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A matheuristic for passenger service optimization through timetabling with free passenger route choice

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Abstract

Designing a public transport timetable that maximizes passenger service, measured in weighted travel time, is an intricate problem. The weighted travel time depends on the free route choice of passengers. Passenger route choice depends on the timetable. In turn, the timetable that minimizes weighted travel time depends on the route choice of passengers – and therefore requires passenger route choice information. Consequently, a sequential approach where timetables are designed provided pre-fixed passenger assignment to routes, may not find the optimal timetable.

This paper aims to integrate passenger route choice and timetabling. It addresses the problem of designing maximal passenger service public transport timetables in systems with free route choice within a budget for operating costs. Operating costs are defined by the minimal cost vehicle schedule required to operate the timetable.

The proposed methodology integrates a matheuristic for timetabling and vehicle scheduling with a passenger assignment model in an iterative framework, where different forms of integration are evaluated. Focus is on long to medium term timetabling, provided an initial timetable. Results for a realistic case study in the Greater Copenhagen area indicate that our approach consistently leads, at no additional cost, to timetables that represent a reduction in passenger weighted travel time in comparison to both an initial timetable and a non-integrated timetabling method that receives a single passenger assignment as input.

Keywords: Public Transport, Bus Timetabling, Passenger Route Choice, Mixed Integer Linear Programming, Matheuristic

1 Introduction

Timetabling consists of assigning specific points in time to a set of events. In the bus timetabling problem considered in this paper, this set of events follows from the service network designing, determining a set of lines, each with a given ordered list of stops and a target headway. Thus, in the context of our paper, headways between vehicles should be within some small bandwidth. The output of the timetabling phase serves as input to the vehicle scheduling problem, that assigns vehicles to specific services in the timetable. Timetabling generally represents a balance between high quality passenger service, and low operating costs. Passenger service depends on the timetable, while the operating costs depend on the vehicle assignment. Passenger service can be expressed in terms of *weighted travel time* (WTT): a weighted sum of the components of *initial waiting time* (IWT), *in-vehicle time* (IVT), and *transfer time* (TrT) [7]. Operating costs are expressed as a function of the number of required vehicle schedules and dead-heading distance.

Integrating passenger route choice and timetabling is an intricate problem, as there is a co-dependency between passenger route choice and timetable design. Indeed, minor changes in the timetable can have large effects on the WTT of routes with a transfer, especially in networks where several alternative geographical paths exist for a large number of origin-destination pairs. Small changes in the timetable could lead to far larger changes in transfer times, and consequently may cause passengers preferring alternative geographical routes. As a result, providing a fixed passenger assignment as input to the timetabling phase could lead to finding suboptimal timetables. Furthermore, also the vehicle schedules can be strongly affected by small changes in the timetable. To ensure reasonable operating costs of the resulting timetable, it is vital to consider vehicle scheduling as well.

This paper studies the *Integrated Passenger Assignment, Timetabling and Vehicle Scheduling Problem* (IPAT-VSP) at a tactical level. The objective of the IPAT-VSP is to maximize passenger service in terms of WTT within a budget for operating costs and a set of headway constraints. Input consists of a non-cyclical initial timetable with time-dependent service times, and an *Origin-Destination-Time* (ODt) passenger demand matrix. Timetable decisions consist of shifting departure times of trips, or extending dwell time at transfer stops, with respect to the provided initial timetable. In this paper, and similarly to what is done in [10], we define operating costs in terms of number of schedules created and not in terms of number of vehicles, since the number of schedules is a good upper bound on the number of vehicles needed and provides computational benefits in comparison to considering individual vehicles.

Key characteristics of passenger route choice as considered in this study are that (i) passengers have free route choice, and (ii) passengers may have different route preferences. Thus passengers with the same origin, destination, and departure time, may still choose different routes.The first is in contrast to the common setting of a central controller assigning passengers to maximize a social optimum, which would be a natural result of integrating a passenger assignment with the objective to minimize weighted travel time in a mixed-integer linear program often used for solving the combinatorial problems around scheduling and timetabling. The second is in contrast to assuming all passengers choose the minimum weight path, and reflects that passengers may have different preferences. Specifically, it will ensure that when two almost equivalent paths exist connecting an origin and destination, passengers will be assigned to both. This assumption fits with commonly accepted route choice theory [1].

A matheuristic approach for the IPAT-VSP is proposed that consists of an iterative framework between a passenger route choice model, and an integrated timetabling and vehicle scheduling model. Different forms of integration are evaluated. The implementation of this modular framework combines the integrated timetabling and vehicle scheduling model of Fonseca et al. [10] with the passenger route choice model of Briem et al. [4], that satisfies the above two key characteristics of free passenger route choice, and different preferences for passengers. The latter also serves as a ground-truth for evaluating the passenger service of any timetable.

A realistic case study representing a large part of the multi-modal public transport network of the Greater Copenhagen Area, Denmark, serves to investigate the value of the IPAT-VSP timetabling approach: (i) in comparison with the status-quo reflected by an initial timetable; (ii) in case of a change in the line network; and (iii) in case of a change in the OD matrix. Results indicate that including free passenger route choice results in timetables with higher passenger service in all three situations compared to a fixed passenger route choice approach (represented by a comparison to the state-of-the-art fixed route choice model in Fonseca et al.[10]). Moreover, our computational studies, supported by a simple clarifying example, illustrate that in order to find timetables with high passenger service indicating *potentially* interesting transfers is more important than estimating accurate usage of transfers in a current timetable.

To summarize the contributions of this work: (i) we investigate the maximal passenger service timetabling problem in the context of a free passenger route choice; (ii) we propose a modular matheuristic approach for the IPAT-VSP that, in an iterative framework, combines two state of the art models: (1) the IT-VSP matheuristic of Fonseca et al. [10], which maximizes passenger service through minimizing excess transfer time under the assumption of fixed passenger route choice, with (2) the stochastic passenger route choice model of Briem et al. [4], which represents free route choice of passengers; and (iii) we find that the inclusion of free passenger route choice results in timetables with higher passenger service for a realistic case study of the Greater Copenhagen area. Thereby the current study is different from [10] by (a) indicating the value of integrating passenger route choice and timetabling, where [10] assumed the passenger route choice as fixed input; (b) demonstrating the value of this approach for a larger, more complex network case study, and contrasting this against the approach on [10], and (c) the evaluation of the value of the timetabling approach not only in comparison to the status-quo, but also in case of a small network re-design, and a change in passenger demand. Both (b) and (c) would lead to a change in (expected) passenger flows, which this model demonstratively is better capable of handling than the model of [10].

2 Literature Review

Recent years saw an increase in research output that integrates passenger decisions into the optimization models, especially in line planning, timetabling, and delay management models. In general, there are two ways of formulating these models. One way is incorporating the decision making problem with passenger behaviour description into one integrated model, which generates simultaneously the optimized decisions and the corresponding passenger flows. Another way is establishing two separate models with one for decision making and another one for passenger assignment. The decision making model optimizes the decisions based on the passenger flows from the passenger assignment model in an iterative way until specific termination criteria are reached. In the following, we give a review on the integrated methods and the iterative methods, respectively.

2.1 Integrated methods of incorporating decision making with passenger behaviour

The literature of incorporating decision making problems with passenger behaviour into one integrated model is usually based on simplified descriptions of passenger behaviour. Schmidt [20] integrates delay management with passenger routing assuming that passengers with the same origins, destinations, and planned departure times always choose the same routes. The same assumption is used by Schmidt and Schöbel [21] and Schiewe [19], who both integrate timetabling with passenger routing. To reduce computational complexities, the model of [21] is simplified in a way that the information about the specific vehicle which a passenger plans to board/alight at the origin/destination is given as known input, which significantly reduces the choice sets of passengers. Schiewe [19] also simplifies the problem by proposing a pre-processing algorithm to reduce passengers' choice sets. Two heuristic approaches are developed to provide lower and upper bounds on the objective. Gattermann et al. [11] present a boolean satisfiability problem (SAT) model that integrates periodic timetabling with passenger routing, distributing OD pairs temporally using time slices to make the problem tractable. To reduce the required constraints, passengers' path choice sets are reduced by imposing a detour factor of 1.2. Only the paths that do not deviate from the fastest paths by the given detour factor are considered in their model. Wang et al. [25] propose a mixed integer nonlinear program to deal with the integration of train scheduling and rolling stock circulation planning under time-varying passenger demand. The target case is a single metro line, where passengers are assumed to always take the first coming train. Borndörfer et al. [3] integrate periodic timetabling with passenger routing allowing a variety of passenger choice models to be integrated. All OD pairs for which a lower bound route is a direct connection are removed in their model as they optimize the timetable with fixed dwell and driving times and thus the timetable affects only passengers who transfer. Robenek et al. [18] also assume fixed dwell and driving times, who integrate train timetabling design with a probabilistic demand forecasting model. The integrated model is applied to a network of Israeli Railways considering the network layout of 2008 and real-life passenger demand of 2008 and 2014, respectively. The results show that pricing strategies and passenger-centric timetabling can together result in up to 15% revenue increase, although a part of this increase might also be explained by other operational changes between 2008 and 2014. Chu [6] presents a mixed integer program (MIP) to integrate network design and timetabling. All feasible paths for OD pairs are generated beforehand by a procedure based on the breadth-first search and path enumeration algorithms. The passenger generalized cost of each path is then calculated and given as a known parameter to the MIP, which aims to minimize the weighted sum of bus operating cost, passenger generalized cost, and penalty for unsatisfied demand. Canca et al. [5] present a mixed integer non linear program to optimize line frequencies (minimizing operating costs and fleet acquisition costs) and simultaneously compute passenger assignments (minimizing average trip time and number of transfers). To reduce the size of the variable sets, they use a pre-processing step to sort out the k-shortest paths for each OD pair. Binder et al. [2] propose an integer linear program to integrate timetable rescheduling with passenger routing considering limited vehicle capacity. Passengers are assigned to different vehicles in an operator-controlled way without free path choices. Wagenaar et al. [24] propose an mixed integer linear program for rolling stock rescheduling considering adjusted passenger demand. Passengers are not allowed to detour but only wait for the next train with the same origin and destination if their planned trains are cancelled.

Different from the reviewed integrated models, our paper deals with timetabling with passenger routing in an iterative way. We consider more realistic passenger behaviours that (i) passengers are free to choose any available routes, (ii) passengers may have different route preferences meaning that passengers with the same origin, destination, and departure time can choose different routes, and (iii) the behaviours of all types of passengers (with or without transfers) are explicitly described in our model. A recent paper of integrated timetabling with passenger distribution by Hartleb and Schmidt [13] also considers free route choices and different passenger preferences. Their target case (8 lines and 9 stops) is smaller than the case (8 lines and 54 stops) considered in our paper, in which vehicle scheduling is also handled besides timetabling.

2.2 Iterative methods of incorporating decision making with passenger behaviour

The literature of handling decision making and passenger assignment in an iterative way usually formulates the problem as a bi-level model. Dollevoet and Huisman [8] develop an iterative heuristic to handle delay management with passenger routing. In each iteration, the classical delay management model is first solved, and then passengers are re-routed. A case study in a part of the Dutch railways shows that on average, the solutions by the iterative algorithm are only 0.4% worse than the exact solutions. However, one should note delay management is different for two key reasons: a) it is generally not possible to shift backward in time, and b) minimal cost of the fleet is not an objective. Zhu et al. [27] present a bi-level model for single line timetabling with passenger routing. The first level model determines the headways to minimize total passenger costs (perceived travel time and travel penalties), and the second level model determines the passenger arrival times given the headways. The authors use a two stage genetic algorithm to solve hypothetical examples of the problem. As their target case is a single line, passengers' transfer behaviour between different lines are not formulated in the model. Focusing on a network with several lines, transfer behaviour is considered by Wu et al. [26]. They present a bi-level program to coordinate timetabling and consider passengers' behavior to the timetable modifications. The first level uses a mixed integer non-linear program to design the timetable, minimizing system cost composed by operating and user costs. The second level is a passenger route choice model to describe passenger behaviour in reactions to the timetable designed by the first level. Only passengers with missed transfers are re-routed. The authors use a heuristic algorithm to solve the problem and show results for two small examples: one with 3 lines and 3 transfer stops, and another with 4 lines and 4 transfer stops. Liu and Ceder [15] propose a bi-objective, bi-level IP formulation to deal with timetabling and vehicle scheduling considering passenger demand. The authors allow timetable modifications by shifting departure times and initial vehicle schedules are given as input. Deadheading is not allowed, meaning that vehicles are assigned to a single line and can only service trips belonging to that line, which significantly reduces the complexity of the problem. They propose a deficit function based sequential search to solve small examples with up to 4 unidirectional lines, 4 transfer stops, and one hour of operations. Polinder et al. [17] deal with passenger-centric timetabling by an iterative approach. They first obtain an ideal timetable of minimized average perceived travel times assuming passengers always choose the shortest routes. Operational constraints are not considered in the ideal timetable, which thus may not be operational feasible. Therefore, they propose a Lagrangian heuristic to modify the ideal timetable to make it feasible. The modified timetable is assessed by an evaluation function from the perspective of passengers, based on which feedback is given to the Lagrangian heuristic for generating better feasible timetables in the next iteration. The process continues until no improvements are obtained, and manual inspections are required to find a good feedback option. Veelenturf et al. [23] embed a timetable rescheduling model and a passenger assignment model into an iterative framework. At each iteration an extra stop will be added to the timetable if it reduces the total passenger inconvenience as evaluated by the passenger assignment model. Kroon et al. [14] study the integration of free passenger flows in a real-time rolling stock rescheduling model for disruption management. The authors present a heuristic approach that iterates between a simulation model for passenger flows and an optimization model for the rolling stock, updating the objective function of the optimization model at each iteration according to the current passenger flows. Van der Hurk et al. [22] propose an advice optimization module and a rolling stock rescheduling module. Both optimization modules are supported by a passenger assignment model to iteratively feed back the passenger responses to the above modules to generate better solutions for passengers.

Our paper uses a bi-level model to handle integrated timetabling and vehicle scheduling considering dynamic passenger reactions in an iterative way. Different from the reviewed iterative papers, our model focuses on a larger network with 8 bi-directional lines and 54 stops. We allow a wider set of timetable modification including changes in the starting time of trips (shifts), and addition of dwell time (stretches) at transfer stops. Trips from different lines can be included in the same vehicle schedule and thus allowing deadheading between consecutive trips in a schedule. It should also be clear that we are dealing with aperiodic timetabling, which is less complex to solve than periodic timetabling.

