A Synthetic median control chart for monitoring process mean with measurement errors

Kim Phuc Tran, Phuong Hanh Tran, Athanasios Rakitzis

A Synthetic median control chart for monitoring process mean with measurement errors

Kim Phuc Tran[*1,2], **Phuong Hanh Tran**[3], **and Athanasios C. Rakitzis**[4]

[1]Division of Data Science, Ton Duc Thang University, Ho Chi Minh City, Vietnam
[2]Faculty of Information Technology, Ton Duc Thang University, Ho Chi Minh City, Vietnam
[3]Division of Artificial Intelligence, Dong A University Research Institute, Dong A University, Danang, Vietnam
[4]University of Aegean, Karlovasi, Greece

July 15, 2018

Abstract

Recent studies show that Shewhart median chart is widely used for detecting shifts in a process, but it is often rather inefficient in detecting small or moderate process shifts. In order to overcome this problem, a Synthetic chart can be used. This chart outperforms the Shewhart type chart because it uses the information about the time interval between two consecutive nonconforming samples. In this paper, we propose and study the Phase II Synthetic median control chart. A Markov chain methodology is used to evaluate the statistical performance of the proposed chart. Moreover, its performance is investigated in the presence of measurement errors, which are modelled by a linear covariate error model. We provide the results of an extensive numerical analysis with several tables and figures in order to show the statistical performance of the investigated chart, for both cases of measurement errors and no measurement errors. Finally, an example illustrates the use of the Synthetic median chart.

Keywords: Synthetic chart, Markov chain, Median, Measurement errors.

[*]trankimphuc@tdt.edu.vn (corresponding author)

1 Introduction

Control charts are the simplest type of on-line statistical process control (SPC) procedures. In recent years, many authors focus on developing univariate and multivariate control charts for monitoring shifts in process mean and/or changes in process standard deviation (covariance matrix). For further details see for instance Chen and Hsieh [9], Castagliola and Figueiredo [6], Frisen [18], Faraz et al. [17], Aslam et al. [3]. In the SPC literature, median ($\tilde{X}$) charts have already been investigated and used to detect shifts in a process. Many authors have focused on developing their properties and design stategies, for further details see for instance Castagliola [5], Khoo [24], Sheu and Yang [36], Castagliola and Figueiredo [6], Ahmad et al. [1], Ahmad et al. [2], Castagliola et al. [7], Hu and Castagliola [19] and Lin et al. [27]. It is well known that Shewhart type control charts are very easy to design and to interpret. However, they are rather slow in the detection of small or moderate process shifts. For this reason several methods / strategies have been proposed in SPC literature to overcome this problem. Among these methods, Synthetic control charts are widely used to detect shifts in a process. Wu and Spedding [42] were the first to introduce the Synthetic $\bar{X}$ chart to the field of SPC; then, its properties and design stategies have been thoroughly investigated by many authors. For further details see, for instance, Davis and Woodall [15], Chen and Huang [8], Huang and Chen [22], Costa and Rahim [13], Costa et al. [12], Wu et al. [41], Khoo et al. [25]. Recently, Zhang et al. [43] investigated the effect of estimated process parameters on the performance of the Synthetic chart using a Markov chain model and they shown that the run length (RL) performance of the Synthetic chart is quite different in the known and in the estimated process parameters cases.

However, as far as we know, the Synthetic median (Synthetic $\tilde{X}$) control chart has never been considered in the SPC literature. Therefore, the goal of this paper is to investigate the performance of the Synthetic $\tilde{X}$ control chart. Furthermore, in many industrial scenarios, there often exist significant measurement errors that affect the performance of control charts. Since Bennet [4] investigated the effect of measurement errors on the Shewhart $\bar{X}$ chart, the consequences of the measurement errors on the performance of various control charts have been studied by a number of authors, see, for example, Kanazuka [23], Linna and Woodall [28], Linna et al. [29], Maravelakis [31], Costa and Castagliola [11], Maravelakis [32], Hu et al. [20], Noorossana and Zerehsaz [35], Hu et al. [21], Tran et al. [39], Tran et al. [38], Cheng and Wang [10], Maleki et al. [30] and Tran [37]. We examine here the performance of the Synthetic $\tilde{X}$ control chart in the presence of measurement errors by assuming the measurement error model as in Linna and Woodall [28].

The remainder of the paper is organized as follows: in Section 2, the

Synthetic $\tilde{X}$ chart and its run length properties are defined; in Section 3, the statistical performance of the Synthetic $\tilde{X}$ chart is presented and simple guidelines are proposed; in Section 4, the linear covariate error model for the sample median is defined; Section 5 provides the formulas for the control limits and the performance metrics of the Synthetic $\tilde{X}$ control chart in the presence of a measurement errors; in Section 6, the effects of measurement errors on the Synthetic $\tilde{X}$ control chart performance are investigated. Section 7 presents an illustrative example and, finally, some concluding remarks and recommendations are made in Section 8.

2 Design and implementation of the Synthetic $\tilde{X}$ control chart

Let $\{X_{i,1}, \ldots, X_{i,n}\}$, $i = 1, 2, \ldots$, be a Phase II sample of n independent normal random variables, more precisely, $N(\mu_0 + \delta\sigma_0, \sigma_0)$, where i is the subgroup number, μ_0 is the in-control mean value, σ_0 is the in-control standard deviation and δ is the magnitude of the standardized mean shift. If $\delta = 0$ the process is in-control and, when $\delta \neq 0$, the process is out-of-control. Let $\tilde{X}_i$ be the sample median of n independent normal random variables $\{X_{i,1}, \ldots, X_{i,n}\}$ corresponding to subgroup $i = 1, 2, \ldots$, i.e.

$$\tilde{X}_i = \begin{cases} X_{i,((n+1)/2)} & \text{if } n \text{ is odd} \\ \dfrac{X_{i,(n/2)} + X_{i,(n/2+1)}}{2} & \text{if } n \text{ is even} \end{cases} \tag{1}$$

where $\{X_{i,(1)}, X_{i,(2)}, \ldots, X_{i,(n)}\}$ is the ordered sample of the mean values for subgroup $i = 1, 2, \ldots$. Without loss of generality, we assume that the sample size n is an odd value in this paper. This makes the sample median easier and faster to compute. Like in Castagliola and Figueiredo[6], the c.d.f. (cumulative distribution function) $F_{\tilde{X}}(x|n)$ of the sample median $\tilde{X}_i$ can be written as

$$\begin{aligned} F_{\tilde{X}_i}(x|n) &= F_\beta\left(\Phi\left(\frac{x-(\mu_0+\delta\sigma_0)}{\sigma_0}\right)\middle|\frac{n+1}{2}, \frac{n+1}{2}\right) \\ &= F_\beta\left(\Phi\left(\frac{x-\mu_0}{\sigma_0} - \delta\right)\middle|\frac{n+1}{2}, \frac{n+1}{2}\right) \end{aligned} \tag{2}$$

where $\Phi(x)$ is the c.d.f. of the standard normal distribution and $F_\beta(x|a, b)$ is the c.d.f. of the beta distribution with parameters (a, b). Here $a = b = \frac{n+1}{2}$.

The Synthetic $\tilde{X}$ chart consists of two sub-charts: a $\tilde{X}$ sub-chart and a conforming run length (CRL) sub-chart. The CRL is defined as the number of inspected samples between two consecutive nonconforming samples, inclusive of the nonconforming sample at the end. A sample is declared as nonconforming if $\tilde{X}_i$, $i = 1, 2, \ldots$, falls outside predetermined control limits of the $\tilde{X}$ sub-chart. Therefore, the control flow of the Synthetic $\tilde{X}$ control chart can be summarized as follows:

Step 1 Determine the sample size n, the lower control limit H of the CRL sub-chart and the control limits LCL and UCL of the $\tilde{X}$ sub-chart (see (3) and (4) below).

Step 2 At each sampling point $i = 1, 2, \ldots$, take a sample of size n from the quality characteristic X and evaluate the sample median $\tilde{X}_i$ as in (1).

Step 3 If $LCL < \tilde{X}_i < UCL$, this sample is considered as a conforming sample in the CRL sub-chart and the control flow goes back to step 2 to take the next sample. Otherwise, the sample is a nonconforming one and the control flow goes to the next step.

Step 4 If $CRL > H$, the process is deemed to be in control and the control flow goes back to step 2. Otherwise, the process is declared as out-of-control and the control flow goes to the next step.

Step 5 Signal an out-of-control status to indicate a process mean shift. Find and remove potential assignable cause(s). Then move back to Step 2.

The control limits of the $\tilde{X}$ sub-chart of the Synthetic $\tilde{X}$ are

$$\begin{aligned} LCL &= \mu_0 - K\sigma_0 && (3)\\ UCL &= \mu_0 + K\sigma_0. && (4) \end{aligned}$$

where $K > 0$ is a control chart constant. In order to obtain the run length properties of the Synthetic $\tilde{X}$ control chart, similarly to Davis and Woodall[15], we use a Markov chain where the $(H+2, H+2)$ transition probability matrix $\mathbf{P}$ is equal to

$$\mathbf{P} = \begin{pmatrix} \mathbf{Q} & \mathbf{r} \\ \mathbf{0}^\intercal & 1 \end{pmatrix} = \left(\begin{array}{cccccc|c} 1-\theta & \theta & 0 & \cdots & \cdots & 0 & 0 \\ 0 & 0 & 1-\theta & \ddots & & 0 & \theta \\ \vdots & & \ddots & \ddots & \ddots & \vdots & \vdots \\ \vdots & & & \ddots & 1-\theta & 0 & \vdots \\ 0 & \cdots & \cdots & \cdots & 0 & 1-\theta & \theta \\ 1-\theta & 0 & \cdots & \cdots & \cdots & 0 & \theta \\ \hline 0 & \cdots & \cdots & \cdots & \cdots & 0 & 1 \end{array}\right), \qquad (5)$$

where $\mathbf{0}^\intercal = (0, 0, \ldots, 0)$ is a $(1, H+1)$ row vector, $\mathbf{Q}$ is a $(H+1, H+1)$ transition probability matrix for the transient states, the $(H+1, 1)$ column vector $\mathbf{r}$ satisfies $\mathbf{r} = \mathbf{1} - \mathbf{Q1}$ with $\mathbf{1} = (1, 1, \ldots, 1)^\intercal$ and $\theta = P(\tilde{X}_i \notin [LCL, UCL])$ is the probability of a nonconforming sample on the $\tilde{X}$ sub-chart, i.e. using (3) and (4)

$$\begin{aligned}
\theta &= 1 - F_{\tilde{X}_i}(UCL|n) + F_{\tilde{X}_i}(LCL|n) \\
&= 1 - F_\beta\left(\Phi\left(\frac{\mu_0 + K\sigma_0 - \mu_0}{\sigma_0} - \delta\right)\middle|\frac{n+1}{2}, \frac{n+1}{2}\right) \\
&\quad + F_\beta\left(\Phi\left(\frac{\mu_0 - K\sigma_0 - \mu_0}{\sigma_0} - \delta\right)\middle|\frac{n+1}{2}, \frac{n+1}{2}\right) \\
&= 1 - F_\beta\left(\Phi\left(K - \delta\right)\middle|\frac{n+1}{2}, \frac{n+1}{2}\right) + F_\beta\left(\Phi\left(-K - \delta\right)\middle|\frac{n+1}{2}, \frac{n+1}{2}\right)
\end{aligned} \tag{6}$$

where $F_{\tilde{X}_i}(.|n)$ is the c.d.f. of $\tilde{X}_i$ as defined in (2).

