expected, $\hat{D}_i$ decreases as the unitary price $p_i(t)$ increases.

2. *Block tariffs*. Block tariffs are enforced to discourage the overuse of a scarce resource such as water. Let us consider the simplest case with two blocks. At any time t, there is a threshold $B_i(t)$ such that the buyer i pays a unit price $p_{i,1}(t)$ for each unit of water below the threshold $B_i(t)$ while paying a higher unit price $p_{i,2}(t)$ for water consumption above the threshold $B_i(t)$. The $B_i(\cdot)$ are established by the public authority and therefore represent given exogenous parameters.[2]

$$Z_i(t, D_i(t)) := \begin{cases} p_{i,1}(t)D_i(t) & \text{if } D_i(t) \leq B_i(t); \\ p_{i,1}(t)B_i(t) + p_{i,2}(t)\,(D_i(t) - B_i(t)) & \text{if } D_i(t) > B_i(t); \end{cases}$$

where $p_{i,1}(t) \leq p_{i,2}(t)$ are the unit price paid by the i-th user for blocks 1 and 2, respectively.
The optimal demand is:

$$\hat{D}_i(t) = \begin{cases} \dfrac{\alpha_i \phi_i(t) - p_{i,2}(t)}{2\beta_i \phi_i(t)} > B_i(t), & \text{if } p_{i,1}(t) \leq p_{i,2}(t) < \sigma_i(B_i(t)); \\ B_i(t), & \text{if } p_{i,1}(t) \leq \sigma_i(B_i(t)) \leq p_{i,2}(t); \\ \dfrac{\alpha_i \phi_i(t) - p_{i,1}(t)}{2\beta_i \phi_i(t)} < B_i(t), & \text{if } \sigma_i(B_i(t)) < p_{i,1}(t) \leq p_{i,2}(t); \end{cases}$$

where

$$\sigma_i(B_i(t)) := \phi_i(t)(\alpha_i - 2\beta_i B_i(t)).$$

Remark 1 . Note that

$$p_{i,h}(t) < \sigma_i(B_i(t)) \iff B_i(t) < \frac{\alpha_i \phi_i(t) - p_{i,h}(t)}{2\beta_i \phi_i(t)}; \quad h = 1, 2.$$

Given $p_{i,1}(t)$ and $p_{i,2}(t)$ we have that:

- if the block threshold $B_i(t)$ is large enough, then it is optimal to buy a quantity of water $\hat{D}_i(t) < B_i(t)$ at the lowest price $p_{i,1}(t)$ and the block tariff provides exactly the same solution as the linear fare scheme;
- if the block threshold $B_i(t)$ is small enough, then it is optimal to buy a quantity of water $\hat{D}_i(t) > B_i(t)$ by paying the quantity $B_i(t)$ at the lower price and the excess quantity at the higher price; moreover, the demand is lower than in the case of the linear price;
- if the block threshold $B_i(t)$ is neither too high nor too low, then it is optimal to buy a quantity of water equal to $B_i(t)$ at the cheapest price $p_{i,1}(t)$ and the block tariff reduces the water demand compared to the linear price case.

[2] It is certainly realistic to have different tariffs for different types of users. For example, in the document https://www.arera.it/allegati/docs/17/251-17.pdf from the (Italian) Authority for electricity, gas and the water system (ARERA) we read that: “The water tariff system is divided into user groups: the tariff is differentiated according to the uses of the resource (for example, domestic, industrial, agricultural, public, etc.).”

4 The feedback Stackelberg equilibrium

In this section we characterize the feedback Stackelberg equilibrium, assuming that the agency is the leader. We are looking for interior solutions which mean, among other things, that the water basin never empties during the time horizon. A discussion of corner solutions will take place in a later section. Therefore here we set

$$W_i(t) = D_i(t); \;\; t \in [0, 1]; \quad i = 1, ..., n.$$

The game is a leader-follower game. The agency determines its tariff schemes $Z_i(t, W_i(t))$ maximizing its overall payoff J which is given by

$$J(Z_1, ..., Z_n) = J_A + \rho \sum_{i=1}^{n} J_{B_i} + \mu(S(1)),$$

where

$$J_A = \int_0^1 \pi(t)dt, \qquad J_{B_i} = \int_0^1 \pi_i(t)dt,$$

subject to the state variable constraints (3) and (4). π and π_i; $i = 1, ..., n$ are defined in (1) and (2), respectively.

The J payoff of the agency is the sum of three contributions: the first addendum is given by the current value of the agency's profits (revenue minus costs), the second addendum takes into account the overall payoff of all users and is due to the social role that a public agency plays, the third addendum (*scrap value*) takes into account the value for the agency of the water stock at the end of the time horizon considered. The coefficient $\rho \in [0, 1]$ weighs the social role of the agency. In particular, $\rho = 0$ corresponds to the case of a fully private agency with no social obligations. We assume that the scrap value $\mu(\cdot)$ is a quadratic function:

$$\mu(x) = \mu_1 x^2 + \mu_2 x; \quad \mu_1 \geq 0; \;\; \mu_2 \geq 0; \;\; \mu_1 + \mu_2 > 0.$$

We solve the game by backward induction. At the second stage, a buyer i, having observed the price scheme $Z_i(t, \cdot)$ determines the quantity of water $W_i(t)$ to demand.

At the first stage, the seller, foreseeing the buyer's response, solves the following optimal control problem

$$\max_{Z_1,...,Z_n} J(Z_1, ..., Z_n)$$

subject to the state variable constraints (3) and (4).

The special tariff schemes introduced in Sect. 3 must be analyzed separately.

4.1 Linear pricing schemes

The agency, foreseeing the buyers' response, calculates its optimal pricing strategy by choosing water unit prices as a function of time t and the observed state of the stock S:

$$p_i = p_i(t, S); \;\; i = 1, ..., n.$$

The following proposition holds:

Proposition 1 *Assuming interior equilibria, there exists a unique feedback Stackelberg equilibrium where the optimal pricing strategies of the (leader) agency are, for any* $i = 1, ..., n$,

$$p_i^*(t, S) = \frac{[\phi_i(t)(1-\rho)\alpha_i + c_0 + c_1 S_{max} + y(t)] + [2x(t) - c_1]\, S}{2-\rho}; \tag{5}$$

where the functions $x(\cdot)$ *and* $y(\cdot)$ *are the solutions of the following system of ordinary differential equations*

$$\dot{x} = -\gamma(t)\frac{(2x - c_1)^2}{4(2-\rho)};$$

$$\dot{y} = a(t, x)y + b(t, x);$$

and

$$a(t, x) := -\gamma(t)\frac{(2x - c_1)}{2(2-\rho)};$$

$$b(t, x) := -2R(t)x + (\delta - \gamma(t)\,(c_0 + c_1 S_{max}))\,\frac{(2x - c_1)}{2(2-\rho)};$$

$$\gamma(t) := \sum_{j=1}^{n} \frac{1}{\beta_j \phi_j(t)}; \quad \delta := \sum_{j=1}^{n} \frac{\alpha_j}{\beta_j}.$$

The terminal conditions are

$$x(1) = \mu_1; \quad y(1) = \mu_2.$$

Moreover, for any buyer $i = 1, ..., n$, *the optimal quantity of water demanded (and supplied) is*

$$W_i^*(t) = \frac{\alpha_i \phi_i(t) - p_i^*(t, S(t))}{2\beta_i \phi_i(t)}.$$

Proof See Appendix. □

Remark 2 The solution of the equation $\dot{x} = -\gamma(t)\frac{(2x - c_1)^2}{4(2-\rho)}$ with terminal condition $x(1) = \mu_1$ is

$$x(t) = \begin{cases} \dfrac{c_1}{2} + \left[\dfrac{2}{2\mu_1 - c_1} - \dfrac{\Gamma(t)}{2-\rho}\right]^{-1}, & \text{if } \mu_1 \neq \dfrac{c_1}{2}; \\ \mu_1, & \text{if } \mu_1 = \dfrac{c_1}{2}; \end{cases} \tag{6}$$

where

$$\Gamma(t) := \sum_{j=1}^{n} \frac{1}{\beta_j} \int_t^1 \frac{1}{\phi_j(s)} ds.$$

Note that the solution is bounded on interval [0, 1] if $\mu_1 < \frac{c_1}{2} + \frac{2-\rho}{\Gamma(0)}$. Moreover, if $\mu_1 = \frac{c_1}{2}$ then $p_i^*(t, S) = \frac{[\phi_i(t)(1-\rho)\alpha_i + c_0 + c_1 S_{max} + y(t)]}{2-\rho}$ is independent of S.