3 Formal definition of the IPAT-VSP

The objective of the IPAT-VSP is to find the maximum passenger service timetable $\mathscr{T}^*$ of all feasible timetables $\mathscr{T} \in \mathbf{T}$, which we define as the timetable with minimum total weighted travel time. A feasible timetable is defined as a timetable that respects the budget constraint, constraints regarding headways, shifts, stretches, and depot capacities. The budget is calculated as the costs of the *vehicle schedules*, where the number of vehicle schedules provides an upperbound to the number of required vehicles.

The IPAT-VSP is a bi-level optimization problem in which: (i) the operator aims to find the maximum passenger service timetable within an operating budget, and (ii) the passengers aim to find their individual best paths in this timetable. Here we assume that (ii) represents free route choice of passengers, where passengers may have different preferences. The latter implies that when two paths exist of (almost) equal in-vehicle time (IVT), initial waiting time (IWT) and transfer time (TR), some passengers will prefer the one path, others the other path. Thus the resulting assignment will be different from an assignment to minimum WTT paths for passengers according to a single WTT function.

Let L be the set of directed lines, where each line $l \in L$ is defined by a sequence of stops $s \in S$, with S the set of all stops. Let a trip i represent a vehicle servicing all stops of a line l once. Each line l is associated with a set T_l of trips in the timetable for this line. The timetable is defined by the set of all trips $T = \bigcup_{l \in L} T_l$ and $T_{l'} \cap T_{l''} = \emptyset$ for all $l', l'' \in L, l' \neq l''$. Moreover, let ODt be the origin-destination-departure time matrix where $odt \in$ ODt represents the number of passengers desiring to travel from origin station o to destination station d, $o, d \in S$, departing at or after time t. The timetabling problem is to assign arrival and departure times to all stops $s \in S_i$, for all trips $i \in T$. We refer to the resulting timetable with assigned times as $\mathscr{T}$.

3.1 The objective

Let $f_{PAM}(odt, \mathscr{T})$ be a function of the odt and the timetable $\mathscr{T}$ that returns the weighted travel time (WTT) of an $odt \in$ ODt multiplied by the weight (number of passnegers) of this odt. The WTT results from a stochastic passenger assignment model *PAM*, that may distribute the passengers of an odt over multiple paths. Let

P_{odt} be the set of all paths of the odt's passenger assignment. A path $p \in P_{odt}$ represents an ordered list of trips $i \in T$ that connect the origin to the destination of the passengers of the odt in time and space, where the departure time of the first trip, from the origin stop of the passenger, is not before the departure time of the passenger. Paths are selected such that when a passenger disembarks a trip i, they have either arrived at their destination, or will transfer to a trip of another line $l \in L$. Then the resulting WTT of the assignment of the ODt is defined as:

$$f_{PAM}(odt, \mathscr{T}) = \mathrm{WTT}_{odt} = \sum_{p \in P_{odt}} w_p \cdot (\beta_1 \cdot \mathrm{IWT}_p + \mathrm{IVT}_p + \beta_2 \cdot \mathrm{TrT}_p) \quad (1)$$

where WTT_{odt} is the total weighted travel time of the passenger assignment, w_p the number of passengers of the odt that will select path p, IWT_p the initial waiting time of path p, IVT_p the in-vehicle time of path p, and TrT_p the transfer penalty of path p. We assume $\beta_1, \beta_2 \geq 1$.

3.2 Formulation of the IPAT-VSP

The input to the IPAT-VSP consists of the set of all to be timetabled trips $i \in T$, the ODt matrix, a budget for the operating costs, and costs and parameters related to the case study, such as allowed headways, minimum and maximum dwell times, or minimum and maximum turnaround times, and the passenger assignment function representing the passenger's individually optimal route choice, function f_{PAM}.

Decision variables consist of:

a) binary assignment variables $x_{ijk} \in \{0,1\}$ storing which vehicles are assigned to which trips, with the triplet (i, j, k) representing a trip j serviced immediately after a trip i with the same vehicle from depot k

b) departure and arrival time variables τ^d_{is} and $\tau^a_{is} \in \mathbb{Z}^+_0$ for each trip $i \in T$ and each stop $s \in S_i$

c) dwell time variables $\delta_{is} \in \mathbb{Z}^+_0$, which store the number of minutes of dwell time added to trip $i \in T$ at stop $s \in J_i$

The turnaround time should generally be in the interval $[q^-, q^+]$. Buffer time added to the trip in the form of dwell time is subtracted from the minimum turnaround time q^-. Each vehicle used in a feasible solution covers a sequence of compatible trips and must return to the depot from which it departed. Two trips $i, j \in T$ are compatible if the following three conditions hold: (a) $Dist(et_i, st_j)$ is smaller than u; (b) The sum of a^-_{i,et_i}, q^-, and b_{ij} is smaller or equal to d^+_{j,st_j}; and (c) the sum of

a^+_{i,et_i}, q^+, and b_{ij} is greater or equal to d^-_{j,st_j}. For reference, a complete list of all sets, parameters and decision variables is presented in Appendix 1.

The MINLP formulation for the IPAT-VSP is:

$$\min \sum_{odt \in ODt} f_{PAM}(odt, \mathscr{T}) \tag{2}$$

$$\text{s.t.} \sum_{(i,j,k)\in Q} c_{ijk} x_{ijk} + \sum_{i \in T} \sum_{s \in J_i} c^{DW} \delta_{is} \leq B \tag{3}$$

$$\sum_{(i,j,k)\in Q} x_{ijk} = 1 \qquad i \in T \tag{4}$$

$$\sum_{(i,j,k)\in Q} x_{ijk} - \sum_{(j,i,k)\in Q} x_{jik} = 0 \qquad k \in K \quad j \in V_k \tag{5}$$

$$\sum_{(i,j,k)\in Q^O} x_{ijk} \leq v_k \qquad k \in K \tag{6}$$

$$d^-_{i,st_i} \leq \tau^d_{i,st_i} \leq d^+_{i,st_i} \qquad i \in T \tag{7}$$

$$0 \leq \tau^d_{is} - \tau^a_{is} - w^-_{is} \leq w^+_{is} \qquad i \in T \quad s \in J_i \tag{8}$$

$$\sum_{s \in J_i} \delta_{is} \leq w \qquad i \in T \tag{9}$$

$$\delta_{is} = \tau^d_{i,s} - \tau^a_{i,s} - w^-_{is} \qquad i \in T \quad s \in J_i \tag{10}$$

$$h^-_{is} \leq \tau^d_{is} - \tau^d_{i-1,s} \leq h^+_{is} \qquad l \in L \quad i \in T_l : i \notin T^1 \quad s \in J_i \cup \{st_i\} \tag{11}$$

$$\tau^a_{i,et_i} + b_{ij} + q^- - \sum_{s \in J_i} \delta_{is} - M(1 - \sum_{(i,j,k)\in Q} x_{ijk}) \leq \tau^d_{j,st_j} \qquad (i,j) \in T(Q) \tag{12}$$

$$x_{ijk} \in \{0,1\} \qquad (i,j,k) \in Q \tag{13}$$

$$\tau^d_{is} \in \mathbb{Z}_+ \qquad i \in T \quad s \in J_i \cup \{st_i\} \tag{14}$$

$$\tau^a_{is} \in \mathbb{Z}_+ \qquad i \in T \quad s \in J_i \cup \{et_i\} \tag{15}$$

$$\delta_{is} \in \mathbb{Z}_+ \qquad i \in T \quad s \in J_i \tag{16}$$

$$\alpha_{ijs} \in \{0,1\} \qquad (i,j,s) \in W \tag{17}$$

The above MINLP relates to [10], and follows largely the same notation, in that it considers the same integrated timetabling and vehicle scheduling problem. However, the IPAT-VSP has a *different objective*: the objective function (2) minimizes a weighted sum of passengers' WTT in the timetable, as defined in equation 1, under *stochastic route choice*; while [10] considered the route choice of passengers to be fixed and minimized the excess transfer time and additional dwell time of the new timetable compared to an initial timetable. The advantages of the new objective function are illustrated in Section 3.3. Note that the representation of passengers in [10] was already a step forward in comparison to other papers on integrated timetabling and vehicle scheduling, such as [16].

Constraints (3) are budget constraints imposing an upper bound on the operating costs. The first term considers driving operating costs for deadhead, pull out, and pull in trips, and the second term considers operating costs associated with additional dwell times. Constraints (4) - (6) model classical MDVSP constraints: assignment constraints (4) guarantee coverage of each trip $i \in T$ by including it in exactly one vehicle schedule; flow conservation constraints (5) on trip and depot nodes guarantee the continuity of the vehicle schedules created; and capacity constraints (6) limit the number of pull-out trips to the maximum number of schedules that can depart from each depot $k \in K$.

Constraints (7) - (11) model timetable modifications: constraints (7) force lower and upper shift bounds on the departure time from the first stop of each trip; constraints (8) impose a maximum added dwell time at each stop of a trip; constraints (9) bound the total added dwell time to all intermediate stops of a trip; constraints (10) define the δ_{is} variables to the added dwell time in the corresponding intermediate stop $s \in J_i$ of trip $i \in T$; and constraints (11) model the minimum and maximum headways between each trip $i \in T$ and its precedent trip in the same directed line at each stop $s \in J_i \cup \{st_i\}$ (at all stops of each line). Linking constraints (12) relate the vehicle scheduling and the timetable modification parts of the problem. These guarantee that if trips i and j are serviced consecutively by the same vehicle, then the vehicle has time to deadhead from et_i to st_j without violating the minimum turnaround time q^-.

Constraints (13)-(17) define the range of all sets of decision variables.

We are not aware of a tractable approach to solve the above nonlinear problem. Therefore we will propose a heuristic approach that will iteratively solve a passenger assignment heuristic and a integrated timetabling and vehicle scheduling problem. The following example will illustrate that such a split is not straightforward.

3.3 Example of the IPAT-VSP

Consider the example in Figure 1, with three bus lines (1, 2, 3) and four stops (A, B, C, D), that may be considered to be part of a larger network. Notice that this example does not include the vehicle scheduling component of the problem. Consider that all bus lines have a headway of 20 minutes. Information on travel times is indicated along the edges, and a minimum transfer time of 4 minutes is required to guarantee a successful transfer, as indicated by the curved arrows. Three *transfer opportunities* exist in this network: at B where lines 1 and 3 meet, at C where lines 1 and 2 meet, and at D where lines 2 and 3 meet.

We propose an iterative approach for solving the IPAT-VSP, where the general concept of a split between passenger assignment and operational planning has been

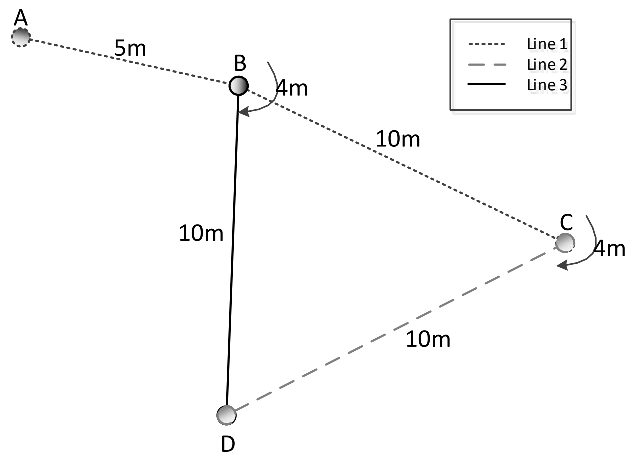

Figure 1: Example of a public transport network with three lines and four stops

followed in several previous other papers (Section 2.2). This example illustrates (1) that when the timetabling model receives a PA input that is different from the PA of the optimal timetable, the timetabling model need not find the optimal timetable; and (2) that although given a fixed PA, minimizing the weighted sum of (excess) transfer time, as in [10], will lead to minimizing total WTT, this is not true when the final PA is different from the input PA. Even more, timetables with a *higher* weighted sum of (excess) transfer time could be associated to timetables with a *lower* total WTT, and thus higher passenger service. It is fair to assume that an initial PA for a previous timetable is likely to exist (i) when estimating demand per transfer from automated fare collection systems, (ii) in large networks that typically are not designed from scratch, but may be only gradually extended or changed, (iii) an initial timetable can always be generated from the information of the target headway per line, which we assume to be input, and therefore the sequential method can also work for new networks for which no initial timetable exists yet.

Passengers traveling from A to D have two routing options: (i) traveling with line 1 from A to C, then transfer to line 2 to travel towards D; and (ii) traveling with line 1 from A to B, then transferring to line 3 to travel towards D. The passenger service is reflected in the weighted travel time, which is calculated as WTT = $\beta_1 \cdot$IWT + IVT + $\beta_2 \cdot$TrT, where $\beta_1, \beta_2 \geq 1$. We may focus on IVT and TrT alone, as for both (i) and (ii) the IWT (dependent on the target headway of the first line) is equal. The IVT of (i) is 5+10+10=25, which is longer than the IVT of (ii) 5+10=15. Passengers will however prefer the longer IVT of route (i) when 25+$\beta_2 \cdot \mathrm{TrT}_{(i)} <$

$15 + \beta_2 \cdot \text{TrT}_{(ii)}$.

Consider a timetable where the transfer from line 1 to 2 is perfectly synchronized at C, with no excess transfer time. Moreover, line 3 departs from B three minutes after the arrival of line 1 at B, resulting in 23 minutes of transfer time at this location. For a value of $\beta_2 = 3$ even under free route choice all passengers will prefer route (i) with a WTT=25+3·4=37 over route (ii) with a WTT=15+3·23=84. However, passenger service could be increased in this example if timetabling decisions would reduce the transfer time from line 1 to line 3 at B in route (ii) such that $15+\beta_2 \cdot \text{TrT}_{(ii)} < 25+\beta_2 \cdot 4$. This alternative has much lower IVT in exchange for a transfer penalty and some additional waiting time, as required for the transfer.

(1): Indeed, in our example reducing the transfer time at stop B could improve passenger service by attracting passengers to route (ii). However, as in the current PA no passengers are using this transfer location. Therefore a timetabling model aimed at reducing excess transfer time based on the current estimate of transferring passengers, like [10], has no incentive to improve the synchronization of the two lines at this location. This shows that a PA different from the optimal PA could prevent finding the optimal timetable.