Let $\mathbf{q}$ be the $(H+1, 1)$ vector of initial probabilities associated with the $H+2$ transient states, i.e., $\mathbf{q} = (q_0, q_1, \ldots, q_{H+1})^\intercal$. As proposed by Neuts [34] and Latouche and Ramaswami [26], since the number of steps, say Run Length or RL, until the process reaches the absorbing state is a *Discrete PHase-type* (or DPH) random variable of parameters $(\mathbf{Q}, \mathbf{q})$, the mean (ARL) and the standard-deviation $(SDRL)$ of RL of the Synthetic $\tilde{X}$ control chart are equal to

$$ARL = \nu_1, \tag{7}$$

$$SDRL = \sqrt{\nu_2 - \nu_1^2 + \nu_1}, \tag{8}$$

with

$$\nu_1 = \mathbf{q}^\intercal(\mathbf{I} - \mathbf{Q})^{-1}\mathbf{1}, \tag{9}$$

$$\nu_2 = 2\mathbf{q}^\intercal(\mathbf{I} - \mathbf{Q})^{-2}\mathbf{Q1}. \tag{10}$$

and $\mathbf{q} = (q_0, q_1, \ldots, q_{H+1})^\intercal$ which the solution of $(\mathbf{I} - \mathbf{Q})\mathbf{q} = \mathbf{1}$ yields the zero-state ARL (see Davis and Woodall [16]).

It is important to note that, if the process is running for some time in an in-control condition, it will reach quite quickly the steady-state mode. In order to study the long term properties of the Synthetic $\tilde{X}$ control chart, it is appropriate to investigate the steady-state ARL. Using the Markov Chain approach, the *cyclical* steady state mean $(SARL)$ and the standard-deviation $(SSDRL)$ of the run length RL of the Synthetic $\tilde{X}$ control chart are found as follows

$$SARL = \nu_{s1}, \tag{11}$$

$$SSDRL = \sqrt{\nu_{s2} - \nu_{s1}^2 + \nu_{s1}} \tag{12}$$

with

$$\begin{aligned} \nu_{s1} &= \boldsymbol{\psi}^{\intercal}(\mathbf{I}-\mathbf{Q})^{-1}\mathbf{1}, & (13)\\ \nu_{s2} &= 2\boldsymbol{\psi}^{\intercal}(\mathbf{I}-\mathbf{Q})^{-2}\mathbf{Q}\mathbf{1}, & (14)\end{aligned}$$

where the vector $\boldsymbol{\psi}$ is the *cyclical* steady state distribution. Following Darroch and Seneta [14] we conclude that the *cyclical* steady-state vector is given by $\boldsymbol{\psi} = \frac{(\mathbf{I}-\mathbf{Q}^{\intercal})^{-1}\mathbf{q}}{\mathbf{1}^{\intercal}(\mathbf{I}-\mathbf{Q}^{\intercal})^{-1}\mathbf{q}}$, where $\mathbf{q}$ is the $(H+1,1)$ vector utilized in (7).

The statistical design of the Synthetic $\tilde{X}$ control chart is a nonlinear optimization problem aimed at selecting the optimal couple of chart parameters (H^*, K^*) such that

$$(H^*, K^*) = \underset{(K,H)}{\operatorname{argmin}}\, ARL(n, K, H, \delta), \tag{15}$$

subject to

$$ARL(n, K, H, \delta = 0) = ARL_0, \tag{16}$$

or

$$ARL(n, K, H, \delta = 0) = SARL_0, \tag{17}$$

where, for $\delta \neq 0$, $ARL(n, K, H, \delta)$ is either the zero state or the *cyclical* steady state ARL of the Synthetic $\tilde{X}$ control chart; ARL_0 and $SARL_0$ are the predefined "in-control" zero state and *cyclical* steady state ARL value, respectively. The optimization procedure can be summarized as follows:

Step 1 Set n, δ and $ARL_0/SARL_0$. Set $ARL_{opt} = +\infty$;

Step 2 Initialize $H = 1$;

Step 3 Compute K through constraint (17);

Step 4 Calculate ARL from the current design solution (H, K) by using equation (7) or (11);

Step 5 If $ARL < ARL_{opt}$, then $ARL_{opt} = ARL$ and $(H^*, K^*) = (H, K)$. Set $H = H + 1$ and go back to Step 3. Otherwise, go to the next step;

Step 6 Take the current solution (H^*, K^*) as the optimal set of design parameters for the Synthetic $\tilde{X}$ control chart and compute the optimal control limits (LCL, UCL) of the Synthetic $\tilde{X}$ sub-chart by using equations (3) and (4).

In this study, like in Tran and Tran [40], in order to find these optimal combinations (H^*, K^*) we simultaneously use a non-linear equation solver jointly with an optimization algorithm developed in Scicoslab software.

3 The Performance of the Synthetic $\tilde{X}$ control chart

In this Section, we will use the ARL, $SDRL$ to evaluate the performance of the Synthetic $\tilde{X}$ chart. Recall that the "in-control" zero state and *cyclical* steady state ARL values are denoted by ARL_0 and $SARL_0$, respectively; and here we set $ARL_0 = SARL_0 = 370.4$.

The zero state ARL and $SDRL$ when the process is out-of-control (denoted by ARL_1 and $SDRL_1$) of the Synthetic $\tilde{X}$ control chart and the optimal set of design parameters K^*, H^* (when $ARL_0 = 370.4$) are shown in Table 1 for different combinations of $\delta \in \{0.1, 0.2, 0.3, 0.5, 0.7, 1.0, 1.5, 2.0\}$ and $n \in \{3, 5, 7, 9\}$.

INSERT TABLE 1 ABOUT HERE

In general, the proposed control chart has an attractive performance compared to the Shewhart-$\tilde{X}$ chart, especially for small magnitude shifts and small sample sizes. For instance, when $n = 3$ and $\delta = 0.2$, we have $ARL_1 = 258.3$ and $SDRL_1 = 257.8$ for the Shewhart-$\tilde{X}$ chart; $ARL_1 = 217.5$ and $SDRL_1 = 282.9$ for the Synthetic $\tilde{X}$ control chart, see Table 1.

The steady state ARL and $SDRL$ when the process is out-of-control (denoted by $SARL_1$ and $SSDRL_1$) of the Synthetic $\tilde{X}$ control chart and the optimal set of design parameters K^*, H^* (when $SARL_0 = 370.4$) are shown in Table 2 for different combinations of $\delta \in \{0.1, 0.2, 0.3, 0.5, 0.7, 1.0, 1.5, 2.0\}$ and $n \in \{3, 5, 7, 9\}$. For instance, when $n = 3$ and $\delta = 0.2$, we have $K^* = 37$, $H^* = 1.7225$, $ARL_1 = 235.3$ and $SDRL_1 = 235.6$ for the Synthetic $\tilde{X}$ control chart.

INSERT TABLE 2 ABOUT HERE

4 Linear covariate error model for sample median

In this section, the linear covariate error model for the sample median $\tilde{X}$ is defined. Let us assume that, at time $i = 1, 2, \ldots$, the quality characteristic X of $n \geq 1$ consecutive items is equal to $\{X_{i,1}, X_{i,2}, \ldots, X_{i,n}\}$. We assume that these $X_{i,j}$'s are *independent* normal $(\mu_0 + \delta\sigma_0, \sigma_0)$ random variables. As suggested by Linna and Woodall [28], we assume that the quality characteristic $X_{i,j}$ is not directly observable, but can only be assessed from the results $\{X^*_{i,j,1}, X^*_{i,j,2}, \ldots, X^*_{i,j,m}\}$ of a set of $m \geq 1$ measurement operations with each $X^*_{i,j,k}$ being equal to (linear covariate error model)

$$X^*_{i,j,k} = A + BX_{i,j} + \varepsilon_{i,j,k}, \tag{18}$$

where A and B are two known constants and $\varepsilon_{i,j,k}$ is a normal $N(0, \sigma_M)$ random error term due to the measurement inaccuracy, which is independent of $X_{i,j}$. The smaller σ_M is, the higher the measure *precision* is.

For subgroup $i = 1, 2, \ldots$, as $j = 1, 2, \ldots, n$ and $k = 1, 2, \ldots, m$, we have $m \times n$ observations $X_{i,j,k}$ and the mean $\bar{X}^*_{i,j}$ of the observable quantities $\{X^*_{i,j,1}, X^*_{i,j,2}, \ldots, X^*_{i,j,m}\}$ is equal to

$$\begin{aligned} \bar{X}^*_{i,j} &= \frac{1}{m}\sum_{k=1}^{m} X^*_{i,j,k} \\ &= \frac{1}{m}\sum_{k=1}^{m}(A + BX_{i,j} + \varepsilon_{i,j,k}) \\ &= A + BX_{i,j} + \frac{1}{m}\sum_{k=1}^{m}\varepsilon_{i,j,k}. \end{aligned} \tag{19}$$

It can then easily be shown that the mean $\mu^* = E(\bar{X}^*_{i,j})$ and the standard deviation $\sigma^* = \sigma(\bar{X}^*_{i,j})$ of $\bar{X}^*_{i,j}$ are equal to

$$\mu^* = A + B(\mu_0 + \delta\sigma_0), \tag{20}$$

$$\sigma^* = \sqrt{B^2\sigma_0^2 + \frac{\sigma_M^2}{m}}. \tag{21}$$

Let $\tilde{X}^*_i$ be the sample median of the mean values $\{\bar{X}^*_{i,1}, \bar{X}^*_{i,2}, \ldots, \bar{X}^*_{i,n}\}$ corresponding to subgroup $i = 1, 2, \ldots$, i.e.,

$$\tilde{X}^*_i = \begin{cases} \bar{X}^*_{i,((n+1)/2)} & \text{if } n \text{ is odd} \\ \dfrac{\bar{X}^*_{i,(n/2)} + \bar{X}^*_{i,(n/2+1)}}{2} & \text{if } n \text{ is even} \end{cases}, \tag{22}$$

where $\{\bar{X}^*_{i,(1)}, \bar{X}^*_{i,(2)}, \ldots, \bar{X}^*_{i,(n)}\}$ is the ordered sample of the mean values for subgroup $i = 1, 2, \ldots$. In this case, the c.d.f. (cumulative distribution function) $F_{\tilde{X}^*}(x|n)$ of the sample median $\tilde{X}^*_i$ can be expressed as

$$\begin{aligned} F_{\tilde{X}^*_i}(x|n) &= F_\beta\left(\Phi\left(\frac{x-\mu^*}{\sigma^*}\right)\middle|\frac{n+1}{2}, \frac{n+1}{2}\right) \\ &= F_\beta\left(\Phi\left(\frac{x - A - B(\mu_0 + \delta\sigma_0)}{\sqrt{B^2\sigma_0^2 + \frac{\sigma_M^2}{m}}}\right)\middle|\frac{n+1}{2}, \frac{n+1}{2}\right). \end{aligned} \tag{23}$$

5 Implementation of the Synthetic $\tilde{X}$ chart with measurement errors

If the in-control values for the mean μ_0, the standard deviation σ_0 and the constants A, B, m, σ_M are all known, the control limits of the Synthetic $\tilde{X}$

sub-chart in the presence of measurement errors are simply equal to

$$\begin{aligned} LCL^* &= A + B\mu_0 - K\sqrt{B^2\sigma_0^2 + \frac{\sigma_M^2}{m}}, \\ UCL^* &= A + B\mu_0 + K\sqrt{B^2\sigma_0^2 + \frac{\sigma_M^2}{m}}, \end{aligned} \tag{24}$$

where $K > 0$ is a constant that depends on n, H and on the desired in-control performance. The run length of the Synthetic $\tilde{X}$ control chart with measurement errors can be obtained from (7) and (11) by simply replacing the probabilities in (25) by θ^* with:

$$\begin{aligned} \theta^* &= 1 - F_{\tilde{X}_i^*}(UCL^*|n) + F_{\tilde{X}_i^*}(LCL^*|n) && (25) \\ &= 1 - F_\beta\left(\Phi\left(K - \frac{\delta B\sigma_0}{\sqrt{B^2\sigma_0^2 + \frac{\sigma_M^2}{m}}}\right)\middle|\frac{n+1}{2}, \frac{n+1}{2}\right) && (26) \\ &+ F_\beta\left(\Phi\left(-K - \frac{\delta B\sigma_0}{\sqrt{B^2\sigma_0^2 + \frac{\sigma_M^2}{m}}}\right)\middle|\frac{n+1}{2}, \frac{n+1}{2}\right) && (27) \end{aligned}$$

where $F_{\tilde{X}_i^*}(.|n)$ is the c.d.f. of $\tilde{X}_i^*$ as defined in (23).