The optimal state variable $S^*(t)$ is the solution of the linear differential equation

$$\dot{S}^*(t) = \begin{cases} R(t) - \dfrac{\delta}{2(2-\rho)} + \gamma(t)\dfrac{c_0 + c_1 S_{max} + y(t)}{2(2-\rho)} + \chi(t)S^*(t), & \text{if } \mu_1 \neq \dfrac{c_1}{2}; \\ R(t) - \dfrac{\delta}{2(2-\rho)} + \gamma(t)\dfrac{c_0 + c_1 S_{max} + y(t)}{2(2-\rho)}, & \text{if } \mu_1 = \dfrac{c_1}{2}; \end{cases} \quad (7)$$

where

$$\chi(t) := \frac{\gamma(t)}{2-\rho}\left[\frac{2}{2\mu_1 - c_1} - \frac{\Gamma(t)}{2-\rho}\right]^{-1}.$$

4.2 Block tariffs

Block rates are a generalization of linear rates. The latter is a particular case of the block rates if the two prices (of the first and second block) coincide. The following proposition shows that it is never optimal to choose two different prices for the two blocks and therefore the seller adopts a linear price scheme.

Proposition 2 *Assume block tariffs and that interior feedback Stackelberg equilibria exist. Let*

$$\left(p^*_{i,1}(t,S),\, p^*_{i,2}(t,S)\right); \quad i = 1, ..., n$$

be an interior equilibrium. Let $t \in [0,1]$ *and*

$$E_1(t) = \left\{ i \in \{1, ..., n\} \,|\, p^*_{i,1}(t,S) < \sigma_i(B_i(t)) \right\};$$

$$E_2(t) = \left\{ i \in \{1, ..., n\} \,|\, p^*_{i,1}(t,S) = \sigma_i(B_i(t)) \right\};$$

$$E_3(t) = \left\{ i \in \{1, ..., n\} \,|\, p^*_{i,1}(t,S) > \sigma_i(B_i(t)) \right\}.$$

1. *If* $i \in E_1(t)$*, then* $p^*_{i,2}(t,S) = p^*_{i,1}(t,S)$ *and* $W^*_i(t) > B_i(t)$*; the demand exceeds the threshold but all the water demanded is sold at the same price.*
2. *If* $i \in E_2(t)$*, then* $p^*_{i,2}(t,S)$ *is any price greater or equal to* $p^*_{i,1}(t,S)$ *and* $W^*_i(t) = B_i(t)$*; the demand equals the threshold, all the water demanded is sold at the same price* $p^*_{i,1}(t,S)$ *and the price* $p^*_{i,2}(t,S)$ *does not matter.*
3. *If* $i \in E_3(t)$*, then then* $p^*_{i,2}(t,S)$ *is any price greater or equal to* $p^*_{i,1}(t,S)$ *and* $W^*_i(t) < B_i(t)$*; the demand is below the threshold, all the water demanded is sold at the same price* $p^*_{i,1}(t,S)$ *and the price* $p^*_{i,2}(t,S)$ *does not matter.*

Proof See Appendix. □

Remark 3 For simplicity, let us call the price applied to the quantity of water below the threshold "basic price" and the price applied to the quantity of water above the threshold "excess price". In essence, in the case of block tariffs, either it is optimal to set a high basic price that corresponds to a demand lower than the threshold (in this case any excess price does not intervene at all and has no importance), or it is optimal to set a low basic price which corresponds to a demand above the threshold (in this case the optimal excess price

Table 1 Model parameters

Parameter	Description
S_0	Initial level of the aquifer
S_{max}	Maximum capacity
c_0	Fixed pumping cost
c_1	Marginal pumping cost
k_r	Average value of the Natural Recharge
A_r	Amplitude of Natural Recharge cycle
φ_r	Phase of Recharge cycle
α_i	Linear coefficient of buyers utility
β_i	Quadratic coefficient of buyers utility
φ_i	Phase of ϕ_i cycle
k_i	Average value of the seasonal factor of buyer's utility
A_i	Amplitude of buyer preferences' cycle
μ_1	Quadratic term of the scrap value
μ_2	Linear term of the scrap value
ρ	Social role of the agency

coincides with the basic price). In summary, the optimal solution in the case of block tariffs substantially coincides with the optimal solution in the case of linear tariffs.

5 Numerical results

In this section, we propose several numerical exercises to discuss the implications of the analytical results obtained above. To this end, we assume, without loss of generality, the presence in the market of only two buyers ($n = 2$).

To obtain the optimal pricing strategies, we numerically calculate the functions x, y and the state variable S (see Prop. 1). Note that x is the solution of a Riccati equation that can be solved analytically by (6). After calculating the function x, the function y is the solution of a linear differential equation that can be easily integrated numerically. The optimal state variable S is also the solution of a linear differential equation (7). From x, y and S, the optimal pricing strategies are given by formula (5).

Furthermore, both rainfall and consumption by agricultural or domestic users have an essentially seasonal temporal trend. Therefore, we have specified both the natural recharge function $R(t)$ and the parameters $\phi_i(t)$ of the functions $\psi_i(t)$ as follows:

$$R(t) = k_r + A_r \sin(2\pi\, t + \varphi_r), \tag{8}$$

$$\phi_i(t) = k_i + A_i \sin(2\pi\, t + \varphi_i), \qquad \text{with } \ i = 1, 2. \tag{9}$$

where $k_r, k_i > 0$, $A_r, A_i \geq 0$ and $-\pi \leq \varphi_r, \varphi_i \leq \pi$.[3] In order to make the following numerical exercises more understandable, we summarize in Table 1 the set of model parameters in the case where all the functions are specified.

In all the numerical exercises we perform in this section, we adopt the following parameters configuration: $S_0 = 100$, $S_{max} = 500$, $c_0 = 10$, $c_1 = 0.1$, $k_r = 30$, $A_r = 15$, $\varphi_r = 0$, $\alpha_i =$

[3] The functions defined in (8) and (9) have the same period, $T = 1$.

$10, k_i = 20$.[4] Recalling the functions defined in (8) and (9), we can notice that both the natural recharge of the basin and buyers' preferences are driven by seasonal cycles, and these cycles reflect on the dynamics of state and control variables. Having assigned to the natural recharge $R(t)$ the seasonal cycles determined by $k_r = 30$ and $A_r = 15$, two degrees of heterogeneity can be discussed in the model: the first concerning the possible difference in the demand saturation (defined by β_i) that buyers experience toward the resource, and the second related to the possible heterogeneous seasonal factors in buyers' utilities (defined by φ_i and A_i).

5.1 The effect of water demand saturation

A first numerical investigation can be made on the *demand saturation effect*. Assuming the demand saturation effect of buyer 2 as fixed, we analyze what happens to the optimal price and demand levels for buyer 1 as β_1 varies,[5] and in addition we observe the dynamics of the water stock in the basin. Specifically, as shown in Fig. 1, while an increase in β_1 has no effects on the optimal price dynamics[6], it drastically affects both the dynamics of optimal demand $W_1^*(t)$ and the evolution of $S^*(t)$. Recalling that the higher the β_1 the less the amount of water that saturates the needs of buyer 1 (the same would be true for buyer 2), we note that as β_1 increases, the dynamics of optimal demand appears increasingly squeezed to low values (see Panel (b) in Fig. 1). Panel (c) in Fig. 1 then allows to observe how as β_1 increases, the optimal demand $W_1^*(t)$ exhibits increasingly lower values than $W_2^*(t)$. Concerning the dynamics of the water stock in the basin, an increase in β_1 induces that $S^*(t)$ expresses higher levels. Moreover, both the natural recharge and the reduced amount of water purchased by the buyer 1 contribute to transforming multimodal basin dynamics to monotonically increasing ones (see Panel (d) in Fig. 1). From an economic point of view, an example of the scenario with $\beta_1 > \beta_2$ can be represented by the coexistence of two companies that produce two different types of food goods (e.g., Christmas cakes and pasta). Both the production activities have seasonal water needs, but while the former will constantly saturate its need with a lower level of water the latter will constantly have more need for water (lower β).

5.2 The effect of seasonal factors

Even the seasonal factors of buyers' preferences are prone to dictate different dynamics for the optimal price and the optimal demand, and then for the water stock in the basin. Specifically, we can observe how the dynamics of optimal values of the model changes when (i) A_i or (ii) φ_i vary. In case (i), by setting $A_2 = 10$, we can notice that starting from the case $A_1 = 0$ (i.e., no seasonal cycle in water preferences for buyer 1) in which the optimal demand $W_1^*(t)$ is constant over time, we have that as A_1 increases, the optimal dynamics can significantly change. In particular, although the optimal price $p_1^*(t)$ negligibly increases as A_1 changes (see Panel (a) in Fig. 2), an increase in A_1 makes the optimal demand's peaks

4 The parameters that have not been assigned a value are the parameters against which the numerical simulations will be performed.