(2): The reduction in transfer time at B could improve total WTT already at a *positive excess transfer time* (when $15+\beta_2 \cdot \text{TrT}_{(ii)} < 25+\beta_2 \cdot 4$, with $\beta_2 \geq 1$). However, this would lead to an increase in total weighted excess transfer time of the model in comparison to passengers only using the perfect synchronized transfer at C. In fact, passengers only transferring at the perfect synchronized transfer in C leads to a minimal objective of 0 minutes excess transfer time, which could suggest in the formulation of [10] that the optimal timetable is found. This is true if route choice was fixed. However, as it is not fixed, the total WTT would be lower when there is a low transfer time at B, even if the transfer is not perfectly synchronized. We are not aware of a tractable model for integrated timetabling, vehicle scheduling, and stochastic passenger route choice that would circumvent this problem.

4 Solution method

We propose to compute good quality solutions to the IPAT-VSP heuristically as a bi-level optimization problem through the `MHeuPA` matheuristic. The objective is to maximize passenger service in terms of minimizing WTT by modifying an initial timetable under the assumption of free passenger route choice and respecting a budget on operating costs. At the one level, a stochastic passenger assignment model will provide input on expected passenger behaviour, specifically in terms of the number of expected passengers planning to transfer from a specific trip $i \in T$

at a specific stop s to the first arriving trip of a different line $l' \in L, i \notin T_l$, under a minimum transfer time. The second level represents a matheuristic for timetabling.

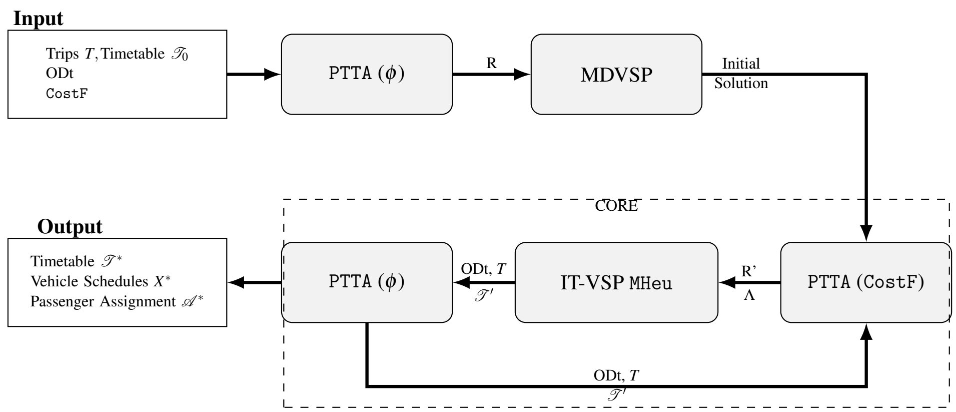

Figure 2: Flow diagram of the `MHeuPA` proposed for solving the IPAT-VSP. The arcs (A,B) indicate what output from A is used as input in B.

Figure 2 depicts the bi-level matheuristic approach `MHeuPA` proposed for the IPAT-VSP. Input consists of the set of all trips T, an initial timetable $\mathscr{T}_0$, an ODt matrix ODt, and a passenger waiting-time cost function `CostF`. An initialization computes a reference WTT for the initial timetable under the ground-truth WTT cost function ϕ, and computes a budget for the vehicle operating costs in the *Multi Depot Vehicle Scheduling Problem* (MDVSP). The core of the algorithm iteratively computes a passenger assignment (PA) for cost function `CostF` in the `PTTA`; and next provides the PA as input to the timetabling and vehicle scheduling matheuristic IT-VSP. The PA consists of a set of passenger transfer demands R' and a vector Λ of passengers on board at each stop, the IT-VSP returns a timetable $\mathscr{T}'$. Only timetable modifications are preserved that represent a non-worsening of the WTT under the `PTTA` with ground-truth WTT cost function ϕ. The ODt and set of trips T remain fixed throughout the algorithm. The approach stops after a computation time limit is reached. Output is a new timetable $\mathscr{T}^*$, the set of vehicle schedules X^*, and a passenger assignment $\mathscr{A}^*$.

The consequences of free and stochastic route choice when solving the IPAT-VSP are:

1. passenger volumes per transfer location in the current timetable need not represent the passenger volumes per transfer location in the maximal passenger service timetable

2. higher objective values of the model of Fonseca et al. [10] could actually be

associated with lower overall WTT, and thus higher passenger service.

To address these two points:

- different passenger routings, by changing the waiting cost parameter in the PTTA model of Briem et al. [4], are tested in the integration;
- timetable quality in terms of WTT is *always* calculated by the passenger route choice model.

Any passenger assignment and timetabling model could be used in the above framework. The ones selected in the implementation are discussed in the following sections.

From a computational point of view, the possible sources of sub-optimality in our approach stem from three main factors: i) the fact that we solve the IT-VSP heuristically with randomized selection of 350 trips to change at each iteration, ii) the fact that we impose a computational time limit on the solution approach and iii) that the proposed heuristic integration between passenger assignment and timetabling need not converge to the optimal passenger timetable. Furthermore, the quality of the initial timetable may entail the risk of missing a global optimum. Our modeling assumptions in terms of minimum and maximum turnaround times, and maximum driving distances are other sources of potential sub-optimality. Unfortunately we are unable to compute the exact solution for the problems in our case study. However, we will compare results obtained by the IPAT-VSP to several other benchmarks.

4.1 Passenger route choice model – the PTTA model

We measure passenger service using the PTTA of Briem et al. [4] to evaluate the WTT of a timetable. Furthermore, the PTTA provides input to the MHeu in terms of the set of transfer opportunities R and the number of passengers on board Λ.

The input to the passenger route choice model PTTA is a timetable $\mathscr{T}$, a set of possible transfer locations, the minimum required transfer time for a transfer to be feasible, an ODt matrix, and a cost function that specifies the relative weights of the travel time components. Output of the PTTA are the set of transfer opportunities R, the number of passengers on board for each trip $i \in T$ and each stop $s \in S_i$, Λ_{is}, and the WTT of the resulting passenger assignment.

To evaluate the quality of a journey, the representative utility is set to use *perceived arrival times* (PAT), which are a linear combination of: the actual arrival time, the waiting time, and the number of transfers. Building upon this, the random utility of a journey is the sum of its PAT and a random variable, which captures the uncertainty in the passengers route choice. When the PTTA model serves as input to the timetabling heuristic IT-VSP, different weights for the waiting time component are evaluated. While in theory every possible journey has to be considered when computing the assignment, this is not feasible in practice. Even if all journeys were to be considered, most of them would only be used by a negligible portion of passengers. Thus, we do not consider all possible journeys for the route choice. In particular, the choice set used by the PTTA consists of the journey that minimizes the PAT and all journeys with a PAT that does not differ from the optimal PAT by more than Δ_{max}, which is a tuning parameter of the PTTA. Depending on the probability distribution of the random variable within the random utility, a different decision model arises. In this work we evaluate our results for the Logit model [9] as well as the linear model proposed in [4].

The PTTA model is used to estimate which routes are likely to be chosen by passengers in a system with free route choice, as well as the expected number of passengers per route. The underlying model and algorithm to compute these routes were first presented in [4]. Conceptually, the PTTA model is a sequential route choice model, as proposed in [12]. This means, that decisions are not made based on complete routes, but one journey leg at a time. Given a passenger, a current location, and a destination, the model specifies which step is probably taken next by the passenger in order to reach the destination. The probability for every possible next step is determined using a random utility model, with the utility being influenced by several cost functions, such as travel time, waiting time, and number of transfers. The PTTA model iteratively repeats this process until every passenger reached its destination, thereby compiling the complete routes used by the passengers.

For a given destination and journey leg, the random utility model in the PTTA characterizes the likelihood of the journey leg being used as next leg of a route leading to the destination. Thus, the first step of computing the overall passenger assignment in the PTTA model consists of computing the utilities for all pairs of possible journey legs and destinations. The PAT at the destination when using the specific leg in turn determines the utility of a leg (for a given destination). The PAT is a linear combination, which besides the actual arrival time, factors in all criteria that effect the route choice. For this work, the PAT is the weighted sum of the actual arrival time, the number of transfers, and the time spent waiting for the next trip. An important aspect of the PTTA model is the algorithm that allows for an efficient computation of PATs for all pairs of journey legs and destinations. To this end, PAT values are computed for one destination at a time. For a given destination, the PATs of all possible journey legs are computed iteratively, sorted by time in decreasing order. A detailed description of the process can be found in [4]. The benefit of processing the journey legs in decreasing order of time is that for a given leg the PATs of all possible journey continuations are already known. Thus, the PAT of every leg can be computed quite efficiently. If the leg itself ends at the destination then the PAT of this leg is given by its actual arrival time (since a single leg does not comprise transfers by definition). If the leg does not end at the destination, then the route has to be continued with another leg. In this case the PAT is given as the PAT of the following leg (which is already known), plus the additional cost (transfer time, waiting time) to connect the legs.

After all PATs have been computed, the actual passenger route choice is determined using a simulation approach. For every passenger, a sequence of decisions is made based on a random utility model. Each of these decisions determines the next journey leg the passenger takes towards the destination. The choice set for this decision is determined on basis of the PAT values. It consists of the leg with the lowest PAT as well as all other legs, such that the difference of PATs in the choice set does not surpass a certain, user-defined limit ($\Delta_{\max}$). The utility of each leg ℓ in the choice set is then defined as $\max(0, \min_{\ell' \neq \ell}(\mathrm{PAT}_{\ell'}) - \mathrm{PAT}_{\ell} + \Delta_{\max})$. Finally, the probability of each leg in the choice set can be obtained using a random utility model. Proportional to these probabilities one leg is chosen, e.g. the passenger is assigned to this leg as part of his route. This process is repeated until all passengers have been assigned to full routes, reaching their destinations. In order to obtain a distribution of several routes that could be used by a passenger (alongside with their respective probabilities), several virtual passenger can be simulated for every actual passenger.

4.2 Integrated timetabling and vehicle scheduling – the IT-VSP formulation

The IPAT-VSP model (Section 3.2, (2) - (17)) can be formulated as a mixed-integer *linear* programming problem thanks to the split into a PAT and a timetabling model. The mathematical formulation for the IT-VSP is an extended version of the model in Fonseca et al. [10]. This extended version allows to explicitly include the effect of extended dwell time for on-board passengers, and uses passenger route choice information computed with the *Public Transport Traffic Assignment* (`PTTA`) model (Section 4.1). The model calculates operating costs in terms of vehicle schedules rather than the total number of vehicles, for the sake of tractability. The number of vehicle schedules forms an upper bound to the required number of vehicles. Secondly, our timetabling model's objective includes transfer time and extended in-vehicle time only, and not initial waiting time. All solutions are, however, evaluated under the (`PTTA`), that includes IWT. Under the (light) assumption that a preprocessing step can assign the two trips defining the initial waiting time for each origin-destination-time passenger triple, the extension of the model to include IWT in the objective, when estimated as half the headway between the two most recent trips, is trivial. Still, it would not reflect the exact same objective as the (`PTTA`) objective, as the latter could also have passengers select an alternative route. Finally, our timetabling model investigates the benefit of small changes in the timetable in the form of shifts and stretches, that could with zero costs to the operator improve passenger service, and may result in uneven headways. The discussions we have had with the public transport service provider Movia indicate that such a timetable can be interesting to them in practice, as they come at zero cost and also do not require new negotiations with all stakeholders on change of frequencies.

The input to the IT-VSP consists of an initial timetable for the set of all trips $i \in T$, passenger route choice information, a budget for the operating costs, and costs and parameters related with the case study, such as allowed headways, minimum and maximum dwell times, or minimum and maximum turnaround times. The passenger route choice information consists of a set of transfer opportunities R, where each $r \in R$ defines a transfer stop, a transfer-from trip $i \in T$, a desired line l to transfer to, and a number of passengers that are expected to make this transfer. Furthermore, Λ_{is} contains the expected on-board passengers per trip $i \in T$ at stop $s \in S_i$.

The objective of the IT-VSP is to minimize a weighted sum of passenger costs incurred by extending dwell times at stops for passengers on board, and passenger costs incurred when transferring. Passengers incur transfer time costs when transfers are above the minimum transfer time. Transfers below the minimum transfer time are infeasible, meaning that we predetermine which transfers at the end of the day should be feasible and cut off timetable modifications that would make them infeasible.

Decision variables consist of (changes in comparison to (2) - (17) marked in **bold**):

a) binary assignment variables $x_{ijk} \in \{0,1\}$ storing which vehicles are assigned to which trips, with the triplet (i,j,k) representing a trip j serviced immediately after a trip i with the same vehicle from depot k

b) departure and arrival time variables τ^d_{is} and $\tau^a_{is} \in \mathbb{Z}^+_0$ for each trip $i \in T$ and each stop $s \in S_i$

c) excess transfer time variables $\gamma_r \in \mathbb{R}^+_0$, which store the amount of excess transfer time for passengers using each transfer location $r \in R$

d) **binary transfer variables $\alpha_{ijs} \in \{0,1\}$, which indicate which trip $j \in T$ passengers of transfer location $r = (i, l_j, s) \in R$ embark**

e) dwell time variables $\delta_{is} \in \mathbb{Z}^+_0$, which store the number of minutes of dwell time added to trip $i \in T$ at stop $s \in J_i$

Passengers are assumed to transfer to the earliest feasible trip $j \in T_l$. A transfer r from trip i to line l at stop s, $r = (i,l,s) \in R$, is feasible when the minimum transfer time for transfer $r \in R$, e_r, is not greater than the difference between the departure time of trip $j \in T_l$ from stop s and the arrival time of trip i at stop s. The α_{ijs} variables indicating transfer opportunities are defined only for a set $W = \{(i,j,s)|i,j \in T, s \in S : r = (i,l,s) \in R, j \in T_l, i \neq j, a^-_{is} + e_r \leq d^+_{js}, a^+_{is} + e_r + 1.5h_l \geq d^-_{js}\}$, where h_l is the largest target headway observed for line $l \in L$ throughout the day. The value 1.5 is used such that we consider enough trips $j \in T_l$ for the transfer to be feasible, but not more than needed. This improves the tractability of the model by reducing the number of α_{ijs} variables created, without imposing any practical constraints, since at least one transfer to a trip in $l \in L$ will be available given the timetable modifications. For reference, a complete list of all sets, parameters and decision variables is presented in Appendix 1.