6 The effects of measurement errors on Synthetic $\tilde{X}$ chart

From Section 2, for fixed values of m, n, B and η, we can obtain the (H^*, K^*) values and the coresponding to the zero state ARL (ARL_1) values. Similarly, we can obtain the (H^*, K^*) values and the coresponding to the steady state ARL ($SARL_1$) values of the Synthetic $\tilde{X}$ chart with linear covariate error model. We set $ARL_0 = SARL_0 = 370.4$. These values are presented in Table 3 and Table 4 for different combinations of the *precision* error ratio $\eta \in \{0, 0.1, 0.2, 0.3, 0.5, 1.0\}$, $\delta \in \{0.1, 0.2, 0.3, 0.5, 0.7, 1.0, 1.5, 2.0\}$ and $n \in \{3, 5, 7, 9\}$ when $m = 1$ and $B = 1$.

INSERT TABLE 3 ABOUT HERE

INSERT TABLE 4 ABOUT HERE

The obtained results show that, for fixed values of n, δ, $m = 1$ and $B = 1$, the smaller the *precision* error ratio η is, the faster the control charts are in the detection of the out-of-control condition, demonstrating the negative effect of the measurement errors on the performance of the Synthetic $\tilde{X}$ chart.

For instance, when $n = 3$, $B = 1$, $m = 1$ and $\delta = 0.2$, we have $ARL_1 = 217.5$ and $SARL_1 = 235.3$ for $\eta = 0$ (process is free of measurement error) and $ARL_1 = 221.4$ and $SARL_1 = 156.6$ for $\eta = 0.2$ (see Table 3 and Table 4).

Table 5 and Table 6 show the performance of the Synthetic $\tilde{X}$ charts under linear covariate error model for different combinations of $B \in \{1, 2, 3, 4, 5\}$, $\delta \in \{0.1, 0.2, 0.3, 0.5, 0.7, 1.0, 1.5, 2.0\}$ and $n \in \{3, 5, 7, 9\}$ when $m = 1$ and $\eta = 0.28$. The specific value of $\eta = 0.28$ is motivated by assuming an acceptable value for the signal-to-noise ratio

$$SNR = \sqrt{\frac{\frac{2}{1+\eta^2}}{1 - \frac{1}{1+\eta^2}}} = \frac{\sqrt{2}}{\eta}, \tag{28}$$

which is a measure of performance of the measurement system *precision* adequacy. The SNR is defined by the Automotive Industry Action Group (AIAG) for the execution of a Gauge R&R analysis (see Montgomery[33]). The value $\eta = 0.28$ corresponds to $SNR = 5$, which is the lower bound value to get an acceptable *precision* of the measurement system.

INSERT TABLE 5 ABOUT HERE

INSERT TABLE 6 ABOUT HERE

It can be noted from Table 5 and Table 6 that, for fixed values of n, δ, η and m, as the value of B increases, the negative effect of the measurement errors on the performance of the Synthetic $\tilde{X}$ charts decrease. For instance, when $n = 3$, $\eta = 0.28$, $m = 1$ and $\delta = 0.2$, we have $ARL_1 = 225.0$ and $SARL_1 = 242.2$ for $B = 1$ and $ARL_1 = 218.0$ and $SARL_1 = 235.7$ for $B = 4$ (see Table 5 and Table 6).

The performance of the Synthetic $\tilde{X}$ chart under linear covariate error model is shown in Table 7 and Table 8 for different combinations of $m \in \{1, 3, 5, 7, 10\}$, $\delta \in \{0.1, 0.2, 0.3, 0.5, 0.7, 1.0, 1.5, 2.0\}$ and $n \in \{3, 5, 7, 9\}$ when $B = 1$ and $\eta = 0.28$.

INSERT TABLE 7 ABOUT HERE

INSERT TABLE 8 ABOUT HERE

We can directly deduce that, for fixed values of n, δ, B and η, as the number m of measurements per item increases, the values of ARL_1 and $SARL_1$ both decrease, demonstrating the positive effect of the number of repeated measurements m per item on the performance of the Synthetic $\tilde{X}$

chart. Furthermore, from Tables 3, 4, 7 and 8, we can immediately note that, for fixed value of n, with $m = 5$ measurements per item, the values of ARL_1 and $SARL_1$ in the presence of measurement errors are approximately the same as the values of ARL_1 and $SARL_1$ without measurement errors (i.e., $\eta = 0$) when $\eta \leq 0.28$. For instance, when $n = 3$, $\eta = 0.28$ and $\delta = 0.2$, we have $ARL_1 = 219.1$ and $SARL_1 = 242.2$ for $m = 1$, $ARL = 219.1$ and $SARL_1 = 236.7$ for $m = 5$ and $ARL = 217.5$ $SARL_1 = 235.3$ when process is free of measurement errors. We can conclude that the *precision* error does not affect significantly the performance of the Synthetic $\tilde{X}$ control chart for the usual levels of accuracy errors provided by calibrated gauges for the case of $m = 5$ measurements per item. In general, we can also note that the effect of measurement errors on the performance of Synthetic $\tilde{X}$ chart is reduced by taking multiple measurements $m = 5$.

7 Illustrative example

In order to illustrate the use of the Synthetic $\tilde{X}$ chart in the presence of measurement error, let us consider a production process of 500 ml milk bottles where the quality characteristic X of interest is the weight (in ml) of each bottle. The context of the example presented here is similar to the one introduced in Castagliola et al.[7]. We assume that, from the Phase I data , the following quantities have been estimated: $\mu_0 = 500.023$ and $\sigma_0 = 0.9616$. According to the quality practitioner in charge of this process, a shift of $0.5\sigma_0$ (i.e. $\delta = 0.5$) in the mean should be interpreted as a signal that something is going wrong in the production. Concerning the parameters of the linear covariate error model, we assume $\eta = 0.28$, $B = 1$, $A = 0$, $m = 1$ and $n = 5$. We set $ARL_0 = 370.4$ for Synthetic $\tilde{X}$ control chart. By using the optimization procedure, we have $K^* = 1.3552$, $H^* = 22$.

INSERT TABLE 9 ABOUT HERE

INSERT FIGURE 1 ABOUT HERE

Based on (24), the control limits of the Synthetic $\tilde{X}$ sub-chart in the presence of measurement errors are:

$$\begin{aligned} LCL^* &= 500.023 - 1.3552 \times \sqrt{0.9616^2 + (0.9616 \times 0.28)^2} = 498.6698, \\ UCL^* &= 500.023 + 1.3552 \times \sqrt{0.9616^2 + (0.9616 \times 0.28)^2} = 501.3762. \end{aligned}$$

The first 10 subgroups are supposed to be in-control while the last 10 subgroups are supposed to have less milk weight, and thus, to be out-of-control. The corresponding sample median values $\tilde{Y}_i$ are presented in Table 9 and plotted in the $\tilde{X}$ sub-chart in Figure 1, respectively. The chart does not trigger any signal during in-control production. On the other hand, at sample #16 a point is plotted above UCL^* and a conforming run length $CRL_1 = 16 < H^* = 22$ is recorded. Therefore, the Synthetic $\tilde{X}$ control chart triggers an alarm signalling at sample #16.

8 Concluding Remarks

In this paper we proposed a Synthetic $\tilde{X}$ control chart and investigated its statistical properties via a Markov chain methodology. We also studied the effects of measurement errors on the performance of the Synthetic $\tilde{X}$ control chart by assuming a linear covariate error model. Based on the presented results, it is obvious that measurement errors greatly affect the performance of Synthetic $\tilde{X}$ chart compared to no measurement errors case. The performance of the Synthetic $\tilde{X}$ chart deteriorates when the measurement errors increase. As a result, increasing the coefficient B in the linear covariate model can reduce the negative effect of measurement errors on Synthetic $\tilde{X}$ chart. Furthermore, measuring each item several times can also reduce the efffects of measurement errors on the performance of Synthetic $\tilde{X}$ chart, but increasing at the same time the cost of monitoring and control.

Investigation of the effect of measurement errors on the performance of other Synthetic-type control charts along similar lines, as well as their economic-statistical design will be of great interest. For both, research is currently in progress.

References

[1] Ahmad, S., Riaz, M., Abbasi, S. and Lin, Z. 2014*a*, 'On efficient median control charting', *Journal of the Chinese Institute of Engineers* **37**(3), 358–375.

[2] Ahmad, S., Riaz, M., Abbasi, S. and Lin, Z. 2014*b*, 'On median control charting under double sampling scheme', *European Journal of Industrial Engineering* **8**(4), 478–512.

[3] Aslam, M., Azam, M., Khan, N. and Jun, C. 2015, 'A control chart for an exponential distribution using multiple dependent state sampling', *Quality & Quantity* **49**(2), 455–462.

[4] Bennet, C. 1954, 'Effect of Measurement Error on Chemical Process Control', *Industry Quality Control* **10**(4), 17–20.

[5] Castagliola, P. 2001, ‘An $\bar{X}/R$-EWMA Control Chart For Monitoring the Process Sample Median’, *International Journal of Reliability, Quality and Safety Engineering* **8**(2), 123–135.

[6] Castagliola, P. and Figueiredo, F. 2013, ‘The Median Chart with Estimated Parameters’, *European Journal of Industrial Engineering* **7**(5), 594–614.

[7] Castagliola, P., Maravelakis, P. E. and Figueiredo, F. O. 2016, ‘The ewma median chart with estimated parameters’, *IIE Transactions* **48**(1), 66–74.

[8] Chen, F. and Huang, H. 2005, ‘A Synthetic Control Chart for Monitoring Process Dispersion with Sample Range’, *International Journal of Advanced Manufacturing Technology* **26**(7-8), 842–851.

[9] Chen, Y. and Hsieh, K. 2007, ‘Hotellings T^2 Charts with Variable Sample Size and Control Limit’, *European Journal of Operational Research* **182**, 1251–1262.

[10] Cheng, X. and Wang, F. K. 2017, ‘The performance of ewma median and cusum median control charts for a normal process with measurement errors’, *Quality and Reliability Engineering International* . DOI: 10.1002/qre.2248 (to appear).

[11] Costa, A. and Castagliola, P. 2011, ‘Effect of Measurement Error and Autocorrelation on the $\bar{X}$ Chart’, *Journal of Applied Statistics* **38**(4), 661–673.

[12] Costa, A., de Magalhaes, M. and Epprecht, E. 2009, ‘Monitoring the Process Mean and Variance Using a Synthetic Chart with Two-stage Testing’, *International Journal of Production Research* **47**(18), 5067–5086.

[13] Costa, A. and Rahim, M. 2006, ‘A Synthetic Control Chart for Monitoring the Process Mean and Variance’, *Journal of Quality in Maintenance Engineering* **12**(1), 81–88.

[14] Darroch, J. and Seneta, E. 1965, ‘On quasi-stationary distributions in absorbing discrete-time finite markov chains’, *Journal of Applied Probability* **2**(1), 88–100.

[15] Davis, R. and Woodall, W. 2002*a*, ‘Evaluting and Improving the Synthetic Control Chart’, *Journal of Quality Technology* **34**(2), 200–208.

[16] Davis, R. and Woodall, W. H. 2002*b*, ‘Evaluating and improving the synthetic control chart’, *Journal of Quality Technology* **34**(2), 200.