5 In Panels of Fig. 1, for a higher readability of the graphs (but without loss of generality of the results), only a few (but representative) number of numerical simulations are shown. The same approach is applied also in the successive figures.

6 As highlighted by Equation (5), the optimal prices are not affected by β_i with $i = 1, 2$. Therefore, the dynamics of p_i^* are not affected by variations in β_i and p_1^* and p_2^* are identical if the buyers differ only for β_i.

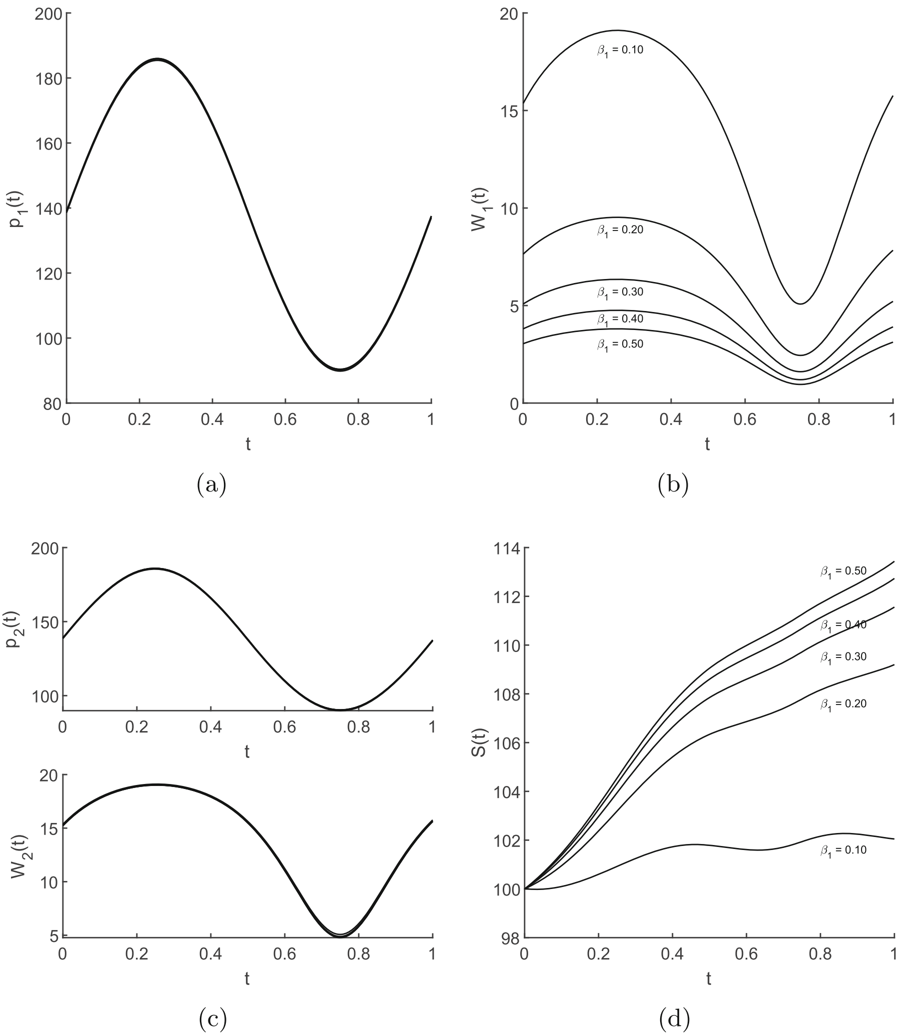

Fig. 1 Parameter set: $A_1 = A_2 = 10, \varphi_1 = \varphi_2 = 0, \rho = 0.1, \mu_1 = 0.1, \mu_2 = 10, \beta_2 = 0.1$. Panels **a**, **b**, **c** show, respectively, the dynamics of $p_1^*(t)$, $W_1^*(t)$, $S^*(t)$ as β_1 varies. Differently, Panel **c** displays the dynamics of $p_2^*(t)$ and $W_2^*(t)$ exclusively for $\beta_2 = 0.1$

more suddenly. Consequently, the stock dynamics $S^*(t)$ becomes increasingly multimodal (for a given optimal demand $W_2^*(t)$ as shown in Panel (c) of Fig. 2). The latter phenomenon can be directly observed in Panel (d) of Fig. 2.

In case (ii), by setting $\varphi_2 = 0$, we explore how the optimal dynamics vary as the phase of the cycles in buyer 1's seasonal preference changes. Specifically, we note that as φ_1 increases, the cycles related to the dynamics of optimal price and optimal demand of buyer 1 shift more and more to the left (see Panels (a) and (b) in Fig. 3), inducing the loss of multimodality in the stock dynamics $S^*(t)$ and a gradual rise in the positive peak of available water that the basin experiences immediately after the negative peak of optimal demand $W_1^*(t)$ is reached (see

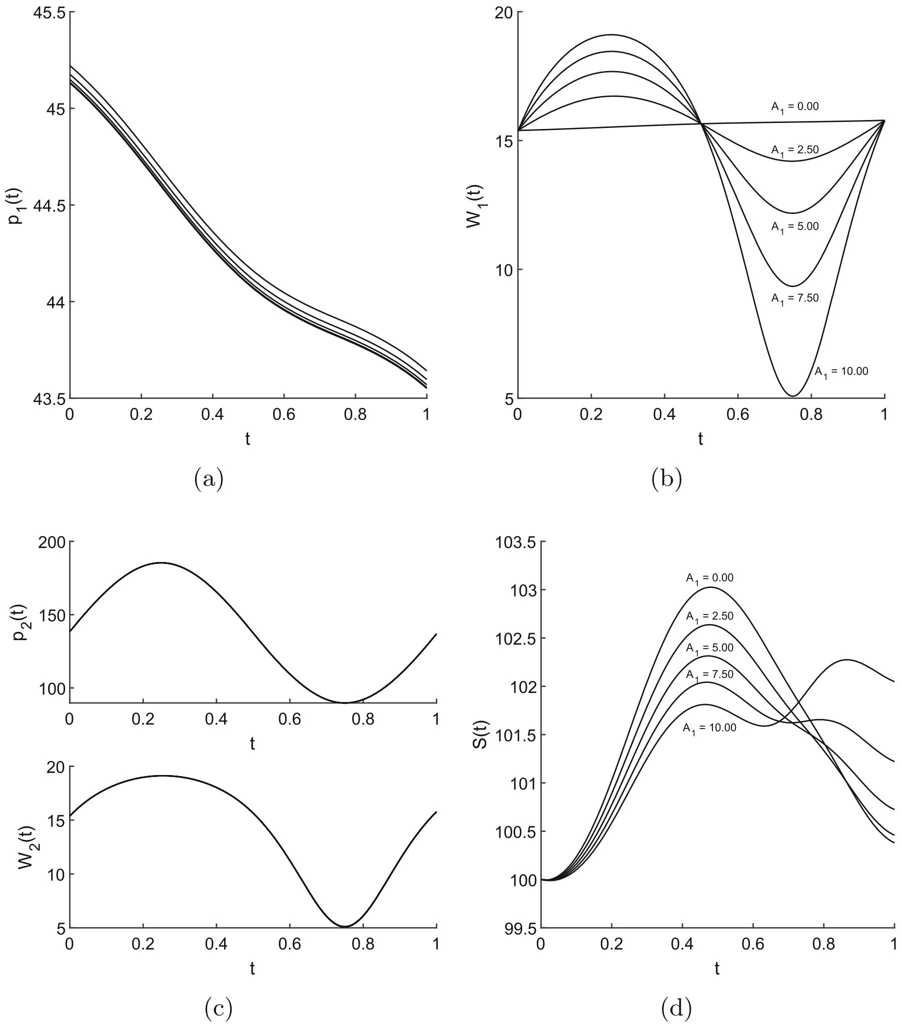

Fig. 2 Parameter set: $\beta_1 = \beta_2 = 0.1, \varphi_1 = \varphi_2 = 0, A_2 = 10, \rho = 0.1, \mu_1 = 0.1, \mu_2 = 10$. Panels **a**, **b**, **d** show, respectively, the dynamics of $p_1^*(t)$, W_1^* and $S^*(t)$ as A_1 varies. Differently, Panel **c** displays the dynamics of $p_2^*(t)$ and $W_2^*(t)$ exclusively for $A_2 = 10$

Panel (d) in Fig. 3).[7] This phenomenon becomes extreme when $\varphi_1 = 3.14$, and the resultant sequence of cycles in the optimal demand is reversed. In such a case, the interplay between an initial saving of water (buyer 1 initially reduces its water demand) and natural recharge ends up producing an even greater peak in water availability that will then be gradually eroded by the increasing demand for water made by buyer 1 in the second half of period.