The MILP formulation for the IT-VSP is:

$$\min \sum_{i \in T} \sum_{s \in J_i} c^{OB} \Lambda_{is} \delta_{is} + \sum_{r \in R} c^{TR} f_r \gamma_r \tag{18}$$

$$\text{s.t.} \quad (3) - (12)$$

$$M \sum_{\substack{k \in T_l : (i,k,s) \in W, \\ k \leq j}} \alpha_{iks} \geq \tau^d_{js} - \tau^a_{is} - e_r \; r \in R \quad (i,j,s) \in W \tag{19}$$

$$\tau^d_{js} - \tau^a_{is} - e_r \geq M(\alpha_{ijs} - 1) \qquad r \in R \quad (i,j,s) \in W \tag{20}$$

$$\sum_{j \in T_l : (i,j,s) \in W} \alpha_{ijs} = 1 \qquad r = (i,l,s) \in R \tag{21}$$

$$\tau^d_{js} - \tau^a_{is} - e_r - M(1 - \alpha_{ijs}) \leq \gamma_r \quad r \in R \quad (i,j,s) \in W \tag{22}$$

$$\gamma_r \in \mathbb{R}_+ \qquad r \in R \tag{23}$$

$$(13) - (17)$$

The objective function (18) minimizes a weighted sum of passengers' costs. The first term refers to on-board passenger costs incurred when adding dwell time to trips. The second term refers to costs associated with excess transfer times. Constraints (3) - (12) are the same as in the IPAT-VSP (Section 3.2). Linking constraints (19) and (20) relate the transfer variables α_{ijs} and the departure and arrival times of trips: constraints (19) ensure that passengers arriving from trip i at stop s transfer to one of the trips j, such that $(i, l_j, s) \in R$, if the arrival and departure times allow the transfer to take place; constraints (20) prevent variable α_{ijs} from taking value 1 whenever passengers do not have enough time to transfer from trip i to trip j at stop s, where $(i, l_j, s) \in R$. Constraints (21) impose that each transfer location is performed by transferring to exactly one trip $j \in T_l$. Constraints

(22) define the values of γ_r variables to the excess transfer times, determining this value for each transfer location based on the selected transfers. Constraints (23), (13)-(17) define the range of all sets of decision variables.

The resulting IT-VSP is solved by a matheuristic approach `MHeu` based on the MILP formulation described above. A heuristic is necessary since solving real-life instances of the IT-VSP directly with a general solver is not feasible, due to the size of the instances making the problem intractable. The proposed matheuristic is also based on Fonseca et al. [10] and will next be described in detail in the context of the algorithm description.

4.3 Algorithm description

Algorithm 1 presents the pseudo code for the `MHeuPA` matheuristic proposed to find good quality solutions for the IPAT-VSP.

Steps 1-5 are the initialization procedure. The algorithm starts by calculating an assignment for the initial timetable $\mathscr{T}_0$ in step 1, using the `PTTA` with ϕ. The set of transfer opportunities R_0 and the vehicle occupancy Λ_0 computed by the `PTTA` are used as input to solve the MDVSP in step 2 without allowing timetable modifications, thus the departure and arrival times of all trips $i \in T$ from/to stop $s \in S_i$, τ^d_{is}, τ^a_{is}, are fixed to $\mathscr{T}_0$ (meaning that these trips will have arrival and departure times at all stops visited equal to the ones in the initial timetable $\mathscr{T}_0$). This is done both for creating initial vehicle schedules and for enabling a comparison between the initial solution in an unchanged timetable and the timetables we find later on. An initial solution $\mathscr{S}_0$ is defined in step 3, composed by vehicle schedules X_0, the initial timetable $\mathscr{T}_0$, and the initial assignment $\mathscr{A}_0$, and since this is the only solution so far, in step 4 it is also saved as the current best solution in terms of WTT. The iterative procedure is described in steps 6 - 16, which runs until the stop criterion *stopCriterion* is met.

Each iteration η starts by selecting in step 8 the subset of trips $T' \subset T$ to modify, being κ the number of trips selected. In this paper, T' is a set of 350 randomly selected trips at each iteration, which extensive tests in [10] found to be the preferred selection strategy. We direct the reader to [10] for more details on the parameter tuning experiments and comparison of different algorithm settings. Trips in T' are allowed modifications in arrival and departure times (shifts and stretches), while all other trips $i \in T \setminus T'$ remain fixed to the timetable in solution $\mathscr{S}_{\eta-1}$. In step 9, an assignment of passengers is calculated for the current timetable $\mathscr{T}_{\eta-1}$ according to `CostF`, generating transfer opportunities R' and vehicle occupancy Λ'. A new timetable $\mathscr{T}_\eta$ and the vehicle schedules X_η are calculated in step 10, solving the restricted IT-VSP(T'), with τ^d_{is}, τ^a_{is} fixed to $\mathscr{T}_{\eta-1}$ for all $i \in T \setminus T'$, optimizing the transfer opportunities R' considering the vector Λ' of passengers on board. The

Algorithm 1 : `MHeuPA`

Input: set of all trips T, initial timetable $\mathscr{T}_0$, origin-destination-time matrix ODt, stop criterion $stopCriterion$, realistic waiting cost value ϕ, a waiting cost function `CostF`

Initialization procedure:

$(\mathscr{A}_0, R_0, \Lambda_0) \leftarrow$ `PTTA`$(\mathscr{T}_0, ODt, \phi)$

$(\mathrm{X}_0,\ \mathscr{T}_0) \leftarrow$ solve IT-VSP`(18)-(23) + (3)-(17)` using R_0 and Λ_0 and with departure and arrival times $(\tau^d_{is},\ \tau^a_{is})$ fixed to $\mathscr{T}_0$ for all $i \in T$

$\mathscr{S}_0 = (\mathrm{X}_0,\ \mathscr{T}_0, \mathscr{A}_0)$

$\mathscr{S}^* = \mathscr{S}_0$

$\eta = 0$

Iterative procedure:

while $stopCriterion$ not reached **do**

  $\eta = \eta + 1$

  $T' \leftarrow$ `selectTrips`$(\mathscr{S}_{\eta-1})$

  $(\mathscr{A}', R', \Lambda') \leftarrow$ `PTTA`$(\mathscr{T}_{\eta-1}, ODt,$ `CostF`$)$

  $(\mathrm{X}_\eta,\ \mathscr{T}_\eta) \leftarrow$ solve IT-VSP`(18)-(23) + (3)-(17)` using R' and Λ' and with departure and arrival times τ^d_{is}, τ^a_{is} fixed to $\mathscr{T}_{\eta-1}$ for all $i \in T \setminus T'$

  $(\mathscr{A}_\eta, R_\eta, \Lambda_\eta) \leftarrow$ `PTTA`$(\mathscr{T}_\eta, ODt, \phi)$

  $\mathscr{S}_\eta = (\mathrm{X}_\eta,\ \mathscr{T}_\eta, \mathscr{A}_\eta)$

  if `WTT`$(\mathscr{S}_\eta) <$ `WTT`$(\mathscr{S}^*)$ **then**

    $\mathscr{S}^* \leftarrow \mathscr{S}_\eta$

  end if

end while

return $\mathscr{S}^*$

Output: Best solution found $\mathscr{S}^*$, composed by vehicle schedules X^*, timetable $\mathscr{T}^*$, and passenger assignment $\mathscr{A}^*$

realistic assignment (obtained using ϕ) of passengers $\mathscr{A}_\eta$ to the new timetable is calculated in step 11 by running the realistic PTTA. The iteration solution $\mathscr{S}_\eta$ is set in step 12. Steps 13-15 save $\mathscr{S}_\eta$ as the best solution $\mathscr{S}^*$ if the weighted travel time associated with it is lower than the weighted travel time associated with the current best solution. The IPAT-VSP concludes in step 17 by returning the best solution $\mathscr{S}^*$ found once the stop criterion *stopCriterion* is met.

Waiting cost functions As demonstrated in the example in Section 3.3, the objective is to find all *potentially beneficial transfer locations*, that is, transfer locations that, with a good synchronization of the transfer, could be used by passengers. Whether the transfer location will be used by passengers, depends on the quality of the set of alternative paths available, and therefore cannot be determined per transfer location independently. Different waiting cost functions are used in a desire to find all potentially beneficial transfer locations.

Each run of the `MHeuPA` uses exactly one of five different waiting cost functions `CostF`. These functions change the weight attributed to the waiting costs when computing a new passenger assignment to serve as input to the IT-VSP `MHeu`.

- Realistic (`Realistic`): this waiting cost function runs the `PTTA` model with the realistic value ϕ for the waiting costs at every iteration. It reflects a base-case where the PA of a current timetable is provided as input to the timetabling module.

- No waiting costs (`NoCosts`): this waiting cost function runs the `PTTA` model without waiting costs at every iteration. This will lead to passengers selecting the path with the minimal IVT.

- Linear ascending costs (`LinAsc`): this waiting cost function increases the waiting costs in iteration i, $WC(i)$, in the `PTTA` model. Assuming a total running time of *maxT* seconds, the `PTTA` iterations in the first $maxT/10$ seconds use a waiting costs parameter of 0. In the remainder of the iterations, the waiting costs increase linearly until the realistic value is achieved by the end of the experiment. The waiting costs at each iteration can be calculated using

$$WC(i) = \frac{\phi \cdot t(i)}{maxT}$$

  where $t(i)$ is the cumulative current total running time up to iteration i.

- Random waiting costs (`Random`): this waiting cost function runs the `PTTA` model with random waiting costs at every iteration of the `MHeuPA`, with a value between 0 and the realistic waiting costs $\phi = 2$, with a uniform distribution.

- Random and linear ascending costs (`RandLinAsc`): this waiting cost function combines the `Random` and `LinAsc` waiting cost functions. In the initial two thirds of the computational time, the `PTTA` model iterations use a random value between 0 and the realistic value for the waiting costs. In the last third, it uses linear ascending costs, calculated using

$$WC(i) = \frac{\phi(t(i) - 2maxT/3)}{maxT - 2maxT/3}$$

where $t(i)$ is the current total running time at iteration i.

The advantage of a lower waiting time costs is that transfer locations that currently have high waiting time, but would provide low in-vehicle time paths, will attract passengers – and thus provide an incentive to the timetabling model to improve the synchronization of trips at these transfer locations. The downside is that in dense networks that contain many connections between lines it is unlikely that *all* transfer locations will be able to provide perfect transfers. Thus, the real passenger assignment is likely to be different from the zero or small waiting cost assignment; and therefore the trade-offs made in the timetable may be non-optimal due to erroneous numbers of expected passengers per transfer location, and expected passengers on board.

Additionally, we define `Route_Fixed` as running the `MHeuPA` with a single, fixed passenger assignment based on an initial timetable. Thus in `Route_Fixed` the number of transferring passengers per transfer location is fixed during the timetabling phase, and no in-between calls are made to the passenger assignment model. First the PTTA is run on the initial timetable, next IT-VSP is run without any updates on passenger flows. Finally, the resulting timetable is evaluated based on the PTTA. This represents the approach of [10] and has the same assumptions about passenger behaviour as e.g. [16]. It thus provides a baseline to compare the importance of acknowledging the dynamics between passenger route choice and the provided timetable. To allow a fair comparison between the approach proposed in this paper and these previous approaches, we do evaluate the final timetable again under the passenger route choice model `PTTA`, our best representation of realistic route choice, and compare results based on total WTT.

5 Case study

5.1 Case study specific information

For the experimental section, we focus on a subset of the public transport network in the Greater Copenhagen area. We consider 8 bi-directional express-bus lines, referred to as S-Bus lines. In comparison to regular bus lines, these are faster

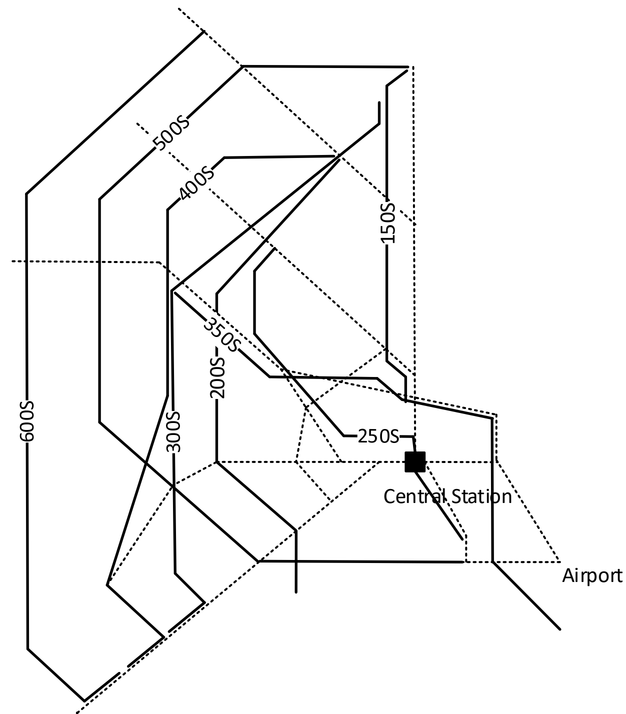

Figure 3: Geographic representation of the case study network. The thick lines represent the S-Bus network, while the dashed lines represent the S-Train, train, Metro and fixed bus lines

and with fewer stops, acting mainly as a complement to the local urban trains (S-Train) across and radially. Figure 3 depicts a geographical representation of the network, which includes not only the S-Bus and S-Train lines but also two bi-directional Metro lines, one bi-directional train line, and one high-frequency bus line that connects the city center to the airport. The public transport service provider Movia, which is responsible for the planning of buses in the eastern part of Denmark, provided timetable data for a generic weekday in November 2016. We allow timetable modifications by shifts and stretches to the trips in the S-Bus lines, while all other lines in the case study operate according to a fixed timetable. The vehicle scheduling component of the IPAT-VSP is solved solely for the S-Bus trips.