[17] Faraz, A., Chalaki, K. and Moghadam, M. 2011, 'On the properties of the hotellings t^2 control chart with variable sampling intervals', *Quality & Quantity* **45**(3), 579–586.

[18] Frisen, M. 2011, 'On Multivariate Control Charts', *Production* **21**(2), 235–241.

[19] Hu, X. and Castagliola, P. 2017, 'Guaranteed conditional design of the median chart with estimated parameters', *Quality and Reliability Engineering International* .

[20] Hu, X., Castagliola, P., Sun, J. and Khoo, M. 2014, 'The effect of measurement errors on the synthetic $\bar{X}$ chart', *Quality and Reliability Engineering International* **31**(8), 1769–1778.

[21] Hu, X., Castagliola, P., Sun, J. and Khoo, M. 2016, 'Economic design of the upper-sided synthetic chart with measurement errors', *International Journal of Production Research* **54**(19), 5651–5670.

[22] Huang, H. and Chen, F. 2005, 'A Synthetic Control Chart for Monitoring Process Dispersion with Sample Standard Deviation', *Computers & Industrial Engineering* **49**(2), 221–240.

[23] Kanazuka, T. 1986, 'The Effect of Measurement Error on the Power of the $\bar{X}$ and R Charts', *Journal of Quality Technology* **18**(2), 91–95.

[24] Khoo, M. 2005, 'A control chart based on sample median for the detection of a permanent shift in the process mean', *Quality Engineering* **17**(2), 243–257.

[25] Khoo, M., Lee, H., Wu, Z., Chen, C. and Castagliola, P. 2011, 'A Synthetic Double Sampling Control Chart for the Process Mean', *IIE Transactions* **43**(1), 23–38.

[26] Latouche, G. and Ramaswami, V. 1999, *Introduction to Matrix Analytic Methods in Stochastic Modelling*, Series on Statistics and Applied Probability. SIAM, Philadelphia, PA.

[27] Lin, Y., Chou, C. and Chen, C. 2017, 'Robustness of the ewma median control chart to non-normality', *International Journal of Industrial and Systems Engineering* **25**(1), 35–58.

[28] Linna, K. and Woodall, W. 2001, 'Effect of Measurement Error on Shewhart Control Chart', *Journal of Quality Technology* **33**(2), 213–222.

[29] Linna, K., Woodall, W. and Busby, K. 2001, 'The performance of multivariate control charts in the presence of measurement error', *Journal of Quality Technology* **33**(3), 349.

[30] Maleki, M., Amiri, A. and Castagliola, P. 2017, ‘Measurement errors in statistical process monitoring: A literature review’, *Computers & Industrial Engineering* **103**, 316–329.

[31] Maravelakis, P. 2004, ‘EWMA Chart and Measurement Error’, *Journal of Applied Statistics* **31**(4), 445–455.

[32] Maravelakis, P. 2012, ‘Measurement Error Effect on the CUSUM Control Chart’, *Journal of Applied Statistics* **39**(2), 323–336.

[33] Montgomery, D. 2013, *Introduction to Statistical Quality Control, 7th Edn*, Wiley, New York.

[34] Neuts, M. 1981, *Matrix-Geometric Solutions in Stochastic Models: an Algorithmic Approach*, Johns Hopkins University Press, Baltimore, MD.

[35] Noorossana, R. and Zerehsaz, Y. 2015, ‘Effect of measurement error on phase II monitoring of simple linear profiles’, *The International Journal of Advanced Manufacturing Technology* **79**(9-12), 2031–2040.

[36] Sheu, S. and Yang, L. 2006, ‘The generally weighted moving average control chart for monitoring the process median’, *Quality Engineering* **18**(3), 333–344.

[37] Tran, K. 2017, ‘Run Rules median control charts for monitoring process mean in manufacturing’, *Quality and Reliability Engineering International* **33**(8), 24372450.

[38] Tran, K., Castagliola, P. and Balakrishnan, N. 2017, ‘On the performance of shewhart median chart in the presence of measurement errors’, *Quality and Reliability Engineering International* **33**(5), 1019–1029.

[39] Tran, K., Castagliola, P. and Celano, G. 2016, ‘The performance of the Shewhart-RZ control chart in the presence of measurement error’, *International Journal of Production Research* **54**, 7504–7522.

[40] Tran, P. and Tran, K. P. 2016, ‘The Efficiency of CUSUM schemes for monitoring the Coefficient of Variation’, *Applied Stochastic Models in Business and Industry* **32**(6), 870–881.

[41] Wu, Z., Ou, Y., Castagliola, P. and Khoo, M. 2010, ‘A Combined Synthetic&X Chart for Monitoring the Process Mean’, *International Journal of Production Research* **48**(24), 7423–7436.

[42] Wu, Z. and Spedding, T. 2000, ‘A Synthetic Control Chart for Detecting Small Shifts in the Process Mean’, *Journal of Quality Technology* **32**(1), 32–38.

[43] Zhang, Y., Castagliola, P., Wu, Z. and Khoo, M. 2011, 'The Synthetic $\bar{X}$ Chart with Estimated Parameters', *IIE Transactions* **43**(9), 676–687.

δ	Zero state ARL			
	$n=3$	$n=5$	$n=7$	$n=9$
0.1	$(100, 1.8533)$	$(95, 1.4771)$	$(91, 1.2613)$	$(88, 1.1178)$
	$(319.3, 413.2)$	$(295.0, 382.1)$	$(272.9, 354.0)$	$(253.2, 328.8)$
0.2	$(79, 1.8305)$	$(67, 1.4498)$	$(59, 1.2320)$	$(53, 1.0872)$
	$(217.5, 282.9)$	$(170.2, 221.8)$	$(136.4, 178.1)$	$(111.8, 146.2)$
0.3	$(58, 1.7996)$	$(45, 1.4172)$	$(36, 1.1968)$	$(31, 1.0528)$
	$(131.6, 171.9)$	$(87.9, 115.0)$	$(62.7, 82.0)$	$(47.1, 61.4)$
0.5	$(30, 1.7300)$	$(20, 1.3466)$	$(15, 1.1302)$	$(12, 0.9878)$
	$(46.1, 60.1)$	$(25.3, 32.6)$	$(16.1, 20.4)$	$(11.2, 14.0)$
0.7	$(17, 1.6664)$	$(11, 1.2912)$	$(8, 1.0792)$	$(6, 0.9370)$
	$(18.3, 23.4)$	$(9.4, 11.4)$	$(5.9, 6.8)$	$(4.1, 4.6)$
1.0	$(8, 1.5773)$	$(5, 1.2141)$	$(4, 1.0201)$	$(3, 0.8834)$
	$(6.3, 7.4)$	$(3.3, 3.5)$	$(2.2, 2.0)$	$(1.7, 1.4)$
1.5	$(4, 1.4907)$	$(3, 1.1617)$	$(2, 0.9579)$	$(2, 0.8508)$
	$(2.1, 1.8)$	$(1.4, 0.8)$	$(1.2, 0.5)$	$(1.1, 0.3)$
2.0	$(2, 1.3995)$	$(2, 1.1187)$	$(2, 0.9579)$	$(2, 0.8508)$
	$(1.3, 0.7)$	$(1.1, 0.3)$	$(1.0, 0.1)$	$(1.0, 0.1)$

Table 1: (H^*, K^*) values (first row) and zero states $(ARL_1,\ SDRL_1)$ values (second row) of the Synthetic $\tilde{X}$ control chart control chart for different values of $\delta \in\ [0.1, 2]$, $n \in \{3, 5, 7, 9\}$ and $ARL_0 = 370.4$

δ	Steady state ARL			
	$n=3$	$n=5$	$n=7$	$n=9$
0.1	$(42, 1.7347)$	$(43, 1.3881)$	$(43, 1.1881)$	$(42, 1.0535)$
	$(326.6, 329.7)$	$(305.3, 307.9)$	$(285.8, 287.9)$	$(268.0, 269.6)$
0.2	$(37, 1.7225)$	$(31, 1.3627)$	$(27, 1.1570)$	$(24, 1.0201)$
	$(235.3, 235.6)$	$(190.4, 189.5)$	$(157.3, 155.7)$	$(132.4, 130.3)$
0.3	$(26, 1.6875)$	$(20, 1.3271)$	$(17, 1.1244)$	$(14, 0.9858)$
	$(152.4, 150.7)$	$(107.2, 104.8)$	$(79.8, 77.2)$	$(62.0, 59.4)$
0.5	$(14, 1.6227)$	$(10, 1.2672)$	$(8, 1.0676)$	$(6, 0.9278)$
	$(60.7, 58.1)$	$(35.6, 33.3)$	$(23.8, 21.6)$	$(17.3, 15.4)$
0.7	$(8, 1.5602)$	$(6, 1.2202)$	$(5, 1.0299)$	$(4, 0.8982)$
	$(26.8, 24.6)$	$(14.7, 12.7)$	$(9.6, 7.9)$	$(7.1, 5.4)$
1.0	$(5, 1.5050)$	$(3, 1.1526)$	$(3, 0.9869)$	$(2, 0.8450)$
	$(10.3, 8.5)$	$(5.7, 4.3)$	$(4.0, 2.5)$	$(3.2, 1.8)$
1.5	$(3, 1.4420)$	$(2, 1.1110)$	$(2, 0.9513)$	$(2, 0.8450)$
	$(3.8, 2.3)$	$(2.6, 1.2)$	$(2.2, 0.7)$	$(2.0, 0.5)$
2.0	$(2, 1.3899)$	$(2, 1.1110)$	$(4, 1.0113)$	$(6, 0.9278)$
	$(2.4, 0.9)$	$(2.0, 0.4)$	$(1.9, 0.4)$	$(1.9, 0.4)$

Table 2: (H^*, K^*) values (first row) and steady states $(SARL_1,\ SSDRL_1)$ values (second row) of the Synthetic $\tilde{X}$ control chart control chart for different values of $\delta \in\ [0.1, 2]$, $n \in \{3, 5, 7, 9\}$ and $ARL_0 = 370.4$