[7] This scenario occurs for the given dynamics of $W_2^*(t)$ described by Panel (c) in Fig. 3.

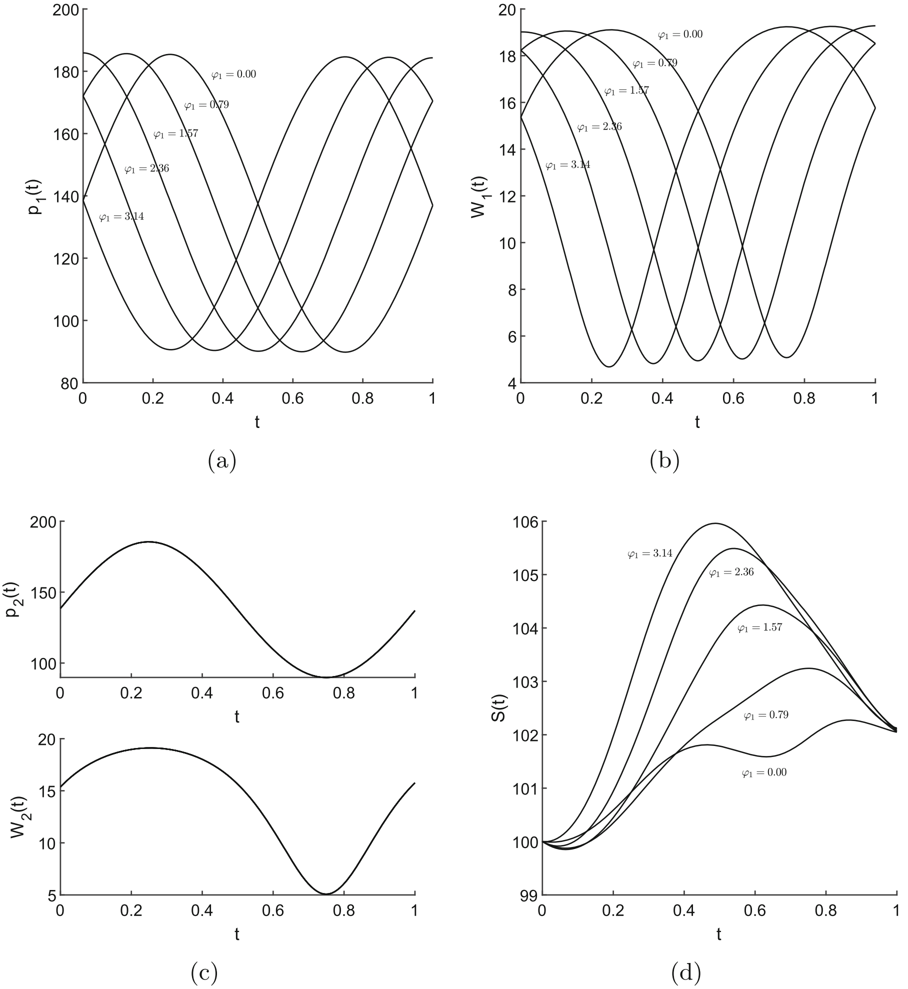

Fig. 3 Parameter set: $\beta_1 = \beta_2 = 0.1, \varphi_2 = 0, A_1 = A_2 = 10, \rho = 0.1, \mu_1 = 0.1, \mu_2 = 10$. Panels **a**, **b**, **d** show, respectively, the dynamics of $p_1^*(t)$, W_1^* and $S^*(t)$ as φ_1 varies. Differently, Panel **c** displays the dynamics of $p_2^*(t)$ and $W_2^*(t)$ exclusively for $\varphi_2 = 0$

5.3 The effect of agency's social role

Assuming, without loss of generality, only the heterogeneity related to the seasonal factors, we can observe the role in the dynamics dictated by the parameter ρ, i.e., the degree of the agency's social involvement to water needs of buyers. The numerical exercises in Panels (a), (b) and (c) of Fig. 4 show that as ρ increases, optimal prices decrease, optimal demands increase, and the basin level decreases, respectively. This effect is due to the social feature of ρ. Indeed, if a higher level of ρ results in a higher agency interest in maximizing buyers' needs, this will be reflected in more affordable optimal pricing. It will therefore follow that buyers will choose a higher quantity of the resource to purchase, and as a result, regardless of the natural recharge capacity, the basin will be more eroded.

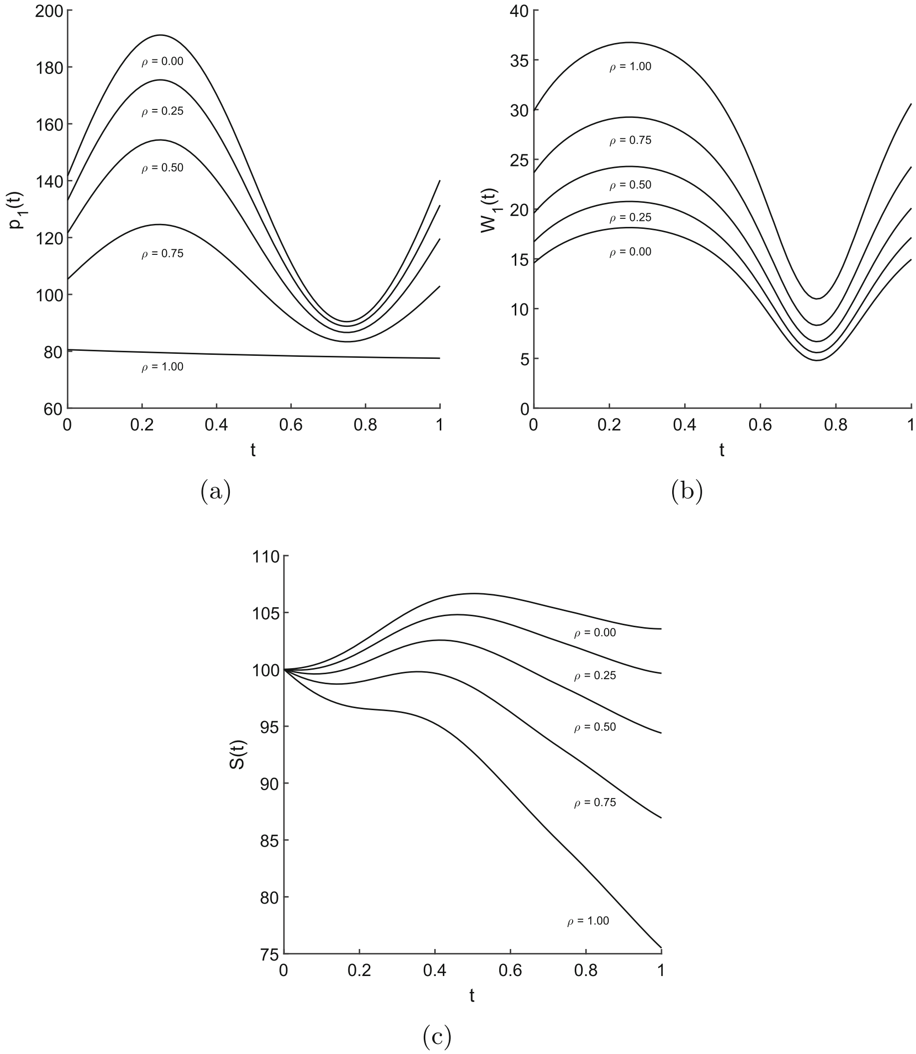

Fig. 4 Parameter set: $A_1 = A_2 = 10, \beta_1 = \beta_2 = 0.1, \varphi_1 = 0, \varphi_2 = \pi, \mu_1 = 0.1, \mu_2 = 10$. As ρ increases, **a** dynamics of $p_1^*(t)$ assume lower values (the same phenomenon occurs for $p_2^*(t)$); **b** dynamics of $W_1^*(t)$ assume higher values (the same phenomenon occurs for $W_2^*(t)$); **c** dynamics of $S(t)$ assume lower values

This phenomenon can be further observed by simulating the payoffs J_{B_1} and J_{B_2} of the two heterogeneous buyers as ρ varies. Indeed, we note that as ρ increases, the payoffs grow monotonically (see Panel (a) in Fig. 5). Differently, the simulations show that ρ plays an *ambiguous* role in the agency payoff J. Indeed, as shown in Panel (b) of Fig. 5, the agency's optimal payoff exhibits an inverted U-shaped behavior with respect to ρ and reaches its maximum point at an intermediate level ($\rho \simeq 0.3$) of social involvement. This result shows that a scenario in which an intermediate degree of altruism for buyers is assumed may be advantageous for the agency in terms of its payoff, compared to the extreme case $\rho = 0$ (e.g., no social obligations).