The data input for the IPAT-VSP is composed by: (i) a *initial timetable* for all S-Bus lines; (ii) a *fixed timetable* for all other lines in the network; (iii) a *distance matrix* with all distances between stops and depots; (iv) an *ODt matrix* describing passenger demand for the full network; (v) *costs and parameters* specific to the case study: minimum and maximum turnaround times, minimum transfer time at stops, vehicle operating costs, fixed costs per vehicle schedule created, travel time and waiting time costs for passengers, driving speed for vehicles while deadheading, maximum deadhead distance, possible changes in headways, maximum shifts, maximum added dwell time per trip and per stop, and depot capacities. We did not obtain data for the minimum dwell times per stop, so we assume that the dwell times in the base timetable are the minimum dwell times, as most of them are equal to 0 minutes. Existing dwell times can reflect the need to switch driver, for example, or locations where it is known that the boarding and unboarding of passengers takes longer. Thus, including these ensured feasibility. However, this is a pure input decision as the model could easily also re-adjust these dwell times.

5.2 Timetabling and Vehicle Scheduling related parameters

The timetables used in input components (i) and (ii) are publicly available and the distance matrix (iii) was obtained using geographical data. The ODt matrix (iv) was provided by Rapidis[1], and it describes hourly demands for each OD pair. To generate the required minute-by-minute ODt information, we distributed the hourly OD demand evenly over each hour. The ODt contains 164,333 entries representing 170,117 passengers. As for costs and parameters (v), these were estimated together with Movia. We use estimates of operating waiting time, distance, and schedule costs expressed in Danish kroner (DKK, 1 Euro is equivalent to approximately 7.5 DKK), which together define the operating costs of a solution. Travel times are weighted by an hourly value of time (VOT) factor of 100 DKK, while initial waiting times and transfer times are weighted by an hourly VOT factor of 300 DKK. The fixed cost for creating a new vehicle schedule is 1,100 DKK, and was calculated based on the yearly fixed cost of using a vehicle, provided by Movia. (The minimum transfer time is considered to be 4 minutes in all stops of the network.) We used value of time studies developed at the Center for Transport Analytics at the Technical University of Denmark as inspiration for these values[2].

Table 1 shows information about all lines included in the case study network. The first column indicates the name of the line, followed by the mode of transport, the number of stops with transfer opportunities, the number of trips, an indication of whether the trips in the line are part of the timetabling design or not, the minimum headway in the initial timetable, and the maximum headway in the initial timetable. The same line may have different target headways during the day,

[1]Rapidis is a Danish company that develops tools for planning in Transportation and Logistics. website: `http://www.rapidis.com/`

[2]Center for Transport Analytics website: `http://www.cta.man.dtu.dk/`

Table 1: Lines in the case study network

Line name	Mode	Stops	Trips	Timetabling	Min scheduled headway	Max scheduled headway
150S	S-Bus	5	256	yes	4	21
200S	S-Bus	7	186	yes	6	23
250S	S-Bus	7	161	yes	8	24
300S	S-Bus	8	230	yes	5	23
350S	S-Bus	12	304	yes	4	22
400S	S-Bus	7	140	yes	7	21
500S	S-Bus	8	167	yes	7	31
600S	S-Bus	7	141	yes	4	34
A	S-Train	10	202	no	10	20
B	S-Train	8	202	no	10	20
Bx	S-Train	4	8	no	20	20
C	S-Train	8	205	no	10	20
E	S-Train	7	200	no	9	21
F	S-Train	3	374	no	5	10
H	S-Train	12	115	no	20	20
KB	Train	6	246	no	3	32
M1	Metro	7	487	no	2	12
M2	Metro	6	450	no	2	12
5C	Bus	8	542	no	4	4

Table 2: Allowed headway variations based on scheduled headways

Scheduled headway (m)	Minimum and maximum headway variation (m)
$= 4$	+/- 1
≤ 12	+/- 2
≤ 20	+/- 3
≥ 21	+/- 4

and the stopping pattern of some lines also changes during the day (for example, only servicing a part of the line during peak-hour and short turn the buses off-peak hour).

For each trip $i \in T_l$ at each stop $s \in J_i \cup st_i$, minimum and maximum headways, h_{is}^- and h_{is}^+, are calculated based on the scheduled headways between trip i and its immediate precedent trip in the line, trip $i-1$, as indicated in Table 2. Notice that in our case study the minimum scheduled headway is 4 minutes. In this case, the minimum and maximum headways allowed will be 3 and 5 minutes respectively.

The maximum dwell time added at each stop is 3 minutes (i.e., $w_{is}^+ = 3, i \in T, s \in J_i$), and a maximum of 10 minutes of dwell time can be added in total to a trip (i.e., $w = 10$). The added dwell time is deducted from the buffer in the turnaround time at the end of the trip, i.e the turnaround time in excess to the minimum turnaround time. For example, assume that a vehicle is supposed to service two trips and has 15 minutes of turnaround time between them. If we add 2 minutes of extra dwell time in the first trip, then instead of 15 minutes of turnaround time it will have 13 minutes. However, if instead of 2 minutes of added dwell time we added 4 minutes, the turnaround time will be 12 minutes and not 11 minutes, since the minimum turnaround time is 12 minutes.

The shifts allowed in each trip departure time were created based on the initial timetable for each bus line. Considering consecutively timetabled trips $(i-1), i, (i+1) \in T_l$ and with departure time from the first stop $d_{i-1,st_i}, d_{i,st_i}, d_{i+1,st_i}$ respectively, the lower and upper shift limits for trip i are calculated with the expressions

$$d^-_{i,st_i} = d_{i,st_i} - \left\lfloor \frac{d_{i,st_i} - d_{i-1,st_i} - 1}{2} \right\rfloor \quad , \quad d^+_{i,st_i} = d_{i,st_i} + \left\lfloor \frac{d_{i+1,st_i} - d_{i,st_i}}{2} \right\rfloor$$

ensuring that trips can never overtake each other in the timetable.

As they are not part of the input, vehicle schedules that cover the initial timetable for the S-Bus trips are calculated using an MDVSP model. The solution consists of 205 vehicle schedules that cover the 1585 S-Bus trips. It uses constraints (4)-(6) of the mathematical model in Section 4.2. The initial timetable considers time dependent service times, but the mathematical model uses constant deadhead speeds along the day. Trips from different lines can be included in the same vehicle schedule, which is known as *interlining*, thus allowing deadheading between consecutive trips in a schedule. The maximum deadhead distance is 15 kilometers (i.e., $u = 15$), the minimum turnaround time is 12 minutes (i.e., $q^- = 12$), and the maximum turnaround time is 30 minutes (i.e., $q^+ = 30$).

5.3 Passenger Assignment related parameters

Several parameters affect the route choice of the passengers in the PTTA model. In particular, these are the coefficients of the linear combination that define the PAT and the parameter for the composition of the choice set. All of these parameters have to be estimated on empirical data in order to reflect passenger behaviour. However, since our approach works for arbitrary choices of these parameters, we simple use the values proposed in [4]. To this end, the PAT is a linear combination where the arrival time is weighted with a factor of 1, the waiting time is weighted with a factor of 2 (which is the waiting cost WC we refer to in the *Waiting Cost Functions* described in 4.3), and the number of transfers is weighted with a factor of 5 min. Finally, for pruning the choice set, we use the cutoff value $\Delta_{\max} = 15$ min.

6 Computational experiments

This section evaluates the performance of the MHeuPA through a set of computational experiments for the case study of the Greater Copenhagen Area described in Section 5. Our paper considers a much larger real life network, compared to the related literature of [26] and [15], in which the largest cases reported are only 4 lines and 4 transfer stops. Results of the MHeuPA for different waiting cost functions (Section 4.3) are compared to the fixed passenger route choice model of Fonseca et al. [10], the IT-VSP MHeu. Note that results between this paper and [10] cannot be directly compared as the current case study represents a larger network, with new detailed passenger demand information that was not available yet during the Fonseca et al. [10] study; and secondly due to a different measure of passenger service, which in this paper is represented as WTT calculated by the passenger route choice model of Briem et al. [4]. Route_Fixed is an exact representation of the model of Fonseca et al. [10] in this new setting. Thus, the comparison between the MHeuPA and the Route_Fixed demonstrates the value of including free passenger route choice.

We evaluate our approach in the following three situations:

- *In comparison to an initial timetable representing the current timetable for our case study area* (Section 6.1). This case study is similar to the setting of [10], and therefore allows the most direct comparison between fixed and free passenger route choice. This section presents a detailed analysis of the results for the different components of weighted travel time, benefits specifically for transferring passengers, and the resulting vehicle schedules.

- *In case of a change in the public transport network* (Section 6.2). A change in the public transport network results in a timetabling situation where one would expect a change in passenger route choice. This situation is simulated by offsetting the timetables of one, or a set, of public transport lines in the network, such that headway constraints and time-dependent vehicle travel times are still respected, but transfers are likely offset.

- *In case of a change in passenger demand* (Section 6.3). A change in the passenger demand matrix could lead to a change the relevance of transfer opportunities: making some transfer opportunities more important than others; for instance in case of a special event. This could also make it important to consider free passenger route choice. We assume that, whatever the change in demand, sufficient capacity is available, as measures to increase capacity on routes (e.g. longer vehicles, or higher frequencies) are not part of the timetabling decisions considered in this paper.

The algorithm is implemented in C++ and uses CPLEX version 12.6 to solve the mathematical program at each iteration. All experiments were conducted on HPC servers, using Intel Xeon E5-2660 v3 2.60GHz processors, and 8 computation cores. Each iteration uses CPLEX warm-start to start from the previous solution. Presented are average results over five runs with each setting, with a 3 hour computation time limit. All computational times reported in this section are wall-clock times. Fixed parameters are the number of trips selected per iteration $\kappa = 350$, the maximum running time per iteration $\psi = 30$, and the realistic value for waiting costs $\phi = 2$. The values for the parameters κ and ψ are based on the computational results of [10] and taking into account that the current case study is larger both in terms of network and OD matrix, while the value for ϕ is based on the findings of [4].

The solution quality is expressed in terms of weighted travel time (WTT) and its components: in-vehicle time (IVT), initial waiting time (IWT), and transfer time (TrT). We also compare the operating costs (OpC) across experiments. To compare the solutions obtained with the `MHeuPA` with the initial timetable and with the solutions obtained with the `Route_Fixed`, we use percentage improvements to initial and percentage improvements to `Route_Fixed`. For $x = \{\texttt{WTT}, \texttt{IVT}, \texttt{IWT}, \texttt{TrT}, \texttt{OpC}\}$ and $\overline{f}_x(S)$ being the x-type average cost of a solution S over n runs, we calculate the percentage improvement to the initial timetable as

$$\frac{\overline{f}_x(\mathscr{S}_{\texttt{MHeuPA}}) - f_x(\mathscr{S}_{\texttt{Initial}})}{f_x(\mathscr{S}_{\texttt{Initial}})} \times 100\%$$

and the percentage improvement to the solutions obtained with the `Route_Fixed` as

$$\frac{\overline{f}_x(\mathscr{S}_{\texttt{MHeuPA}}) - \overline{f}_x(\mathscr{S}_{\texttt{Route\_Fixed}})}{\overline{f}_x(\mathscr{S}_{\texttt{Route\_Fixed}})} \times 100\%$$

since there is only one solution for the initial timetable but n solutions for the `Route_Fixed` (one for each run). A negative percentage for x corresponds to a reduction of costs in the `MHeuPA` solution in comparison to the initial timetable or to the `Route_Fixed` solutions. Since we use the budget version of the IT-VSP `MHeu` in all experiments, we keep the operating costs under a budget. We consider as budget the operating costs obtained by solving the MDVSP for the initial timetable.

6.1 Results for the base scenario

In this section, we present the results for the base scenario: initial timetable and base ODt matrix. We present results in terms of WTT and different WTT compo-

Table 3: WTT results for the base scenario for 1) the full ODt matrix and 2) for the transferring passengers only

	Full ODt Matrix			Zoom on transferring passengers		
Solution	WTT (DKK)	Improv. to Base (%)	Improv. to Route_Fixed (%)	WTT (DKK)	Improv. to Base (%)	Improv. to Route_Fixed (%)
initial timetable	14,952,021	-	-	11,281,416	-	-
Route_Fixed	14,844,516	-0.72	-	11,153,787	-1.13	-
Realistic	14,832,916	-0.80	-0.08	11,138,733	-1.26	-0.13
NoCosts	**14,805,563**	**-0.98**	**-0.26**	**11,110,820**	**-1.51**	**-0.39**
LinAsc	14,810,788	-0.94	-0.23	11,114,746	-1.48	-0.35
Random	14,811,079	-0.94	-0.23	11,113,400	-1.49	-0.36
RandomLinAsc	14,811,457	-0.94	-0.22	11,112,630	-1.50	-0.37

nents, operating costs, timetable modifications and characteristics of vehicle schedules. Additionally we show a convergence analysis for the different waiting cost functions considered and histograms of WTT variations.

Table 3 shows the WTT results for the different waiting cost functions in absolute WTT values, percentage improvement to initial timetable, and percentage improvement to Route_Fixed solution. We present results both for the full ODt matrix and for a version of the ODt matrix that considers only passengers that, given the public transport network lines, will have to transfer at least one time.

The results in Table 3 support the hypothesis that inclusion of free route choice leads to timetables with higher passenger service, and that using alternative cost models, to find potentially beneficial transfer locations, also enable finding timetables with higher passenger service. Indeed, all MHeuPA solutions improve passenger service in comparison to Route_Fixed solutions by 0.08% to 0.26% in the full ODt matrix and by 0.13% to 0.39% in the zoom on transferring passengers. The NoCosts is the best performing waiting cost function, both for the full ODt matrix and for transferring passengers only, while all alternative cost functions improve on input from the Realistic assignment model. All approaches improve passenger service in terms of WTT in relation to the initial timetable. The improvement in passenger service stems mainly from improved WTT for passengers with a transfer in their path, which is observed when comparing the results for the full ODt matrix with the results for transferring passengers only.

Figure 4 shows the convergence of WTT for the three best CostF functions LinAsc, NoCosts, and RandomLinAsc. The horizontal axis shows the algorithm total computational time in minutes and the vertical axis shows the percentage reduction in WTT in comparison to the initial timetable. The figure shows that the LinAsc waiting cost function has a steeper initial decline in WTT, while NoCosts finds the overall minimum WTT from around 120 minutes of computational time.