δ	$\eta=0$	$\eta=0.1$	$\eta=0.2$	$\eta=0.3$	$\eta=0.5$	$\eta=1$
			$n=3$			
0.1	$(100, 1.8533, 319.3)$	$(100, 1.8533, 319.8)$	$(100, 1.8533, 321.1)$	$(101, 1.8542, 323.1)$	$(102, 1.8552, 328.6)$	$(105, 1.8579, 343.4)$
0.2	$(79, 1.8305, 217.5)$	$(79, 1.8305, 218.5)$	$(80, 1.8318, 221.4)$	$(81, 1.8330, 226.0)$	$(83, 1.8354, 239.1)$	$(92, 1.8453, 278.4)$
0.3	$(58, 1.7996, 131.6)$	$(58, 1.7996, 132.6)$	$(59, 1.8014, 135.7)$	$(60, 1.8031, 140.7)$	$(64, 1.8096, 155.3)$	$(76, 1.8267, 205.5)$
0.5	$(30, 1.7300, 46.1)$	$(30, 1.7300, 46.7)$	$(31, 1.7336, 48.4)$	$(32, 1.7370, 51.3)$	$(35, 1.7467, 60.3)$	$(48, 1.7801, 99.0)$
0.7	$(17, 1.6664, 18.3)$	$(17, 1.6664, 18.6)$	$(17, 1.6664, 19.4)$	$(18, 1.6730, 20.8)$	$(20, 1.6849, 25.3)$	$(30, 1.7300, 47.3)$
1.0	$(8, 1.5773, 6.3)$	$(8, 1.5773, 6.4)$	$(9, 1.5916, 6.7)$	$(9, 1.5916, 7.2)$	$(10, 1.6042, 8.8)$	$(16, 1.6595, 17.8)$
1.5	$(4, 1.4907, 2.1)$	$(4, 1.4907, 2.1)$	$(4, 1.4907, 2.2)$	$(4, 1.4907, 2.3)$	$(5, 1.5191, 2.8)$	$(7, 1.5610, 5.3)$
2.0	$(2, 1.3995, 1.3)$	$(2, 1.3995, 1.3)$	$(3, 1.4534, 1.3)$	$(3, 1.4534, 1.3)$	$(3, 1.4534, 1.5)$	$(4, 1.4907, 2.4)$
			$n=5$			
δ	$\eta=0$	$\eta=0.1$	$\eta=0.2$	$\eta=0.3$	$\eta=0.5$	$\eta=1$
0.1	$(95, 1.4771, 295.0)$	$(95, 1.4771, 295.6)$	$(95, 1.4771, 297.4)$	$(96, 1.4779, 300.3)$	$(97, 1.4787, 308.0)$	$(101, 1.4818, 329.4)$
0.2	$(67, 1.4498, 170.2)$	$(68, 1.4510, 171.2)$	$(68, 1.4510, 174.4)$	$(70, 1.4533, 179.4)$	$(73, 1.4566, 194.1)$	$(84, 1.4677, 241.0)$
0.3	$(45, 1.4172, 87.9)$	$(45, 1.4172, 88.8)$	$(46, 1.4191, 91.4)$	$(47, 1.4209, 95.7)$	$(51, 1.4276, 108.6)$	$(64, 1.4461, 157.4)$
0.5	$(20, 1.3466, 25.3)$	$(21, 1.3510, 25.6)$	$(21, 1.3510, 26.7)$	$(22, 1.3552, 28.5)$	$(25, 1.3666, 34.3)$	$(36, 1.3983, 61.7)$
0.7	$(11, 1.2912, 9.4)$	$(11, 1.2912, 9.5)$	$(11, 1.2912, 9.9)$	$(12, 1.2994, 10.7)$	$(14, 1.3139, 13.1)$	$(21, 1.3510, 26.0)$
1.0	$(5, 1.2141, 3.3)$	$(5, 1.2141, 3.4)$	$(6, 1.2324, 3.5)$	$(6, 1.2324, 3.7)$	$(7, 1.2476, 4.5)$	$(11, 1.2912, 9.1)$
1.5	$(3, 1.1617, 1.4)$	$(3, 1.1617, 1.4)$	$(3, 1.1617, 1.4)$	$(3, 1.1617, 1.5)$	$(3, 1.1617, 1.7)$	$(5, 1.2141, 2.8)$
2.0	$(2, 1.1187, 1.1)$	$(2, 1.1187, 1.1)$	$(2, 1.1187, 1.1)$	$(2, 1.1187, 1.1)$	$(2, 1.1187, 1.1)$	$(3, 1.1617, 1.5)$
			$n=7$			
δ	$\eta=0$	$\eta=0.1$	$\eta=0.2$	$\eta=0.3$	$\eta=0.5$	$\eta=1$
0.1	$(91, 1.2613, 272.9)$	$(91, 1.2613, 273.7)$	$(92, 1.2620, 275.9)$	$(92, 1.2620, 279.4)$	$(94, 1.2634, 289.0)$	$(100, 1.2674, 315.9)$
0.2	$(59, 1.2320, 136.4)$	$(59, 1.2320, 137.4)$	$(60, 1.2332, 140.5)$	$(62, 1.2355, 145.5)$	$(65, 1.2387, 160.2)$	$(77, 1.2502, 210.2)$
0.3	$(36, 1.1968, 62.7)$	$(37, 1.1988, 63.4)$	$(37, 1.1988, 65.6)$	$(39, 1.2026, 69.1)$	$(42, 1.2080, 80.1)$	$(56, 1.2284, 124.2)$
0.5	$(15, 1.1302, 16.1)$	$(16, 1.1353, 16.3)$	$(16, 1.1353, 17.0)$	$(17, 1.1400, 18.2)$	$(19, 1.1487, 22.2)$	$(28, 1.1782, 42.1)$
0.7	$(8, 1.0792, 5.9)$	$(8, 1.0792, 5.9)$	$(8, 1.0792, 6.2)$	$(9, 1.0890, 6.7)$	$(10, 1.0976, 8.2)$	$(16, 1.1353, 16.5)$
1.0	$(4, 1.0201, 2.2)$	$(4, 1.0201, 2.3)$	$(4, 1.0201, 2.3)$	$(4, 1.0201, 2.5)$	$(5, 1.0395, 2.9)$	$(8, 1.0792, 5.7)$
1.5	$(2, 0.9579, 1.2)$	$(2, 0.9579, 1.2)$	$(2, 0.9579, 1.2)$	$(2, 0.9579, 1.2)$	$(3, 0.9947, 1.3)$	$(4, 1.0201, 2.0)$
2.0	$(2, 0.9579, 1.0)$	$(2, 0.9579, 1.0)$	$(2, 0.9579, 1.0)$	$(2, 0.9579, 1.0)$	$(2, 0.9579, 1.0)$	$(2, 0.9579, 1.2)$
			$n=9$			
δ	$\eta=0$	$\eta=0.1$	$\eta=0.2$	$\eta=0.3$	$\eta=0.5$	$\eta=1$
0.1	$(88, 1.1178, 253.2)$	$(88, 1.1178, 254.0)$	$(88, 1.1178, 256.5)$	$(89, 1.1184, 260.5)$	$(91, 1.1197, 271.5)$	$(98, 1.1240, 303.2)$
0.2	$(53, 1.0872, 111.8)$	$(53, 1.0872, 112.7)$	$(54, 1.0883, 115.6)$	$(55, 1.0895, 120.4)$	$(59, 1.0938, 134.5)$	$(72, 1.1059, 184.8)$
0.3	$(31, 1.0528, 47.1)$	$(31, 1.0528, 47.7)$	$(32, 1.0549, 49.4)$	$(33, 1.0569, 52.3)$	$(36, 1.0626, 61.5)$	$(49, 1.0823, 100.4)$
0.5	$(12, 0.9878, 11.2)$	$(12, 0.9878, 11.4)$	$(13, 0.9935, 11.9)$	$(13, 0.9935, 12.8)$	$(15, 1.0035, 15.7)$	$(23, 1.0329, 30.7)$
0.7	$(6, 0.9370, 4.1)$	$(6, 0.9370, 4.2)$	$(7, 0.9485, 4.4)$	$(7, 0.9485, 4.7)$	$(8, 0.9584, 5.7)$	$(13, 0.9935, 11.6)$
1.0	$(3, 0.8834, 1.7)$	$(3, 0.8834, 1.7)$	$(3, 0.8834, 1.8)$	$(4, 0.9060, 1.9)$	$(4, 0.9060, 2.2)$	$(6, 0.9370, 4.0)$
1.5	$(2, 0.8508, 1.1)$	$(2, 0.8508, 1.1)$	$(2, 0.8508, 1.1)$	$(2, 0.8508, 1.1)$	$(2, 0.8508, 1.1)$	$(3, 0.8834, 1.6)$
2.0	$(2, 0.8508, 1.0)$	$(2, 0.8508, 1.0)$	$(2, 0.8508, 1.0)$	$(2, 0.8508, 1.0)$	$(2, 0.8508, 1.0)$	$(2, 0.8508, 1.1)$

Table 3: (H^*, K^*, ARL_1) values of the Synthetic $\tilde{X}$ chart control chart in the presence of measurement errors for different values of η, δ, n, $B = 1$, $m = 1$