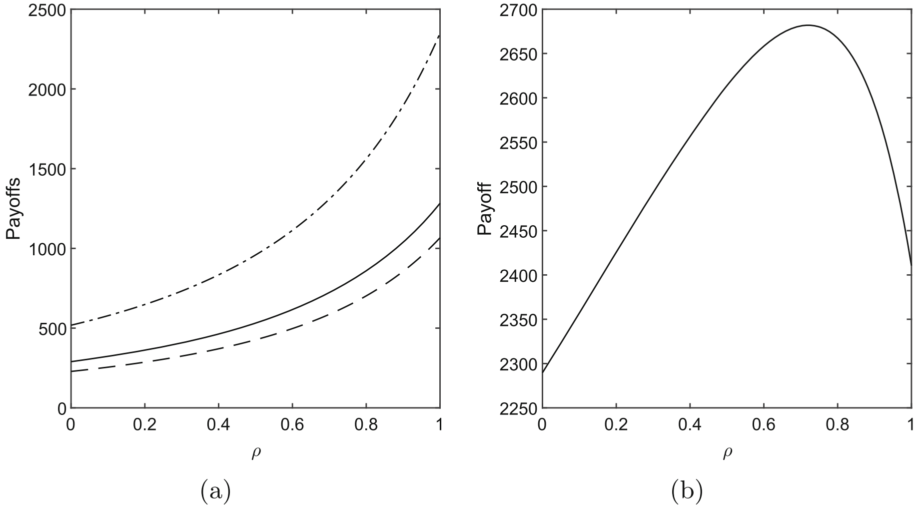

Fig. 5 Parameter set: $A_1 = 10, A_2 = 0, \beta_1 = \beta_2 = 0.1, \varphi_1 = 0, \varphi_2 = \pi, \mu_1 = 0.2, \mu_2 = 20$. **a** As ρ increases, the payoffs of the two buyers increase. The solid curve and the dashed one depict the payoffs J_{B_1} and J_{B_2}, respectively. The dash-dotted curve describes the sum of buyers' payoffs. **b** As ρ increases, the payoff of the agency shows an inverted U-shaped behavior

5.4 Linear vs. block tariffs

To complete the numerical analysis that highlights the analytical results obtained in the previous sections, we provide some numerical simulations on the sub-optimal dynamics generated by the employment of block tariffs. Indeed, in Fig. 6 we show the payoffs of the buyers (Panel (a)) and the payoff of the agency (Panel (b)) in the case of block tariffs. For simplicity, we set $B_1 = B_2$. Starting from the linear optimal price (i.e. basic and excess prices are the same, $p^*_{i,1} = p^*_{i,2}$), we calculated the optimal quantity of water demanded and the corresponding payoffs, setting $p_{i,2} = (1+\eta)p^*_{i,1}$, where $\eta \geq 0$. As η increases, the payoffs decrease up to η equal to approximately 30%. After this value the payoffs became flat. This indicates that the quantity of water demanded is below the threshold B_i and therefore only the basic price $p^*_{i,1}$ is applied. According to Proposition 3, block tariffs lead to lower payoffs than those obtained with linear pricing. Aiming also to provide insights on the buyer's perspective in the case of block tariffs, we show what happens to the optimal demand $W^*_i(t)$ as η varies. Recalling that as η increases the price $p^*_{i,2}$ grows, the consumption of water above the threshold consequently decreases. Indeed, the buyer's optimal demand will experience (net of the seasonal preferences towards the resource) longer phases in which it is given by threshold B. Panel (c) in Fig. 6 highlights this phenomenon for the buyer 1 (the same applies to buyer 2).

6 Corner solutions

In the previous sections, we have characterized and studied the solution of the differential game assuming that the solution is interior, i.e. that the water stock remains strictly positive and below the maximum capacity of the aquifer for the entire time interval. In general, it is difficult to identify cases in which the solution is actually interior. The problem should

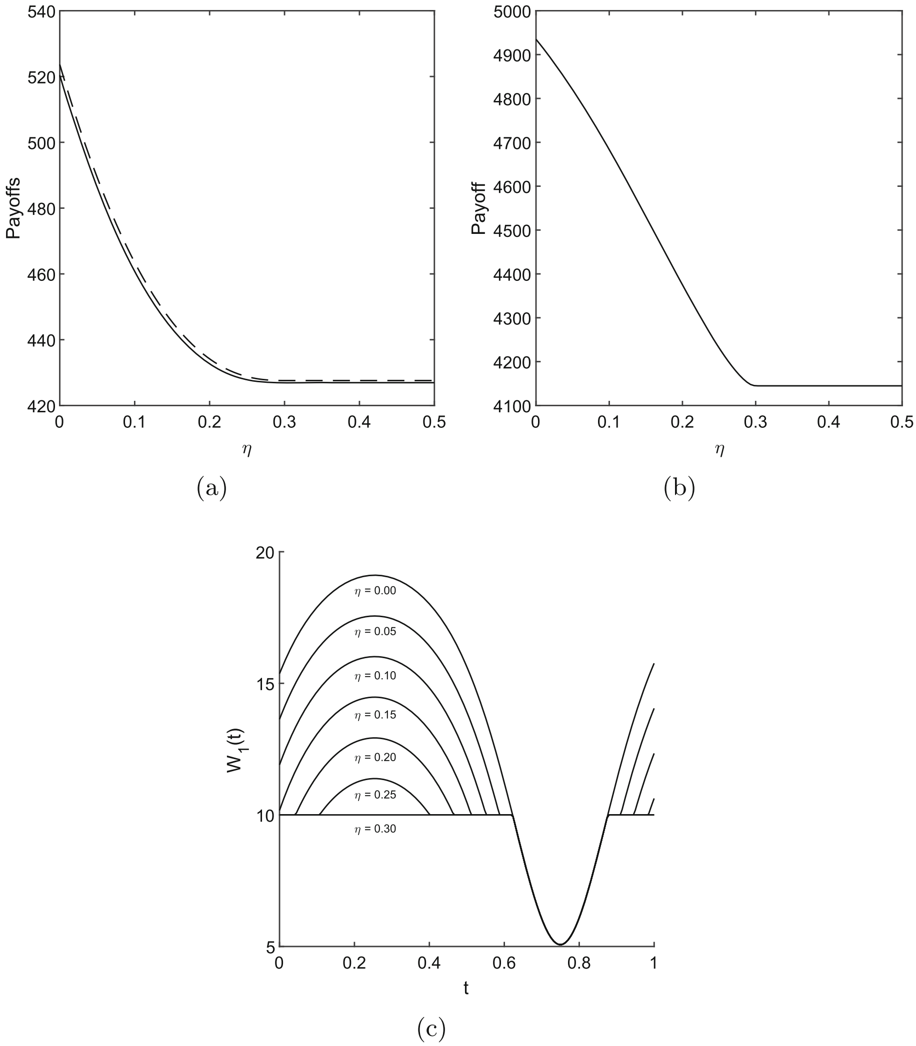

Fig. 6 Parameter set: $A_1 = A_2 = 10$, $\beta_1 = \beta_2 = 0.1$, $\varphi_1 = 0$, $\varphi_2 = \pi$, $\mu_1 = 0.1$, $\mu_2 = 10$, $B_1 = B_2 = 10$. **a** As η increases, the payoffs of the two buyers decrease. The solid curve and the dashed one depict the payoffs J_{B_1} and J_{B_2}, respectively. **b** As η increases, the payoff of the agency decreases. **c** Different optimal water demand dynamics $W_1^*(t)$ as η varies

be formulated as an optimal control problem with pure inequality constraints in the state variable. In this case, necessary and/or sufficient conditions are known in literature but are generally very difficult to apply (see eg Seierstad and Sydsæter 1987).

In this section, we simply address the problem of corner solutions by studying a static version of the proposed model. The purpose of this analysis is to establish the role of parameters with respect to the existence of corner solutions. From a practical point of view, the question is relevant as regards the possible scarcity of the water reserve compared to the needs due to, for example, too low rainfall.