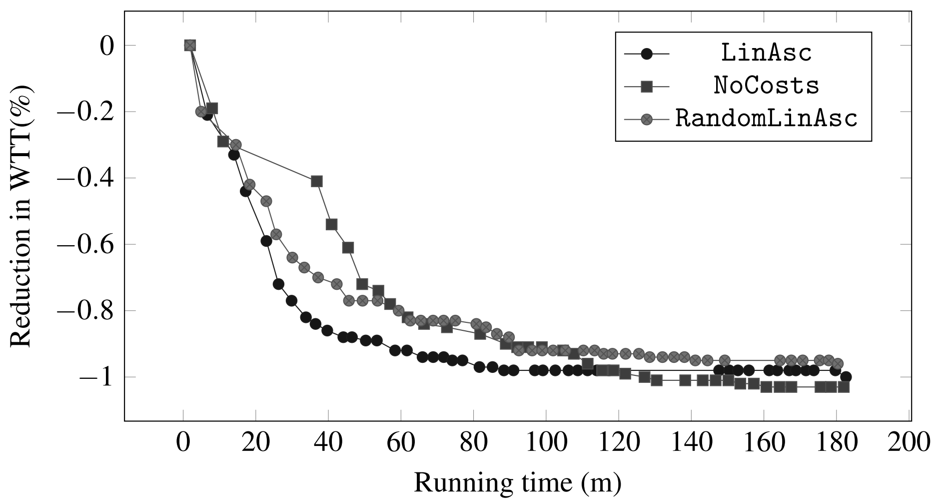

Figure 4: WTT convergence of the `LinAsc`, `NoCosts`, and `RandomLinAsc` solutions over time

Table 4: Passenger WTT improvements for the base scenario

Solution	Avg. improvement (DKK)	Pax better off (%)	Pax worse off (%)	Perc of pax with WTT reduction >10 DKK (%)	Perc of pax with WTT increase >10 DKK (%)
`Route_Fixed`	-1.12	34.37	28.52	10.23	6.84
`Realistic`	-1.21	35.41	29.64	12.43	8.17
`NoCosts`	-1.45	35.20	31.19	13.22	9.64
`LinAsc`	-1.40	35.72	30.79	13.40	9.28
`Random`	-1.39	35.86	30.94	13.54	9.40
`RandomLinAsc`	-1.39	36.03	30.94	13.74	9.59

This indicates that the `LinAsc` waiting cost function allows to find good timetables fast, but in the long run it is better to use the `NoCosts` waiting cost function. For all waiting cost functions, the improvements in WTT decline after the first 1.5 hours of computational time.

Table 4 shows for all waiting cost functions the average improvement in WTT expressed in DKK, the percentages of passengers better and worse off, the percentage of passengers better off by more than 10 DKK of WTT, and the percentage of passengers worse off by more than 10 DKK of WTT. The value of 10 DKK of WTT is equivalent to 6 minutes of in-vehicle time or 2 minutes of excess transfer time or initial waiting time.

Table 4 shows that the `NoCosts` waiting cost function achieves the best average improvement in WTT per passenger, with a value of -1.45 DKK, which follows from the results in Table 3. The percentage of passengers better and worse off is similar across waiting cost functions, with more passengers being better off than worse off when compared with the initial timetable. Specifically for differences in WTT larger than 10 DKK, the percentages of passengers better off are higher than the percentages of passengers worse off, as evidenced by the last two columns in Table 4. Although the `NoCosts` waiting cost function achieves the best average improvement, it is not the one that achieves the highest percentage of passengers better off, with the `RandomLinAsc` surpassing the 36% mark. If the objective is to maximize the percentage of passengers better off by a certain threshold or to minimize the percentage of passengers worse off it might be preferable to use other waiting cost functions than the `NoCosts` waiting cost function for this instance.

For better understanding how the `MHeuPA` solutions improve the WTT for passengers, Figures 5 and 6 show histograms of changes in WTT in the best solutions obtained by the `Route_Fixed` and by the `MHeuPA`. The horizontal axis shows the change in WTT experienced by passengers and the vertical axis shows the absolute number of passengers that experience changes in each interval. The histogram is divided into two figures due to the difference in magnitude of the number of passengers.

Figures 5 and 6 show that in the `MHeuPA` solution more passengers experience high decreases in WTT, especially in the interval $[-70,-20[$. From Figure 6, it is clearly observed that the amount of passengers on the left hand side of the histogram (passengers better off) is larger than the one on the right hand side (passengers worse off), which is linked to the results in Table 4. However, Figure 5 shows that in the `Route_Fixed` solution more passengers experience smaller changes in WTT than in the `MHeuPA` solution, between -10 DKK and 10 DKK. Additionally, Figures 5 and 6 show that the `Route_Fixed` solution has less passengers worse off than the `MHeuPA` solution, which is explained by existing in general less changes in the timetable. The timetables of the `MHeuPA` represent a different trade-off between passenger groups, and although not a strict improvement for all passengers; the disbenefits for some passengers are offset by the benefits for a larger group of other passengers.

Table 5 shows the results in terms of each of the components of WTT, for the same set of experiments as in Tables 3 and 4. The table contains the absolute values in DKK of in-vehicle time (IVT), initial waiting time (IWT), and transfer time (TrT), along with their percentage improvements in relation to the initial timetable.

The results in Table 5 show that all components of WTT are improved in relation to the initial timetable, as evidenced by the negative percentages. Most of the improvement in WTT comes from the improvement in transfer times, with decreases ranging from -2.59% to -3.17% compared to decreases of -0.13% to -0.34% in in-vehicle time and of -0.04% to -0.40% in initial waiting time. This is

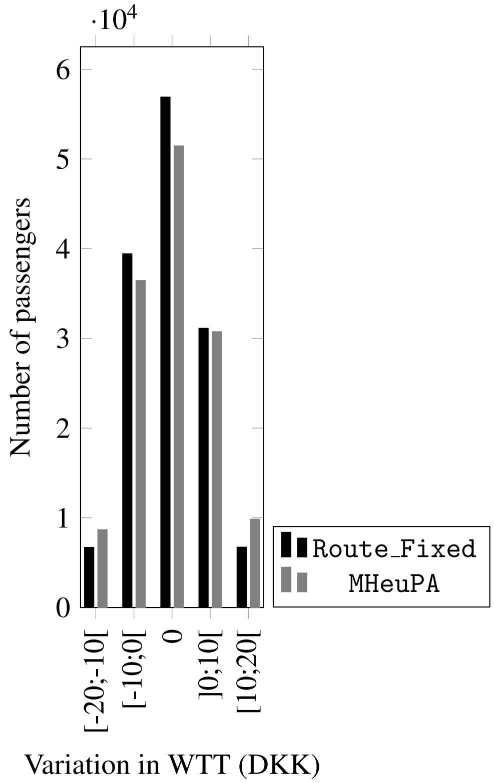

Figure 5: Histogram of variation in WTT for the Route_Fixed solution and for the best solution obtained with our algorithm

Table 5: WTT details for the base scenario

Solution	IVT (DKK)	Improv to Base (%)	IWT (DKK)	Improv to Base (%)	TrT (DKK)	Improv to Base (%)
initial timetable	7,399,710	-	4,043,178	-	3,509,133	-
Route_Fixed	7,390,300	-0.13	4,036,102	-0.18	3,418,115	-2.59
Realistic	7,389,179	-0.14	4,041,434	-0.04	3,402,302	-3.04
NoCosts	7,374,506	-0.34	4,026,984	-0.40	3,404,074	-2.99
LinAsc	7,378,596	-0.29	4,034,440	-0.22	3,397,752	-3.17
Random	7,378,806	-0.28	4,033,539	-0.24	3,398,735	-3.15
RandomLinAsc	7,379,542	-0.27	4,033,954	-0.23	3,397,962	-3.17

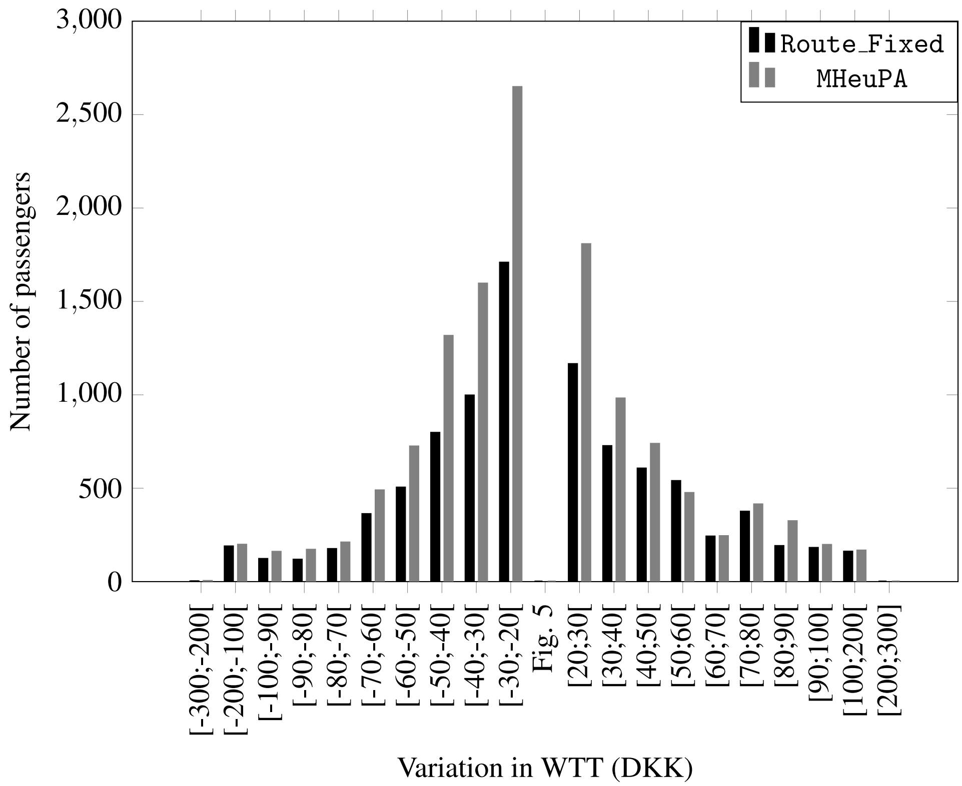

Figure 6: Histogram of variation in WTT for the Route_Fixed solution and for the best solution obtained with our algorithm

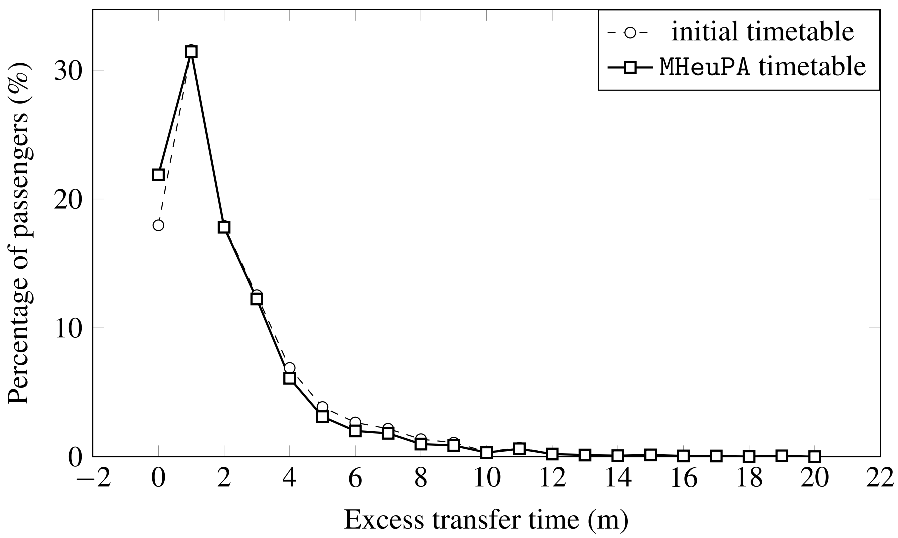

Figure 7: Comparison of excess transfer time in the initial timetable and in the best MHeuPA solution timetable

expected, since transfer time is the WTT component that is specifically considered in the IT-VSP objective function. Among the MHeuPA results, the NoCosts waiting cost function obtains the smallest reduction in TrT, with a value of -3.00%, but obtains the highest reductions in IVT and IWT, respectively -0.34% and -0.40%. Improvements in IVT show that passenger routes actually change in comparison to the initial timetable, because trips can only have additional dwell time and not less dwell time than in the initial timetable. This indicates that passengers are able to find better routes to travel from origin to destination, spending less time in-vehicle. Improvements in IWT are incidental since the IT-VSP objective does not include the effect of timetable modifications on IWT. Reductions in IWT can be explained by: 1) a reduction in headway at a time where more passengers are waiting to be served, 2) a route with a higher frequency (lower headway) has attracted more passengers: e.g. due to it now providing a better transfer, or a previous selected route providing a worse transfer. Trips are shifted to cater for transfer synchronization, but in the process also IWT could be reduced.

Figure 7 visualizes the excess transfer time (transfer time minus the minimum required transfer time for a feasible transfer) experienced by passengers in the initial timetable and in the best MHeuPA solution. The horizontal axis shows the excess transfer time in minutes and the vertical axis shows the percentage of transferring passengers that experiences that value of excess transfer time.

Table 6: Operating costs, timetable and vehicle schedules for the 8 line case and for the base OD matrix

Solution	OpC (DKK)	OpC improv to Base (%)	Trips with shifts only (%)	Trips with stretches only (%)	Trips with shifts and stretches (%)	Avg added dwell (m)	Avg added shift (m)	Number Schedules	Avg schedule duration (m)
initial timetable	323,744	-	-	-	-	-	-	205	784
Route_Fixed	317,417	-1.95	48.3	3.8	15.4	2.3	1.7	200	802
Realistic	317,608	-1.90	52.8	3.3	14.2	2.4	2.0	201	800
NoCosts	318,124	-1.74	55.1	3.8	14.0	2.2	1.9	201	798
LinAsc	318,437	-1.64	56.6	2.6	13.6	2.3	2.0	201	798
Random	316,754	-2.16	54.0	4.0	17.1	2.3	2.0	200	802
RandomLinAsc	316,649	-2.19	54.4	3.8	16.6	2.4	2.1	200	803

Figure 7 shows that in the MHeuPA timetable there are significantly more passengers experiencing perfectly synchronized transfers, with 0 minutes of excess transfer time. A total of 22% of transferring passengers experience perfectly synchronized transfers in the MHeuPA solution, while in the initial timetable this value is 18%. For all other excess transfer time values, the initial timetable has more passengers experiencing each value. Furthermore, the average excess transfer time decrease from 2.29 m to 2.06 m in the MHeuPA solution, for more than 100,000 transferring passengers.