δ	$\eta = 0$	$\eta = 0.1$	$\eta = 0.2$	$\eta = 0.3$	$\eta = 0.5$	$\eta = 1$
			$n = 3$			
0.1	$(42, 1.7347, 326.6)$	$(42, 1.7347, 327.0)$	$(42, 1.7347, 328.2)$	$(42, 1.7347, 329.9)$	$(41, 1.7324, 334.7)$	$(37, 1.7225, 347.5)$
0.2	$(37, 1.7225, 235.3)$	$(37, 1.7225, 236.2)$	$(38, 1.7250, 238.9)$	$(38, 1.7250, 243.2)$	$(39, 1.7276, 255.2)$	$(42, 1.7347, 290.6)$
0.3	$(26, 1.6875, 152.4)$	$(27, 1.6913, 153.5)$	$(27, 1.6913, 156.6)$	$(28, 1.6950, 161.5)$	$(30, 1.7019, 176.0)$	$(36, 1.7198, 224.0)$
0.5	$(14, 1.6227, 60.7)$	$(14, 1.6227, 61.4)$	$(14, 1.6227, 63.4)$	$(14, 1.6227, 66.7)$	$(16, 1.6371, 77.0)$	$(22, 1.6705, 118.9)$
0.7	$(8, 1.5602, 26.8)$	$(8, 1.5602, 27.1)$	$(8, 1.5602, 28.2)$	$(9, 1.5737, 29.9)$	$(10, 1.5856, 35.6)$	$(14, 1.6227, 62.1)$
1.0	$(5, 1.5050, 10.3)$	$(5, 1.5050, 10.4)$	$(5, 1.5050, 10.8)$	$(5, 1.5050, 11.5)$	$(6, 1.5267, 13.9)$	$(8, 1.5602, 26.1)$
1.5	$(3, 1.4420, 3.8)$	$(3, 1.4420, 3.8)$	$(3, 1.4420, 4.0)$	$(3, 1.4420, 4.2)$	$(3, 1.4420, 4.9)$	$(4, 1.4778, 8.8)$
2.0	$(2, 1.3899, 2.4)$	$(2, 1.3899, 2.4)$	$(2, 1.3899, 2.5)$	$(2, 1.3899, 2.5)$	$(2, 1.3899, 2.8)$	$(3, 1.4420, 4.3)$
			$n = 5$			
δ	$\eta = 0$	$\eta = 0.1$	$\eta = 0.2$	$\eta = 0.3$	$\eta = 0.5$	$\eta = 1$
0.1	$(43, 1.3881, 305.3)$	$(43, 1.3881, 305.9)$	$(43, 1.3881, 307.5)$	$(43, 1.3881, 310.0)$	$(43, 1.3881, 316.8)$	$(41, 1.3845, 335.4)$
0.2	$(31, 1.3627, 190.4)$	$(32, 1.3652, 191.4)$	$(32, 1.3652, 194.5)$	$(33, 1.3676, 199.3)$	$(34, 1.3700, 213.3)$	$(39, 1.3806, 256.8)$
0.3	$(20, 1.3271, 107.2)$	$(20, 1.3271, 108.2)$	$(21, 1.3312, 110.9)$	$(21, 1.3312, 115.5)$	$(23, 1.3387, 129.0)$	$(30, 1.3601, 178.1)$
0.5	$(10, 1.2672, 35.6)$	$(10, 1.2672, 36.1)$	$(10, 1.2672, 37.4)$	$(10, 1.2672, 39.7)$	$(11, 1.2757, 46.8)$	$(16, 1.3084, 78.6)$
0.7	$(6, 1.2202, 14.7)$	$(6, 1.2202, 14.9)$	$(6, 1.2202, 15.5)$	$(6, 1.2202, 16.5)$	$(7, 1.2347, 19.8)$	$(10, 1.2672, 36.6)$
1.0	$(3, 1.1526, 5.7)$	$(3, 1.1526, 5.8)$	$(3, 1.1526, 6.0)$	$(4, 1.1812, 6.4)$	$(4, 1.1812, 7.6)$	$(6, 1.2202, 14.3)$
1.5	$(2, 1.1110, 2.6)$	$(2, 1.1110, 2.6)$	$(2, 1.1110, 2.7)$	$(2, 1.1110, 2.8)$	$(2, 1.1110, 3.1)$	$(3, 1.1526, 5.0)$
2.0	$(2, 1.1110, 2.0)$	$(2, 1.1110, 2.0)$	$(2, 1.1110, 2.0)$	$(2, 1.1110, 2.1)$	$(2, 1.1110, 2.2)$	$(2, 1.1110, 2.8)$
			$n = 7$			
δ	$\eta = 0$	$\eta = 0.1$	$\eta = 0.2$	$\eta = 0.3$	$\eta = 0.5$	$\eta = 1$
0.1	$(43, 1.1881, 285.8)$	$(43, 1.1881, 286.4)$	$(43, 1.1881, 288.4)$	$(43, 1.1881, 291.5)$	$(43, 1.1881, 300.0)$	$(43, 1.1881, 323.7)$
0.2	$(27, 1.1570, 157.3)$	$(28, 1.1595, 158.3)$	$(28, 1.1595, 161.4)$	$(29, 1.1619, 166.4)$	$(31, 1.1664, 180.8)$	$(37, 1.1782, 228.5)$
0.3	$(17, 1.1244, 79.8)$	$(17, 1.1244, 80.6)$	$(17, 1.1244, 83.0)$	$(18, 1.1285, 86.9)$	$(19, 1.1324, 98.8)$	$(26, 1.1544, 145.0)$
0.5	$(8, 1.0676, 23.8)$	$(8, 1.0676, 24.1)$	$(8, 1.0676, 25.1)$	$(8, 1.0676, 26.7)$	$(9, 1.0767, 31.8)$	$(13, 1.1047, 56.1)$
0.7	$(5, 1.0299, 9.6)$	$(5, 1.0299, 9.8)$	$(5, 1.0299, 10.1)$	$(5, 1.0299, 10.8)$	$(5, 1.0299, 13.0)$	$(8, 1.0676, 24.5)$
1.0	$(3, 0.9869, 4.0)$	$(3, 0.9869, 4.1)$	$(3, 0.9869, 4.2)$	$(3, 0.9869, 4.4)$	$(3, 0.9869, 5.2)$	$(4, 1.0113, 9.4)$
1.5	$(2, 0.9513, 2.2)$	$(2, 0.9513, 2.2)$	$(2, 0.9513, 2.2)$	$(2, 0.9513, 2.3)$	$(2, 0.9513, 2.5)$	$(3, 0.9869, 3.6)$
2.0	$(4, 1.0113, 1.9)$	$(4, 1.0113, 1.9)$	$(4, 1.0113, 1.9)$	$(3, 0.9869, 2.0)$	$(2, 0.9513, 2.0)$	$(2, 0.9513, 2.3)$
			$n = 9$			
δ	$\eta = 0$	$\eta = 0.1$	$\eta = 0.2$	$\eta = 0.3$	$\eta = 0.5$	$\eta = 1$
0.1	$(42, 1.0535, 268.0)$	$(42, 1.0535, 268.8)$	$(42, 1.0535, 271.1)$	$(42, 1.0535, 274.6)$	$(43, 1.0549, 284.5)$	$(44, 1.0562, 312.6)$
0.2	$(24, 1.0201, 132.4)$	$(24, 1.0201, 133.4)$	$(25, 1.0226, 136.3)$	$(26, 1.0250, 141.2)$	$(28, 1.0295, 155.5)$	$(34, 1.0411, 204.6)$
0.3	$(14, 0.9858, 62.0)$	$(14, 0.9858, 62.7)$	$(15, 0.9903, 64.7)$	$(15, 0.9903, 68.0)$	$(17, 0.9984, 78.4)$	$(23, 1.0175, 120.6)$
0.5	$(6, 0.9278, 17.3)$	$(6, 0.9278, 17.5)$	$(7, 0.9387, 18.2)$	$(7, 0.9387, 19.4)$	$(8, 0.9480, 23.3)$	$(11, 0.9698, 42.4)$
0.7	$(4, 0.8982, 7.1)$	$(4, 0.8982, 7.1)$	$(4, 0.8982, 7.4)$	$(4, 0.8982, 7.9)$	$(4, 0.8982, 9.5)$	$(6, 0.9278, 17.8)$
1.0	$(2, 0.8450, 3.2)$	$(2, 0.8450, 3.2)$	$(2, 0.8450, 3.3)$	$(3, 0.8765, 3.5)$	$(3, 0.8765, 4.0)$	$(4, 0.8982, 6.9)$
1.5	$(2, 0.8450, 2.0)$	$(2, 0.8450, 2.0)$	$(2, 0.8450, 2.1)$	$(2, 0.8450, 2.1)$	$(2, 0.8450, 2.2)$	$(2, 0.8450, 2.9)$
2.0	$(6, 0.9278, 1.9)$	$(6, 0.9278, 1.9)$	$(6, 0.9278, 1.9)$	$(6, 0.9278, 1.9)$	$(4, 0.8982, 1.9)$	$(2, 0.8450, 2.1)$

Table 4: $(H^*, K^*, SARL_1)$ values of the Synthetic $\tilde{X}$ chart control chart in the presence of measurement errors for different values of η, δ, n, $B = 1$, $m = 1$

δ	$B=1$	$B=2$	$B=3$	$B=4$	$B=5$
			$n=3$		
0.1	$(101, 1.8542, 322.6)$	$(100, 1.8533, 320.2)$	$(100, 1.8533, 319.7)$	$(100, 1.8533, 319.5)$	$(100, 1.8533, 319.5)$
0.2	$(80, 1.8318, 225.0)$	$(79, 1.8305, 219.4)$	$(79, 1.8305, 218.4)$	$(79, 1.8305, 218.0)$	$(79, 1.8305, 217.8)$
0.3	$(60, 1.8031, 139.5)$	$(58, 1.7996, 133.6)$	$(58, 1.7996, 132.5)$	$(58, 1.7996, 132.1)$	$(58, 1.7996, 131.9)$
0.5	$(32, 1.7370, 50.6)$	$(30, 1.7300, 47.2)$	$(30, 1.7300, 46.6)$	$(30, 1.7300, 46.4)$	$(30, 1.7300, 46.3)$
0.7	$(18, 1.6730, 20.5)$	$(17, 1.6664, 18.9)$	$(17, 1.6664, 18.6)$	$(17, 1.6664, 18.5)$	$(17, 1.6664, 18.4)$
1.0	$(9, 1.5916, 7.0)$	$(9, 1.5916, 6.5)$	$(8, 1.5773, 6.4)$	$(8, 1.5773, 6.3)$	$(8, 1.5773, 6.3)$
1.5	$(4, 1.4907, 2.3)$	$(4, 1.4907, 2.2)$	$(4, 1.4907, 2.1)$	$(4, 1.4907, 2.1)$	$(4, 1.4907, 2.1)$
2.0	$(3, 1.4534, 1.3)$	$(2, 1.3995, 1.3)$	$(2, 1.3995, 1.3)$	$(2, 1.3995, 1.3)$	$(2, 1.3995, 1.3)$
			$n=5$		
δ	$B=1$	$B=2$	$B=3$	$B=4$	$B=5$
0.1	$(96, 1.4779, 299.6)$	$(95, 1.4771, 296.2)$	$(95, 1.4771, 295.5)$	$(95, 1.4771, 295.3)$	$(95, 1.4771, 295.2)$
0.2	$(69, 1.4522, 178.3)$	$(68, 1.4510, 172.3)$	$(68, 1.4510, 171.1)$	$(68, 1.4510, 170.7)$	$(67, 1.4498, 170.5)$
0.3	$(47, 1.4209, 94.7)$	$(45, 1.4172, 89.6)$	$(45, 1.4172, 88.7)$	$(45, 1.4172, 88.3)$	$(45, 1.4172, 88.2)$
0.5	$(22, 1.3552, 28.1)$	$(21, 1.3510, 26.0)$	$(21, 1.3510, 25.6)$	$(21, 1.3510, 25.4)$	$(21, 1.3510, 25.4)$
0.7	$(12, 1.2994, 10.5)$	$(11, 1.2912, 9.6)$	$(11, 1.2912, 9.5)$	$(11, 1.2912, 9.4)$	$(11, 1.2912, 9.4)$
1.0	$(6, 1.2324, 3.7)$	$(5, 1.2141, 3.4)$	$(5, 1.2141, 3.3)$	$(5, 1.2141, 3.3)$	$(5, 1.2141, 3.3)$
1.5	$(3, 1.1617, 1.5)$	$(3, 1.1617, 1.4)$	$(3, 1.1617, 1.4)$	$(3, 1.1617, 1.4)$	$(3, 1.1617, 1.4)$
2.0	$(2, 1.1187, 1.1)$	$(2, 1.1187, 1.1)$	$(2, 1.1187, 1.1)$	$(2, 1.1187, 1.1)$	$(2, 1.1187, 1.1)$
			$n=7$		
δ	$B=1$	$B=2$	$B=3$	$B=4$	$B=5$
0.1	$(92, 1.2620, 278.6)$	$(91, 1.2613, 274.4)$	$(91, 1.2613, 273.6)$	$(91, 1.2613, 273.3)$	$(91, 1.2613, 273.2)$
0.2	$(61, 1.2343, 144.4)$	$(60, 1.2332, 138.4)$	$(59, 1.2320, 137.3)$	$(59, 1.2320, 136.9)$	$(59, 1.2320, 136.7)$
0.3	$(38, 1.2008, 68.3)$	$(37, 1.1988, 64.1)$	$(37, 1.1988, 63.4)$	$(37, 1.1988, 63.1)$	$(37, 1.1988, 63.0)$
0.5	$(17, 1.1400, 17.9)$	$(16, 1.1353, 16.5)$	$(16, 1.1353, 16.3)$	$(16, 1.1353, 16.2)$	$(16, 1.1353, 16.1)$
0.7	$(9, 1.0890, 6.6)$	$(8, 1.0792, 6.0)$	$(8, 1.0792, 5.9)$	$(8, 1.0792, 5.9)$	$(8, 1.0792, 5.9)$
1.0	$(4, 1.0201, 2.4)$	$(4, 1.0201, 2.3)$	$(4, 1.0201, 2.3)$	$(4, 1.0201, 2.2)$	$(4, 1.0201, 2.2)$
1.5	$(2, 0.9579, 1.2)$	$(2, 0.9579, 1.2)$	$(2, 0.9579, 1.2)$	$(2, 0.9579, 1.2)$	$(2, 0.9579, 1.2)$
2.0	$(2, 0.9579, 1.0)$	$(2, 0.9579, 1.0)$	$(2, 0.9579, 1.0)$	$(2, 0.9579, 1.0)$	$(2, 0.9579, 1.0)$
			$n=9$		
δ	$B=1$	$B=2$	$B=3$	$B=4$	$B=5$
0.1	$(89, 1.1184, 259.6)$	$(88, 1.1178, 254.8)$	$(88, 1.1178, 253.9)$	$(88, 1.1178, 253.6)$	$(88, 1.1178, 253.4)$
0.2	$(55, 1.0895, 119.3)$	$(53, 1.0872, 113.7)$	$(53, 1.0872, 112.6)$	$(53, 1.0872, 112.2)$	$(53, 1.0872, 112.1)$
0.3	$(32, 1.0549, 51.6)$	$(31, 1.0528, 48.2)$	$(31, 1.0528, 47.6)$	$(31, 1.0528, 47.4)$	$(31, 1.0528, 47.3)$
0.5	$(13, 0.9935, 12.6)$	$(13, 0.9935, 11.6)$	$(12, 0.9878, 11.4)$	$(12, 0.9878, 11.3)$	$(12, 0.9878, 11.3)$
0.7	$(7, 0.9485, 4.6)$	$(6, 0.9370, 4.3)$	$(6, 0.9370, 4.2)$	$(6, 0.9370, 4.2)$	$(6, 0.9370, 4.2)$
1.0	$(4, 0.9060, 1.9)$	$(3, 0.8834, 1.8)$	$(3, 0.8834, 1.7)$	$(3, 0.8834, 1.7)$	$(3, 0.8834, 1.7)$
1.5	$(2, 0.8508, 1.1)$	$(2, 0.8508, 1.1)$	$(2, 0.8508, 1.1)$	$(2, 0.8508, 1.1)$	$(2, 0.8508, 1.1)$
2.0	$(2, 0.8508, 1.0)$	$(2, 0.8508, 1.0)$	$(2, 0.8508, 1.0)$	$(2, 0.8508, 1.0)$	$(2, 0.8508, 1.0)$