Consider a two-stage game with a single buyer: in the first phase the agency, by predicting the buyer's demand for water, chooses the price. In the second stage, the buyer decides the amount of water to be consumed and the agency provides a quantity of water not exceeding the needs. We limit ourselves to the linear tariff scheme. Let

- $S_0 > 0$ and S_1 be the stocks of the aquifer at time $t = 0, 1$, respectively.
- S_{max} the capacity of the aquifer.
- $p \geq 0$ the sale price of water per unit of volume at time $t = 0$.
- $c = c_0 + c_1(S_{max} - S_0) \geq 0$ the pumping cost of water per unit of volume at time $t = 0$, where $c_0 > 0$ and $c_1 \geq 0$.
- $D = D(p) \geq 0$ the water demanded at time $t = 0$.
- $W \in [0, D]$ the water supplied at time $t = 0$.
- $R \geq 0$ the natural recharge of the aquifer at time $t = 1$.

The dynamics of the stock of the aquifer is given by the following equation:

$$S_1 = \min\{S_{max}, S_0 + R - W\}.$$

The net income of the seller is

$$\pi = (p - c)W.$$

The buyer derives a utility $\psi(D) = \phi[\alpha - \beta D]D;\ \ \phi > 0, \alpha > 0, \beta > 0$ from the demanded water.
The payoff for the buyer is

$$\pi_1 = \psi(D) - pD.$$

The agency set the price p and the supply W to maximize its payoff

$$J(p, W) = \pi + \rho\pi_1 + \mu(S_1), \quad 0 \leq \rho < 1,$$

The agency solves the following optimization problem

$$\max_{p \geq 0;\ 0 \leq W \leq D} J(p, W),$$

subject to the state constraint[8]

$$S_1 = \min\{S_{max}, S_0 + R - W\} \geq 0.$$

As we are mainly interested in the water shortage situation, we assume in the following that

$$R < S_{max} - S_0,$$

so that $S_0 + R - W < S_{max}$ and

$$S_1 = S_0 + R - W; \quad S_0 > 0.$$

We assume that $\mu(x) = kx, k > 0$. The buyer, having observed $p \geq 0$ but not S_0, determines the quantity of water D to consume, solving the following optimization problem

$$\max_{D \geq 0} \pi_1(D).$$

The model therefore consists of a Stackelberg game. Solving the game by backward induction, we have the following results.

[8] One way to make $S_1 \geq 0$ is to provide the user, in the case of excessive demand W, with a quantity of water $D = \delta W, 0 \leq \delta < 1$, such that $S_1 = 0$. However, this strategy is not necessarily optimal.

Proposition 3 *Let*

$$\sigma = \alpha\phi - k - c_0 - c_1 (S_{max} - S_0).$$
$$M_1 = \frac{2(2-\rho)\beta\phi S_0 - \alpha\phi + k + c_0 + c_1 (S_{max} - S_0)}{2\beta\phi(2-\rho)}; \quad M_2 = \frac{\alpha - 2\beta S_0}{2\beta}.$$
$$p_1 = \phi [\alpha - 2\beta(S_0 + R)]; \quad \hat{p} = \frac{k + c_0 + c_1 (S_{max} - S_0) + \alpha\phi(1-\rho)}{2-\rho}.$$

We distinguish two cases:

1. $\sigma > 0$. *In this case we have:*
   (a) *If* $0 \leq R \leq M_1$, *then* J *is decreasing as* $\tilde{p} \leq p \leq \alpha\phi$ *so that the optimal price is* $p^* = p_1$. *The quantity of water demanded and consumed is* $W^* = S_0 + R$. *The aquifer after consumption is empty, that is* $S_1 = 0$.
   (b) *If* $M_1 < R$, *then* J *is increasing in the interval* $\tilde{p} \leq p \leq \hat{p}$ *and decreasing for* $\hat{p} \leq p \leq \alpha\phi$. *The optimal price is* $p^* = \hat{p}$. *The quantity of water demanded and consumed is*
   $$W^* = \frac{\sigma}{2\beta\phi(2-\rho)} = \frac{\alpha\phi - k - c_0 - c_1 (S_{max} - S_0)}{2\beta\phi(2-\rho)} > 0.$$
   The volume of water that remains in the aquifer after consumption is $S_1 = S_0 + R - W^* > 0$.
2. $\sigma \leq 0$. *In this case we have:*
   (a) *If* $0 \leq R < M_2$, *then* $\tilde{p} = p_1$, J *is decreasing as* $\tilde{p} \leq p \leq \alpha\phi$ *so that the optimal price is* $p^* = p_1$. *The quantity of water demanded and consumed is* $W^* = S_0 + R$. *The aquifer after consumption is empty, that is* $S_1 = 0$.
   (b) *If* $M_2 \leq R$, *then* $\tilde{p} = 0$, J *is decreasing as* $\tilde{p} \leq p \leq \alpha\phi$ *so that the optimal price is* $p^* = 0$. *The quantity of water demanded and consumed is* $W^* = \frac{\alpha}{2\beta}$. *The volume of water that remains in the aquifer after consumption is* $S_1 = S_0 + R - \frac{\alpha}{2\beta} > 0$.

Proof See Appendix. □

Remark 4 Prop. 3 states that if the amount of rainfall (i.e. the recharge of the aquifer) is below a given threshold (M_1 when $\sigma > 0$ and $M_2 > M_1$ when $\sigma \leq 0$), then the aquifer will be empty at the end of the period (and therefore we have a corner solution). Thresholds $M_i,\ i = 1, 2$ depend on the initial water stock and the other parameters of the problem.

$$M_1 = M_1(S_0; S_{max}, \alpha, \beta, \phi, k, \rho, c_0, c_1); \quad M_1 = M_2(S_0; \alpha, \beta).$$

1. Let $\sigma > 0$.
   - M_1 decreases compared to $k, c_0, c_1, S_{max}, \beta$. In other words, the more the agency is attentive to the water stock at the end of the period or the extraction costs are high or the capacity of the basin is high, the lower the amount of rain necessary to ensure that the aquifer will not be empty at the end of the period.
   - M_1 increases with respect to ϕ, ρ, α. In other words, the more the water is important to the user or the agency cares about the immediate well-being of the buyer, the higher the quantity of rainfall necessary to guarantee that the aquifer will not be empty at the end of the period.

- The dependence of M_1 on S_0 varies according to the value of all the other parameters. Precisely M_1 increases with respect to S_0 if $c_1 > 2\beta\phi(2-\rho)$, it decreases otherwise.

2. Let $\sigma \leq 0$.

- M_2 decreases with respect to S_0, β.
- M_2 increases with respect to α.

7 Conclusions

Water is an increasingly scarce resource and it is necessary to distribute it where it creates more benefits for society. At the same time, the collection, abstraction and distribution of water impose significant costs and it is recommended that these costs are covered by the tariffs paid by the users. Water is a basic necessity and must remain accessible, at least in indispensable quantities, even to the poorest sections of the population. One of the tools to achieve the objectives mentioned above is pricing. In this paper, we considered the case of a water supplier who determines sales rates pursuing not only the usual goal of maximizing profits but also the protection of consumer welfare and the need to preserve adequate water supplies for the future. Buyers are assumed to be heterogeneous and can use the water for domestic, agricultural, and industrial purposes. Different prices are allowed (as happens in real life) for different types of users (price discrimination). We calculated the water demand of the various customers assuming that their choices are determined by the price offered, as well as by their needs, but that they are not conditioned by the available water stocks, either because they are not observable or simply because the buyers do not take care of it (myopic behavior).