Table 6 shows results for the same set of experiments from an operating perspective. The table contains information on absolute value of operating costs in DKK, percentage improvement in relation to the initial timetable, percentage of trips modified by shifts only, stretches only, and both, average added stretches, average added shifts, number of vehicle schedules, and average schedule duration in minutes.

The results in Table 6 show that all waiting cost functions use less operating costs than the budget of the initial timetable, but also considerably decrease them, with percentages between 1.64% and 2.19%. The solutions obtained with the MHeuPA shift 52.8% to 56.6% of the trips, add stretches to 2.6% to 4.0% of the trips, and add both shifts and stretches to 13.6% to 17.1% of the trips. The values of added dwell time and added shifts are averages over the total number of trips with added stretches and added shifts respectively. On average, just 2 minutes of dwell time are added to modified trips, and in Appendix 2 we show a histogram of added dwell time in a representative solution obtained with our approach. Regarding the vehicle schedules, the Route_Fixed and MHeuPA solutions use 200 to 201 schedules to cover all trips, while the base solution uses 205. Furthermore, schedules are on average longer in the Route_Fixed and MHeuPA solutions with values ranging between 798 and 803 minutes, compared to the 784 minutes in the initial timetable. This means that the Route_Fixed and MHeuPA solutions use resources

Table 7: Number of timetabled trips and number of vehicle schedules assigned to each of the four depots in the initial timetable and in the MHeuPA timetable

	Original schedules		MHeuPA schedules	
Depot	Trips	Schedules	Trips	Schedules
1	595	75	616	75
2	475	56	458	55
3	377	54	373	52
4	138	20	138	20

more efficiently, with schedules covering on average more trips.

Figure 8 and Tables 7 and 8 contain additional information on the obtained vehicle schedules for the initial timetable and the best MHeuPA timetable. In Figure 8, we present a histogram of the number of timetabled trips per vehicle schedule created in both cases. The MHeuPA solution approach is able to create vehicle schedules with a larger number of timetabled trips, with an average number of 7.84, while in the initial timetable the average number of timetabled trips per vehicle schedule is 7.73.

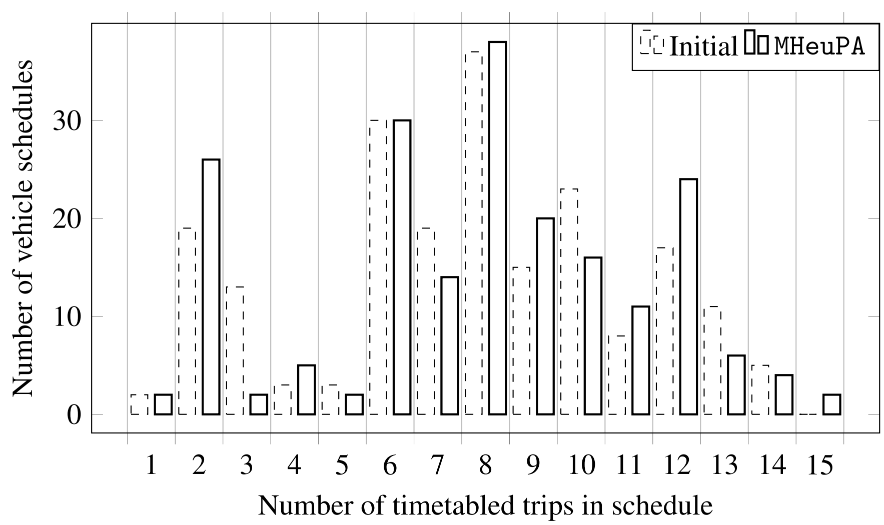

Figure 8: Histogram of number of of trips in schedules in the original solution and in the MHeuPA best solution

Table 7 outlines how many timetable trips and vehicle schedules are assigned to each of the four depots in the case study, both for the initial and for the best

Table 8: Distribution of timetabled trips per depot in the initial and in the `MHeuPA` schedules

		`MHeuPA` timetable			
	Depot	1	2	3	4
Initial Timetable	1	377 (23.8%)	127 (8.0%)	71 (4.5%)	20 (1.3%)
	2	146 (9.2%)	251 (15.8%)	69 (4.4%)	9 (0.6%)
	3	76 (4.8%)	69 (4.4%)	212 (13.4%)	20 (1.3%)
	4	17 (1.1%)	11 (0.7%)	21 (1.3%)	89 (5.6%)

`MHeuPA` solution. Both cases see depot 1 being the most utilized, and in both cases it is at its maximum capacity of 75 schedules, reinforcing that this depot is located at a more convenient geographical location than the other three depots. The number of timetabled trips and schedules of depot 4 remains unchanged in the `MHeuPA` solution when comparing to the initial solution. Depots 2 and 3 experience a decrease in both number of timetabled trips and number of schedules, showing that those timetabled trips were able to be serviced by one of the schedules of depot 1 without increasing the number of schedules assigned to the first depot.

Table 8 shows additional information on how timetabled trips change between depots in the initial and in the best `MHeuPA` solution, both in terms of absolute and percentage values of the total number of timetabled trips. As an example, timetabled trips in entry (a,b) indicate that these trips were assigned to depot a in the initial solution and to depot b in the `MHeuPA` solution. A total of 58.6% of all timetabled trips, which corresponds to 929 trips (sum of the diagonal values in the table), do not change between depots in the initial and in the `MHeuPA` solution. Nevertheless, the remaining 41.4% of timetabled trips switch their depot, which indicates that timetabled modifications have a significant impact on the vehicle schedules created.

The overall improvement in WTT in comparison to the initial timetable is approximately 1%, of which 0.25% is due to the inclusion of free passenger route choice. The 0.25% is equivalent to a daily reduction of approximately 40,000 DKK when expressed as value of time. Due to the budget constraints, these savings come at no additional operating costs, and in fact allow a reduction of operating costs, as demonstrated in Table 6.

6.2 Designing timetables - changes in the public transport network

In this section, we analyze if our model is able to find good timetabling solutions when new geographical lines are introduced to the network. In case of changes

in the physical network, we would also expect changes in the passengers route choices. Thus the combination between passenger route choice and timetabling is essential to find timetables with high passenger service. As we would like to compare these results to the situation where a good integration between lines and a high quality initial timetable already exists, we simulate the introduction of new lines by the following procedure.
For a new line, we assume that a target headway and a set of trips for this line is predetermined. Next, an initial timetable is generated under the assumption of even headways. This timetable would be optimal for direct travellers when demand is evenly distributed over time. Next we apply our modelling framework to illustrate that indeed our model is able to integrate such lines into the already existing network.

We test four different scenarios:

1. introduction of 1 ”new” line: 350S only, which is the line that transports the largest volume of passengers in the network;
2. introduction of 3 ”new” lines: lines 250S, 300S, and 400S, which are the lines with largest volumes of passenger transfers for all modes;
3. introduction of 3 ”new” lines: lines 350S, 500S, and 600S, which are the lines with largest volumes of passenger transfers involving a bus trip;
4. introduction of a fully ”new” S-Bus network.

This section demonstrates that also provided such a ”bad” starting point, the `MHeuPA` can be used for constructing a timetable. Thus in principle it is suitable for designing new timetables. Results for the best performing waiting cost functions from the previous section (`NoCosts`, `LinAsc`, and `RandomLinAsc`) are compared to solutions obtained with the `Route_Fixed` (fixed passenger route choice).

The creation of the initial timetables for the ”new” lines may also change the vehicle schedules obtained when solving the MDVSP. In order to enable a fair comparison between results in this section and in the previous section, we use as budget for the operating costs the same budget used before, i.e. the operating costs obtained by solving the MDVSP for the initial timetable. Table 9 shows results for all four scenarios in terms of absolute WTT and DKK savings in comparison to the `Route_Fixed` solutions. Each scenario is associated with an χ value, which is the percentage increase in WTT of the new timetable in comparison to the initial timetable.

The results in Table 9 show that all `MHeuPA` solutions have a lower WTT than the solutions obtained with the `Route_Fixed`, similarly to what is observed in Section 6.1. Again, the `NoCosts` waiting cost function is the one that shows the largest

Table 9: WTT results for the four scenarios of designing timetables

Offset lines (α)	350S (0.07)		250S, 300S, 400S (0.16)		350S, 500S, 600S (0.13)		All S-Bus network (0.39)	
Solution	WTT (DKK)	Savings ict Route_Fixed (DKK)	WTT (DKK)	Savings ict Route_Fixed (DKK)	WTT (DKK)	Savings ict Route_Fixed (DKK)	WTT (DKK)	Savings ict Route_Fixed (DKK)
Route_Fixed	14,858,155	-	14,862,350	-	14,886,838	-	14,928,154	-
NoCosts	14,813,879	-44,276	14,817,685	-44,665	14,840,451	-46,388	14,879,730	-48,424
LinAsc	14,818,344	-39,811	14,832,089	-30,261	14,843,607	-43,231	14,882,466	-45,689
RandomLinAsc	14,822,558	-35,596	14,832,355	-29,995	14,840,764	-46,074	14,894,508	-33,646

decreases in WTT. By comparing Tables 3 and 9, we see that the solutions in Table 9 also have a lower WTT than the initial timetable of Section 6.1, despite starting from a worse timetable (evidenced by all χ values being positive). Solutions obtained in the previous sections with the same waiting cost functions are better in terms of WTT by 0.05% to 0.56% than solutions obtained in this section, since they start from a timetable with more transfer synchronization and therefore lower WTT.

6.3 Changes in passenger demand

In this section, we test the MHeuPA for a change in ODt matrix in comparison to the base matrix. We test two different scenarios:

1. a random variation in the base ODt matrix (Random $\pm$10%);
2. an event simulation ODt matrix (Event Simulation).

One ODt matrix is generated for each scenario. The random variation scenario was generated by varying OD hourly demand in the base ODt matrix randomly by a value between -10% and 10%. For the Special Event Simulation scenario, we selected three stations in the city center and simulated a two hour event happening between 6p.m. and 8p.m. Consequently, we increase by 50% all OD pairs in the base ODt towards these three stations with departure time during the two hours prior to the event. We also increase by 50% all OD pairs in the base ODt originating from these three stations in the two hours after the event.

Table 10 shows the results for the two scenarios, in absolute WTT and percentage improvement in relation to the Route_Fixed solutions. Similarly to Section 6.2, each scenario is associated with an χ value, indicating the percentage increase of the initial solution in comparison to the initial timetable for the base ODt matrix.

From Table 3, the initial timetable has a WTT of 14,952,021 DKK. We observe that, despite the increase in passenger demand, the MHeuPA is able to obtain solu-

Table 10: WTT results for the two scenarios of changing the OD matrices

OD Variation (χ)	Random ± 10% (0.03)		Special Event Simulation (0.35)		initial timetable (Table 3) (14,952,021 DKK)	
Solution	WTT (DKK)	Improv. to Route_Fixed (%)	WTT (DKK)	Improv. to Route_Fixed (%)	Improv. in Random (%)	Improv. in Event (%)
Route_Fixed	14,857,357	-	14,906,759	-	-0.63	-0.30
NoCosts	14,814,032	-0.29	14,861,172	-0.31	-0.92	-0.61
LinAsc	14,825,444	-0.21	14,867,111	-0.27	-0.85	-0.57
RandomLinAsc	14,822,207	-0.24	14,865,578	-0.28	-0.87	-0.58

tions that have lower WTT than the initial timetable. Furthermore, the integration with a PTTA model proved to be beneficial, evidenced by the solutions with lower WTT obtained with the MHeuPA solutions in comparison to the Route_Fixed solutions. The NoCosts waiting cost function once again outperforms the other cost functions, with a reduction in WTT of -0.29% in the random ODt scenario and of -0.31% in the event simulation scenario, in comparison to the Route_Fixed solutions. Furthermore, the NoCosts is also the best performing waiting cost function when comparing with the initial timetable, with WTT reductions of -0.92% in the random ODt scenario and of -0.61% in the special event simulation scenario.

We acknowledge that, in general, in case of a large event it is important to evaluate if there is capacity to transport the higher volumes of passengers. Since our approach does not take into account capacity constraints, it is out of the scope of the current work to consider the analysis of capacity restrictions. The purpose of the above described cases is to demonstrate new timetables can *also* be found in case of a change in demand scenarios. Thus, the MHeuPA is suitable to use in a wide range of situations: improving on the current timetable, dealing with a change in the network, dealing with a change in passenger demand.

6.4 Experiments using the Logit passenger assignment model

The experiments presented in the previous subsections use the passenger assignment model of [4], which is a linear model. To evaluate if the reported results still hold under a different passenger assignment model the same experiments as in Table 3 were conducted this time using a Logit model with parameter $\beta = 0.01$. Table 11 summarizes these results.

Similarly to Table 3, the results in Table 11 indicate that timetables with higher passenger service are obtained when including free route choice. Furthermore, the conclusion that using alternative cost functions to calculate the passenger assignment at each iteration leads to timetables with lower WTT still holds. The best performing cost function is still the NoCosts function, with an improvement to

Table 11: WTT results for the base scenario using the Logit model

Solution	WTT (DKK)	Improv. to Base (%)	Improv. to Route_Fixed (%)
initial timetable	14,931,957	-	-
Route_Fixed	14,824,743	-0.72	-
Realistic	14,812,647	-0.80	-0.08
NoCosts	**14,782,247**	**-1.00**	**-0.29**
LinAsc	14,785,044	-0.98	-0.27
Random	14,785,558	-0.98	-0.26
RandomLinAsc	14,787,633	-0.97	-0.25

the initial timetable equal to 1% and an improvement to the Route_Fixed solution of 0.26%. Overall, all solutions obtained with our approach improve the initial timetable, and we can say that the conclusions drawn in previous experiments are still valid when using the Logit model. Decreasing the value of the Logit β parameter (we tried 0.001 and 0.003) still holds the conclusions, but with improvement values smaller than the ones reported in Table 11.

6.5 Feedback from the transport service provider

The results obtained from the case study were presented to the public transport service provider Movia. The results were found to be encouraging and confirmed Movia in their interest into experimenting with changing the timetable by shifting and stretching to enable an increase in potential transfers. It has not been possible to implement the presented method into the planning framework of Movia since a third-party software system is used for all timetable planning activities which would means that intensive work would be required to integrate with the front-end and back-end of the planning software system used. However, the findings from the case study have lead to further internal discussions within Movia and may inspire a specific request to the third-party software provider to implement a similar method.