Table 5: (H^*, K^*, ARL_1) values of the Synthetic $\tilde{X}$ chart control chart in the presence of measurement errors for different values of B, δ, n, $\eta = 0.28$, $m = 1$

	$n=3$				
δ	$B=1$	$B=2$	$B=3$	$B=4$	$B=5$
0.1	$(42, 1.7347, 329.5)$	$(42, 1.7347, 327.4)$	$(42, 1.7347, 327.0)$	$(42, 1.7347, 326.8)$	$(42, 1.7347, 326.8)$
0.2	$(38, 1.7250, 242.2)$	$(37, 1.7225, 237.1)$	$(37, 1.7225, 236.1)$	$(37, 1.7225, 235.7)$	$(37, 1.7225, 235.5)$
0.3	$(28, 1.6950, 160.4)$	$(27, 1.6913, 154.5)$	$(27, 1.6913, 153.3)$	$(27, 1.6913, 152.9)$	$(26, 1.6875, 152.8)$
0.5	$(14, 1.6227, 66.0)$	$(14, 1.6227, 62.1)$	$(14, 1.6227, 61.3)$	$(14, 1.6227, 61.1)$	$(14, 1.6227, 60.9)$
0.7	$(9, 1.5737, 29.5)$	$(8, 1.5602, 27.5)$	$(8, 1.5602, 27.1)$	$(8, 1.5602, 26.9)$	$(8, 1.5602, 26.9)$
1.0	$(5, 1.5050, 11.4)$	$(5, 1.5050, 10.5)$	$(5, 1.5050, 10.4)$	$(5, 1.5050, 10.3)$	$(5, 1.5050, 10.3)$
1.5	$(3, 1.4420, 4.1)$	$(3, 1.4420, 3.9)$	$(3, 1.4420, 3.8)$	$(3, 1.4420, 3.8)$	$(3, 1.4420, 3.8)$
2.0	$(2, 1.3899, 2.5)$	$(2, 1.3899, 2.4)$	$(2, 1.3899, 2.4)$	$(2, 1.3899, 2.4)$	$(2, 1.3899, 2.4)$
	$n=5$				
δ	$B=1$	$B=2$	$B=3$	$B=4$	$B=5$
0.1	$(43, 1.3881, 309.4)$	$(43, 1.3881, 306.4)$	$(43, 1.3881, 305.8)$	$(43, 1.3881, 305.6)$	$(43, 1.3881, 305.5)$
0.2	$(32, 1.3652, 198.3)$	$(32, 1.3652, 192.4)$	$(32, 1.3652, 191.3)$	$(32, 1.3652, 190.9)$	$(32, 1.3652, 190.7)$
0.3	$(21, 1.3312, 114.4)$	$(21, 1.3312, 109.0)$	$(20, 1.3271, 108.0)$	$(20, 1.3271, 107.7)$	$(20, 1.3271, 107.5)$
0.5	$(10, 1.2672, 39.2)$	$(10, 1.2672, 36.5)$	$(10, 1.2672, 36.0)$	$(10, 1.2672, 35.9)$	$(10, 1.2672, 35.8)$
0.7	$(6, 1.2202, 16.2)$	$(6, 1.2202, 15.1)$	$(6, 1.2202, 14.8)$	$(6, 1.2202, 14.8)$	$(6, 1.2202, 14.7)$
1.0	$(4, 1.1812, 6.3)$	$(3, 1.1526, 5.9)$	$(3, 1.1526, 5.8)$	$(3, 1.1526, 5.8)$	$(3, 1.1526, 5.8)$
1.5	$(2, 1.1110, 2.7)$	$(2, 1.1110, 2.6)$	$(2, 1.1110, 2.6)$	$(2, 1.1110, 2.6)$	$(2, 1.1110, 2.6)$
2.0	$(2, 1.1110, 2.1)$	$(2, 1.1110, 2.0)$	$(2, 1.1110, 2.0)$	$(2, 1.1110, 2.0)$	$(2, 1.1110, 2.0)$
	$n=7$				
δ	$B=1$	$B=2$	$B=3$	$B=4$	$B=5$
0.1	$(43, 1.1881, 290.8)$	$(43, 1.1881, 287.1)$	$(43, 1.1881, 286.3)$	$(43, 1.1881, 286.1)$	$(43, 1.1881, 286.0)$
0.2	$(29, 1.1619, 165.2)$	$(28, 1.1595, 159.3)$	$(28, 1.1595, 158.2)$	$(28, 1.1595, 157.8)$	$(27, 1.1570, 157.6)$
0.3	$(17, 1.1244, 86.0)$	$(17, 1.1244, 81.3)$	$(17, 1.1244, 80.5)$	$(17, 1.1244, 80.2)$	$(17, 1.1244, 80.0)$
0.5	$(8, 1.0676, 26.3)$	$(8, 1.0676, 24.4)$	$(8, 1.0676, 24.1)$	$(8, 1.0676, 24.0)$	$(8, 1.0676, 23.9)$
0.7	$(5, 1.0299, 10.7)$	$(5, 1.0299, 9.9)$	$(5, 1.0299, 9.7)$	$(5, 1.0299, 9.7)$	$(5, 1.0299, 9.7)$
1.0	$(3, 0.9869, 4.4)$	$(3, 0.9869, 4.1)$	$(3, 0.9869, 4.1)$	$(3, 0.9869, 4.0)$	$(3, 0.9869, 4.0)$
1.5	$(2, 0.9513, 2.3)$	$(2, 0.9513, 2.2)$	$(2, 0.9513, 2.2)$	$(2, 0.9513, 2.2)$	$(2, 0.9513, 2.2)$
2.0	$(3, 0.9869, 1.9)$	$(4, 1.0113, 1.9)$	$(4, 1.0113, 1.9)$	$(4, 1.0113, 1.9)$	$(4, 1.0113, 1.9)$
	$n=9$				
δ	$B=1$	$B=2$	$B=3$	$B=4$	$B=5$
0.1	$(42, 1.0535, 273.8)$	$(42, 1.0535, 269.5)$	$(42, 1.0535, 268.7)$	$(42, 1.0535, 268.4)$	$(42, 1.0535, 268.3)$
0.2	$(25, 1.0226, 140.1)$	$(25, 1.0226, 134.3)$	$(24, 1.0201, 133.2)$	$(24, 1.0201, 132.9)$	$(24, 1.0201, 132.7)$
0.3	$(15, 0.9903, 67.3)$	$(14, 0.9858, 63.3)$	$(14, 0.9858, 62.6)$	$(14, 0.9858, 62.3)$	$(14, 0.9858, 62.2)$
0.5	$(7, 0.9387, 19.2)$	$(6, 0.9278, 17.8)$	$(6, 0.9278, 17.5)$	$(6, 0.9278, 17.4)$	$(6, 0.9278, 17.4)$
0.7	$(4, 0.8982, 7.8)$	$(4, 0.8982, 7.2)$	$(4, 0.8982, 7.1)$	$(4, 0.8982, 7.1)$	$(4, 0.8982, 7.1)$
1.0	$(3, 0.8765, 3.4)$	$(2, 0.8450, 3.3)$	$(2, 0.8450, 3.2)$	$(2, 0.8450, 3.2)$	$(2, 0.8450, 3.2)$
1.5	$(2, 0.8450, 2.1)$	$(2, 0.8450, 2.0)$	$(2, 0.8450, 2.0)$	$(2, 0.8450, 2.0)$	$(2, 0.8450, 2.0)$
2.0	$(6, 0.9278, 1.9)$	$(6, 0.9278, 1.9)$	$(6, 0.9278, 1.9)$	$(6, 0.9278, 1.9)$	$(6, 0.9278, 1.9)$

Table 6: $(H^{*}, K^{*}, SARL_1)$ values of the Synthetic $\tilde{X}$ chart control chart in the presence of measurement errors for different values of B, δ, n, $\eta = 0.28$, $m = 1$