The model we studied is a leader-follower differential game with a finite horizon. The leader (the water supplier) determines the selling price and the followers (consumers) react by requesting the resulting amount of water. From an analytical point of view, we calculated a Stackelberg feedback equilibrium assuming that all user demand is satisfied (interior equilibrium). Subsequently, by applying periodic functions for the natural recharge of water (thinking it is dependent on seasons) and for the seasonal coefficient in the buyers' utility, we presented several numerical simulations to better illustrate how the model solutions are sensitive to the various parameters included. The most important result expressed by the analysis of the model and also highlighted by the numerical simulations is the comparison of two different tariff schemes: linear tariffs (the price paid is a multiple of the volume of water purchased), and increasing block tariffs (the unit price is lower for quantities of water that do not exceed a fixed threshold). What we showed is that although block tariffs are increasingly popular in many countries and usually justified by determining larger equity to heterogeneous consumers, it is never optimal for the seller to propose this type of pricing (in fact, linear pricing is always preferred). This result, seemingly peculiar in a model in which the agency takes into account both the scarcity of the resource and the welfare of consumers, turns out instead to be consistent with both theoretical (see Monteiro and Roseta-Palma 2011; Sibly and Tooth 2014) and empirical (see Martins et al. 2013; Al-Saidi 2017) growing literature. Numerical simulations then showed how buyer heterogeneity can spill over into oscillating dynamics of optimal prices and the onset of monotonic or multimodal trajectories for the stock of water in the basin. Such results may represent an important policy suggestion that an agency may consider to avoid situations in which total demand fully empties the basin or in which the demand cannot be completely satisfied. In this regard, limiting ourselves

to a static version of the game, we analyzed the possible existence of corner solutions, i.e. solutions that imply the total emptying of the water basin. As expected, we demonstrated that corner solutions exist if the amount of precipitation is excessively low. Finally, we studied the sensitivity of the optimal values to changes in the social role of the agency. What emerged is that although an increasing agency's social altruism is reflected in lower prices and thus higher demands (for a scarce resource this phenomenon cannot always be seen as positive), excessive agency's altruism may result in unsatisfactory agency performance (the relationship between altruism and agency payoff is de facto U-shaped). The latter result therefore provides an additional policy suggestion to the agency, pointing out that a positive (but sufficiently low) level of social altruism can simultaneously ensure affordable prices, adequate stock levels, and satisfactory performance.
This research can be extended in several directions. We would like to point out here two questions that seem to us of greater interest. The hypothesis that the water demand can be fully satisfied is somewhat restrictive and contrasts with what actually happens, for example during long periods of drought. However, the study of solutions that make it impossible to satisfy the demand due to the emptying of the aquifer basin as well as the determination of equilibria that preserve a minimum stock of water present considerable difficulties (non-regularity of the state equation and of the HJB equation). Another interesting question is to consider more general tariff schemes: in fact, consumers pay bills made up not only of volumetric prices but also of fixed components, determined by the type of user and independent of consumption. Furthermore, an alternative way to meet the needs of the poorest part of the population can be to provide them with government subsidies. Our future research will go in the directions indicated above.

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Appendix

Proof of Proposition 1 Let $V(t, S)$ be the value of the game for the agency at the time t with the state S. Then $p_i(t, S);\ i = 1, ..., n$ solve the agengy-HJB equation

$$-\frac{\partial V}{\partial t} = \max_{p_1,\dots,p_n} RHS, \quad V(1, S) = \mu(S), \tag{10}$$

where

$$RHS := \pi + \rho \sum_{j=1}^{n} \pi_j + \frac{\partial V}{\partial S}(R - W), \quad W = \sum_{j=1}^{n} W_j,$$

$$\pi = \sum_{j=1}^{n} p_j W_j - C(S)W,$$

$$\pi_j = \phi_j \left[\alpha_j - \beta_j W_j\right] W_j - p_j W_j.$$

Let $V_i(t, S)$ be the value of the game for the buyer $i \in \{1, ..., n\}$ at the time t with the state S. Then $V_i(t, S)$ solves the HJB equation

$$-\frac{\partial V_i}{\partial t} = \max_{W_i} RHS_i, \quad V_i(1, S) = 0, \tag{11}$$

where

$$\begin{aligned} RHS_i &:= \pi_i + \frac{\partial V_i}{\partial S}(R - W), \quad W = \sum_{j=1}^{n} W_j, \\ \pi_i &= \phi_i\,[\alpha_i - \beta_i W_i]\, W_i - p_i W_i. \end{aligned}$$

We solve the game by backward induction. At the second stage the buyers, having observed the prices decided by the agency, determine the water demands W_i.
First order conditions give that the maximum of RHS_i is attained at $\hat{W}_i$, where

$$\hat{W}_i = \frac{\alpha_i \phi_i - p_i - \dfrac{\partial V_i}{\partial S}}{2\beta_i \phi_i}$$

We look for a solution of the HJB equation that doesn't depend on S, i.e.

$$V_i(t, S) = z_i(t); \quad i = 1, ..., n.$$

It is[9]

$$\frac{\partial V_i}{\partial t} = \dot{z}_i(t); \quad \frac{\partial V_i}{\partial S} = 0.$$

Substituting in (11) we obtain that $z_i(t)$ is the unique solution of the ODE

$$\dot{z}_i = -\frac{(\alpha_i \phi_i - p_i)^2}{4\beta_i \phi}; \quad z_i(1) = 0.$$

Consequently, it is

$$\hat{W}_i = \frac{\alpha_i \phi_i - p_i}{2\beta_i \phi_i}.$$

Let's go back to the first stage of the game.
Note that

$$RHS = \frac{\partial V}{\partial S} R + \sum_{j=1}^{n} \omega_j,$$

where

$$\begin{aligned} \omega_j &:= \left[(1-\rho)p_j - C(S) + \rho\phi_j \left(\alpha_j - \beta_j \hat{W}_j\right) - \frac{\partial V}{\partial S}\right] \hat{W}_j, \\ \hat{W}_j(t) &:= \frac{\alpha_j \phi_j(t) - p_j(t, S)}{2\beta_j \phi_j(t)}. \end{aligned} \tag{12}$$

[9] As usual the superscript $\dot{}$ denotes the time derivative.

First-order conditions give that the maximum of RHS is attained at [10]

$$\tilde{p}_j = \frac{\alpha_j \phi_j (1-\rho) + C(S) + \dfrac{\partial V}{\partial S}}{2-\rho}. \tag{13}$$

Substituting (13) in (12) we obtain

$$\tilde{W}_j = \frac{\alpha_j \phi_j - C(S) - \dfrac{\partial V}{\partial S}}{2\beta_j \phi_j (2-\rho)}.$$

Substituting in (10), the HJB equation becomes

$$-\frac{\partial V}{\partial t} = R\tilde{H}S, \quad V(1, S) = \mu(S), \tag{14}$$

where

$$R\tilde{H}S = \frac{\partial V}{\partial S} R + \sum_{j=1}^{n} \tilde{\omega}_j,$$

and

$$\tilde{\omega}_j := \frac{\left[\alpha_j \phi_j - C(S) - \dfrac{\partial V}{\partial S}\right]^2}{4\beta_j \phi_j (2-\rho)}.$$

Due to the structure of the problem, we look for a quadratic solution of the HJB equation with respect to S, i.e.

$$V(t, S) = x(t)S^2 + y(t)S + z(t).$$

It is[11]

$$\frac{\partial V}{\partial t} = \dot{x}(t)S^2 + \dot{y}(t)S + \dot{z}(t); \quad \frac{\partial V}{\partial S} = 2x(t)S + y(t).$$

Substituting in (14) we obtain

$$-\dot{x}S^2 - \dot{y}S - \dot{z} - [2xS + y]R - \sum_{j=1}^{n} \left\{ \frac{\left[2xS + y + c_0 + c_1(S_{max} - S) - \alpha_j \phi_j\right]^2}{4\beta_j \phi_j \, (2-\rho)} \right\} = 0 \tag{15}$$

[10] Note that $\dfrac{\partial^2 \omega_j}{\partial p_j^2} = \dfrac{\rho - 2}{2\beta_j \phi_j} < 0$ iff $\rho < 2$. Hence, since $\rho \leq 1$, first-order conditions give maxima.

[11] As usual the superscript $\dot{}$ denotes the time derivative.

The left side of (15) is a second degree polynomial in S which must be zero for every value of S. Therefore all its coefficients must be zero. You get

$$\dot{x} = -\gamma \frac{(2x - c_1)^2}{4(2-\rho)};$$

$$\dot{y} = a(t, x)y + b(t, x);$$

$$\dot{z} = -Ry - \left(\sum_{j=1}^{n} Q_j(y)\right) \frac{1}{4(2-\rho)};$$

where

$$\gamma := \sum_{j=1}^{n} \frac{1}{\beta_j \phi_j};$$

$$a(t, x) := -\gamma \frac{(2x - c_1)}{2(2-\rho)};$$

$$b(t, x) := -2Rx + (\delta - \gamma (c_0 + c_1 S_{max})) \frac{(2x - c_1)}{2(2-\rho)};$$

$$\delta := \sum_{j=1}^{n} \frac{\alpha_j}{\beta_j};$$

$$Q_j(y) := \frac{\left[c_0 + c_1 S_{max} + y - \alpha_j \phi_j\right]^2}{\beta_j \phi_j}.$$

The terminal conditions are

$$x(1) = \mu_1; \quad y(1) = \mu_2; \quad z(1) = 0.$$

□

Proof of Proposition 2 Suppose, for simplicity of exposition, that $n = 1$. The proof extends trivially to the case $n > 1$ following the approach of the proof of the previous proposition. We denote by

$$p_1(t, S) := p_{1,1}(t, S); \quad p_2(t, S) := p_{1,2}(t, S).$$

Assume that an interior feedback equilibrium

$$\left(p_1^*(t, S), p_2^*(t, S)\right)$$

exists.