7 Conclusions and future research

This paper addresses the problem of maximizing passenger service through timetabling under the assumptions of free passenger route choice within a fixed budget for operating costs at a tactical level. Free route choice implies that passengers follow their individually preferred path, rather than one that optimizes a social optimum, and that passengers with the same origin, destination, and departure time may have different preferences. The latter ensures that in case two equivalent routes exist, passengers are assumed to use both.

The proposed matheuristic for the IPAT-VSP combines two state-of-the-art models: the integrated timetabling and vehicle scheduling model of [10] with the passenger route choice model of [4]. Provided an initial timetable and an ODt matrix describing passenger demand over time, the objective of the `MHeuPA` is to maximize passenger service, expressed as weighted travel time, through modifications of the timetable. These modifications consist of changes in the starting time of trips (shifts), and addition of dwell time (stretches) at transfer stops, in comparison to the initial timetable within a set of headway constraints and a budget on operating costs. Operating costs are defined by the minimum cost vehicle schedules for a timetable, which problem is simultaneously solved during the timetabling procedure.

A realistic case study focused on timetabling bus lines in the context of the multi-modal network of the Greater Copenhagen area illustrates that (i) including free passenger route choice leads to timetables with higher passenger service than assuming fixed passenger route choice such as in [10], (ii) that the indication of *potentially* interesting transfers for passengers results in timetables with a higher passenger service than providing the timetabling model information on the precise passenger route choice on the current timetable, and (iii) that benefits of including free passenger route choice can be found in comparison to the current timetable of our case study area, in case of a change in the network, and in case of a change in passenger demand. The latter also suggests that the proposed `MHeuPA` approach could be used to design new timetables in case of changes in the network, e.g. due to planned maintenance, or in case of an expected change in the demand matrix, e.g. due to special events.

Although the higher passenger service in our case study results from a trade-off between passenger groups, the increase in service results foremost from a sizable decrease in WTT for a large group of passengers that offsets the increase in WTT for others. Overall improvement in WTT in comparison to the initial timetable is approximately 1%, of which 0.25% is due to the inclusion of free passenger route choice. The 0.25% is equivalent to a daily reduction of approximately 40,000 DKK when expressed as value of time. Due to the budget constraints, these savings come at no additional operating costs.

In summary, this paper contributes to the field of timetabling and public transport planning by studying integrated maximal passenger service timetabling and vehicle scheduling in the context of a realistic free passenger route choice model representing free route choice of passengers; demonstrating that the inclusion of free passenger route choice leads to timetables with higher passenger service and that the indication of potential important transfers for passengers is more important than providing a timetabling model with accurate information on the passenger route choice in a current, initial timetable.

Future research may focus on a further integration of passenger route choice decisions into the timetabling and vehicle scheduling model; or on extending the timetabling procedure to include decisions on stops per line and target headway, which have a major influence on passenger service but are currently generally fixed in the previous planning stage of line planning and network design. Other forms of cost functions for waiting costs could also be considered, for example depending on the headway of the destination line. Moreover, future research could focus on finding exact lower bounds for the maximal passenger service timetabling problem.

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Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] Ben-Akiva, M., Ramming, M., Bekhor, S.: Route choice models. Human Behaviour and Traffic Networks pp. 23–45 (2004)

[2] Binder, S., Maknoon, Y., Bierlaire, M.: The multi-objective railway timetable rescheduling problem. Transportation Research Part C: Emerging Technologies **78**, 78–94 (2017)

[3] Borndörfer, R., Hoppmann, H., Karbstein, M.: Passenger routing for periodic timetable optimization. Public Transport **9**(1-2), 115–135 (2017)

[4] Briem, L., Buck, S., Ebhart, H., Mallig, N., Strasser, B., Vortisch, P., Wagner, D., Zündorf, T.: Efficient traffic assignment for public transit networks. In: 16th International Symposium on Experimental Algorithms (SEA 2017), vol. 75. Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik (2017)

[5] Canca, D., Barrena, E., De-Los-Santos, A., Andrade-Pineda, J.L.: Setting lines frequency and capacity in dense railway rapid transit networks with simultaneous passenger assignment. Transportation Research Part B: Methodological **93**, 251–267 (2016)

[6] Chu, J.C.: Mixed-integer programming model and branch-and-price-and-cut algorithm for urban bus network design and timetabling. Transportation Research Part B: Methodological **108**, 188–216 (2018)

[7] Comi, A., Nuzzolo, A., Crisalli, U., Rosati, L.: A new generation of individual real-time transit information systems. In: A. Nuzzolo, W.H. Lam (eds.) Modelling intelligent multi-modal transit systems, pp. 80–107. CRC Press, Taylor & Francis Group (2017)

[8] Dollevoet, T., Huisman, D.: Fast heuristics for delay management with passenger rerouting. Public Transport **6**(1-2), 67–84 (2014)

[9] Domencich, T.A., McFadden, D.: Urban travel demand - A Behavioral Analysis. North-Holland Publishing co. Ltd. (1975)

[10] Fonseca, J., van der Hurk, E., Roberti, R., Larsen, A.: A matheuristic for transfer synchronization through integrated timetabling and vehicle scheduling. Transportation Research. Part B: Methodological **109**, 128–149 (2018)

[11] Gattermann, P., Großmann, P., Nachtigall, K., Schöbel, A.: Integrating passengers' routes in periodic timetabling: A sat approach. In: OASIcs-OpenAccess Series in Informatics, vol. 54. Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik (2016)

[12] Gentile, G., Papola, A.: An Alternative Approach to Route Choice Simulation: The Sequential Models. In: Proceedings of the European Transport Conference. Association for European Transport (2006)

[13] Hartleb, J., Schmidt, M.: Railway timetabling with integrated passenger distribution. Available at SSRN 3505167 (2019)

[14] Kroon, L., Maróti, G., Nielsen, L.: Rescheduling of railway rolling stock with dynamic passenger flows. Transportation Science **49**(2), 165–184 (2014)

[15] Liu, T., Ceder, A.: Integrated public transport timetable synchronization and vehicle scheduling with demand assignment: A bi-objective bi-level model using deficit function approach. Transportation Research Part B: Methodological **117**, 935–955 (2017)

[16] Petersen, H.L., Larsen, A., Madsen, O.B.G., Petersen, B., Ropke, S.: The simultaneous vehicle scheduling and passenger service problem. Transportation Science **47**(4), 603 – 616 (2013)

[17] Polinder, G.J., Cacchiani, V., Schmidt, M., Huisman, D.: An iterative heuristic for passenger-centric train timetabling with integrated adaption times. Tech. rep., Erasmus University Rotterdam, The Netherlands (2020). URL `http://hdl.handle.net/1765/127816`

[18] Robenek, T., Azadeh, S.S., Maknoon, Y., de Lapparent, M., Bierlaire, M.: Train timetable design under elastic passenger demand. Transportation Research Part B: Methodological **111**, 19–38 (2018)

[19] Schiewe, P.: Integrating timetabling and passenger routing. In: Integrated Optimization in Public Transport Planning, pp. 33–64. Springer (2020)

[20] Schmidt, M.: Integrating routing decisions in network problems. Operations Research Proceedings 2012 pp. 21–26 (2014)

[21] Schmidt, M., Schöbel, A.: Timetabling with passenger routing. OR spectrum **37**(1), 75–97 (2015)

[22] Van der Hurk, E., Kroon, L., Maróti, G.: Passenger advice and rolling stock rescheduling under uncertainty for disruption management. Transportation Science **52**(6), 1391–1411 (2018)

[23] Veelenturf, L.P., Kroon, L.G., Maróti, G.: Passenger oriented railway disruption management by adapting timetables and rolling stock schedules. Transportation Research Part C: Emerging Technologies **80**, 133–147 (2017)

[24] Wagenaar, J., Kroon, L., Fragkos, I.: Rolling stock rescheduling in passenger railway transportation using dead-heading trips and adjusted passenger demand. Transportation Research Part B: Methodological **101**, 140–161 (2017)

[25] Wang, Y., D'Ariano, A., Yin, J., Meng, L., Tang, T., Ning, B.: Passenger demand oriented train scheduling and rolling stock circulation planning for an urban rail transit line. Transportation Research Part B: Methodological **118**, 193–227 (2018)

[26] Wu, W., Liu, R., Jin, W., Ma, C.: Stochastic bus schedule coordination considering demand assignment and rerouting of passengers. Transportation Research Part B: Methodological **121**, 275–303 (2019)

[27] Zhu, Y., Mao, B., Bai, Y., Chen, S.: A bi-level model for single-line rail timetable design with consideration of demand and capacity. Transportation Research Part C: Emerging Technologies **85**, 211–233 (2017)

Appendix 1 - Sets, parameters, and decision variables used in the IT-VSP mathematical model

Sets	
S	Set of all stops
L	Set of all directed lines
$T = \{1, \ldots, n\}$	Set of all timetabled trips
$T_l \subseteq T$	Subset of all trips in the directed line $l \in L$
$T^1 \subseteq T$	Set of all trips which are the first in their directed line
$S_i \subseteq S$	Set of all stops visited by trip $i \in T$
$J_i \subseteq S_i$	Set of all intermediate stops visited by trip $i \in T$, i.e., $J_i = S_i \setminus \{st_i, et_i\}$
R	Set of all transfer opportunities, each defined by a triplet (i, l, s): passengers disembarking trip $i \in T$ at stop $s \in J_i \cup \{et_i\}$ with the intent of embarking a trip $j \in T_l$ of line $l \in L$ such that $l \neq l_i$ and $s \in J_j \cup \{st_j\}$
K	Set of all depots
I	Set of all compatible trips, $I = \{(i, j) \| i, j \in T : i \neq j, Dist(et_i, st_j) \leq u, a^-_{i,et_i} + q^- + b_{ij} \leq d^+_{j,st_j}, a^+_{i,et_i} + q^+ + b_{ij} \geq d^-_{j,st_j}\}$
V_k	Set of nodes, which contains a node for each trip $i \in T$, as well as for depot $k \in K$ which is denoted $n+k$, thus $V_k = T \cup \{n+k\}$
A_k	Set of arcs, including deadhead trips, pull-out trips, and pull-in trips, thus $A_k = I \cup (\{n+k\} \times T) \cup (T \times \{n+k\})$
$G_k = (V_k, A_k)$	Graph associated with depot $k \in K$
Q^D	Set of all deadhead triplets $Q^D = \{(i, j, k) : k \in K, (i, j) \in I\}$
Q^O	Set of all pull-out triplets $Q^O = \{(n+k, j, k) : k \in K, j \in T\}$
Q^H	Set of all pull-in triplets $Q^H = \{(i, n+k, k) : i \in T, k \in K\}$
Q	Set of all compatible triplets (i, j, k), representing a vehicle from depot $k \in K$ covering the pair of trips $(i, j) \in A_k$. $Q = Q^D \cup Q^O \cup Q^H$
$T(Q)$	Set of all pairs of trips $i, j \in T$ for which a triplet involving i and j exists, $T(Q) = \{(i, j) \| i, j \in T : \exists (i, j, k) \in Q\}$.
Parameters	
l_i	Directed line of trip $i \in T$
t_i	Total travel time of trip $i \in T$ in the initial timetable
$st_i \in S_i$	Start terminal of trip $i \in T$
$et_i \in S_i$	End terminal of trip $i \in T$

h^-_{is}, h^+_{is}	Minimum and maximum headways, respectively, in relation to the timetabled headways, for each trip $i \in T$ at each stop $s \in J_i \cup \{st_i\}$
$d^-_{i,st_i}, d^+_{i,st_i}$	Minimum and maximum departure shift from the first station for trip $i \in T$, defined in relation to its departure time in the initial timetable
w^-_{is}	Dwell time in the initial timetable of a trip $i \in T$ at stop $s \in J_i$
w^+_{is}	Maximum allowed dwell time of a trip $i \in T$ at stop $s \in J_i$
w	Upper bound on the total added dwell time to all stops of any trip
Λ_{is}	Number of passengers that are on board (and will continue on board) when trip $i \in T$ arrives at stop $s \in J_i$
a^-_{is}, a^+_{is}	Earliest and latest arrival times of trip $i \in T$ at stop $s \in J_i \cup \{et_i\}$, determined by the possible timetable modifications
d^-_{is}, d^+_{is}	Earliest and latest departure times of trip $i \in T$ from stop $s \in J_i \cup \{st_i\}$, determined by the possible timetable modifications
f_r	Number of passengers requesting transfer $r \in R$
e_r	Minimum transfer time for transfer $r \in R$
q^-, q^+	Minimum and maximum turnaround times
b_{ij}	Driving time between et_i and st_j
v_k	Number of schedules that can be created departing from depot $k \in K$
$Dist(i, j)$	Distance between the end terminal of trip $i \in T$, et_i, and the start terminal of trip $j \in T$, st_j
u	Maximum deadhead distance
c_{ijk}	Operating cost associated with servicing triplet $(i, j, k) \in Q$. The cost c_{ijk} of triplet $(i, j, k) \in Q$ is equal to the deadhead time b_{ij} multiplied by a driving cost per time unit; if $(i, j, k) \in Q^O$, c_{ijk} also includes a fixed cost for creating a new schedule, corresponding to the fixed cost for using a vehicle.
c^{DW}	Operating cost per minute of extra dwell time
c^{OB}	Cost per minute of extra dwell time per on-board passenger
c^{TR}	Cost per minute of excess transfer time at transfers per passenger
B	Budget value for the operating costs
M	Big M value. In this problem, it is sufficient to set a big M value higher than the number of minutes in the planning horizon
Decision variables	

$x_{ijk} \in \{0,1\}$	1 if and only if a vehicle from depot k travels from node i directly to node j, 0 otherwise
$\tau_{is}^{d} \in \mathbb{Z}_0^+$	Departure time of trip $i \in T$ from stop $s \in J_i \cup \{st_i\}$
$\tau_{is}^{a} \in \mathbb{Z}_0^+$	Arrival time of trip $i \in T$ at stop $s \in J_i \cup \{et_i\}$
$\gamma_r \in \mathbb{R}_0^+$	Excess transfer time for passengers using transfer location $r \in R$
$\alpha_{ijs} \in \{0,1\}$	1 if and only if passengers of transfer location $r = (i, l_j, s) \in R$ embark trip $j \in T$, 0 otherwise
$\delta_{is} \in \mathbb{Z}_0^+$	Minutes of dwell time added to trip $i \in T$ at stop $s \in J_i$

Appendix 2 - Histogram of added dwell time in a representative solution