	$n = 3$				
δ	$m = 1$	$m = 3$	$m = 5$	$m = 7$	$m = 10$
0.1	$(101, 1.8542, 322.6)$	$(100, 1.8533, 320.5)$	$(100, 1.8533, 320.0)$	$(100, 1.8533, 319.8)$	$(100, 1.8533, 319.7)$
0.2	$(80, 1.8318, 225.0)$	$(79, 1.8305, 220.1)$	$(79, 1.8305, 219.1)$	$(79, 1.8305, 218.6)$	$(79, 1.8305, 218.3)$
0.3	$(60, 1.8031, 139.5)$	$(58, 1.7996, 134.3)$	$(58, 1.7996, 133.2)$	$(58, 1.7996, 132.8)$	$(58, 1.7996, 132.4)$
0.5	$(32, 1.7370, 50.6)$	$(30, 1.7300, 47.6)$	$(30, 1.7300, 47.0)$	$(30, 1.7300, 46.7)$	$(30, 1.7300, 46.5)$
0.7	$(18, 1.6730, 20.5)$	$(17, 1.6664, 19.0)$	$(17, 1.6664, 18.8)$	$(17, 1.6664, 18.6)$	$(17, 1.6664, 18.5)$
1.0	$(9, 1.5916, 7.0)$	$(9, 1.5916, 6.5)$	$(8, 1.5773, 6.4)$	$(8, 1.5773, 6.4)$	$(8, 1.5773, 6.4)$
1.5	$(4, 1.4907, 2.3)$	$(4, 1.4907, 2.2)$	$(4, 1.4907, 2.1)$	$(4, 1.4907, 2.1)$	$(4, 1.4907, 2.1)$
2.0	$(3, 1.4534, 1.3)$	$(2, 1.3995, 1.3)$	$(2, 1.3995, 1.3)$	$(2, 1.3995, 1.3)$	$(2, 1.3995, 1.3)$
	$n = 5$				
δ	$m = 1$	$m = 3$	$m = 5$	$m = 7$	$m = 10$
0.1	$(96, 1.4779, 299.6)$	$(95, 1.4771, 296.6)$	$(95, 1.4771, 296.0)$	$(95, 1.4771, 295.7)$	$(95, 1.4771, 295.5)$
0.2	$(69, 1.4522, 178.3)$	$(68, 1.4510, 172.9)$	$(68, 1.4510, 171.8)$	$(68, 1.4510, 171.4)$	$(68, 1.4510, 171.0)$
0.3	$(47, 1.4209, 94.7)$	$(45, 1.4172, 90.2)$	$(45, 1.4172, 89.3)$	$(45, 1.4172, 88.9)$	$(45, 1.4172, 88.6)$
0.5	$(22, 1.3552, 28.1)$	$(21, 1.3510, 26.2)$	$(21, 1.3510, 25.8)$	$(21, 1.3510, 25.7)$	$(21, 1.3510, 25.5)$
0.7	$(12, 1.2994, 10.5)$	$(11, 1.2912, 9.7)$	$(11, 1.2912, 9.6)$	$(11, 1.2912, 9.5)$	$(11, 1.2912, 9.5)$
1.0	$(6, 1.2324, 3.7)$	$(6, 1.2324, 3.4)$	$(5, 1.2141, 3.4)$	$(5, 1.2141, 3.4)$	$(5, 1.2141, 3.3)$
1.5	$(3, 1.1617, 1.5)$	$(3, 1.1617, 1.4)$	$(3, 1.1617, 1.4)$	$(3, 1.1617, 1.4)$	$(3, 1.1617, 1.4)$
2.0	$(2, 1.1187, 1.1)$	$(2, 1.1187, 1.1)$	$(2, 1.1187, 1.1)$	$(2, 1.1187, 1.1)$	$(2, 1.1187, 1.1)$
	$n = 7$				
δ	$m = 1$	$m = 3$	$m = 5$	$m = 7$	$m = 10$
0.1	$(92, 1.2620, 278.6)$	$(91, 1.2613, 274.9)$	$(91, 1.2613, 274.1)$	$(91, 1.2613, 273.8)$	$(91, 1.2613, 273.5)$
0.2	$(61, 1.2343, 144.4)$	$(60, 1.2332, 139.1)$	$(60, 1.2332, 138.0)$	$(59, 1.2320, 137.6)$	$(59, 1.2320, 137.2)$
0.3	$(38, 1.2008, 68.3)$	$(37, 1.1988, 64.6)$	$(37, 1.1988, 63.8)$	$(37, 1.1988, 63.5)$	$(37, 1.1988, 63.3)$
0.5	$(17, 1.1400, 17.9)$	$(16, 1.1353, 16.7)$	$(16, 1.1353, 16.4)$	$(16, 1.1353, 16.3)$	$(16, 1.1353, 16.2)$
0.7	$(9, 1.0890, 6.6)$	$(8, 1.0792, 6.1)$	$(8, 1.0792, 6.0)$	$(8, 1.0792, 6.0)$	$(8, 1.0792, 5.9)$
1.0	$(4, 1.0201, 2.4)$	$(4, 1.0201, 2.3)$	$(4, 1.0201, 2.3)$	$(4, 1.0201, 2.3)$	$(4, 1.0201, 2.3)$
1.5	$(2, 0.9579, 1.2)$	$(2, 0.9579, 1.2)$	$(2, 0.9579, 1.2)$	$(2, 0.9579, 1.2)$	$(2, 0.9579, 1.2)$
2.0	$(2, 0.9579, 1.0)$	$(2, 0.9579, 1.0)$	$(2, 0.9579, 1.0)$	$(2, 0.9579, 1.0)$	$(2, 0.9579, 1.0)$
	$n = 9$				
δ	$m = 1$	$m = 3$	$m = 5$	$m = 7$	$m = 10$
0.1	$(89, 1.1184, 259.6)$	$(88, 1.1178, 255.4)$	$(88, 1.1178, 254.5)$	$(88, 1.1178, 254.1)$	$(88, 1.1178, 253.8)$
0.2	$(55, 1.0895, 119.3)$	$(53, 1.0872, 114.3)$	$(53, 1.0872, 113.3)$	$(53, 1.0872, 112.9)$	$(53, 1.0872, 112.5)$
0.3	$(32, 1.0549, 51.6)$	$(31, 1.0528, 48.6)$	$(31, 1.0528, 48.0)$	$(31, 1.0528, 47.7)$	$(31, 1.0528, 47.5)$
0.5	$(13, 0.9935, 12.6)$	$(13, 0.9935, 11.7)$	$(13, 0.9935, 11.5)$	$(12, 0.9878, 11.4)$	$(12, 0.9878, 11.4)$
0.7	$(7, 0.9485, 4.6)$	$(7, 0.9485, 4.3)$	$(6, 0.9370, 4.2)$	$(6, 0.9370, 4.2)$	$(6, 0.9370, 4.2)$
1.0	$(4, 0.9060, 1.9)$	$(3, 0.8834, 1.8)$	$(3, 0.8834, 1.8)$	$(3, 0.8834, 1.8)$	$(3, 0.8834, 1.7)$
1.5	$(2, 0.8508, 1.1)$	$(2, 0.8508, 1.1)$	$(2, 0.8508, 1.1)$	$(2, 0.8508, 1.1)$	$(2, 0.8508, 1.1)$
2.0	$(2, 0.8508, 1.0)$	$(2, 0.8508, 1.0)$	$(2, 0.8508, 1.0)$	$(2, 0.8508, 1.0)$	$(2, 0.8508, 1.0)$

Table 7: (H^*, K^*, ARL_1) values of the Synthetic $\tilde{X}$ chart control chart in the presence of measurement errors for different values of B, δ, n, $\eta = 0.28$

δ	$m=1$	$m=3$	$m=5$	$m=7$	$m=10$
			$n=3$		
0.1	$(42, 1.7347, 329.5)$	$(42, 1.7347, 327.6)$	$(42, 1.7347, 327.2)$	$(42, 1.7347, 327.1)$	$(42, 1.7347, 326.9)$
0.2	$(38, 1.7250, 242.2)$	$(37, 1.7225, 237.6)$	$(37, 1.7225, 236.7)$	$(37, 1.7225, 236.3)$	$(37, 1.7225, 236.0)$
0.3	$(28, 1.6950, 160.4)$	$(27, 1.6913, 155.1)$	$(27, 1.6913, 154.1)$	$(27, 1.6913, 153.6)$	$(27, 1.6913, 153.2)$
0.5	$(14, 1.6227, 66.0)$	$(14, 1.6227, 62.5)$	$(14, 1.6227, 61.8)$	$(14, 1.6227, 61.5)$	$(14, 1.6227, 61.3)$
0.7	$(9, 1.5737, 29.5)$	$(8, 1.5602, 27.7)$	$(8, 1.5602, 27.3)$	$(8, 1.5602, 27.2)$	$(8, 1.5602, 27.1)$
1.0	$(5, 1.5050, 11.4)$	$(5, 1.5050, 10.6)$	$(5, 1.5050, 10.5)$	$(5, 1.5050, 10.4)$	$(5, 1.5050, 10.4)$
1.5	$(3, 1.4420, 4.1)$	$(3, 1.4420, 3.9)$	$(3, 1.4420, 3.9)$	$(3, 1.4420, 3.9)$	$(3, 1.4420, 3.8)$
2.0	$(2, 1.3899, 2.5)$	$(2, 1.3899, 2.4)$	$(2, 1.3899, 2.4)$	$(2, 1.3899, 2.4)$	$(2, 1.3899, 2.4)$
			$n=5$		
δ	$m=1$	$m=3$	$m=5$	$m=7$	$m=10$
0.1	$(43, 1.3881, 309.4)$	$(43, 1.3881, 306.7)$	$(43, 1.3881, 306.2)$	$(43, 1.3881, 305.9)$	$(43, 1.3881, 305.7)$
0.2	$(32, 1.3652, 198.3)$	$(32, 1.3652, 193.1)$	$(32, 1.3652, 192.0)$	$(32, 1.3652, 191.6)$	$(32, 1.3652, 191.2)$
0.3	$(21, 1.3312, 114.4)$	$(21, 1.3312, 109.7)$	$(20, 1.3271, 108.7)$	$(20, 1.3271, 108.3)$	$(20, 1.3271, 108.0)$
0.5	$(10, 1.2672, 39.2)$	$(10, 1.2672, 36.8)$	$(10, 1.2672, 36.3)$	$(10, 1.2672, 36.1)$	$(10, 1.2672, 36.0)$
0.7	$(6, 1.2202, 16.2)$	$(6, 1.2202, 15.2)$	$(6, 1.2202, 15.0)$	$(6, 1.2202, 14.9)$	$(6, 1.2202, 14.8)$
1.0	$(4, 1.1812, 6.3)$	$(3, 1.1526, 5.9)$	$(3, 1.1526, 5.9)$	$(3, 1.1526, 5.8)$	$(3, 1.1526, 5.8)$
1.5	$(2, 1.1110, 2.7)$	$(2, 1.1110, 2.6)$	$(2, 1.1110, 2.6)$	$(2, 1.1110, 2.6)$	$(2, 1.1110, 2.6)$
2.0	$(2, 1.1110, 2.1)$	$(2, 1.1110, 2.0)$	$(2, 1.1110, 2.0)$	$(2, 1.1110, 2.0)$	$(2, 1.1110, 2.0)$
			$n=7$		
δ	$m=1$	$m=3$	$m=5$	$m=7$	$m=10$
0.1	$(43, 1.1881, 290.8)$	$(43, 1.1881, 287.5)$	$(43, 1.1881, 286.8)$	$(43, 1.1881, 286.5)$	$(43, 1.1881, 286.3)$
0.2	$(29, 1.1619, 165.2)$	$(28, 1.1595, 160.0)$	$(28, 1.1595, 158.9)$	$(28, 1.1595, 158.5)$	$(28, 1.1595, 158.1)$
0.3	$(17, 1.1244, 86.0)$	$(17, 1.1244, 81.9)$	$(17, 1.1244, 81.0)$	$(17, 1.1244, 80.7)$	$(17, 1.1244, 80.4)$
0.5	$(8, 1.0676, 26.3)$	$(8, 1.0676, 24.6)$	$(8, 1.0676, 24.3)$	$(8, 1.0676, 24.2)$	$(8, 1.0676, 24.1)$
0.7	$(5, 1.0299, 10.7)$	$(5, 1.0299, 10.0)$	$(5, 1.0299, 9.8)$	$(5, 1.0299, 9.8)$	$(5, 1.0299, 9.7)$
1.0	$(3, 0.9869, 4.4)$	$(3, 0.9869, 4.1)$	$(3, 0.9869, 4.1)$	$(3, 0.9869, 4.1)$	$(3, 0.9869, 4.1)$
1.5	$(2, 0.9513, 2.3)$	$(2, 0.9513, 2.2)$	$(2, 0.9513, 2.2)$	$(2, 0.9513, 2.2)$	$(2, 0.9513, 2.2)$
2.0	$(3, 0.9869, 1.9)$	$(4, 1.0113, 1.9)$	$(4, 1.0113, 1.9)$	$(4, 1.0113, 1.9)$	$(4, 1.0113, 1.9)$
			$n=9$		
δ	$m=1$	$m=3$	$m=5$	$m=7$	$m=10$
0.1	$(42, 1.0535, 273.8)$	$(42, 1.0535, 270.0)$	$(42, 1.0535, 269.2)$	$(42, 1.0535, 268.9)$	$(42, 1.0535, 268.6)$
0.2	$(25, 1.0226, 140.1)$	$(25, 1.0226, 135.0)$	$(25, 1.0226, 133.9)$	$(24, 1.0201, 133.5)$	$(24, 1.0201, 133.1)$
0.3	$(15, 0.9903, 67.3)$	$(14, 0.9858, 63.8)$	$(14, 0.9858, 63.0)$	$(14, 0.9858, 62.7)$	$(14, 0.9858, 62.5)$
0.5	$(7, 0.9387, 19.2)$	$(6, 0.9278, 17.9)$	$(6, 0.9278, 17.7)$	$(6, 0.9278, 17.6)$	$(6, 0.9278, 17.5)$
0.7	$(4, 0.8982, 7.8)$	$(4, 0.8982, 7.3)$	$(4, 0.8982, 7.2)$	$(4, 0.8982, 7.2)$	$(4, 0.8982, 7.1)$
1.0	$(3, 0.8765, 3.4)$	$(2, 0.8450, 3.3)$	$(2, 0.8450, 3.2)$	$(2, 0.8450, 3.2)$	$(2, 0.8450, 3.2)$
1.5	$(2, 0.8450, 2.1)$	$(2, 0.8450, 2.1)$	$(2, 0.8450, 2.0)$	$(2, 0.8450, 2.0)$	$(2, 0.8450, 2.0)$
2.0	$(6, 0.9278, 1.9)$	$(6, 0.9278, 1.9)$	$(6, 0.9278, 1.9)$	$(6, 0.9278, 1.9)$	$(6, 0.9278, 1.9)$

Table 8: $(H^*, K^*, SARL_1)$ values of the Synthetic $\tilde{X}$ control chart in the presence of measurement errors for different values of B, δ, n, $\eta = 0.28$