Let $V(t, S)$ be the value of the game at the time t with the state S. Then $\left(p_1^*(t, S), p_2^*(t, S)\right)$ solve the HJB equation

$$-\frac{\partial V}{\partial t} = \max_{0 \le p_1(\cdot) \le p_2(\cdot)} RHS, \quad V(1, S) = \mu(S),$$

where

$$RHS = \pi + \rho\pi_1 + \frac{\partial V}{\partial S}(R - \hat{W}_1),$$
$$\pi = Z_1(t, \hat{W}_1) - C(S)\hat{W}_1,$$
$$\pi_1 = \phi_1 \left[\alpha_1 - \beta_1 \hat{W}_1\right] \hat{W}_1 - Z_1(t, \hat{W}_1),$$
$$Z_1(t, \hat{W}_1(t)) = \begin{cases} p_1(t)\hat{W}_1(t) & \textit{if } \hat{W}_1(t) \le B_1(t); \\ p_1(t)B_1(t) + p_2(t)\left(\hat{W}_1(t) - B_1(t)\right) & \textit{if } \hat{W}_1(t) > B_1(t); \end{cases}$$
$$\hat{W}_1(t) = \begin{cases} \dfrac{\alpha_1\phi_1(t) - p_2(t)}{2\beta_1\phi_1(t)} > B_1(t), & \textit{if } p_1(t) \le p_2(t) < \sigma_1(B_1(t)); \\ B_1(t), & \textit{if } p_1(t) \le \sigma_1(B_1(t)) \le p_2(t); \\ \dfrac{\alpha_1\phi_1(t) - p_1(t)}{2\beta_1\phi_1(t)} < B_1(t), & \textit{if } \sigma_1(B_1(t)) < p_1(t) \le p_2(t); \end{cases}$$

$$\sigma_1(B_1(t)) := \phi_1(t)(\alpha_1 - 2\beta_1 B_1(t)).$$

$\left(p_1^*, p_2^*\right)$ maximizes RHS in the region

$$\mathcal{D} = \{(p_1, p_2)|0 \le p_1 \le p_2\}.$$

The region $\mathcal{D}$ is the union of three subsets $\mathcal{D} = \mathcal{D}_1 \cup \mathcal{D}_2 \cup \mathcal{D}_3$ having no internal points in common.

$$\mathcal{D}_1 = \{(p_1, p_2)|0 \le p_1 \le p_2 \le \sigma_1\};$$

$$\mathcal{D}_2 = \{(p_1, p_2)|0 \le p_1 \le \sigma_1 \le p_2\}.$$

$$\mathcal{D}_3 = \{(p_1, p_2)|\sigma_1 \le p_1 \le p_2\};$$

Assume that an equilibrium $\left(p_1^*, p_2^*\right) \in \mathcal{D}$ exists.
We distinguish three cases:

1. $\left(p_1^*, p_2^*\right) \in \mathcal{D}_1$.
   In $\mathcal{D}_1$ it is
   $$\frac{\partial RHS}{\partial p_1} = (1 - \rho)B_1 > 0.$$
   It follows that $p_1^* = p_2^*$.
2. $\left(p_1^*, p_2^*\right) \in \mathcal{D}_2$.
   In $\mathcal{D}_2$ it is
   $$\frac{\partial RHS}{\partial p_1} = (1 - \rho)B_1 > 0.$$
   It follows that $p_1^* = \sigma_1$. Moreover, since RHS does not depend on p_2, there are infinite equilibria $(p_1^* = \sigma_1, p_2^* = k)\ k \ge \sigma_1$. In particular, $(p_1^* = \sigma_1, p_2^* = \sigma_1)$ is an equilibrium. Note that the value $RHS(p_1^* = \sigma_1, p_2^* = k)$ is the same for any $k \ge \sigma_1$.

3. $\left(p_1^*, p_2^*\right) \in \mathcal{D}_3$.
   In $\mathcal{D}_3$ RHS does not depend on p_2 and

$$p_1^* = \arg\max RHS(p_1).$$

   At each point (p_1^*, k), $k \geq p_1^*$ RHS reaches its maximum. Hence there are infinite equilibria (p_1^*, k) $k \geq p_1^*$. In particular (p_1^*, p_1^*) is an equilibrium.
   Note that the value $RHS(p_1^*, p_2^* = k)$ is the same for any $k \geq p_1^*$.

□

Proof of Proposition 3 At the second stage, first order condition gives the optimal demand for water

$$\hat{D} = \frac{\alpha\phi - p}{2\beta\phi}.$$

At the first stage, the agency, foreseeing the response of the buyer, solves the following optimization problem

$$\max_{p \geq 0;\ 0 \leq W \leq \hat{D}(p)} J(p, W), \tag{16}$$

subject to the state constraint $S_1 = S_0 + R - W \geq 0$ (that is $W \leq S_0 + R$,) where

$$J(p, W) = \pi + \rho\pi_1 + \mu(S_1).$$

Since $\dfrac{\partial J}{\partial p} = (1 - \rho)W \geq 0$, we have that the maximum of J occurs on the set

$$S = \{(p, W) \mid \tilde{p} \leq p \leq \alpha\phi;\ \ W = \hat{D}(p\},$$

where

$$\tilde{p} = \max\{0, p_1\}; \quad p_1 = \phi\,[\alpha - 2\beta(S_0 + R)];$$

p_1 is the price such that $\hat{D}(\tilde{p}) = S_0 + R$. Problem (16) becomes

$$\max_{\tilde{p} \leq p \leq \alpha\phi} J(p, \hat{D}(p)),$$

where

$$J(p) = (p - c_0 - c_1(S_{max} - S_0))\hat{D}(p) + \rho\left[\phi(\alpha - \beta\hat{D}(p))\hat{D}(p) - p\hat{D}(p)\right] + k(S_0 + R - \hat{D}(p)).$$

First order condition $\dfrac{dJ(p)}{dp} = 0$ gives $p = \hat{p}$, where

$$\hat{p} = \frac{k + c_0 + c_1\,(S_{max} - S_0) + \alpha\phi(1 - \rho)}{2 - \rho}.$$

Note that $\hat{p} > 0$ and that $\hat{p} < p_1$ iff $R < M_1$, where

$$M_1 := \frac{2(2 - \rho)\beta\phi S_0 - \alpha\phi + k + c_0 + c_1\,(S_{max} - S_0)}{2\beta\phi(2 - \rho)}.$$

Moreover $\hat{p} < \alpha\phi$ iff $\sigma > 0$, where

$$\sigma := \alpha\phi - k - c_0 - c_1\,(S_{max} - S_0)\,.$$

Note that $p_1 > 0$ iff $R < M_2$ where

$$M_2 := \frac{\alpha - 2\beta S_0}{2\beta}.$$

It is $M_1 < M_2$.
We distinguish two cases:

1.

$$\sigma > 0.$$

(a) If $0 \leq R \leq M_1$, then J is decreasing as $\tilde{p} \leq p \leq \alpha\phi$ so that the optimal price is $p^* = p_1$. The quantity of water demanded and consumed is $W^* = S_0 + R$. The aquifer after consumption is empty, that is $S_1 = 0$.
(b) If $M_1 < R$, then J is increasing in the interval $\tilde{p} \leq p \leq \hat{p}$ and decreasing for $\hat{p} \leq p \leq \alpha\phi$. The optimal price is $p^* = \hat{p}$. The quantity of water demanded and consumed is

$$W^* = \frac{\sigma}{2\beta\phi(2-\rho)} = \frac{\alpha\phi - k - c_0 - c_1\left(S_{max} - S_0\right)}{2\beta\phi(2-\rho)} > 0.$$

The volume of water that remains in the aquifer after consumption is $S_1 = S_0 + R - W^* > 0$.

2.

$$\sigma \leq 0.$$

(a) If $0 \leq R < M_2$, then $\tilde{p} = p_1$, J is decreasing as $\tilde{p} \leq p \leq \alpha\phi$ so that the optimal price is $p^* = p_1$. The quantity of water demanded and consumed is $W^* = S_0 + R$. The aquifer after consumption is empty, that is $S_1 = 0$.
(b) If $M_2 \leq R$, then $\tilde{p} = 0$, J is decreasing as $\tilde{p} \leq p \leq \alpha\phi$ so that the optimal price is $p^* = 0$. The quantity of water demanded and consumed is $W^* = \dfrac{\alpha}{2\beta}$. The volume of water that remains in the aquifer after consumption is $S_1 = S_0 + R - \dfrac{\alpha}{2\beta} > 0$.

□

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